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Prediction of Random Incidence Transmission Loss based on Normal Incidence Four-Microphone Measurements

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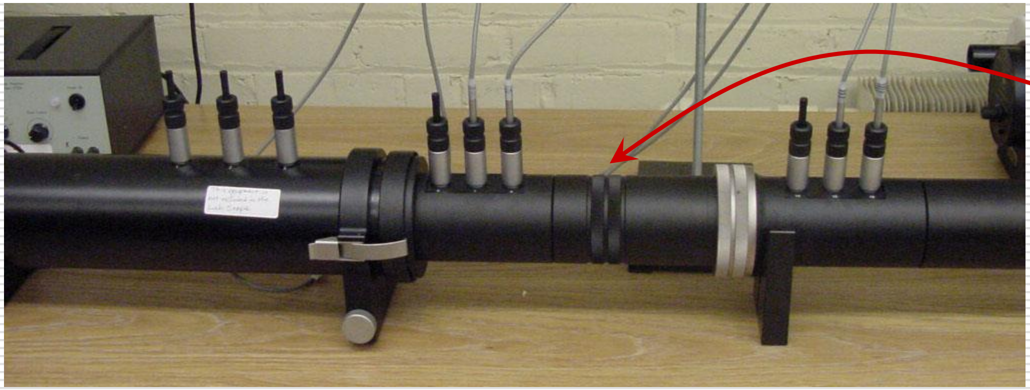
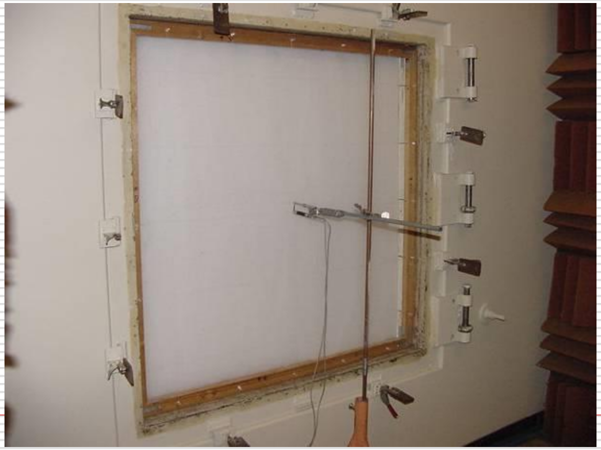
Objectives

- Procedures for measuring the normal incidence transmission loss of porous material are now available

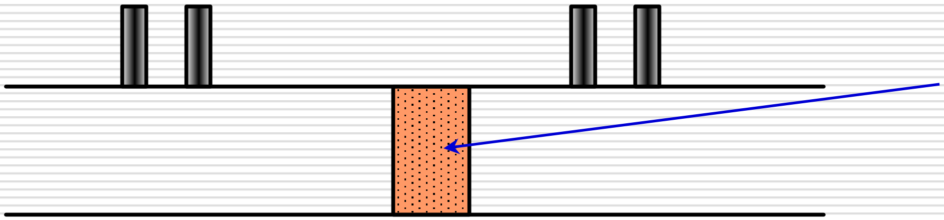


- Can normal incidence information be used to estimate random incidence transmission loss



Overall Approach

1. Use 4-mic TL tube to estimate characteristic properties of porous materials

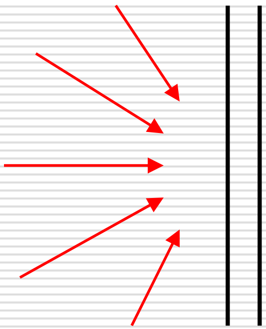


Estimate

-Complex density ρ_p

-Complex wave number k_p

2. Use k_p and ρ_p in theoretical prediction of random incidence TL



* Approach works only for porous materials that can be modeled as effective fluids

Introduction

1. 4-mic transmission loss tube
   - Transfer matrix method and two-load method
   - In this study, the transfer matrix method was applied
   - Mid-size impedance tube (d:63.5 cm) was used

2. Normal impedance prediction
   - Measurements in one configuration used to predict performance in other configurations

3. Random incidence TL prediction
   - Closer to real application
   - No simple relationship with normal incidence TL

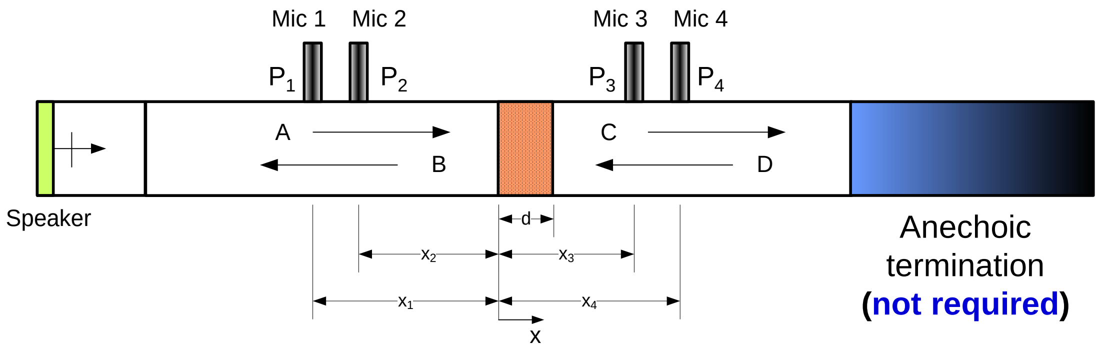
1-1 Transfer Matrix Method
(Song and Bolton JASA 2000)

