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NONLINEAR APPROXIMATION - Final Report for NSF Grant GP-3294OX

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I. ABSTRACT. A summary is given of the research accomplishments for the 5-year period of my grant GP32940X for research on nonlinear approximation. Twenty one papers were written with the support of this grant. Three Ph.D. theses were written which led to three additional papers and one widely used computer program. The two principle accomplishments were (1) the first theoretical analysis and convergence proofs for adaptive quadrature and the exploitation of this idea for adaptive approximation and (2) the demonstration that standard finite differences is an inferior method for solving partial differential equations.

II. RESEARCH PROPOSED FOR GP32940X AT THE BEGINNING OF THE PERIOD 1972-1977.

There were seven areas of activity proposed:

1. Completion of some work in progress nearly complete.
2. Piecewise polynomial approximation with variable knots.

   This was a central topic in the research of the period and considerable progress was made.
3. Degree of convergence for spline approximation with variable knots.

   The main theoretical advances here were made by others, but several results were obtained on the degree of convergence of various constructive methods. New results on the degree of convergence for multivariate approximation are in progress.
4. New approach to fast, reliable, general purpose curve fitting.

   The discovery, theoretical analysis and perfection of adaptive approximation is one of the principle accomplishments of this research program. The theoretical development occured first for adaptive quadrature, then for approximation and is now being extended to multivariate approximation.

5. Applications to differential equations.

   The development, analysis and comparison of approximation theoretical methods for operator equations has been pursued extensively.

6. A systematic organization of the theory of splines.

   Carl de Boor (who was involved in this project for 1 year) pursued this topic with considerable success.

7. Abstract geometric theory of nonlinear approximation.

   No progress was made here.

III. SPECIFIC PROJECTS AND PAPERS UNDER THIS RESEARCH PROGRAM.

The work of Carl de Boor in the early part of this period is not included. The work is subdivided into three categories. Numbers in brackets refer to the publications of John Rice, other references are given in the form of [Author, date].

Piecewise Polynomials, Variable Knots and Adaptive Methods

1. A Metalgorithm for Adaptive Quadrature, J. ACM, 22 (1975) pp. 61-82. This paper [3] gave the first theoretical analysis of adaptive quadrature and a general convergence result. It also analyzes the structure of such algorithms in some detail.
2. Parallel Algorithm for Adaptive Quadrature: Convergence, Proc. IFIP Congress '74, North-Holland Publishing Co., Amsterdam (1974) pp. 600-604.
3. Parallel Algorithms for Adaptive Quadrature II: Metalgorithm Correctness, Acta Informatica 5, (1976) pp. 273-285.
4. Parallel Algorithms for Adaptive Quadrature III: Program Correctness, ACM Trans. Math. Software, 2, (1976) pp. 1-30.
5. Speed-up in Parallel Algorithms for Adaptive Quadrature, to appear. The preceding four papers [2], [7], [8], [17] constitute an in-depth

study of the application of parallel computers to adaptive methods for numerical integration. All aspects are covered from theoretical convergence and speed-up to practical considerations of implementation. The fourth paper [8] presents the first (to my knowledge) formal proof of correctness for a non-trivial algorithm in numerical computation.

6. Adaptive Approximation, J. Approx. Thy., 16 (1976) pp. 329-337.
   This paper [5] presents the first fast methods for computing smooth approximations. The rate of convergence is shown to be optimal for all functions of practical interest.

7. On Adaptive Piecewise Polynomial Approximation, in "Theory of Approximation with Applications" (A. Law and B. Sahney, eds.), Academic Press, (1976) pp. 359-386.

8. ADAPT - Adaptive Smooth Curve Fitting, ACM Trans. Math. Software, to appear.
   The preceding two papers [11], [14] present a Fortran algorithm for adaptive approximation and report on extensive experience with it. It behaves as the theory predicts.

