1992

Free-Piston Stirling Coolers

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A simple linear analysis of the dynamics of free-piston Stirling coolers is presented to describe the behavior of these machines. Useful results for the displacer phase angle and stroke ratio are obtained and it is shown that the linear analysis is able to accurately account for the dissipative losses in a given machine. A vector diagram is used to indicate the balance of forces at steady operation. This analysis has been used to design a number of successful prototype machines. They include non-CFC domestic refrigerator coolers, small electronic coolers and a 100K, 200W lift cryocooler.

NOMENCLATURE

α	Feed back coupling [N/m]	p	Pressure [Pa]
A	Displacer cylinder area [m^2]	P	Power [W]
A_p	Piston area [m^2]	Q	Stored energy / energy dissipated per cycle
A_R	Displacer rod area [m^2]	$\dot{Q}_c$	Conduction heat transfer [W]
D_{ext}	Incidental damping [Ns/m]	t	Time [s]
F	Force [N]	V	Volume [m^3]
H	Regenerator heat loss [W]	x	Displacement [m]
j	$\sqrt{-1}$	X	Amplitude [m]
K_{ext}	Displacer external spring [N/m]	ω_0	Radial frequency [rads/s]
m	Component masses [kg]		

Subscripts

c	Cylinder, compression space	e	Expansion space
d	Displacer	p	Piston

INTRODUCTION

The Stirling cycle has for some years been the closed-cycle of choice in miniature low temperature applications (eg. infrared imaging applications). Recently, interest in higher lift Stirlings operating at warmer temperatures has been rekindled in light of the concern over the destruction of the earth's ozone layer caused by chlorofluorocarbons. This interest is driven mainly by the fact that the Stirling uses environmentally benign gas as the working fluid and that its ideal coefficient of performance (COP) is that of Carnot, the theoretical maximum for any heat pump. Though the cycle itself is easily scaled to higher lifts, conventional crank mechanisms require lubrication oil at higher power levels which tends to contaminate the internal heat exchangers and reduce performance. The free-piston / linear alternator configuration [1] neatly circumvents this problem by using the cycle gas pressure forces and a linear motor to achieve the proper motions of the moving parts. In so doing, a number of important advantages accrue which allow practical machines of higher powers to be built. The most important being:

i) The side loads on the moving parts are very low and it becomes practical to utilize gas bearings and therefore avoid the need for lubricating oil. Since gas bearings operate without contact, there is practically no wear and long life can be expected in addition to low levels of friction.
ii) The linear motor is easily placed within the pressure vessel making it possible to hermetically seal the pressure vessel which avoids the working gas leakage problem.
iii) Modulating the machine becomes a simple matter of adjusting the piston stroke which for a linear motor simply means controlling the input voltage.
iv) The motion of the moving parts are almost pure sinusoids, thus the higher harmonic content in the vibration of the unit is very small. This makes it easy to balance the machine with a simple dynamic absorber to levels of very low residual amplitude and noise.
v) Simplicity of construction. The basic machine has only two moving parts.
vi) Because friction has been reduced to almost zero, the mechanical efficiency of the device is very high and internal heat generation very low.

A number of prototype machines based on the free-piston / linear motor configuration have been built. These include cryocoolers for biological storage, domestic refrigerator coolers and small electronic coolers. Efficiencies have been high and, though life testing has only began, one unit has already accumulated over 3000hrs of operation at high stress levels.

BASICS

The Stirling cycle is a closed cycle in which a fixed mass of gas is alternately expanded, warmed, compressed and cooled to the beginning of the expansion process. Figure 1 shows the two most common configurations for realizing the Stirling cycle. The processes of heating and cooling are appreciably augmented by the regenerator which usually consists of a matrix of fine wires or simply annular gaps made by winding foil on itself. The regenerator serves to store heat as the gas leaves the warm-section and to transfer this heat back to the gas as the gas returns from the cold section. Sensible heat is mainly transferred internally by the regenerator and the compression and expansion processes are therefore able to approach isothermal operation which allows high efficiencies to be obtained. Figure 2 shows an example of a small free-piston Stirling cooler designed for cooling electronics.