$$P_1 = (Ae^{-jkx_1} + Be^{jkx_1})e^{j\omega t}$$

$$P_2 = (Ae^{-jkx_2} + Be^{jkx_2})e^{j\omega t}$$

$$A = \frac{j(P_1 e^{jkx_2} - P_2 e^{jkx_1})}{2\sin k(x_1 - x_2)}$$

$$B = \frac{j(P_2 e^{-jkx_1} - P_1 e^{-jkx_2})}{2\sin k(x_1 - x_2)}$$



$$P_3 = (Ce^{-jkx_3} + De^{jkx_3})e^{j\omega t}$$

$$P_4 = (Ce^{-jkx_4} + De^{jkx_4})e^{j\omega t}$$

$$C = \frac{j(P_3 e^{jkx_4} - P_4 e^{jkx_3})}{2\sin k(x_3 - x_4)}$$

$$D = \frac{j(P_4 e^{-jkx_3} - P_3 e^{-jkx_4})}{2\sin k(x_3 - x_4)}$$

- Sound pressure and velocity relationship $\begin{bmatrix} P \\ V \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix}_{x=d}$

$$T_{11} = \frac{P|_{x=d} V|_{x=d} + P|_{x=0} V|_{x=0}}{P|_{x=0} V|_{x=d} + P|_{x=d} V|_{x=0}} \qquad T_{12} = \frac{P|_{x=0}^2 - P|_{x=d}^2}{P|_{x=0} V|_{x=d} + P|_{x=d} V|_{x=0}}$$

$$T_{21} = \frac{V|_{x=0}^2 - V|_{x=d}^2}{P|_{x=0} V|_{x=d} + P|_{x=d} V|_{x=0}} \qquad T_{22} = \frac{P|_{x=d} V|_{x=d} + P|_{x=0} V|_{x=0}}{P|_{x=0} V|_{x=d} + P|_{x=d} V|_{x=0}}$$

- Symmetric sample $T_{11}=T_{22}$, $T_{11}T_{22}-T_{12}T_{21}=1$
- Transmission loss $\mathrm{TL} = 20\log_{10}(1/|T|)$

where $T_a = \dfrac{2e^{jkd}}{T_{11} + (T_{12}/\rho_0 c) + \rho_0 c T_{21} + T_{22}}$

- Transfer matrix

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos k_p d & j\rho_p c_p \sin k_p d \\ j \sin k_p d / \rho_p c_p & \cos k_p d \end{bmatrix}$$

Property of material — Limp or rigid porous material

$$\Rightarrow \begin{cases} k_p = \dfrac{1}{d}\cos^{-1} T_{11} \\ \rho_p = \dfrac{1}{c_p}\sqrt{\dfrac{T_{12}}{T_{21}}} \end{cases}$$

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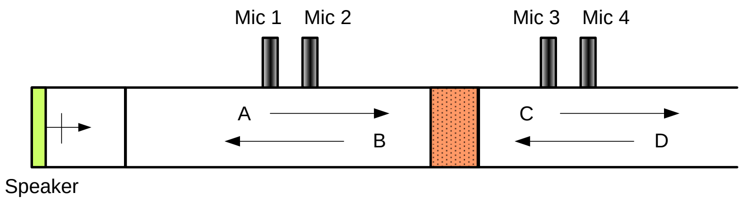
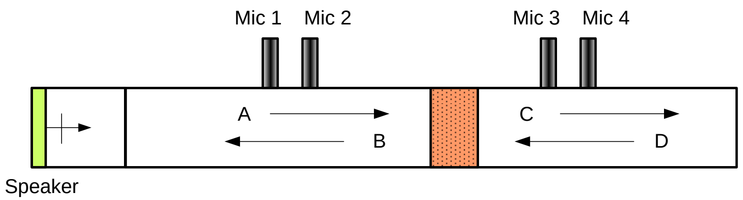
1-2 Two Load Method (Munjal)

- Measurement 1:



Open termination

- Measurement 2:



Hard termination

Measurement 1: P_1, V_1; Measurement 2: P_2, V_2

$$\begin{bmatrix} P_1 \\ V_1 \\ P_2 \\ V_2 \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} & 0 & 0 \\ T_{21} & T_{22} & 0 & 0 \\ 0 & 0 & T_{11} & T_{12} \\ 0 & 0 & T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ V_1 \\ P_2 \\ V_2 \end{bmatrix}_{x=d}$$

4 equations in 4 unknowns

- Solve for $T_{11}, T_{12}, T_{21}, T_{22}$
- Advantage: no requirement that sample be symmetric
- Disadvantage: twice as many measurements

