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A Study of Boundary Conditions Encountered in Reciprocating Compressor Systems

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ABSTRACT

This study showed that the boundary conditions met with in compressor systems can be treated in a unified manner. The boundaries analysed were: sudden changes in pipe cross-sectional area (expansions and contractions), three way junctions, orifices, restrictions and non return valves in a pipeline. To assess the validity of the treatment the analytical results obtained by numerical methods were compared with experimental records obtained from a reciprocating air compressor system into which each of the boundaries was incorporated in turn.

INTRODUCTION

The basic equations which describe one dimensional unsteady flow of a compressible fluid in a duct are well established (1) and a number of techniques are available to obtain solutions to them (2). Whether the small perturbation (acoustic) type of analysis or more complex treatments of the governing non-linear partial differential equations are used, the techniques to obtain solutions for the variables in the field of flow are well understood. Thus the success of the solution to a particular problem will depend largely on the treatment of the conditions to be satisfied at the flow boundaries. In many instances the problem may be transformed into that of solving a set of non linear simultaneous equations. The present paper describes this approach and shows how the underlying philosophy can be applied to obtain solutions to the flow equations when subject to various boundary conditions that may be encountered in positive displacement compressor systems.

Compressor systems contain a variety of components, e.g. cylinders, valves, pipes, storage vessels, orifices, etc., and it is important to be able to describe the behaviour of the fluid as it flows from one component to another. The flow equations describe the behaviour of the fluid as it flows in a pipe. However, additional considerations are required when the fluid reaches the pipe end. In unsteady flow problems the boundary conditions should take account of both the temporal and spatial variation of any variable. i.e. The equations have the form

$$\frac{\partial \bar{V}}{\partial t} + \phi\,(\bar{V})\,\frac{\partial \bar{V}}{\partial x} = f\,(\bar{V})$$

A major simplification is possible if at any boundary it is assumed that the temporal derivative is negligible compared with the spatial derivative.

i.e. $\frac{\partial \bar{V}}{\partial x} \gg \frac{\partial \bar{V}}{\partial t}$

Under these circumstances an instantaneously steady state situation may be considered to exist, i.e. the flow is quasi-steady. Then the boundary conditions result in a set of simultaneous equations. These are frequently non linear and require iterative techniques to effect a solution.

BOUNDARY CONDITIONS ENCOUNTERED IN RECIPROCATING COMPRESSOR SYSTEMS

The boundary conditions encountered in compressor systems are similar to those in internal combustion engines although compressor systems are not subject to the high temperatures consequential to the combustion process in engines. Boundary regions may conveniently be classified into three broad divisions, each being capable of further subdivision for flow description purposes.

Division 1. Junction of a pipe and a large volume (cylinder or receiver)

(a) Open end
(b) Partially restricted end (valve, port, nozzle, vena-contracta)
(c) Closed end (dead end or closed valve or port)

Analyses must be capable of dealing with inflow and outflow situations (with respect to the pipe).

Division 2. Simple pipe junctions (one pipe followed by another pipe or a pipe containing a flow metering or controlling device).

(a) Junction of sections of identical pipes
(b) Sudden enlargement
(c) Sudden contraction
(d) Junction involving flow restriction (orifice, control valve, gauzes)
(e) Sharp bend

Division 3. Multiple branch junctions and junctions with self-actuating elements.

(a) Three-way junctions
(b) Multiple branches junctions involving more than three pipes
(c) Non-return or check valve installations

PREVIOUS STUDIES OF BOUNDARY CONDITIONS

The simplest boundary conditions, i.e. those involving a fully open pipe end or a closed pipe end have been discussed by Shapiro (1). Several authors (2, 3, 4, 5, 6, 24, 25, 26, 27) have examined the flow processes which occur at a partially restricted pipe end in engine and compressor systems.