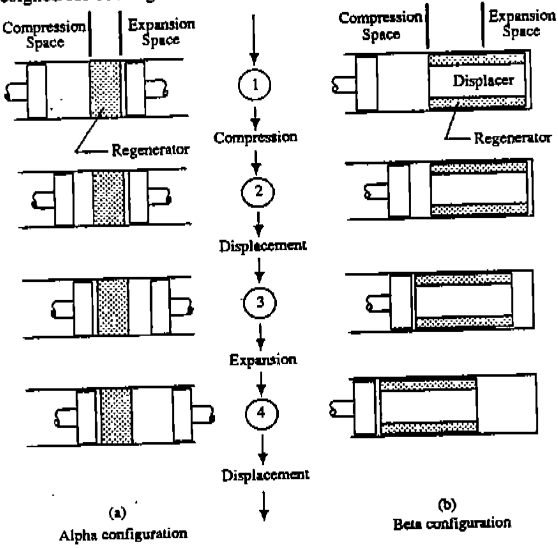

Figure 1 – Piston and displacer motions for two common configurations.

In order to achieve the proper motions, feed-back must be provided to the displacer. Figure 3 shows one method of achieving this by the use of a displacer rod which forms a differential area over which the working gas pressure acts. The operation of a free-piston cooler may be understood by describing the pressure forces as the cycle goes through its motions. Beginning at the compression stage, as the piston moves up it increases the gas pressure. Because of the presence of the rod this creates a force on the displacer which pushes the displacer towards the compression space thus effecting the constant volume displacement of gas to the cold side. The piston now moves out to expand the gas which is mainly in the cold end. In so doing the piston again creates a pressure force across the displacer which eventually forces the displacer towards the expansion space, displacing the gas to the warm end which is the beginning of the cycle. Since the piston and displacers are freely moving components and the appropriate phase angle between their motions is important for the lifting of heat, it is necessary to spring the moving parts so that they operate correctly at the required frequency. Linear dynamics theory [2, 3, 4, 5, 6] has been found to adequately describe the behavior of these machines and allows the determination of the important parameters for optimal operation.

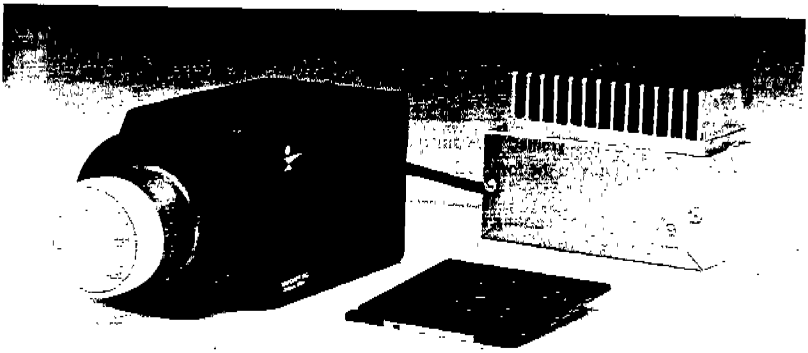

Figure 2 - A small free-piston Stirling cooler capable of lifting 35W at - 50°C

LINEAR MODEL

Referring to Figure 3, piston motion causes changes in working gas pressure p that excite motion of the displacer which transfers working gas across a temperature differential thus changing p and hence also the force on the piston. The piston is driven and its frequency and amplitude are assumed to be known.

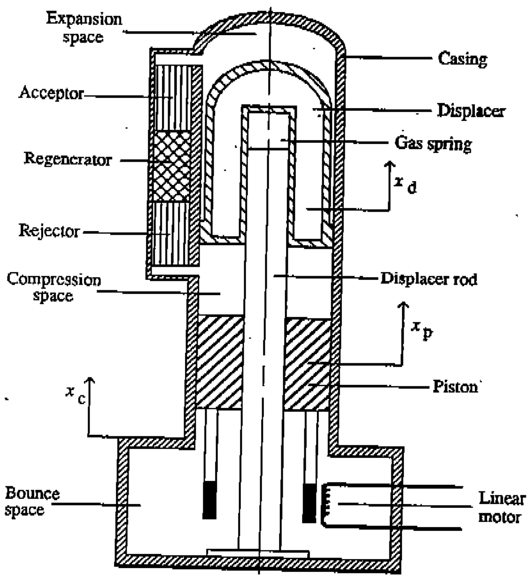

Figure 3 – General arrangement of a free-piston Stirling cooler driven by a linear motor

