1977

NONLINEAR APPROXIMATION - Final Report for NSF Grant GP-3294OX

John R. Rice
Purdue University, jrr@cs.purdue.edu

Report Number:
77-235
NONLINEAR APPROXIMATION
Final Report for NSF Grant GP-32940X

John R. Rice
Purdue University

June 10, 1977
CSD-TR 235
I. ABSTRACT. A summary is given of the research accomplishments for the 5-year period of my grant GP32940X for research on nonlinear approximation. Twenty one papers were written with the support of this grant. Three Ph.D. theses were written which led to three additional papers and one widely used computer program. The two principle accomplishments were (1) the first theoretical analysis and convergence proofs for adaptive quadrature and the exploitation of this idea for adaptive approximation and (2) the demonstration that standard finite differences is an inferior method for solving partial differential equations.


There were seven areas of activity proposed:

1. Completion of some work in progress nearly complete.

2. Piecewise polynomial approximation with variable knots.
   
   This was a central topic in the research of the period and considerable progress was made.

3. Degree of convergence for spline approximation with variable knots.
   
   The main theoretical advances here were made by others, but several results were obtained on the degree of convergence of various constructive methods. New results on the degree of convergence for multivariate approximation are in progress.

4. New approach to fast, reliable, general purpose curve fitting.
   
   The discovery, theoretical analysis and perfection of adaptive approximation is one of the principle accomplishments of this research program. The theoretical development occurred first for adaptive quadrature, then for approximation and is now being extended to multivariate approximation.
5. Applications to differential equations.
   The development, analysis and comparison of approximation
   theoretical methods for operator equations has been pursued
   extensively.

   Carl de Boor (who was involved in this project for 1 year)
   pursued this topic with considerable success.

   No progress was made here.

III. SPECIFIC PROJECTS AND PAPERS UNDER THIS RESEARCH PROGRAM.

   The work of Carl de Boor in the early part of this period is not included.
   The work is subdivided into three categories. Numbers in brackets refer
   to the publications of John Rice, other references are given in the form
   of [Author, date].

   **Piecewise Polynomials, Variable Knots and Adaptive Methods**

   This paper [3] gave the first theoretical analysis of adaptive
   quadrature and a general convergence result. It also analyzes the
   structure of such algorithms in some detail.

2. Parallel Algorithm for Adaptive Quadrature: Convergence, Proc. IFIP

3. Parallel Algorithms for Adaptive Quadrature II: Metalgorithm Correctness,

4. Parallel Algorithms for Adaptive Quadrature III: Program Correctness,

   The preceding four papers [2], [7], [8], [17] constitute an in-depth
study of the application of parallel computers to adaptive methods for numerical integration. All aspects are covered from theoretical convergence and speed-up to practical considerations of implementation. The fourth paper [8] presents the first (to my knowledge) formal proof of correctness for a non-trivial algorithm in numerical computation.

This paper [5] presents the first fast methods for computing smooth approximations. The rate of convergence is shown to be optimal for all functions of practical interest.


The preceding two papers [11], [14] present a Fortran algorithm for adaptive approximation and report on extensive experience with it. It behaves as the theory predicts.

This is an invited paper [12] for the East European congress approximation theory. It focuses on what is currently feasible in the computation of approximations.

This paper [4] announces the main results of items 1, 2, 3 and 4 above.
Computational Complexity


These two papers [1], [6] develop a theory of the complexity of computing various kinds of best approximations. There is the surprising result that there is little (theoretical) difference in the difficulty of computing $L_1$, $L_2$, and $L_\infty$ approximations. It is shown that piecewise polynomial approximations may be computed with optimal efficiency.

Application to Differential Equations


These lecture notes present a systematic framework for the analysis and comparison of approximation theoretic methods (e.g. collocation, least squares) for operator equations. It was never submitted for publication anywhere.


The principle result of these two papers [9], [15] is conclusive evidence that the standard finite difference method is inferior for
solving second order linear elliptic partial differential equations. Detailed computational characteristics of various numerical methods are presented.


These three papers [16], [18], [21] develop the theory and application of a new approach to high order finite difference equations discovered by R. E. Lynch (co-author of the above). This new method is good for ordinary differential equations, but its real potential lies in 2 and 3 dimensional problems (with or without rectangular geometry). See The HODIE method: A Brief Introduction with a Summary of Computational Properties, CSD-TR 170, Purdue University, November 18, 1975.

19. Software for Elliptic Partial Differential Equations. This is now a separate project that arose out of items 13, 14, 15, 17 above. It has been underway since early 1976 and is a collaborative effort with people at Harvard, the University of Texas and, perhaps, other institutions.

Research by Graduate Students. Three of my five Ph.D. Students were supported by this program.


Item 24. below is an outgrowth of this thesis and a computer program developed is now widely used through SPSS (Statistical
Package for Social Scientists) and the Inter. Math. Stat.
Libraries (of computer programs).

21. E. Houstis, Finite Element Methods for solving Initial/Boundary
Results from this thesis have been developed into two
separate papers [Houstis, 1976], [Houstis, 1977] now being
published. Another set of Houstis' results appeared in
the SIAM J. Numer. Anal. by other authors [Baker and Dougalis,
1976].

22. J. Lemme, Speedup in Parallel Algorithms for Adaptive Quadrature,
1976.
Item 5. above is an outgrowth of this thesis as well as a
second paper now being published [Lemme, 1977].

Other Papers

23. The Algorithm Selection Problem, in "Advances in Computers", Vol. 15
This paper [10] gives a systematic treatment of the problem of
selecting the best algorithm for a class of computations. The
parallel with certain parts of approximation theory are developed
in some detail.

This paper [13] shows that random or Monte Carlo search in high
dimensional sets is inferior to systematic search. This contradicts
widely held opinion. This search is applicable to starting computations
in approximation and optimization.
REFERENCES


19. An adaptive algorithm for multivariate approximation with optimal convergence rate (with C. de Boor) to appear.