Benson et al (7) examined the effect of sudden changes of pipe cross sectional area on steady and unsteady compressible gas flow. Satisfactory predictions of the amplitudes of transmitted and reflected pressure waves were obtained by considering the flow at the discontinuity to be quasi-steady. Trengrouse and Soliman (8) also examined the behaviour of finite amplitude waves at an area discontinuity and developed analyses on the assumption that the flow was either adiabatic or isentropic. The results were compared with experimental observations of pressure wave transmission and reflection at such discontinuities. Srivastava (9) performed similar studies for shock waves.

Studies of unsteady flow at junctions in a duct have been reported by various authors. Benson et al (10) derived equations to describe the flow in a three leg junction. It was found that the assumption of isentropic flow with uniform pressure at the junction gave poor results in some cases. An iterative algorithm was presented for the solution of the non-homentropic version of the equations: empirical coefficients based on steady flow tests were employed. Daneshyar and Pearson (11, 12) carried out similar studies for three and four way junctions. Alternative methods of solution of the boundary equations were presented which overcame limitations detected by Benson. Benson (13) later extended the constant pressure homentropic approach to the case of multiple branch junctions. Chelsom, Deckker and Male (14,15,16) reported theoretical and experimental studies of the behaviour of compression and rarefaction waves at pipe junctions for a number of geometries in which the angles formed by the branches were varied. Quasi-steady flow was assumed and semi-empirical coefficients were introduced into the analysis. Ehrlich (17) described a two dimensional finite difference technique to solve the equations which describe pulsatile fluid flow through a bifurcation.

The boundary conditions involved when a flow meter or control device is present in a pipe have been investigated previously. Benson and El-Shafie (18) studied unsteady flow through an orifice by treating it as a sudden contraction followed by a sudden expansion. Quasi-steady flow was assumed and the predicted results were substantiated by experiment. Cheng et al (19) used a two dimensional flow theory to investigate unsteady flow through orifices and past various obstacles.

Kaddah and Woollatt (20) studied the behaviour of a spring loaded non-return valve in a duct in which unsteady flow was occurring. Quasi steady flow was assumed. Empirical relationships were determined from steady flow tests to relate valve displacement, mass flow, and pressure drop across the valve. The differential equations governing the motion of the valve were integrated and coupled with the equations describing wave action in the duct. Benson and Baruah (21) investigated a simple gauze filter in a pipe and established that the pressure drop across the gauze was a function of the Reynolds and Mach numbers describing the flow and the solidity of the gauze.

THEORETICAL BACKGROUND

Although many calculational schemes are available to obtain solutions to the unsteady flow equations at internal points in a pipe, the Method of Characteristics has particular advantages when handling boundary conditions. In the Method of Characteristics a knowledge of the way in which variables change along certain lines (characteristics) enables a solution to be obtained and this knowledge is also used at the boundary together with further assumptions to aid the representation of actual occurrences. The following illustrates this approach applied to various boundary conditions encountered in reciprocating compressor systems.

Whether a simple boundary such as that between two identical sections of pipe or a more complex boundary such as that at a T junction or a valve is being considered the assumption of quasi-steady flow allows formulation of equations describing the conservation of mass, energy and momentum. At most boundaries adiabatic flow may be assumed when formulating the energy equation. When considering complicated boundary regions it is difficult a priori to write the momentum equation(s) since flows are generally multi-direction and pressure distributions are unknown. Momentum equations may be written if empirically determined coefficients are included. Steady flow tests should be conducted for the boundary being studied but for some situations, such as standard T or Y junctions, information for flows at low Mach number is available (22). The assumption that the fluid is an ideal gas allows a simple form of the equation of state to be used. The method of characteristics provides information about some of the variables (velocity, acoustic velocity, entropy) at different points on the boundary and completes the system of equations to be considered.

Although each boundary condition is capable of lengthy examination the present aim was to highlight aspects common to a number of boundary conditions. The system of equations are set out in charts A, B, C, D for flow at

(a) the junction of a pipe and a large volume (receiver or cylinder in which stagnation conditions prevail)

(b) sudden expansion and contraction in a pipe

(c) a three branch pipe junction

The charts contain the basic equations together with any further assumptions used to effect a solution to the particular case. There is an evident similarity in the methods of obtaining solutions at these boundaries: the boundaries become problems which require the solution of simultaneous non-linear equations. The charts owe much to solution techniques developed by other investigators (10), (11), (29), (30).