The equation of motion for the displacer may be written as follows:

$$m_d\ddot{x}_d = (A - A_R)p_c - Ap_e - K_{ext_d}(x_d - x_c) - D_{ext_d}(\dot{x}_d - \dot{x}_c) \quad (1)$$

where K_{ext} is the gas spring constant and D_{ext} accounts for incidental dissipative terms such as friction and gas hysteresis. Note that $(p_c - p_e)$ is simply the pressure drop across the heat exchanger loop, Δp.

From the equation of motion for the centre of mass of the system, the following is obtained:

$$x_c = -\frac{m_d}{m_c}x_d - \frac{m_p}{m_c}x_p \tag{2}$$

Mounting forces have been neglected here since the light springs that are generally used for this purpose contribute negligibly to the force balance.

Substituting (2) into (1) and noting that the Δp term is the result of viscous dissipation in the heat exchangers which is a function of the displacer, piston and to a lesser extent, casing velocities, the linearized form of (1) is:

$$m_d\left(\ddot{x}_d + \frac{\omega_d}{2\pi Q_d}\dot{x}_d + \omega_d^2 x_d\right) + \left[\left(D_{dp} - D_{dc}\frac{m_p}{m_c}\right)\dot{x}_p + \alpha_p x_p\right] = 0 \tag{3}$$

where the quantities ω_d, Q_d and α_p are defined as follows:

$$\omega_d = (\textit{undamped natural frequency}) = \sqrt{K_d/m_d} \tag{4}$$

where

$$K_d = K_{ext_d}\left(1 + \frac{m_d}{m_c}\right) + A_R\left(\frac{\partial p_c}{\partial x_d} - \frac{m_d}{m_c}\frac{\partial p_c}{\partial x_c}\right).$$

$$Q_d = (\textit{stored energy/energy loss per cycle at } \omega_d) \tag{5}$$

$$= \left(\frac{\omega_d m_d}{2\pi D_d}\right)$$

where

$$D_d = D_{dd} - D_{dc}\frac{m_d}{m_c}$$

and

$$D_{dd} = D_{ext_d} - A\frac{\partial \Delta p}{\partial \dot{x}_d}$$

$$D_{dc} = -D_{ext_d} - A\frac{\partial \Delta p}{\partial \dot{x}_c}$$

$$D_{dp} = -A\frac{\partial \Delta p}{\partial \dot{x}_p}$$

$$\alpha_p = (\textit{piston coupling}) \tag{6}$$

$$\alpha_p = A_R\left(\frac{\partial p_c}{\partial x_p} - \frac{m_p}{m_c}\frac{\partial p_c}{\partial x_c}\right) + K_{ext_d}\frac{m_p}{m_c}$$

For practical machines, an order of magnitude analysis reduces K_d, D_d and α_p to the following forms[6]:

$$K_d \approx K_{ext_d} + A_R\frac{\partial p_c}{\partial x_d} \tag{7}$$

$$D_d = D_{dd} \tag{8}$$

$$\alpha_p = A_R \frac{\partial p_c}{\partial x_p} + K_{ext_d} \frac{m_p}{m_c} \tag{9}$$

Note that α_p is still a positive quantity for zero rod area ($A_R = 0$). Thus it is possible to obtain piston coupling for machines without a displacer rod. For example, machines with mechanically sprung displacers.

