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A Transfer Matrix Method for Estimating the Dispersion and Attenuation of Plane Waves in a Standing Wave Tube - Presentation

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A transfer matrix approach for estimating the sound speed and attenuation constant of air in a tube

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Outline

- Sound propagation in a pipe
  - Traces back to the early period of acoustic research
- Standing wave tube technique
  - 2-microphone standard versus 4-microphone standard
- Estimation of complex wave number
  - Iterative procedure based on four-microphone measurement

Sound propagation in the pipe

3M™ Littmann® Classic II Infant Stethoscope

- Classical Problem

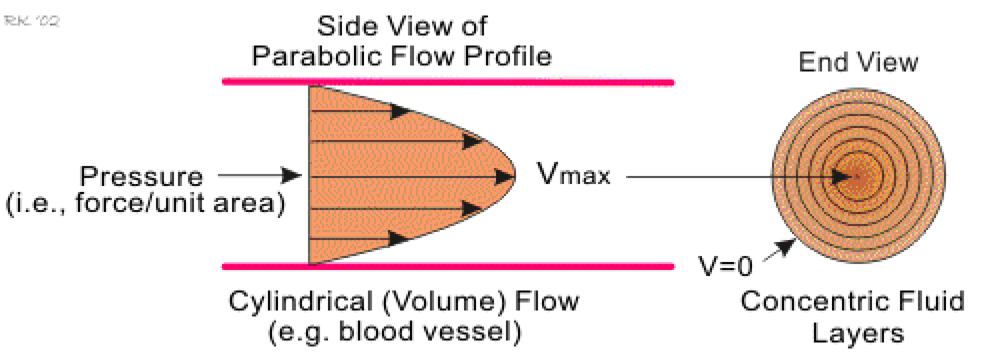
Kirchoff, Zwikker, Kosten, Morse and Ingard

- “Wall Effect”