9. Remarks on Piecewise Polynomial Approximation, Proc. Int. Conf. Thy. Approximation of Functions (B. Steckkin, ed.) (1977).
   This is an invited paper [12] for the East European congress approximation theory. It focuses on what is currently feasible in the computation of approximations.

10. Adaptive Quadrature: Convergence of Parallel and Sequential Algorithms, Bull. Amer. Math. Soc., 80 (1974) pp. 1250-1254.
    This paper [4] announces the main results of items 1, 2, 3 and 4 above.

Computational Complexity

11. On the Computational Complexity of Approximation Operators, in "Approximation Theory" (G. Lorentz, ed.) Academic Press (1973) pp. 449-456.

12. On the Computational Complexity of Approximation Operators, in "Analytical Computational Complexity" (J. Traub, ed.) Academic Press (1976) pp. 191-204.

    These two papers [1], [6] develop a theory of the complexity of computing various kinds of best approximations. There is the surprising result that there is little (theoretical) difference in the difficulty of computing L_1, L_2 and L_∞ approximations. It is shown that piecewise polynomial approximations may be computed with optimal efficiency.

Application to Differential Equations

13. Approximation Theoretic Methods for Operator Equations, lecture presented at the Michigan State Regional Conference on Approximation and Applications, 1972.

    These lecture notes present a systematic framework for the analysis and comparison of approximation theoretic methods (e.g. collocation, least squares) for operator equations. It was never submitted for publication anywhere.

14. Development, Evaluation and Selection of Methods for Elliptic Partial Differential Equations, Ann. Assoc. Inter. Calcul. Anal., 11 (1975) pp. 98-103.

15. Evaluation of Numerical Methods for Elliptic Partial Differential Equations, to appear.

    The principle result of these two papers [9], [15] is conclusive evidence that the standard finite difference method is inferior for solving second order linear elliptic partial differential equations. Detailed computational characteristics of various numerical methods are presented.

16. The HODIE method for ordinary differential equations, to appear.

17. High accuracy finite difference approximation to solutions of elliptic partial differential equations, Proc. Nat. Acad. Sci., (1977).

18. The HODIE method for elliptic partial differential equations, in preparation.

    These three papers [16], [18], [21] develop the theory and application of a new approach to high order finite difference equations discovered by R. E. Lynch (co-author of the above). This new method is good for ordinary differential equations, but its real potential lies in 2 and 3 dimensional problems (with or without rectangular geometry). See The HODIE method: A Brief Introduction with a Summary of Computational Properties, CSD-TR 170, Purdue University, November 18, 1975.

19. Software for Elliptic Partial Differential Equations. This is now a separate project that arose out of items 13, 14, 15, 17 above. It has been underway since early 1976 and is a collaborative effort with people at Harvard, the University of Texas and, perhaps, other institutions.

Research by Graduate Students. Three of my five Ph.D. Students were supported by this program.

20. T. J. Aird, Computational Solution of Global Nonlinear Least Squares Problems (1973).

    Item 24. below is an outgrowth of this thesis and a computer program developed is now widely used through SPSS (Statistical

Package for Social Scientists) and the Inter. Math. Stat. Libraries (of computer programs).

21. E. Houstis, Finite Element Methods for solving Initial/Boundary value Problems (1974).

    Results from this thesis have been developed into two separate papers [Houstis, 1976], [Houstis, 1977] now being published. Another set of Houstis' results appeared in the SIAM J. Numer. Anal. by other authors [Baker and Dougalis, 1976].

22. J. Lemme, Speedup in Parallel Algorithms for Adaptive Quadrature, 1976.

    Item 5. above is an outgrowth of this thesis as well as a second paper now being published [Lemme, 1977].

<u>Other Papers</u>

23. The Algorithm Selection Problem, in "Advances in Computers", Vol. 15 (Rubicoff and Yovits, eds.) Academic Press (1976) pp. 65-118. This paper [10] gives a systematic treatment of the problem of selecting the best algorithm for a class of computations. The parallel with certain parts of approximation theory are developed in some detail.