And (3) reduces to the final form of the displacer motion equation:

$$m_d \left(\ddot{x}_d + \frac{\omega_d}{2\pi Q_d} \dot{x}_d + \omega_d^2 x_d \right) + D_{dp} \dot{x}_p + \alpha_p x_p = 0 \tag{10}$$

If the piston motion is given as follows:

$$x_p = X_p \, e^{j\omega_0 t} \tag{11}$$

where x_p is the reference phase. Then an oscillatory solution to the differential equation is:

$$x_d = \overline{X}_d \, e^{j\omega_0 t}$$

The overbar indicates a complex amplitude. Substituting this into (16) gives

$$\frac{\overline{X}_d}{X_p} = - \frac{\alpha_p + j D_{dp} \omega_0}{K_d \left[1 - \left(\frac{\omega_0}{\omega_d} \right)^2 + j \frac{\omega_0}{\omega_d} \frac{1}{2\pi Q_d} \right]} \tag{12}$$

From which follows the stroke ratio and displacer/piston phase angle.

$$\left| \frac{X_d}{X_p} \right| = \frac{\left(\alpha_p^2 + \omega_0^2 D_{dp}^2 \right)^{\frac{1}{2}}}{K_d} \left\{ \left[1 - \left(\frac{\omega_0}{\omega_d} \right)^2 \right]^2 + \left[\frac{\omega_0}{\omega_d} \frac{1}{2\pi Q_d} \right]^2 \right\}^{-\frac{1}{2}} \tag{13}$$

$$\phi = \tan^{-1} \left\{ \frac{\frac{\omega_0}{\omega_d} \frac{1}{2\pi Q_d} - \frac{\omega_0 D_{dp}}{\alpha_p} \left[1 - \left(\frac{\omega_0}{\omega_d} \right)^2 \right]}{- \left[1 - \left(\frac{\omega_0}{\omega_d} \right)^2 \right] - \frac{\omega_0}{\omega_d} \frac{\omega_0 D_{dp}}{\alpha_p} \frac{1}{2\pi Q_d}} \right\} \tag{14}$$

These results for stroke ratio and displacer/piston phase angle are shown plotted in Figures 4 and 5. Figure 4 shows that the stroke ratio increases for increasing Q_d as would be expected. Maximum amplitude ratio is obtained very close to resonance conditions. In Figure 5, phase angle can be seen to become more sensitive to frequency ratio as Q_d increases. For frequency ratios greater than one, phase angle decreases for increasing Q_d. The inverse holds for frequency ratios less than one. Typically, practical machines operate with phase angles between 45° and 75°. Note that for the displacer/piston viscous coupling equal to zero, ie, $D_{dp} = 0$ the behavior is that of a simple force-damped system and $\phi = 90°$ when $\omega_0 = \omega_d$. Since $D_{dp} \neq 0$ in practical machines, quadrature is not obtained when the operating frequency is equal to the displacer natural frequency. D_{dp} is, in fact, a negative quantity since positive piston motions create a positive pressure drop across the displacer forcing it in a positive direction. The other damping terms resist motion whereas the viscous coupling term actually feeds some energy back to the displacer.

At the point of steady oscillation, the forces balance as shown in Figure 6.

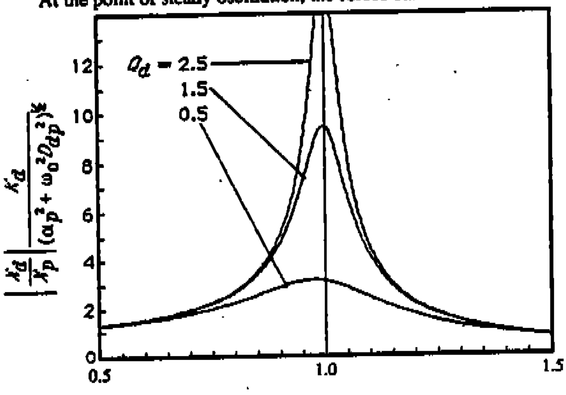

Figure 4 - Effect of Q_d and Resonance on Amplitude Ratio

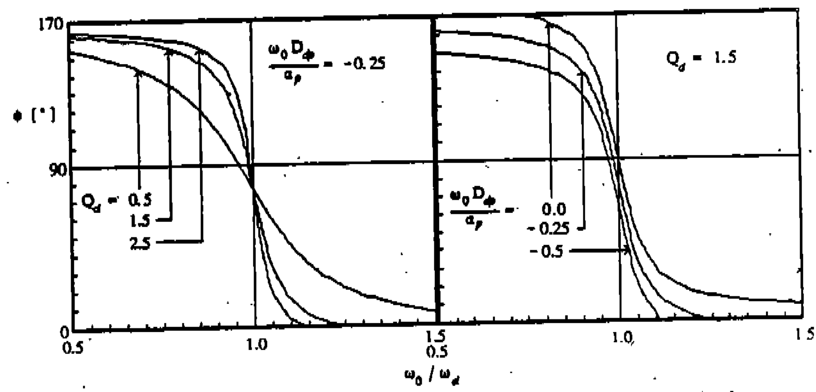

Figure 5 - Effect of Q_d, Viscous Coupling and Resonance on Piston/Displacer Phase Angle