Four boundary conditions have been used for illustrative purposes: the techniques outlined can be extended to other cases. The example of a restriction in a pipeline is obtained by combining the solutions for a sudden expansion and contraction. A sharp bend may be treated in this manner also, if use is made of an empirical form of the momentum equation to account for pressure losses at the bend. Daneshyar and Pearson (12) have shown that four way branch junctions may be analysed in a similar manner to that used here to analyse flow in a three way junction if empirical coefficients used in the momentum equations are obtained from steady flow tests.

Variable orifices (compressor valves, control valves) in which the flow area is a function of the displacement of the valve plate or gate require that the equation of motion of the moving element be integrated to yield successive element positions. The pressure forces acting on the moving element vary continuously but satisfactory predictions of valve displacement have been achieved by assuming that pressure forces are constant during an integration interval. An ordinary differential equation which describes a simple valve element motion may be integrated using standard techniques (Runge-Kutta-Merson): the solutions to the flow equations may be obtained using the methods outlined above and illustrated in charts A, B, C, D.

COMPUTER PROGRAM

The solution at the boundaries was incorporated into the computer program of the compressor simulation model described by MacLaren et al (4). Equations for finite amplitude unsteady flow in the pipes were solved using either of two finite difference schemes, the Two-Step Lax Wendroff method or the Leapfrog method, coupled to the method of characteristics at the boundaries.

EXPERIMENTAL PROGRAMME

To assess the validity of the model developed for the different boundary conditions,a one stage air compressor (6.0 inch bore x 4.5 inch stroke (152 x 114 mm)) was tested. The inlet line of the compressor was modified to incorporate each type of boundary in turn, Figure 1. Compressor speeds were in the range 400 to 600 rev/min. Experimental records of pressure at several points in the system and of compressor valve plate displacement were obtained using a computer controlled high speed data acquisition system (23).

RESULTS

In the experiments conducted with a sudden expansion and sudden contraction in the inlet pipe of the compressor the pipe area ratios used range from 0.25 to 2.21. Figure 2(a) illustrates the results obtained for the particular expansion area ratio shown in Figure 1(a): a comparison can be made between the experimental and analytical pressure records at stations 1, 2, 3 (inlet valve plenum) and cylinder. Stations 1 and 2 were located at mid length of each pipe. Experimental and analytical compressor valve displacements are also compared. Figure 2(b) shows results for the particular contraction area ratio shown in Figure 1 (b).

Figures 3(a) and 3 (b) illustrate results for the pipe branch configurations shown in Figures 1 (c) and 1(d). Stations 1, 2 and 3 were located in the pipes which met at the junction. Analytical predictions were obtained using empirical pressure drop coefficients for the junction as derived by Benson et al (1). Tests were conducted with inlet pipework which included a right angled junction in six different configurations.

Two sets of tests were conducted with the configuration depicted in Figure 1 (e). In the first set an orifice was included in the suction line: the diameter of the orifices used ranged from 0.5 in to 1.13 in. Figure 4 (a) shows the experimental and the analytical results when an orifice 0.5 in diameter was present in the inlet line. In the second set of tests a check valve was included in the inlet line. In the analytical model of a non-return valve, the valve was treated as an orifice of variable area. The flow equations for an orifice were coupled to the dynamic equations for the movement of a spring loaded valve. This approach avoided the need for the empirical relationships introduced by Kaddah and Woollatt (20). Figure 4 (b) illustrates the experimental and analytical pressure histories at three stations with a check valve fitted in the inlet line (configuration as in Figure 1 (e)) together with the displacements of the check valve and compressor suction valve.

CONCLUSIONS

The investigation demonstrated that the boundaries met with in reciprocating compressor systems can be modelled satisfactorily using a uniform approach. Hence compressor installations may be simulated by solving the finite amplitude wave equations for flow within the pipes in conjunction with the solutions presented here for the various boundaries at the pipe ends.