Viscous, thermal effect

- CFD

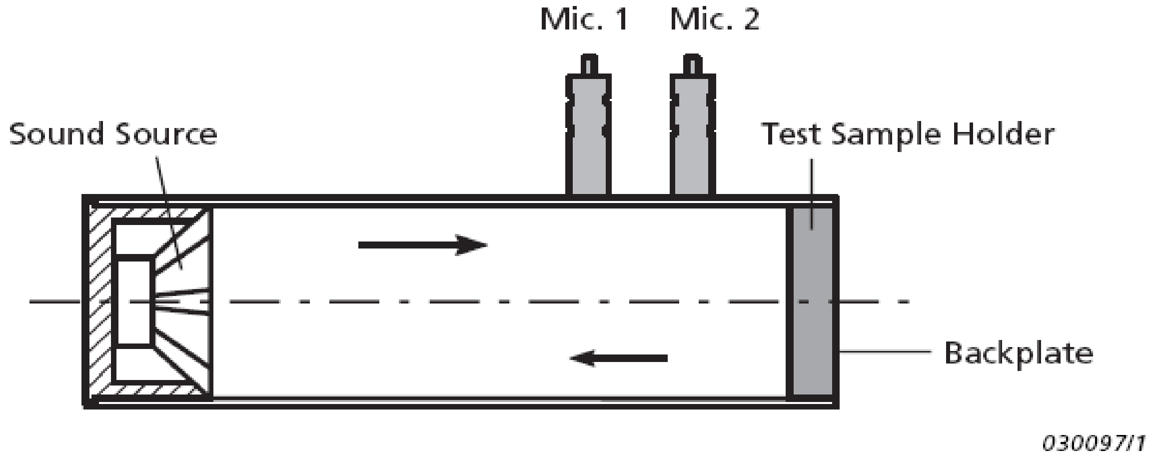
Computational fluid dynamics



Plumbing Industry

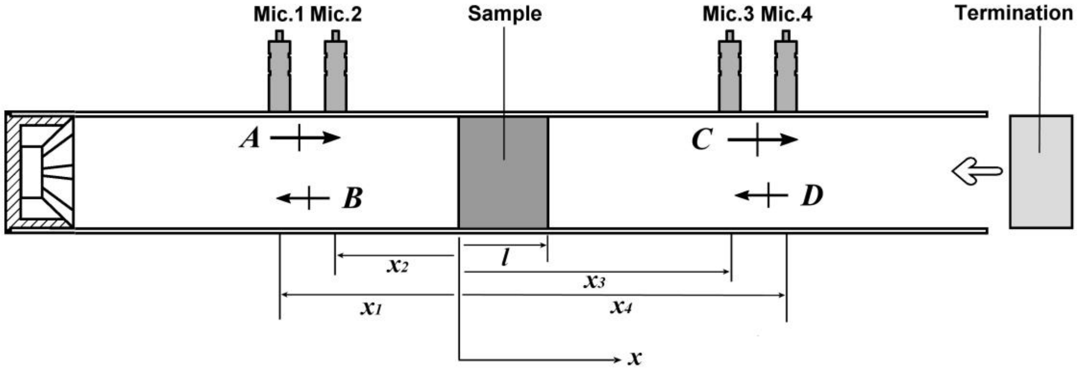
Standing wave tube techniques

- Standard test method for estimating impedance and absorption coefficients of acoustical materials
  - ISO 10534-2
  - ASTM E 1050
- Sources of errors: tube attenuation
- Two-Microphone absorption measurement
  - Newton-Raphson iteration scheme



Transfer matrix approach

- Four-microphone measurement is in the process of standardization by ASTM



$$P|_{x=0} = (A+B)$$
$$V|_{x=0} = (A-B)/\rho_0 c$$

$$P|_{x=d} = (Ce^{-jkd} + De^{jkd})$$
$$V|_{x=d} = (Ce^{-jkd} - De^{jkd})/\rho_0 c$$

Transfer Matrix: $$\begin{bmatrix} P \\ V \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix}_{x=d}$$

Estimation of complex wave number

For acoustic materials, typically foam and fiberglass, the complex wave number can be obtained from the transfer matrix

Transfer Matrix

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos k_p d & j\rho_p c_p \sin k_p d \\ j \sin k_p d / \rho_p c_p & \cos k_p d \end{bmatrix}$$