Power at the piston face may be obtained from

$$P = \frac{\omega_0}{2\pi} \int_0^{2\pi} p_c \, dV \tag{15}$$

Taking pressure and volume to be given by

$$p_c = \langle p \rangle + |p_c| \sin(\theta + \beta') \tag{16}$$

$$V = V_0 - A_p X_p \sin\theta \tag{17}$$

Evaluating the integral, the power becomes

$$P = -\frac{\omega_0}{2} A_p X_p |p_c| \sin\beta' \tag{18}$$

where β' is positive for a heat pump thus making P negative for power *in*.

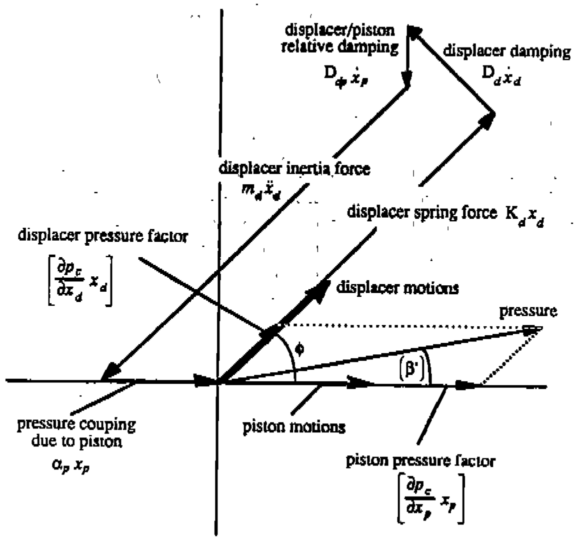

Figure 6 -.Displacer Force Balance at Steady Operation

which from Figure 6 may be written

$$P = -\frac{\omega_0}{2}\alpha_T X_p X_d \sin\phi \tag{19}$$

where

$$\alpha_T = A_p \frac{\partial p_c}{\partial x_d} \tag{20}$$

α_T is referred to as the thermal coupling between the displacer motion and piston force. This quantity is positive for heat pumps.

From (19) it can be seen that the power at the piston face is proportional to the product of the piston and displacer amplitudes and the sin of the included angle. Power input is therefore maximized for $\phi = 90°$, which from Figure 5 occurs when the displacer resonance is slightly greater than the operating frequency. However, efficiency generally improves for lower phase angles owing to reduced gas flow losses through the heat exchangers.

In order to find the mechanical efficiency of the machine it is necessary to evaluate the power required by the displacer to overcome the dissipative losses associated with its motion. From a power balance for the displacer:

$$P_d = \frac{\omega_0}{2\pi}\left(\int_0^{2\pi} p_e\, dV_e + \int_0^{2\pi} p_c\, dV_{cd} + \int_0^{2\pi} D_{ext_d}\left(\dot{x}_d - \dot{x}_c\right) dx_d + \int_0^{2\pi} F_{spring}\, dx_d\right) \tag{21}$$

Substituting for displacements and noting that $\Delta p = p_c - p_e$, this becomes:

$$P_d = \frac{\omega_0}{2\pi}\int_0^{2\pi}\left(A\,\Delta p - A_R p_c + D_{ext_d}\left(\dot{x}_d - \dot{x}_c\right) + K_{ext_d}\left(x_d - x_c\right)\right) dx_d$$

where, as before, x_c is given by (2). Linearizing Δp and p_c as in (3) and using the definition of a_p in (6), the following is obtained:

$$P_d = \frac{\omega_0 X_d}{2}\left(-D_d\,\omega_0 X_d - D_{dp}\,\omega_0 X_p \cos\phi + a_p X_p \sin\phi\right) \tag{22}$$