24. Systematic Search in High-Dimensional Sets, SIAM J. Numer. Anal., This paper [13] shows that random or Monte Carlo search in high dimensional sets is inferior to systematic search. This contradicts widely held opinion. This search is applicable to starting computations in approximation and optimization.

REFERENCES

Baker, G. and G. Dougalis [1976], The effect of quadrature errors on finite element approximations for second order hyperbolic equations, SIAM J. Numer. Anal. 13, pp. 577-598.

Houstis, E. [1976], A collocation method for systems of nonlinear ordinary differential equations, J. Math. Anal. Appl.

Houstis, E. [1977], Application of the method of collocation on lines for solving nonlinear hyperbolic problems, to appear.

Lemme, J. M. [1977], Adaptive quadrature algorithms for ILLIAC-IV, to appear.

1. Rice, John R., On the Computational Complexity of Approximation Operators in "Approximation Theory" (G. G. Lorentz, ed.), Academic Press, (1973), 449-456.

2. __________, Parallel Algorithms for Adaptive Quadrature - Convergence. Proc. IFIP 1974. (1974), 600-604.

3. __________, A metalgorithm for adaptive quadrature, J. Assoc. Comp. Mach., 22 (1975), 61-82.

4. __________, Adaptive quadrature - Convergence of parallel and sequential algorithms. Bull. Amer. Math. Soc., 80 (1974), 1250-1254.

5. __________, Adaptive approximation, J. Approx. Theory, 16 (1976) 329-337.

6. __________, On the Computational Complexity of Approximation Operators II, in "Analytic Computational Complexity" (J. F. Traub, editor), Academic Press, (1975), 191-204.

7. __________, Parallel algorithms for adaptive quadrature II - Metalgorithm correctness. Acta Info., 5 (1975), 273-285.

8. __________, Parallel algorithms for adaptive quadrature III - Program correctness. Assoc. Comp. Mach. Trans. Math. Software, 2 (1976), 1-30.

9. __________, Development, evaluation and selection of methods for elliptic partial differential equations (with E. N. Houstis, R. E. Lynch and T. S. Papatheodorou). Ann. Assoc. Calcul Analog., 11 (1975), 98-105.

10. __________, The Algorithm Selection Problem, in "Advances in Computers," Vol. 15 (Rubicoff and Yovits, eds.), Academic Press, (1976), 65-118.

11. __________, On Adaptive Piecewise Polynomial Approximation, in "Theory of Approximation," (A. Law and B. Sahney, eds.), Academic Press, (1976) 359-386.

12. __________, Remarks on Piecewise Polynomial Approximation, in Proc. Inter. Conf. "Approximation of Functions," (S. Steckin, ed.), Moscow, (1977).

13. ____________ , Systematic search in high dimensional sets (with T. J. Aird). SIAM J. Numer. Anal, 14 (1977) 296-312.

14. ____________ , ALGORITHM XXX: ADAPT - Adaptive Smooth Curve Fitting, ACM Trans. Math. Software.

15. ____________ , Evaluation of Numerical Methods for Elliptic Partial Differential Equations (with E. Houstis, R. E. Lynch and T. S. Papatheodorou). J. Computational Physics.

16. ____________ , The HODIE method for ordinary differential equations (with R. E. Lynch).

17. ____________ , Speedup in parallel algorithms for adaptive quadrature (with J. Lemme), J. Assoc. Comp. Mach.

18. ____________ , High accuracy finite difference approximation to solutions of elliptic partial differential equations (with R. E. Lynch), Proc. Nat. Acad. Science, (1977).

19. ____________ , An adaptive algorithm for multivariate approximation with optimal convergence rate (with C. de Boor) to appear.

20. ____________ , Multivariate piecewise polynomial approximation, to appear.

21. ____________ , The HODIE method for elliptic partial differential equations (with R. E. Lynch).