Numerical Modelling of the Acoustics of Low Density Fibrous Media having a Distribution of Fiber Sizes

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Numerical Modeling of the Acoustics of Low Density Fibrous Media Having a Distribution of Fiber Sizes

Thomas Herdtle, 3M Company
Yutong Xue and J. Stuart Bolton, Purdue University
Focus

• Low density, polymeric, fibrous medium
  – Blown microfibers with broad fiber size distribution
  – Staple fiber component with larger but narrow size distribution

Airflow Resistivity

• Considered the most important macroscopic quantity for acoustics
• Literature typically only considers a single uniform fiber size
• Tarnow, for example, considers uniform size with random 2-D spacing
• Authors extended theory to distributions of fiber size (InterNoise 2017)

Acoustic Properties

• Derived from resistivity using limp fluid models such as JCA
  – Method doesn’t explicitly account for range of fiber sizes
• Present work to determine
  – Reliability of resistivity calculation
  – Applicability for media with broad fiber size distribution (work in progress)
Modeling Approach

Numerical Tools

- **GeoDict** with **FlowDict** by Math2Market (a Fraunhofer Institute spinoff)
  - Fiber geometry generation, including fiber size distributions and 3D orientation
  - Flow resistance calculation (CFD)
  - **AcoustoDict** with **DiffuDict** can also calculate other JCA parameters

- 3M Proprietary Software
  - Import GeoDict geometry and create a COMSOL model

- **COMSOL Multiphysics**
  - Flow resistance calculation (CFD)
  - Diffusion calculation (for remaining JCA parameters)
  - Visco-thermal acoustic calculation (rigid fibers or fluid-structure interaction for limp fibers)
**Inputs**

- Fiber size (radius): $r$
- Fiber bulk density: $\rho_b$
- Solid material density: $\rho$

**Output**

- Solidity: $C = \frac{\rho_b}{\rho}$
- Fiber mean spacing: $b^2 = \frac{\pi r^2}{C}$
- AFR: $\sigma = \frac{4\pi \eta}{b^2[0.640 \ln \left(\frac{1}{C}\right) + C - 0.737]}$ (Based on Tarnow model)

**Steps**

- **Step 1**: $C$ calculation based on $\rho_b$ and $\rho$
- **Step 2**: $b^2$ calculation based on $r$ and $C$
- **Step 3**: $\sigma$ calculation base on $C$ and $b^2$

Tarnow Voronoi Cells: for modeling the mean spacing $b^2$ distributed around each fiber cylinder. Fiber F is relocated to centroid G.
**Tarnow Verification – Uniform fiber size**

**2D Results**
- Regular lattice
  - *Exact match* to equations
- Random fibers in 2D
  - Lower resistivity is for parallel case
- Tarnow assumed fibers centered within Voronoi cells

**3D Extensions**
- Resistivity vs. air flow angle
  - Nonlinear variation (*more data needed*)
- Isotropic 3D distribution
  - Between parallel and perpendicular
- Transverse isotropic 3D distribution
  - Much like perpendicular

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**Random 2D Fibers vs. Angle of Flow**

**Velocity Mag. Parallel**

**Vel. Mag. Perpendicular**

**Resistivity vs. Air Flow Angle**

**GeoDict results**
**Xue – InterNoise 2017**

**Inputs**
- Fiber mean radii: $r_1, r_2$, distribution parameters
- Fiber bulk density: $\rho_b$
- Component weight fractions: $X_1, X_2$
- Solid material densities: $\rho_1, \rho_2$

**Output**
- AFR: $\sigma = \frac{4\pi \eta}{b^2 [0.640 \ln\left(\frac{1}{C}\right) + C - 0.737]}$ (Based on Tarnow model)

**The modifications were made only on Step 1 and Step 2.**

- **Step 1**: $C$ calculation based on $\rho_b, X_1, X_2, \rho_1, \rho_2$
- **Step 2**: $b^2$ calculation based on $r_1, r_2, \rho_1, \rho_2$
- **Step 3**: $\sigma$ calculation base on $C$ and $b^2$
Broad Fiber Size Distribution

**Flow Resistance** for 1 mm³ volume
- Broader distributions have lower flow resistance
  - Larger fibers appear to reduce FR more than small fibers increase it
- Tarnow calculation vs. GeoDict
  - Shows similar trends
  - Tarnow has about 18-20% lower resistance
  - GeoDict is overpredicting dP compared to COMSOL
    (3 cases compared – could be due in part to 1 µm voxels and boundary effects)
Broad Fiber Size Distribution

Measurement Difficulties

• Micro-CT scanning
  – Biased to larger fibers and clumps
  – Algorithm broadens distribution artificially

• SEM Images
  – Automatic fiber counting
    • Need a clean section of fiber
    • May count same fiber several times
    • May count bundles as a single fiber