NOTATION

a	Speed of sound
a_a	Speed of sound after isentropic expansion to p_{ref} i.e. $\frac{a}{a_a} = \left(\frac{p}{p_{ref}}\right)^{\overline{k-1}/2k}$
a_{ref}	Reference speed of sound at p_{ref}
A	a/a_{ref} - non-dimensional speed of sound
A_a	a_a/a_{ref} - non-dimensional entropy parameter
Aa_G	Guessed value of Aa
A^*	$A^* = A/Aa = a/a_a = (p/p_{ref})^{\overline{k-1}/2k}$
C1	Pseudo-Riemann variable $= A + \frac{k-1}{2} U$
C2	Pseudo-Riemann variable $= A - \frac{k-1}{2} U$
$C1_G$	Guessed value of C1
$C2_G$	Guessed value of C2
C1*	$C1^* = C1/Aa$
C2*	$C2^* = C2/Aa$
F	Cross sectional area
k	Isentropic index (ratio of specific heats)
p	Pressure
p_{ref}	Reference pressure
P	Non-dimensional pressure $= p/p_{ref}$
s	Entropy
t	Time
u	Particle velocity
U	Non-dimensional particle velocity
U*	U/Aa
x	Distance
X	Non-dimensional distance $X = \frac{x}{x_{ref}}$
Z	Non-dimensional time $= \frac{t}{(x_{ref}/a_{ref})}$

$\theta_1, \theta_2, \theta_3, \theta_4$, etc. Coefficients used in momentum equation

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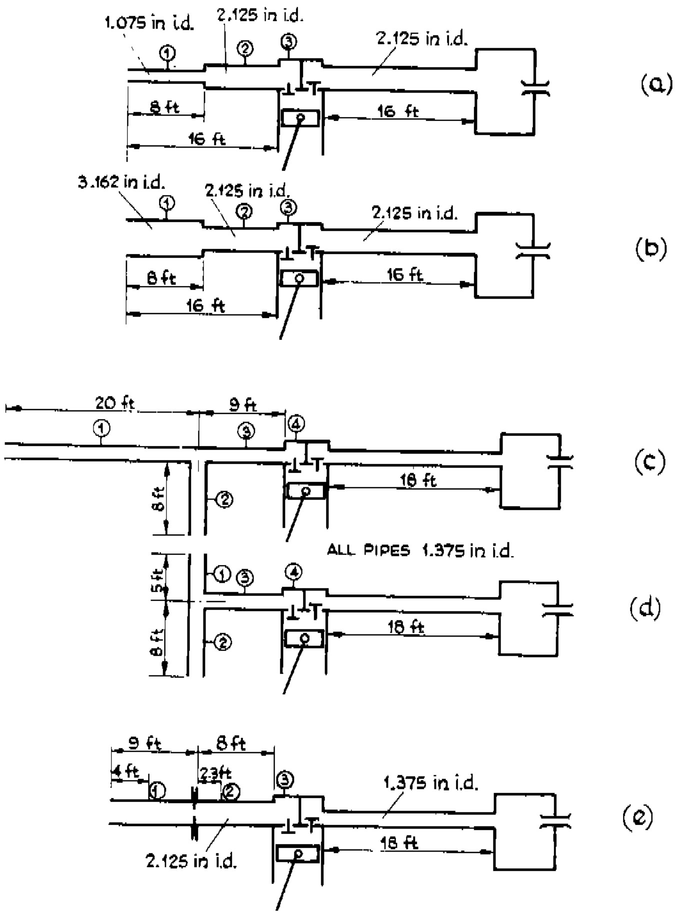