One Load Approach

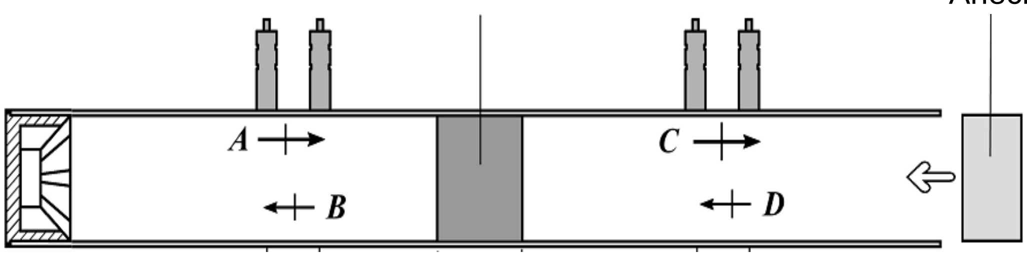
$$T_{11} = T_{22}$$

$$T_{11}T_{22} - T_{12}T_{21} = 1$$

$$k_p = \frac{1}{d} \cos^{-1} T_{11}$$

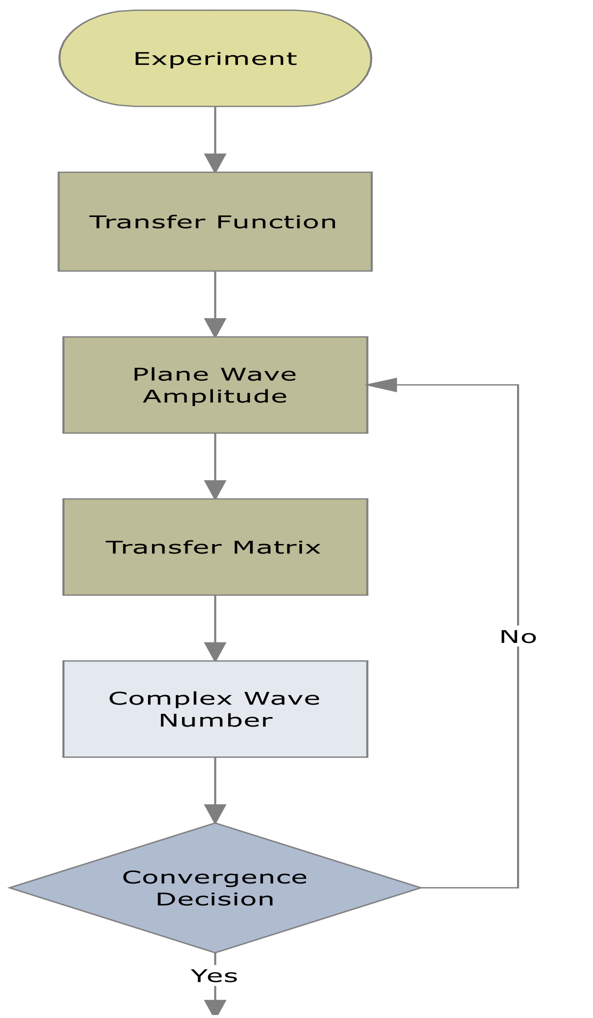
Treat empty air column as tested samples

Anechoic termination



Post-process iterative algorithm

Iterative Algorithm Flow Chart



$$A = \frac{j(P_1 e^{jkx_2} - P_2 e^{jkx_1})}{2\sin k(x_1 - x_2)}, \qquad C = \frac{j(P_3 e^{jkx_4} - P_4 e^{jkx_3})}{2\sin k(x_3 - x_4)}$$

$$B = \frac{j(P_2 e^{-jkx_1} - P_1 e^{-jkx_2})}{2\sin k(x_1 - x_2)}, \qquad D = \frac{j(P_4 e^{-jkx_3} - P_3 e^{-jkx_4})}{2\sin k(x_3 - x_4)}$$

$$P\big|_{x=0} = A + B, \qquad P\big|_{x=d} = Ce^{-jkd} + De^{jkd},$$

$$V\big|_{x=0} = \frac{A - B}{\rho_0 c}, \qquad V\big|_{x=d} = \frac{Ce^{-jkd} - De^{jkd}}{\rho_0 c}$$