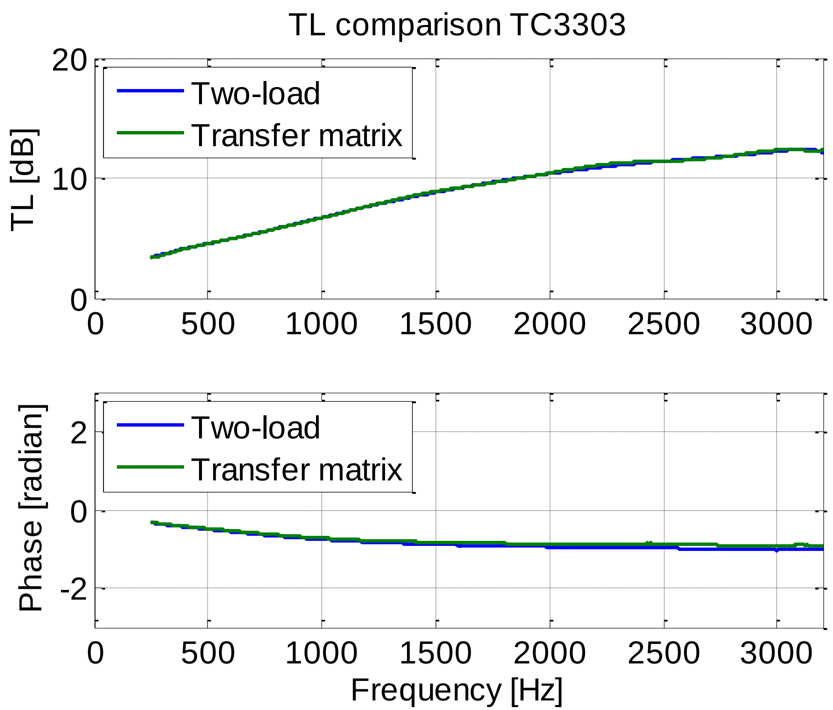
1-3 Materials

- THL3
  - polyester staple fibers
  - Lower density than TC3303
  - Lower TL and absorption coefficient
  - Thinner than TC3303

- TC3303
  - blown micro fibers with mix of polypropylene and polyester staple fibers

	THL3	TC3303
Thickness [cm]	3.95	4.98
Mass per unit area [g/m2]	156	376

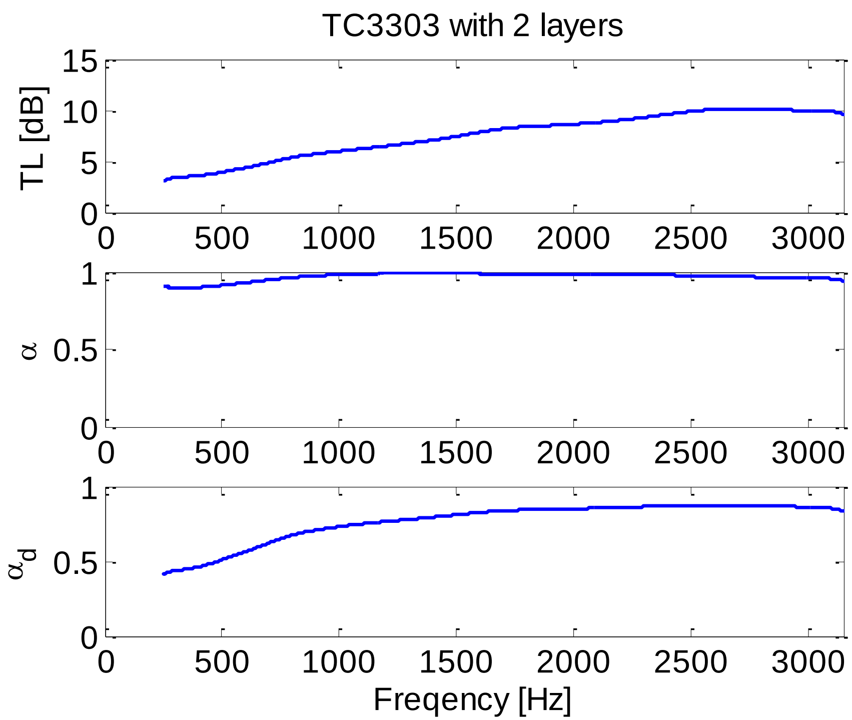
1-4 Two-Load and Transfer Matrix Methods

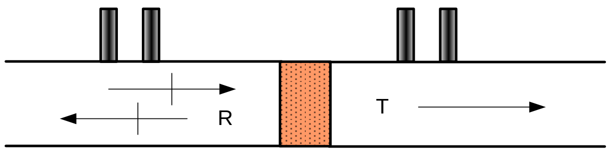


- Two-load and transfer matrix methods show perfect agreement both on magnitude and phase.
- Two-load method needs two different terminations with and without a sample: Total of four measurements are required