FIG. 1. INLET LINE CONFIGURATIONS

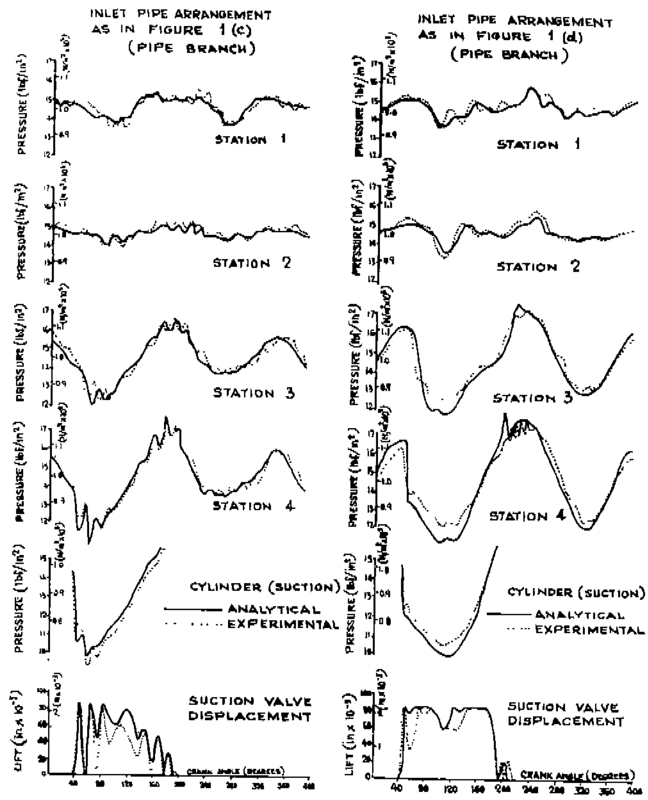


FIGURE 3 COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS
(COMPRESSOR SPEED 600 rev/min)

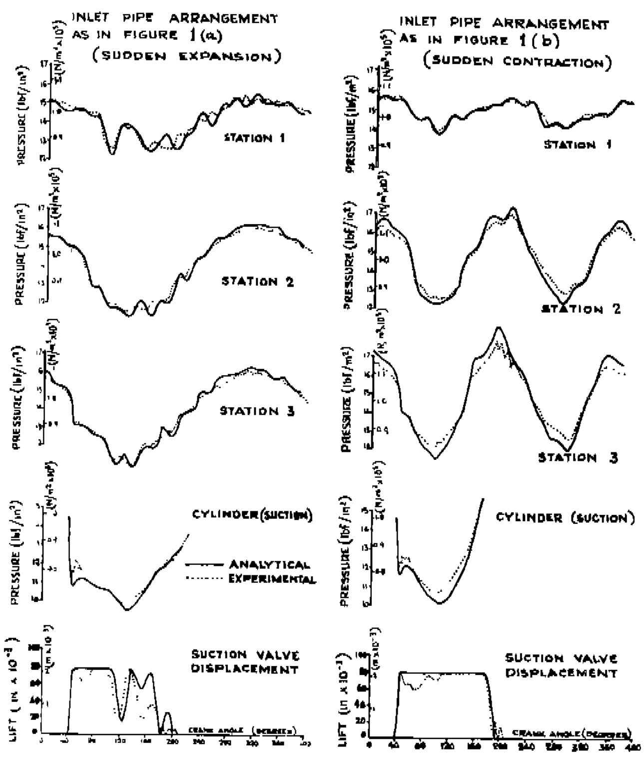


FIGURE 2 COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS
(COMPRESSOR SPEED 600 rev/min)