$$k_p = \frac{1}{d}\cos^{-1} T_{11}$$

Experimental Setup

- Semi-empirical prediction: Temkin formula

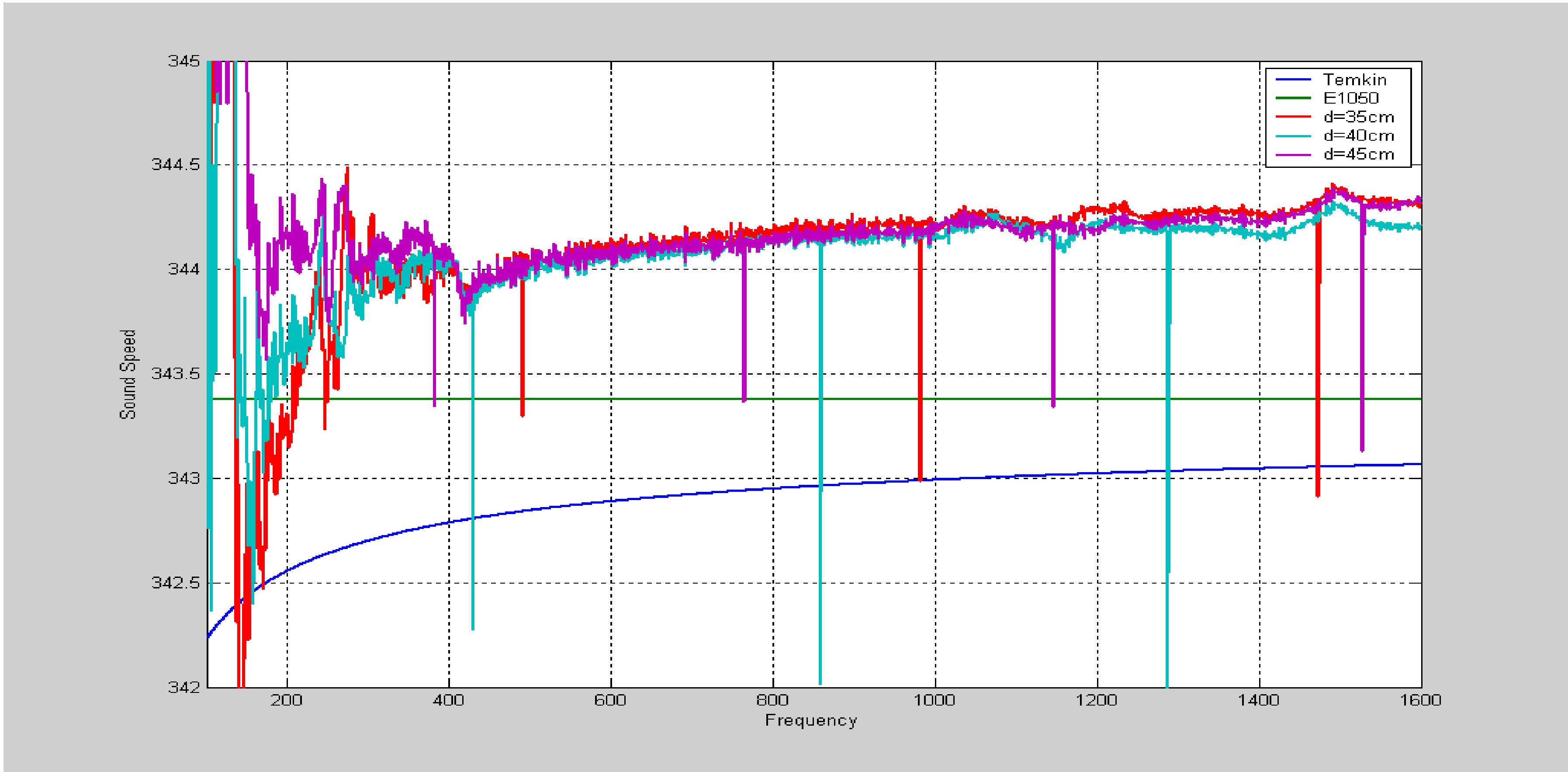
$$k = \frac{\omega}{c} + (1 - i)\delta$$

- Experimental estimation: Four-microphone measurement based on the transfer function combined with our iterative algorithm