The first two terms in this expression is the work done against damping, the last term is the pV work required to overcome the damping. Thus at steady oscillation, (22) evaluates to zero. From Figure 6 it can be seen that at steady oscillation the net damping force is exactly equal to the pressure force term which is to be expected (remember that D_{dp} is negative). The same result could have been determined geometrically from Figure 6. Extracting the pV work term from (22):

$$P_{dd} = \frac{\omega_0}{2} a_p X_p X_d \sin\phi \tag{23}$$

It is now possible to obtain the displacer mechanical efficiency, defined as the power at the piston for a lossless displacer / actual piston input power, or:

$$\eta_m = \frac{P - P_{dd}}{P} \tag{24}$$

where all quantities are assumed to be absolute values.

Substituting for P and P_{dd} yields:

$$\eta_m = 1 - \frac{a_p}{a_T} \tag{25}$$

Minimizing the ratio a_p / a_T will maximize the displacer mechanical efficiency and therefore limit the dissipation of input pV work.

Finally, the heat lift is required which may be approximated by:

$$\dot{Q}_e \approx \frac{\omega_0}{2\pi}\oint p_e \, dV_e \tag{26}$$

which is true for an isothermal expansion space.

Noting that $p_c - p_e = \Delta p$, linearizing as before and substituting for the appropriate motions, (26) becomes:

$$\dot{Q}_e = \frac{\omega_0}{2}\left\{A\,|p_c|\sin(\beta' - \phi) + \left(D_{dd} - D_{ex_d}\right)\omega_0 X_d + D_{dp}\,\omega_0 X_p \cos\phi\right\} X_d \tag{27}$$

Once again, $D_d = D_{dd}$, and from the geometry of Figure 6, (27) may be written more conveniently

$$\dot{Q}_e = -\frac{\omega_0}{2}\left\{a_p X_p\left(\frac{A}{A_R} - 1\right)\sin\phi - K_{ex_d} X_p \frac{A}{A_R}\frac{m_p}{m_c}\sin\phi + D_{ex_d}\,\omega_0 X_d\right\} X_d \tag{28}$$

where a negative value indicates heat to the expansion space.

The cycle COP (heat lift / input power) may now be defined from (28) and (19).

$$COP = \frac{\alpha_p}{\alpha_T}\left(\frac{A}{A_R} - 1\right) - \frac{A}{A_R}\frac{K_{ext_d}}{\alpha_T}\frac{m_p}{m_c} + \frac{D_{ext_d}}{\alpha_T}\frac{\omega_0}{\sin\phi}\frac{X_d}{X_p} \tag{29}$$

which shows that minimizing the ratio α_p/α_T reduces the COP, so optimum performance does not necessarily occur at maximum displacer mechanical efficiency.

In most cases the last two terms are small, particularly the $D_{ext\,d}$ term in machines employing gas bearings. In these cases the COP result is easily determined from the dynamic measurements of displacements and pressure. The main limitations of this result is that it does not, of course, include heat transfer losses. Of the heat transfer losses, conduction and regenerator losses are the most significant. Including these two losses in the above result, (29) becomes:

$$COP = \frac{\alpha_p}{\alpha_T}\left(\frac{A}{A_R} - 1\right) - \frac{A}{A_R}\frac{K_{ext_d}}{\alpha_T}\frac{m_p}{m_c} + \frac{D_{ext_d}}{\alpha_T}\frac{\omega_0}{\sin\phi}\frac{X_d}{X_p} - \frac{|H| + |\dot{Q}_{cond}|}{|P|} \tag{30}$$

where the bars denote absolute values. Determination of the heat transfer losses is beyond the scope of this paper. Good estimations may be found in the following references [4, 6, 7].

