A Transfer Matrix Method for Estimating the Dispersion and Attenuation of Plane Waves in a Standing Wave Tube - Presentation

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A transfer matrix approach for estimating the sound speed and attenuation constant of air in a tube

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Outline

- Sound propagation in a pipe
  - Traces back to the early period of acoustic research

- Standing wave tube technique
  - 2-microphone standard versus 4-microphone standard

- Estimation of complex wave number
  - Iterative procedure based on four-microphone measurement
Sound propagation in the pipe

- Classical Problem
  Kirchoff, Zwikker, Kosten, Morse and Ingard
- “Wall Effect”
  Viscous, thermal effect
- CFD
  Computational fluid dynamics

3M™ Littmann® Classic II Infant Stethoscope

Plumbing Industry
Standing wave tube techniques

- Standard test method for estimating impedance and absorption coefficients of acoustical materials
  - ISO 10534-2
  - ASTM E 1050

- Sources of errors: tube attenuation

- Two-Microphone absorption measurement
  - Newton-Raphson iteration scheme
Transfer matrix approach

- Four-microphone measurement is in the process of standardization by ASTM

Transfer Matrix:

\[
\begin{bmatrix}
P \\
V
\end{bmatrix}_{x=0} = \begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
P \\
V
\end{bmatrix}_{x=d}
\]

\[
P_{x=0} = (A + B)
\]

\[
V_{x=0} = (A - B)/\rho_0 c
\]

\[
P_{x=d} = (Ce^{-jkd} + De^{jkd})
\]

\[
V_{x=d} = (Ce^{-jkd} - De^{jkd})/\rho_0 c
\]
Estimation of complex wave number

For acoustic materials, typically foam and fiberglass, the complex wave number can be obtained from the transfer matrix

\[
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix} =
\begin{bmatrix}
\cos k_p d & j \rho_p c_p \sin k_p d \\
 j \sin k_p d / \rho_p c_p & \cos k_p d
\end{bmatrix}
\]

\[
k_p = \frac{1}{d} \cos^{-1} T_{11}
\]

One Load Approach

\[
T_{11} = T_{22}
\]

\[
T_{11} T_{22} - T_{12} T_{21} = 1
\]

Treat empty air column as tested samples

Anechoic termination

Transfer Matrix

Anechoic termination

One Load Approach

Anechoic termination

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Anechoic termination

Transfer Matrix
Post-process iterative algorithm

Iterative Algorithm Flow Chart

Experiment
→ Transfer Function
→ Plane Wave Amplitude
→ Transfer Matrix
→ Complex Wave Number

No
→ Convergence Decision

Yes

\[ A = \frac{j(P_1e^{jx_2} - P_2e^{jx_1})}{2\sin k(x_1 - x_2)}, \]
\[ B = \frac{j(P_2e^{-jx_1} - P_1e^{-jx_2})}{2\sin k(x_1 - x_2)}, \]
\[ C = \frac{j(P_3e^{jx_4} - P_4e^{jx_3})}{2\sin k(x_3 - x_4)}, \]
\[ D = \frac{j(P_4e^{-jx_3} - P_3e^{-jx_4})}{2\sin k(x_3 - x_4)} \]

\[ P_{x=0} = A + B, \quad P_{x=d} = Ce^{-jkd} + De^{jkd}, \]
\[ V_{x=0} = \frac{A - B}{\rho_0c}, \quad V_{x=d} = \frac{Ce^{-jkd} - De^{jkd}}{\rho_0c} \]

\[ k_p = \frac{1}{d}\cos^{-1}T_{11} \]
Experimental Setup

- Semi-empirical prediction: Temkin formula

\[ k = \frac{\omega}{c} + (1 - i)\delta \]

- Experimental estimation: Four-microphone measurement based on the transfer function combined with our iterative algorithm

- B&K 4206 standard small and large transmission loss tube kit
Measurement results - Large Tube

- **Advantage:** Simple, Fast, Flexible
- **Disadvantage:** Half Wave Length Problem
Corrected Temkin Formula

- Temkin formula can match the experimental results well if we modify the measured temperature

![Graph showing corrected interior temperature and measured ambient temperature](image-url)
Measurement results - Small Tube

Sound Speed

Tube Attenuation
Conclusions

- Semi-empirical Temkin formula for complex wave number estimation can be adjusted to match the experimental results very well.

- The iterative algorithm based on four-microphone technique gives a quick and reliable estimation of sound speed and attenuation in the tube.

- The approach has the potential to be used for the accurate measurements of gases properties and duct lining performance.
Thanks You!