Separation of Closely-Spaced Acoustics Sources in an Under-Determined System with Convex Optimization

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Importance of Sound Source Identification

Concerns in municipal environment

Concerns in occupational health and safety

Eliminate noise source in design phase

Efficient noise control tool

Requirements in laws and regulations
Motivation of Current Work

➢ NAH is a powerful tool to identify sound source
  • Measurements can be taken away from the source and sound field can be visualized in three-dimensional space
  • Large number of measurements is required to avoid different measurement errors: e.g., spatial aliasing, windowing errors, etc.
  • Economically costly, and hard to perform

➢ Motivation
  • Using as few microphone measurements as possible to accurately identify major sound source
  • Encourage wider application of NAH in industry

Figure: LOUD 1020-node microphone array
2. Equivalent source models don’t have strict requirements on microphone array.
3. Mathematically straightforward.
Source Strength Estimation

Matrix formation

\[ \bar{P} = A(\bar{X})\bar{S} \]

\[ \bar{S} = A(\bar{X})^{-1}\bar{P} \]

\[ \bar{S} = (U \Sigma V^H)^{-1}\bar{P} \]

\[ \bar{S} = \sum_{i=1}^{n} \frac{u_i^H \tilde{p}_i}{\sigma_i} v_i \]

= \sum_{i=1}^{n} \frac{u_i^H \tilde{p}_i}{\sigma_i} v_i + \sum_{i=1}^{n} \frac{u_i^H e}{\sigma_i} v_i

\[ \text{Sound from source} \quad \text{noise} \]

Ill-posed problem

Unstable solution without physical sense

\[ \text{Ghost source problem} \]

\[ \text{Strongly under-determined system} \]

Difficultly

• Under-determined System: There are infinitely many solutions \(\bar{S}\)

• Hard to reconstruct “correct” sound source location: Sparsity VS Accuracy
Wideband Acoustical Holography


Source strength vector $q_k$

- Error between reconstruction pressure and measurement pressure is less than $e$
- $D_k$ reach the maximum value

Calculate residual vector

$\text{Residual vector: } r(\bar{q}) \equiv \| \bar{P}_m - A\bar{q} \|^2$

Calculate the step $\Delta q$ to minimize the residual vector

$\Delta q_k = s_k \bar{w}_k$, where $\bar{w}_k$ is the negative gradient vector, $\bar{w}_k \equiv A^H r(q_k) = A^H (\bar{p} - Aq_k)$, $s_k$ is the step length to minimum along that direction $s_k \equiv g_k^H r(q_k) / g_k^H g_k$, the vector $g_k$ defined as $g_k \equiv A\bar{w}_k$

Turn off the sources below threshold

Get the next source strength candidate

Check the threshold condition

If does not reach the threshold condition

Take source strength as zero at first iteration

$q_{k+1,i} = \begin{cases} \tilde{q}_{k+1,i} & \text{if } |\tilde{q}_{k+1,i}| \geq T_k, \\ 0 & \text{otherwise} \end{cases}, \text{ where } T_k \equiv 10^{-\frac{D_k}{20}} \max_{i} |\tilde{q}_{k+1,max}|$

$q_{k+1} = q_k + \alpha \Delta q$
Studies on Wideband Acoustical Holography

1) Monopole-based Wideband Acoustical Holography (WBH) is able to identify sound source locations when the system is under-determined.

2) When there is only one major sound source present, the WBH can localize the sound source location and reconstruct the sound field. The reconstructed source distribution is more localized than the true source distribution at high frequency.

3) WBH has difficulty in separating two closely-positioned sound sources, the algorithm tends to identify two sources as one large source located in between the two sources, this is especially true at low frequency.

4) WBH with zero initial guess develops solution around strongest source and tends to ignore weaker sources.