APPLICATION

The power of this simple analysis is demonstrated by applying it to the four sample machines listed in Table 1. The errors associated with the measurements are less than 5%. The heat transfer losses are calculated and there could be a larger error associated with those quantities, more so for the regenerator loss. In any event, the correlation can be seen to be remarkably good. Since the dynamics of a free-piston Stirling machine are determined by the damping and resonances, it follows that the dissipation of work by the damping forces must equal the sum dissipation associated with gas flow losses, spring hysteresis, friction and other work degrading processes. Thus for given dynamics, α_T and α_p are uniquely determined which leads to a precise result for all the cycle dissipative losses. By then including the heat transfer losses, it is possible to closely approach the actual performance of a machine. There are, however, limitations. The most important being that the heat lift is calculated assuming an isothermal expansion space. This is not a bad assumption if the regeneration is good and if the compression ratio is such that the temperatures in the working spaces do not vary much. In most practical machines the regenerator effectiveness approaches 0.99 and the working space temperatures do not vary by more than 5% of their absolute values. Another limitation concerns the assumption of sinusoidal displacements. If there are significant higher harmonics in the motions, then the analysis begins to break down. Typically, this is not the case, the motions are almost always very close to being pure sinusoids. Note that higher harmonics in the pressure have no effect on the dynamics provided that the motions remain first order (sinusoidal).

TABLE 1 Parameters taken from various free-piston Stirling coolers

Machine	P_c [Pa] X 10^5	θ' [°]	X_d [mm]	X_p [mm]	ϕ [°]	A_p [m²] X 10^{-4}	A [m²] X 10^{-4}	A_R [m²] X 10^{-4}	f [Hz]	H [W]	$\dot{Q}_c$ [W]	Lift [W]
Electronics cooler -50°C	1.85	18.7	4.67	5.50	75.1	6.137	4.337	0.332	60.0	8.15	7.89	36.9
Refrigerator cooler -26°C	2.14	10.1	6.03	8.00	59.6	24.63	16.05	0.801	52.0	19.5	35.0	197.0
Cryocooler 110K	1.95	41.8	7.5	12.0	73.5	45.50	44.90	7.07	34.79	85.32	160.6	47.33
Cryocooler* 77K	7.51	18.8	16.3	16.0	37.4	273.5	69.40	16.00	45.0	808.5	673.7	1510

* from simulation, other parameters from actual machines. The regenerator and conduction losses are not measured, they have been determined by calculation.

TABLE 2 Comparison between linear and actual results

Machine	α_p $\times 10^3$	α_T $\times 10^3$	$\frac{A}{A_R}$	$\frac{m_p}{m_c}$ $\times 10^{-3}$	K_{ad} [N/m] $\times 10^3$	$\frac{\text{Regenerator loss}}{\text{pV power}}$	$\frac{\text{Conduction loss}}{\text{pV power}}$	Linear COP	Actual COP	η_m
Electronics cooler -50°C	0.979	8.502	13.06	2.95	5.5	0.216	0.209	0.94	0.92	0.885
Refrigerator cooler -26°C	1.892	17.77	20.03	0.325	7.8	0.161	0.290	1.57	1.54	0.894
Cryocooler 110K	6.30	82.24	6.353	small (?)	-	0.110	0.207	0.093	0.061	0.923
Cryocooler* 77K	39.44	668.7	4.337	small	-	0.054	0.045	0.098	0.100	0.941

Although linear analysis is particularly powerful in the analysis of running machines, it is also an excellent tool for design. The accuracy of the analysis is really only limited by the determination of the dynamic parameters and α_T and α_p. These being easily evaluated from any of the published comprehensive thermodynamic analyses [4, 8]. On the other hand, useful indications may be obtained by using the ideal Schmidt analysis for the thermodynamics [4, 7]. Note that many of the dynamic parameters are not constant with displacements or frequency. The damping coefficients, in particular, are usually functions of the velocities of the moving parts (except in the case of laminar flow, when they are actually constant). Thus, the linearization is only valid for the operating point under investigation and for small changes around that point. Of course, the design of free-piston Stirling machines includes two other dynamic elements, namely the piston and the linear motor, both of which are amenable to linear analysis. The piston resonance is a vital parameter and has strong bearing on the design of the motor. These considerations are beyond the scope of this paper but they have, to some extent, been described by de Jonge [2].

CONCLUSIONS

It has been shown that simple linear analysis is able to produce useful results for the dynamic analysis and design of free-piston Stirling coolers. In particular, for known dynamics, it is possible to exactly evaluate the cycle dissipative losses for a given machine operating at a given point.

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