1-5 Typical Results

For anechoically terminated sample





$$TL = 10\log_{10}\left(\frac{1}{|T_a|^2}\right)$$

$$T_a = \frac{2e^{jkd}}{T_{11} + (T_{12}/\rho_0 c) + \rho_0 c T_{21} + T_{22}}$$

Expressed in terms of T-matrix elements

$$R_a = \frac{T_{11} + (T_{12}/\rho_0 c) - \rho_0 c T_{21} - T_{22}}{T_{11} + (T_{12}/\rho_0 c) + \rho_0 c T_{21} + T_{22}}$$

$$\alpha = 1 - |R_a|^2$$

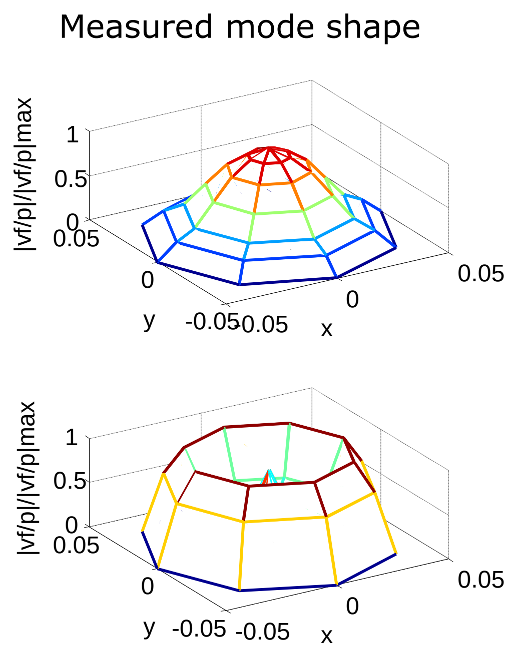
$$\alpha_d = 1 - |R_a|^2 - |T_a|^2$$

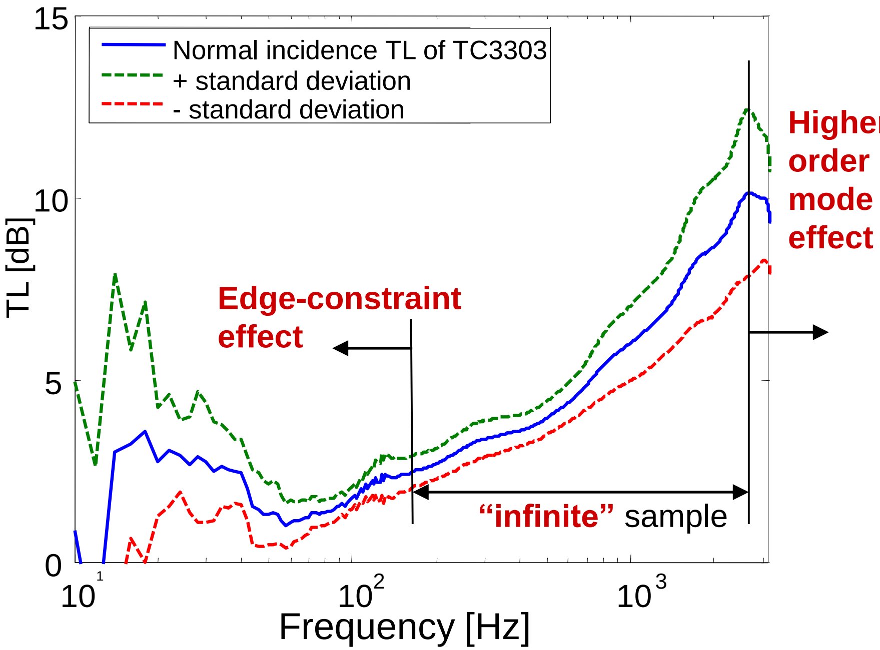
- α_d represents the fraction of the incident energy dissipated within the sample

1-6 Normal incidence TL (average of 10 measurements)

1st mode

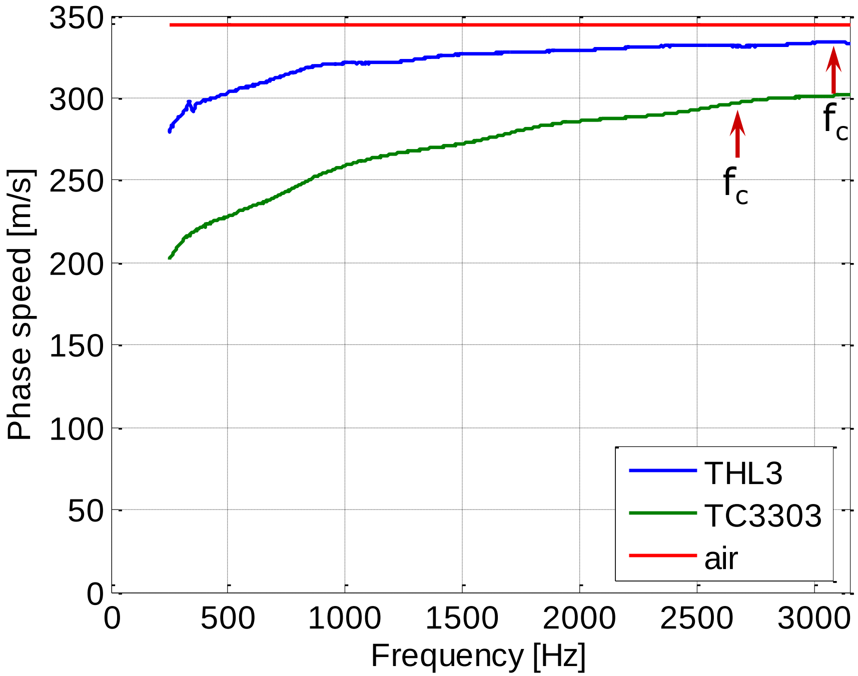
2nd mode





- Because of leakage problem, two layers were used for normal incidence TL test.
- Edge-constraint effects appear at low frequencies in the measurements
- Higher duct modes appear in sample around 3 kHz and cause increased standard deviation

