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Separation of Closely-Spaced Acoustics Sources in an Under-Determined System with Convex Optimization

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Importance of Sound Source Identification

Motivation of Current Work

- NAH is a powerful tool to identify sound source
 - Measurements can be taken away from the source and sound field can be visualized in three-dimensional space
 - Large number of measurements is required to avoid different measurement errors: e.g., spatial aliasing, windowing errors, etc.
 - Economically costly, and hard to perform

- Motivation
 - Using as few microphone measurements as possible to accurately identify major sound source
 - Encourage wider application of NAH in industry

Figure: LOUD 1020-node microphone array

Choice of Near-field Acoustical Holography Method

Monopoles Distribution at fixed locations

- Reason:
1. Equivalent model having strongly associated physical meaning. Ex: monopole: volume contraction and expansion, dipole: an oscillating force, lateral quadrupole: rotational torque or a vortex.
 2. Equivalent source models don't have strict requirements on microphone array.
 3. Mathematically straightforward.

Source Strength Estimation

➤ Matrix formation

➤ Difficulty

- Under-determined System: There are infinitely many solutions $\vec{S}$
- Hard to reconstruct “correct” sound source location: Sparsity VS Accuracy

Wideband Acoustical Holography

➤ Jorgen Hald. Wideband Holography. *Inter-Noise, Melbourne, Australia, November 2014.*

Studies on Wideband Acoustical Holography

- 1) Monopole-based Wideband Acoustical Holography (WBH) is able to identify sound source locations when the system is under-determined.
- 2) When there is only one major sound source present, the WBH can localize the sound source location and reconstruct the sound field. The reconstructed source distribution is more localized than the true source distribution at high frequency.
- 3) WBH has difficulty in separating two closely-positioned sound sources, the algorithm tends to identify two sources as one large source located in between the two sources, this is especially true at low frequency.
- 4) WBH with zero initial guess develops solution around strongest source and tends to ignore weaker sources

- T. Shi, Y. Liu, and J. S. Bolton. "The Use of Wideband Acoustical Holography for Noise Source Visualization", *Proceedings of NOISE-CON 2016, Providence, Rhode Island, pp. 479-490, 2016.*
- T. Shi, Y. Liu, and J. S. Bolton. "A Comparison of Two Equivalent Source Methods for Noise Source Visualization", In *INTER-NOISE and NOISE-CON Congress and Conference Proceedings* (Vol. 254, No. 1, pp. 1058-1068). Institute of Noise Control Engineering.
- T. Shi, Y. Liu, J.S. Bolton, F. Eberhardt and W. Frazer. "Diesel Engine Noise Source Visualization with Wideband Acoustical Holography", No. 2017-01-1874. *SAE Technical Paper*, 2017.

Compressive Sensing and Convex Optimization

- Under-determined system
- Low spatial sampling rate

- Nyquist-Shannon sampling theorem
- Compressive Sampling (CS)

$$\min \left\| \vec{P} - A(\vec{X})\vec{S} \right\|^2$$

- Convex function
- Convex Optimization

➤ Convex function

- $f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y)$

➤ Convex optimization problem

minimize $f_0(x)$

subject to $f_i(x) \leq 0, \quad i = 1, \dots, m$

$h_i(x) = 0, \quad i = 1, \dots, m$

Objective Function Formulation Noise Free Case

➤ Objective function

minimize $\|\vec{S}\|_1$

Solution
sparsity

subject to $\vec{\hat{P}}_m = A(\vec{X})\vec{S}$

Solution
accuracy

- M. Grant, S. Boyd, and Y. Ye *CVX: software for disciplined convex programming*

➤ Why l_1 -norm works

- l_1 -norm: $\|\vec{S}\|_1 = \sum_{i=1}^m |s_i|$
- l_2 -norm: $\|\vec{S}\|_2 = \sqrt{s_1^2 + s_1^2 + \dots + s_m^2}$

s_1
 s_2

$$p = \alpha_1 s_1 + \alpha_2 s_2$$

Two Closely-Positioned Simulated Sound Sources

➤ Simulation set up

- Two simulated sources were placed 0.2 m from each other, each source was composed with three unit source strength monopoles
- 54 virtual microphone measurements

➤ Equivalent source plane

- -0.19 - 0.18 m, in x-direction
- -0.10 - 0.10 m, in y-direction
- 0.01 m spacing in both x- and y-direction, 798 monopoles.
- 0.25 m from measurement plane

Reconstruction on Equivalent Source Plane at 300 Hz

- Two separated sources were identified by Convex Optimization

Reconstruction on Equivalent Source Plane at 2000 Hz

frequency 2000 Hz true source strength

True sources distribution
Total source strength 6

frequency 2000 Hz reconstructed source strength

Convex Optimization reconstructed
sources
Total source strength 6

frequency 2000 Hz reconstructed source strength

WBH reconstructed sources
Total source strength 6.52
Error < 10%, D_max = 60

Objective Function Formulation Measurement with Noise

➤ Objective function

- M. Grant, S. Boyd, and Y. Ye *CVX: software for disciplined convex programming*

- Careful choosing the weighting parameter
- l_1 -norm for source strength and l_2 -norm for residual

Two Closely-Positioned Simulated Sound Sources

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- -0.19 - 0.18 m, in x-direction
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- 0.01 m spacing in both x- and y-direction, 798 monopoles.
- 0.25 m from measurement plane

- White Gaussian Random Noise added into the virtual measurement, SNR = 30 dB

Reconstruction on Equivalent Source Plane at 300 Hz

$$\text{minimize } \|\vec{S}\|_1 + 2 \left\| A(\vec{X}_S) \vec{S} - \vec{\hat{P}}_m \right\|_2$$

Error < 10%, D_max = 100

True sources distribution
Total source strength 6

Convex Optimization reconstructed sources
Total source strength 5.78

WBH reconstructed sources
Total source strength 6.83

- Two separated sources were identified by Convex Optimization

Reconstruction on Equivalent Source Plane at 2000 Hz

$$\text{minimize } \|\vec{S}\|_1 + 2 \|A(\vec{X}_S)\vec{S} - \vec{P}_m\|_2$$

Error < 10%, D_max = 100

frequency 2000 Hz true source strength

True sources distribution
Total source strength 6

frequency 2000 Hz reconstructed source strength

Convex Optimization reconstructed sources
Total source strength 5.78

frequency 2000 Hz reconstructed source strength

WBH reconstructed sources
Total source strength 6.60

- Convex Optimization reconstruction result is more concentrated than WBH

Difference between WBH and Convex Optimization

- Wideband holography method **roughly** equivalent to

minimize $\left\| A(\vec{X}_S) \vec{S} - \overrightarrow{\hat{P}_m} \right\|_2$ ○ Steepest gradient method

subject to $\mathbf{card}(\vec{S}) \leq \tilde{t}_k$ ○ Adjust $\tilde{t}_k$ at each iteration through T_k

- Convex formulation

minimize $\left\| \vec{S} \right\|_1 + \lambda \left\| A(\vec{X}_S) \vec{S} - \overrightarrow{\hat{P}_m} \right\|_2$

- Different formulation to create the solution sparsity

Conclusion