- B&K 4206 standard small and large transmission loss tube kit

Measurement results-Large Tube

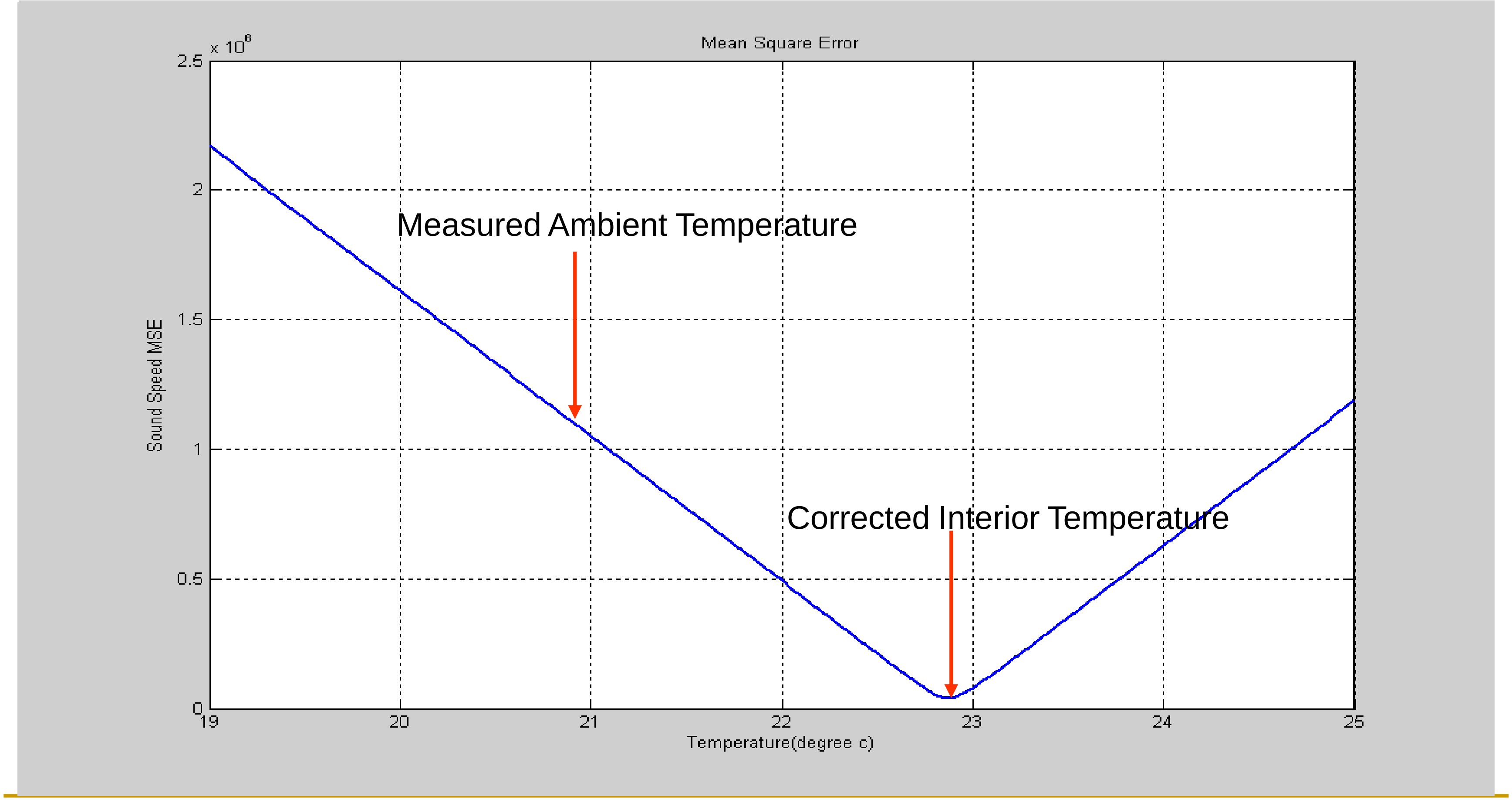
- Advantage: Simple, Fast, Flexible
- Disadvantage: Half Wave Length Problem