Compressive Sensing and Convex Optimization

- Under-determined system
- Low spatial sampling rate
- Nyquisit-Shannon sampling theorem
- Compressive Sampling (CS)

\[
\min \left\| P - A(\bar{X})S \right\|^2
\]

- Convex function
- Convex Optimization

- Convex function

\[
f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y)
\]

- Convex optimization problem

minimize \( f_0(x) \)
subject to \( f_i(x) \leq 0, \quad i = 1, \ldots, m \)
\( h_i(x) = 0, \quad i = 1, \ldots, m \)
Objective Function Formulation Noise Free Case

- **Objective function**

  \[
  \text{minimize } \| \hat{S} \|_1 \\
  \text{subject to } \bar{P}_m = A \left( \hat{X} \right) \hat{S}
  \]

- **Solution sparsity**

- **Solution accuracy**

- **Why }_1\text{-norm works**

  - }_1\text{-norm: } \| \hat{S} \|_1 = \sum_{i=1}^{m} |s_i|

  - }_2\text{-norm: } \| \hat{S} \|_2 = \sqrt{s_1^2 + s_2^2 + \cdots + s_m^2}

- **M. Grant, S. Boyd, and Y. Ye** *CVX: software for disciplined convex programming*

  - \( p = \alpha_1 s_1 + \alpha_2 s_2 \)
Two Closely-Positioned Simulated Sound Sources

➢ Simulation set up
  - Two simulated sources were placed 0.2 m from each other, each source was composed with three unit source strength monopoles
  - 54 virtual microphone measurements

➢ Equivalent source plane
  - -0.19 - 0.18 m, in x-direction
  - -0.10 - 0.10 m, in y-direction
  - 0.01 m spacing in both x- and y-direction, 798 monopoles.
  - 0.25 m from measurement plane
Reconstruction on Equivalent Source Plane at 300 Hz

True sources distribution
Total source strength 6

Convex Optimization reconstructed sources
Total source strength 6

WBH reconstructed sources
Total source strength 6.80
Error < 10%, D_max = 60

• Two separated sources were identified by Convex Optimization
Reconstruction on Equivalent Source Plane at 2000 Hz

True sources distribution
Total source strength 6

Convex Optimization reconstructed sources
Total source strength 6

WBH reconstructed sources
Total source strength 6.52
Error < 10%, D_max = 60
Objective Function Formulation Measurement with Noise

Objective function

\[
\minimize \left\| \hat{S} \right\|_1 + \lambda \left\| A\hat{S} - \bar{P}_m \right\|_2
\]

- Solution sparsity
- Solution accuracy

- Careful choosing the weighting parameter
- \(l_1\)-norm for source strength and \(l_2\)-norm for residual

M. Grant, S. Boyd, and Y. Ye *CVX: software for disciplined convex programming*
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➢ White Gaussian Random Noise added into the virtual measurement, SNR = 30 dB
Reconstruction on Equivalent Source Plane at 300 Hz

True sources distribution
Total source strength 6

Convex Optimization reconstructed sources
Total source strength 5.78

WBH reconstructed sources
Total source strength 6.83

• Two separated sources were identified by Convex Optimization

\[
\minimize \|\tilde{S}\|_1 + 2 \|A(\tilde{X}_S)\tilde{S} - \tilde{P}_m\|_2
\]

Error < 10%, D_max = 100
Reconstruction on Equivalent Source Plane at 2000 Hz

\[
\text{minimize } \|\tilde{S}\|_1 + 2 \|A(\tilde{X}_S)\tilde{S} - \tilde{P}_m\|_2
\]

Error < 10%, D_max = 100

True sources distribution
Total source strength 6

Convex Optimization reconstructed sources
Total source strength 5.78

WBH reconstructed sources
Total source strength 6.60

- Convex Optimization reconstruction result is more concentrated than WBH
Difference between WBH and Convex Optimization

- Wideband holography method roughly equivalent to

  \[
  \text{minimize} \quad \| A \left( \bar{X}_s \right) \hat{S} - \hat{P}_m \|_2 \\
  \text{subject to} \quad \text{card}(\hat{S}) \leq \bar{t}_k
  \]

  - Steepest gradient method
  - Adjust \( \bar{t}_k \) at each iteration through \( T_k \)

- Convex formulation

  \[
  \text{minimize} \quad \| \hat{S} \|_1 + \lambda \| A \left( \bar{X}_s \right) \hat{S} - \hat{P}_m \|_2
  \]

  - Different formulation to create the solution sparsity
Conclusion

- With Compressive Sensing, the least-square solution was formulated as a convex function which balances the solution sparsity and accuracy, then this function was solved by an open source Convex Optimization algorithm.

- From simulation, it was found that with the proposed solution, the reconstructed source is more accurate than WBH. Closely-positioned sources can be separated in space and recovered with appropriate source strength even at low frequency.