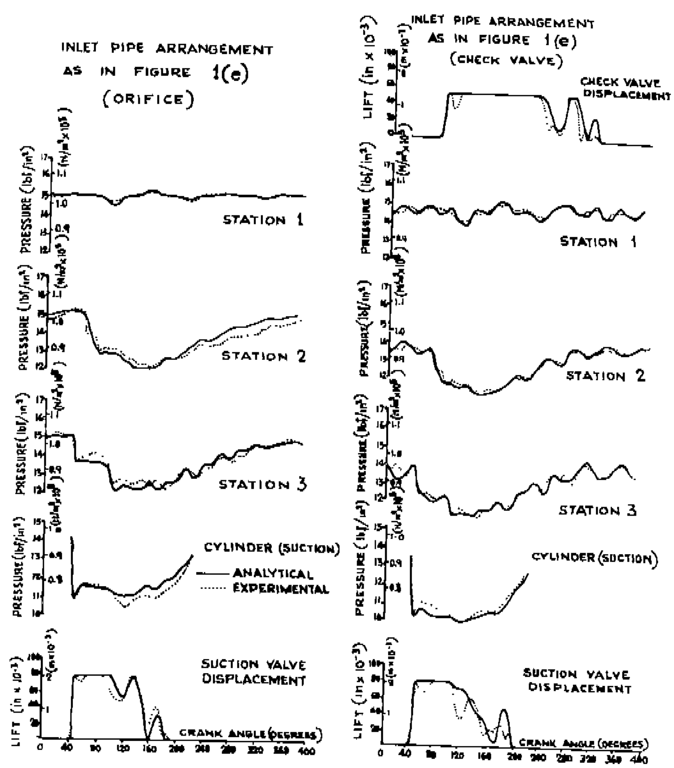
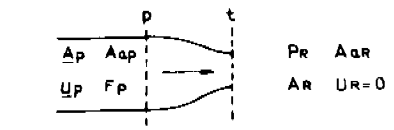
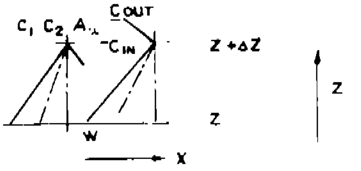
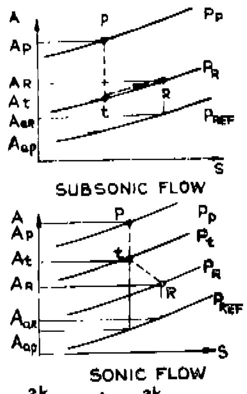


FIGURE 4 COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS
(COMPRESSOR SPEED 600 rev/min)

OUTFLOW FROM PIPE







_ DENOTES UNKNOWN VARIABLES

PRESSURE CONDITION $P_p > P_R$ OR $\left(\frac{A_p}{A_{ap}}\right)^{\frac{2k}{k-1}} > \left(\frac{A_R}{A_{aR}}\right)^{\frac{2k}{k-1}}$

SINCE THE FLOW DIRECTION AND CHARACTERISTIC DIRECTION C_{IN} ARE THE SAME THEN FOR $U_p > 0$ WE HAVE

$$\frac{C_{IN}}{A_p} = 1{\cdot}0 + \frac{k-1}{2}\left(\frac{U_p}{A_p}\right) > 1{\cdot}0 \quad \text{i.e.} \quad \frac{C_{IN}}{A_R\left(\frac{A_{ap}}{A_{aR}}\right)} > 1{\cdot}0$$

THE FLOW IS ASSUMED TO BE ISENTROPIC BETWEEN SECTIONS P AND t AND ADIABATIC BETWEEN SECTION t AND THE ADJACENT VOLUME.

THE ENERGY AND CONTINUITY EQUATIONS MAY BE WRITTEN FOR THE FLOW BETWEEN SECTIONS P AND t

$$A_p^2 + \frac{k-1}{2}U_p^2 = A_t^2 + \frac{k-1}{2}U_t^2 \; ; \quad \rho_p U_p = \phi \rho_t U_t \text{ WHERE } \phi = \frac{F_t}{F_p}$$

ASSUMING ISENTROPIC FLOW BETWEEN SECTIONS P AND t WE MAY WRITE

$$\frac{P_p}{P_t} = \left(\frac{A_p}{A_t}\right)^{2k/k-1} \text{ AND HENCE } \frac{U_t}{A_t} = \frac{U_p}{A_t}\;\frac{1}{\phi}\left(\frac{A_p}{A_t}\right)^{2/k-1}$$