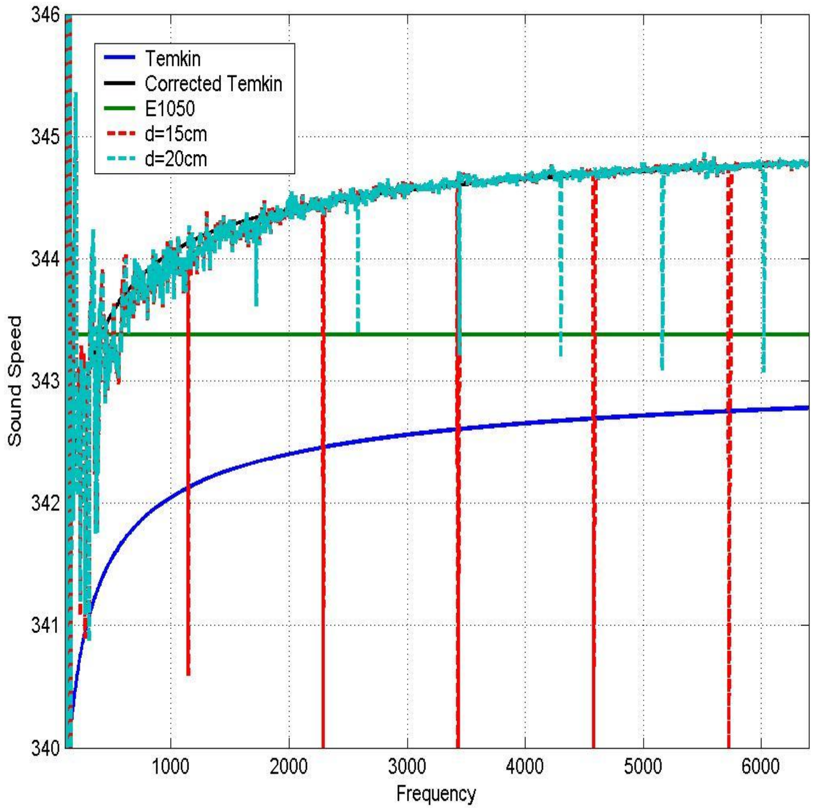
Sound Speed

Corrected Temkin Formula

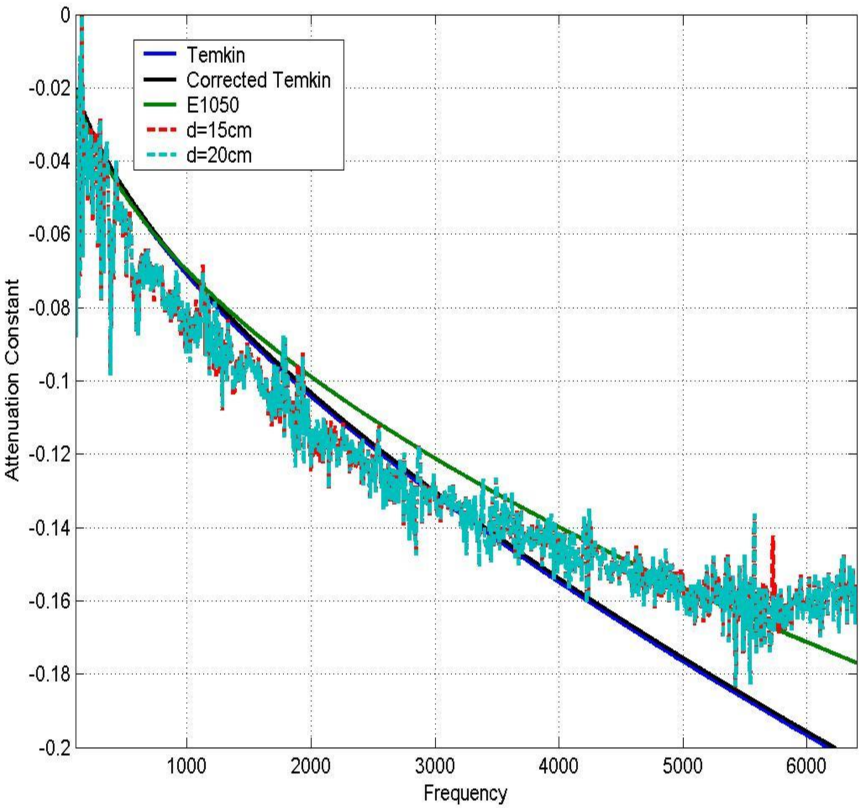
- Temkin formula can match the experimental results well if we modify the measured temperature



Measurement results-Small Tube



Sound Speed



Tube Attenuation

Conclusions

- Semi-empirical Temkin formula for complex wave number estimation can be adjusted to match the experimental results very well

- The iterative algorithm based on four-microphone technique gives a quick and reliable estimation of sound speed and attenuation in the tube

- The approach has the potential to be used for the accurate measurements of gases properties and duct lining performance.

Thanks You!