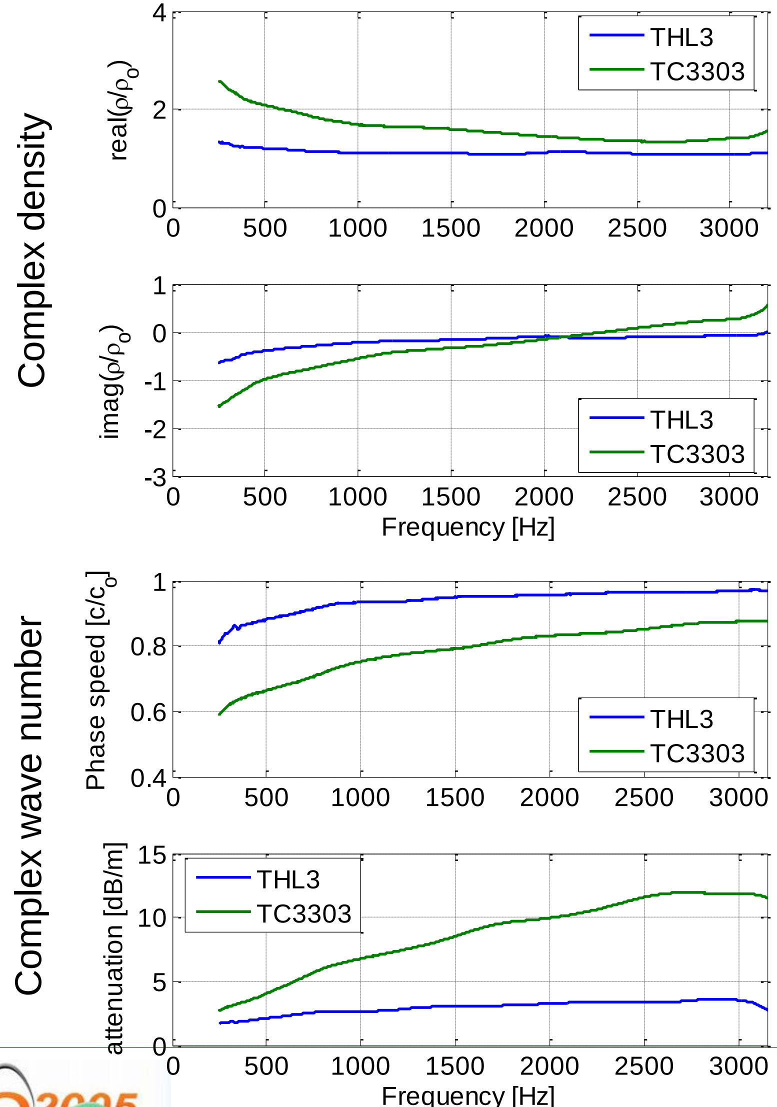
1-7 Higher Order Modes in Samples



$$f_c = \frac{1.84 \times c_p}{2\pi r}$$

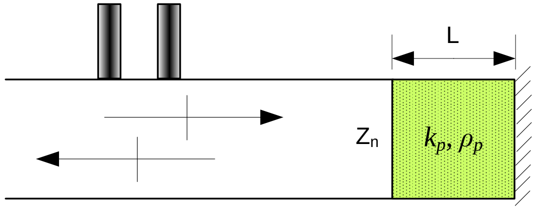
- Since wave speed in porous materials is subsonic, higher order modes may "cut-on" in the sample at lower frequencies than in the tube
  - TC3303 – accurate at frequencies < 2700 Hz
  - THL3 – accurate at frequencies < 3100 Hz
- This effect limits high frequency accuracy of the measurements

1-8 Complex Density and Complex Wave Number



- Normalized complex densities show that TC3303 has higher density
- Phase speed: $\dfrac{\omega}{\mathrm{Re}\{k_p\}}$
- Attenuation per m: $\mathrm{Im}\{k_p\}$
- These values can be used in SEA, FE predictions and plane wave predictions