Fiber size distribution from µCT scan

Tarnow calculation for given fiber distributions

Fiber distribution from SEM images
JCA Parameters from COMSOL

**Geometry**

\[ \Lambda_{\text{therm}} = \frac{2 \int_{\text{Vol}} dV}{\int_{\text{Surf}} dS} = \frac{2V}{S} \]

\[ \phi = \frac{V}{\Delta X \Delta Y \Delta Z} \]

- Porosity
- Thermal characteristic length

**Laplace Equation in Air Space only**

- Inviscid flow, thermal conduction, or electric conduction
  - Apply a potential difference and compute current
- Tortuosity factor and tortuosity
- Viscous characteristic length

\[ J = \int_{\text{Inlet}} J_z dS = \frac{1}{V} \int_{\text{Vol}} J_z dV \]

\[ \alpha_\infty = \frac{\phi \Delta V \Delta X \Delta Y C}{J \Delta Z} \approx \frac{V \int_{\text{Vol}} J \cdot J dV}{(\int_{\text{Vol}} J_z dV)^2} \]

\[ \Lambda_{\text{visc}} = \frac{2 \int_{\text{Vol}} |J|^2 dV}{\int_{\text{Surf}} |J|^2 dS} \]

\[ \tau = \sqrt{\alpha_\infty} \]
Viscous Flow

• Flow resistance and resistivity
• Low frequency tortuosity factor
  \[ \alpha_0 > \alpha_\infty \text{ due to } v = 0 \text{ on fiber surfaces} \]

\[
\begin{align*}
\nu_{ave} &= \frac{1}{\Delta X \Delta Y} \int_{\text{Inlet}} u \, dS \\
\sigma &= \frac{dP}{\phi \Delta Z \nu_{ave}} = \frac{\mu}{\kappa} \\
\alpha_0 &= \frac{V \int_{Vol} \nu \cdot \nu \, dV}{\left( \int_{Vol} \nu_z \, dV \right)^2}
\end{align*}
\]
Direct Modeling of the Thermo-Acoustic fields

• Computes acoustic pressure and velocity fields along with fiber motion.
• 4-mic impedance tube equations (1-load) using virtual pressure points within model.
• Allows computation of wavenumber and complex speed of sound, density, and bulk modulus.
• Requires HPC-system: used one node with 28 cores and 1TB RAM.
GeoDict Fiber Size Distribution

<table>
<thead>
<tr>
<th>Solidity</th>
<th>Uniform Fiber Flow Resistance</th>
<th>Broad Distribution Flow Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>13,303 Rayl</td>
<td>10,746 Rayl</td>
</tr>
<tr>
<td>2%</td>
<td>33,062 Rayl</td>
<td>26,707 Rayl</td>
</tr>
<tr>
<td>3%</td>
<td>57,643 Rayl</td>
<td>46,562 Rayl</td>
</tr>
</tbody>
</table>

Fiber size distribution generated by GeoDict

Tarnow calculation for model fiber distributions
COMSOL Acoustic Model with Structural Motion (FSI)

Compute complex material properties

- 200 × 200 × 1000 µm Volume
- 3 Realizations at each solidity
- Higher solidity shifts curves to higher frequencies
COMSOL Acoustic Model with Structural Motion (FSI)

Compute complex material properties

- Size distribution increases case-to-case variability
- Suggests the need for larger models or averaging many cases
JCA (Limp) vs. Direct Acoustic Properties – Uniform Fibers

Wave Number – **Good**
Density – **Excellent**
Speed of Sound – **Fair**
Bulk Modulus – **Poor**

Solid curves are JCA calculation
Markers are direct model results

2% Solidity
Wave Number – Good
Density – Good
Speed of Sound – Fair
Bulk Modulus – Poor
All are a little worse than uniform fiber size.

Solid curves are JCA calculation
Markers are direct model results

2% Solidity
Wave Number – **Good**
Density – **Good**
Speed of Sound – **Fair**
Bulk Modulus – **Better at high frequencies**

3% Solidity appears to be a bit better behaved, also less variable.

Solid curves are JCA calculation
Markers are direct model results

3% Solidity
**Bonding changes fiber motion significantly**
- Individual fibers move more or less, depending on their diameter and orientation
- Bonded fibers move in unison
- *Simple test case of $500 \times 500 \times 200 \, \mu m$ at 3% solidity*
Conclusions

Results

• Extended Tarnow calculations match CFD calculations reasonably well.
• Fiber size distributions are difficult to obtain.
• JCA parameters can be obtained from finite element models.
• Acoustic equivalent fluid properties can be calculated directly using FEA.
  – Much variability for small systems with low solidity.
  – Results don’t necessarily match JCA calculations.
  – Large computers are needed for modeling fine fibers.
• Intersecting or bonded fibers tend to move in unison.

Next steps

• 2-Load method which doesn’t require/assume a symmetric sample.
• Determine difference between COMSOL and FlowDict pressure drop calculations.
• Determine acoustic effect of individual vs. bonded fibers.
• Currently adiabatic or isothermal fibers, can be extended to include heat transfer in fibers.
• Consideration of fiber length distribution (after C. Perrot’s presentation).