2-1 Prediction of Hard Termination Impedance

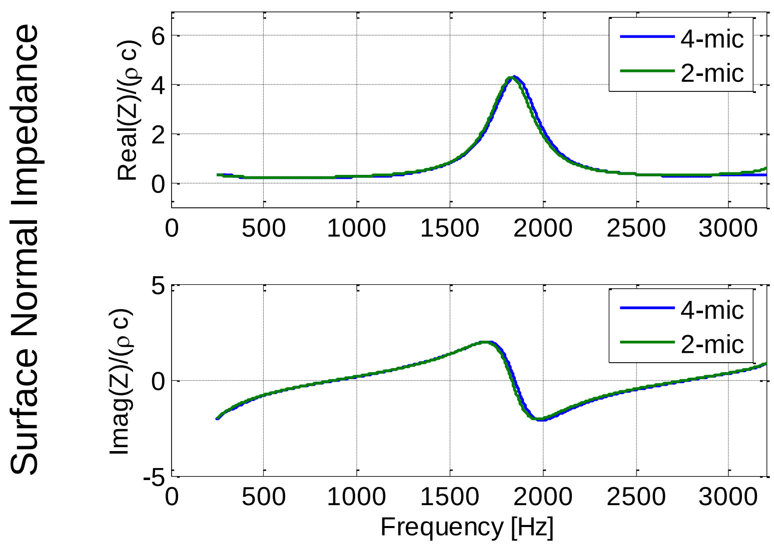


Rigid termination

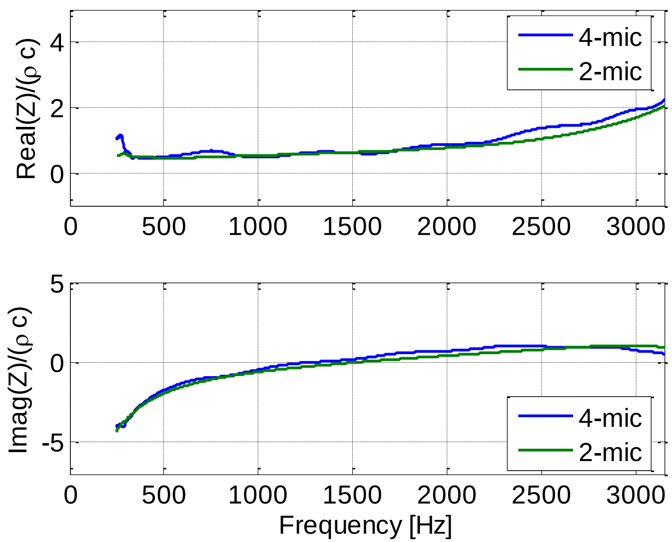
$$Z_n = -j\rho_p c_p \cot k_p L$$

- Use values of k_p and ρ_p measured in 4-microhpone tube to predict properties of same material in different environment (e.g., hard backing) and compare with direct measurement

Noraml Impedance with hard termination of THL3

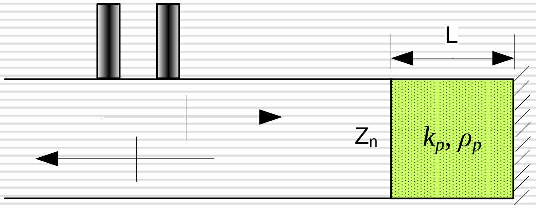


Normal Impedance with hard termination of TC3303



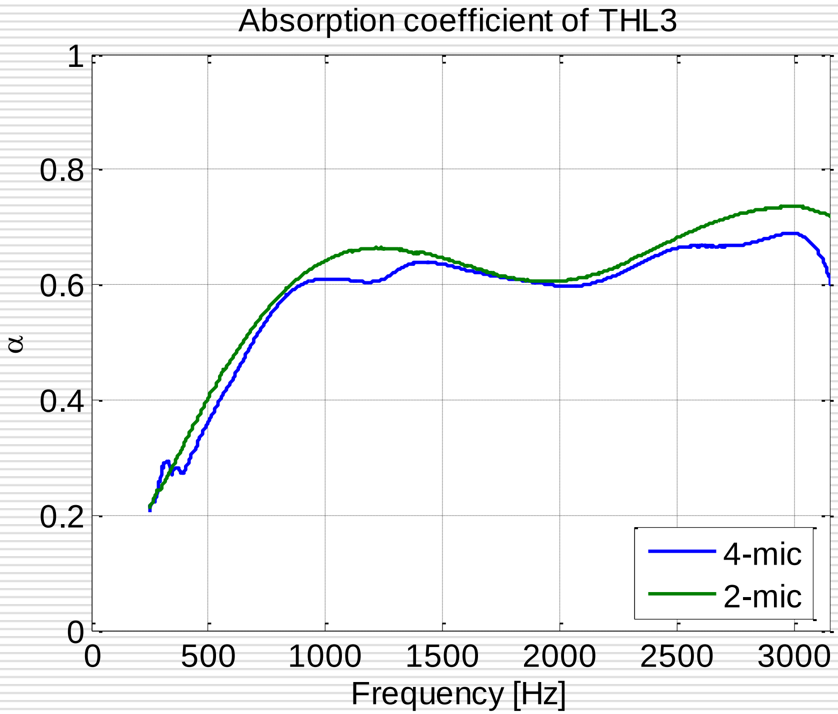
- Any known backing impedances can be accommodated

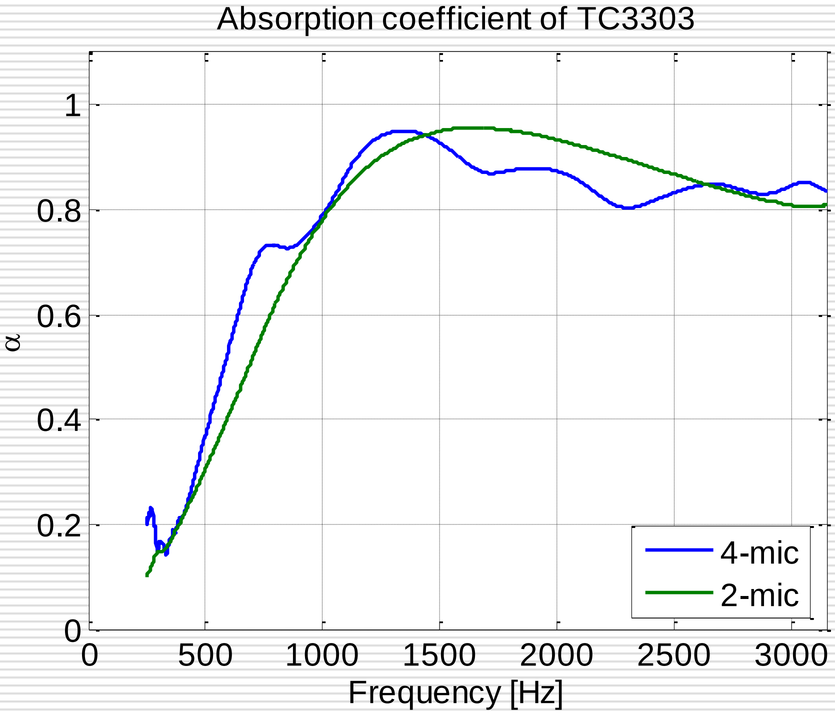
2-2 Prediction of Hard Termination Case Absorption coefficient



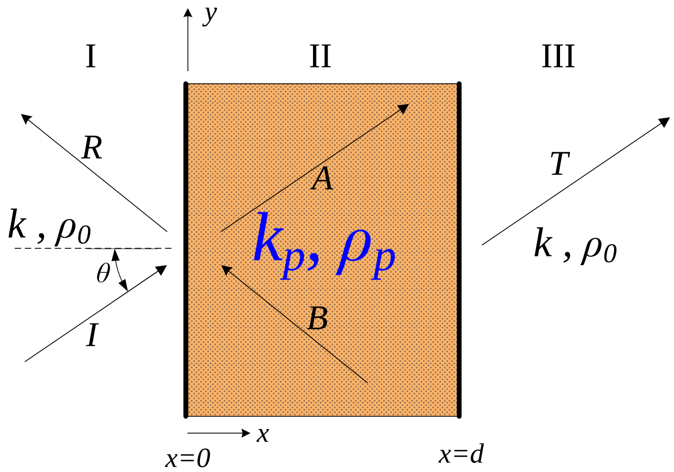
Rigid termination

$$R = \frac{Z_n - \rho_0 c}{Z_n + \rho_0 c} \qquad \alpha = 1 - |R|^2$$





3-1 Random Incidence Transmission Loss (for rigid or limp porous materials)



Region I
$$\begin{cases} P_1 = e^{-jk_x x - jk_y y} + Re^{jk_x x - jk_y y} \\ U_{1x} = -\dfrac{1}{j\omega\rho_0}\dfrac{\partial P_1}{\partial x} = -\dfrac{1}{j\omega\rho_0}\left(-jk_x e^{-jk_x x - jk_y y} + jk_x Re^{jk_x x - jk_y y}\right) \end{cases}$$

Region II
$$\begin{cases} P_2 = Ae^{-jk_{Px} x - jk_{Py} y} + Be^{jk_{px} x - jk_{py} y} \\ U_{2x} = -\dfrac{1}{j\omega\rho_p}\dfrac{\partial P_2}{\partial x} = -\dfrac{1}{j\omega\rho_p}\left(-jk_{px} Ae^{-jk_{px} x - jk_{py} y} + jk_{px} Be^{jk_{px} x - jk_{py} y}\right) \end{cases}$$

Region III
$$\begin{cases} P_3 = Te^{-jk_x x - jk_y y} \\ U_{3x} = -\dfrac{1}{j\omega\rho_0}\dfrac{\partial P_3}{\partial x} = -\dfrac{1}{j\omega\rho_0}\left(-jk_x Te^{-jk_x x - jk_y y}\right) \end{cases}$$

k_p and ρ_p can be acquired from normal incidence TL test

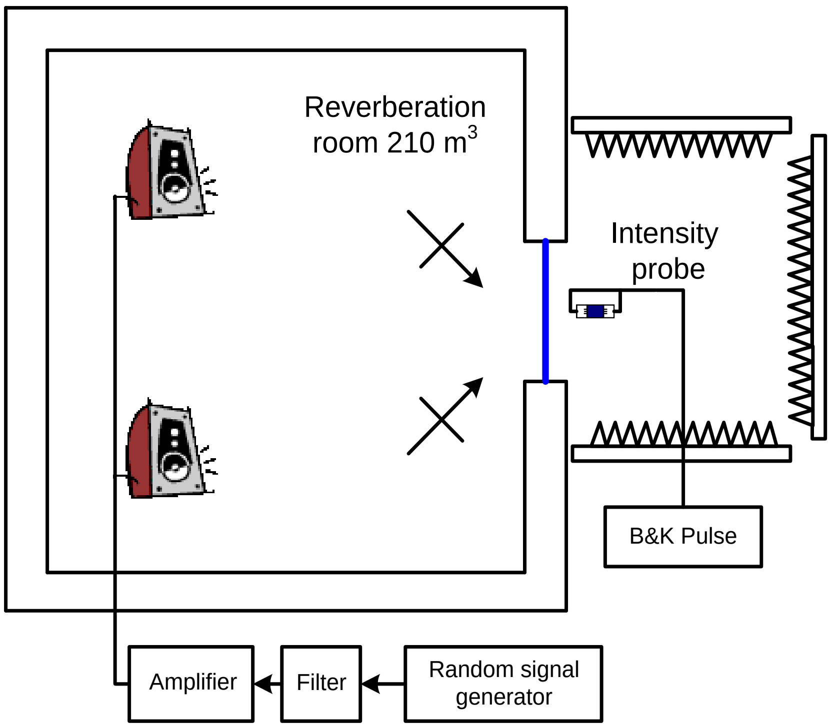
- Transmission coefficient

$$T = \frac{2\dfrac{\rho_p k_x}{\rho_0 k_{px}}}{e^{-jdk_x}\left\{ j\sin dk_{px}\left(\left(\dfrac{\rho_p k_x}{\rho_0 k_{px}}\right)^2 + 1\right) + 2\dfrac{\rho_p k_x}{\rho_0 k_{px}}\cos dk_{px} \right\}}$$

- Transmission loss

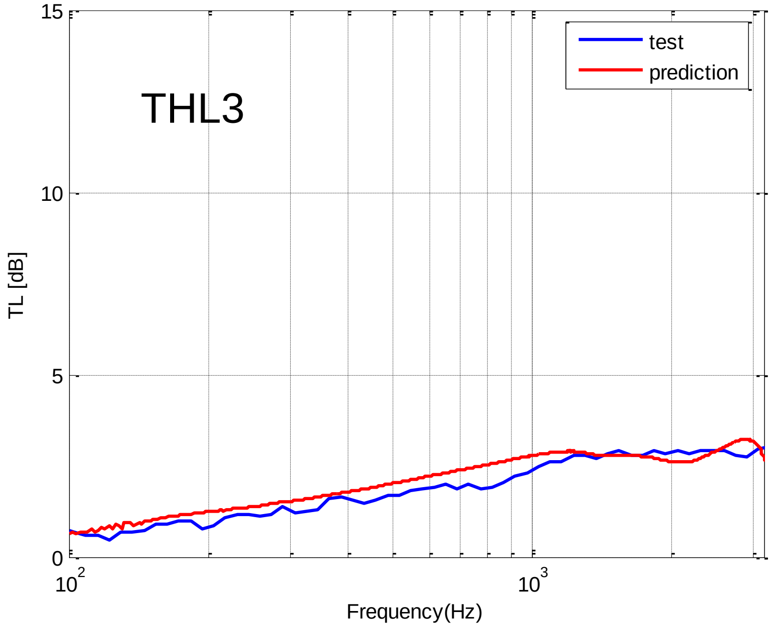
$$\mathrm{TL} = 10\log_{10}\left(1/\bar{\tau}\right)$$ where $\bar{\tau} = \int_0^{\pi/2} |T|^2 \sin 2\theta d\theta$

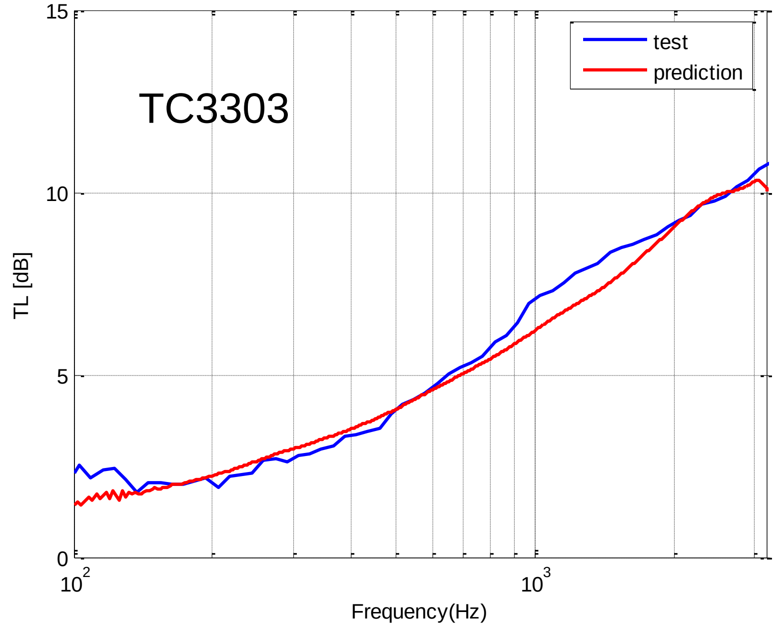
3-2 Test Setup of Random Incidence TL



- Test was performed in reverberation room with intensity probe with two different sized spacers.
- TL was calculated by averaging TL at 25 points over sample
- Two layers of each material were used

3-3 Random Incidence Transmission Loss





- Predictions based on complex density and wave number and direct measurements show excellent agreement

Summary

- Random incidence transmission loss and other acoustical properties (e.g., surface normal impedance) can be predicted by using complex wave number (k_p) and complex density (ρ_p) which can be acquired from normal incidence TL test (materials should be rigid or limp)
- Prediction and measurement of surface normal impedance and random incidence transmission loss showed good agreement with each other
- At low frequencies: measurements affected by edge-constraint effect
- At high frequencies: measurements affected by higher order mode propagation within the sample
- Transfer matrix and two-load method gave same results
- Transfer matrix method is more convenient than two-load method when the material is symmetric because a single measurement is sufficient