SUBSONIC FLOW AT t

$$\left(\frac{U_p}{A_t}\right)^2 = \frac{\left(\frac{2}{k-1}\right)\left(\left(\frac{A_p}{A_t}\right)^2 - 1\right)}{\frac{1}{\phi^2}\left(\frac{A_p}{A_t}\right)^{4/k-1} - 1}$$

FOR SUBSONIC FLOW $P_t = P_R$

HENCE $\frac{A_t}{A_{at}} = \frac{A_R}{A_{aR}}$

BUT FOR ISENTROPIC FLOW

$A_{ap} = A_{at}$

HENCE $A_t = \frac{A_R}{A_{aR}} A_{ap}$

FROM THE DEFINITION OF C_{IN} AND SUBSTITUTING FOR U_p/A_t

$$\frac{C_{IN}}{A_t} = \frac{A_p}{A_t} + \frac{k-1}{2}\;\frac{U_p}{A_t}$$

$$= \frac{A_p}{A_t} + \phi\left[\frac{\left(\frac{k-1}{2}\right)\left\{\left(\frac{A_p}{A_t}\right)^2 - 1\right\}}{\left(\frac{A_p}{A_t}\right)^{4/k-1} - \phi^2}\right]$$

THE VALUE OF A_p/A_t IS FOUND BY ITERATION

SINCE C_{IN}, A_t, ϕ ARE KNOWN

C_{OUT} IS CALCULATED AS

$C_{OUT} = 2A_p - C_{IN} = 2\frac{A_p}{A_t}A_t - C_{IN}$

$C_{OUT} = 2A_{ap}\frac{A_p}{A_t}\cdot\frac{A_R}{A_{aR}} - C_{IN}$

SONIC FLOW AT t

$M = \frac{U_t}{A_t} = 1$ i.e. $U_t = A_t$

FROM THE ENERGY EQUATION

$$\left(\frac{U_p}{A_t}\right)^2 = \frac{k+1}{k-1} - \frac{2}{k-1}\left(\frac{A_p}{A_t}\right)^2$$

FOR SONIC FLOW AT t

$$\frac{U_t}{A_t} = 1 = \frac{U_p}{A_t}\;\frac{1}{\phi}\left(\frac{A_p}{A_t}\right)^{2/k-1}$$

SUBSTITUTION FOR U_p/A_t YIELDS

$$\phi = \left(\frac{A_p}{A_t}\right)^{2/k-1}\left[\frac{k+1}{k-1} - \frac{2}{k-1}\left(\frac{A_p}{A_t}\right)^2\right]^{\frac{1}{2}}$$

WHICH MAY BE SOLVED BY ITERATION FOR A_p/A_t SINCE ϕ IS KNOWN.

NOW $\frac{U_p}{A_t} = \frac{\phi}{(A_p/A_t)^{2/k-1}}$

$C_{IN} = A_p + \frac{k-1}{2}U_p$

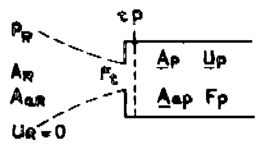
$C_{OUT} = A_p - \frac{k-1}{2}U_p$

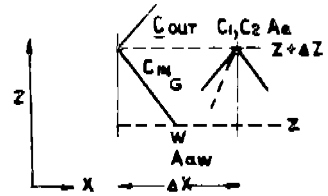
HENCE

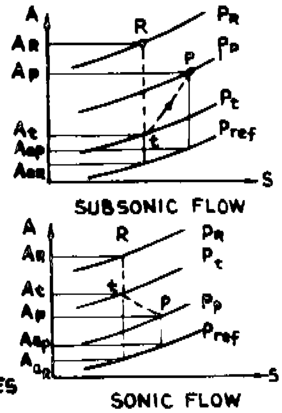
$$C_{OUT} = C_{IN}\left[\frac{\frac{A_p}{A_t} - \frac{k-1}{2}\;\frac{U_p}{A_t}}{\frac{A_p}{A_t} + \frac{k-1}{2}\;\frac{U_p}{A_t}}\right]$$

INFLOW TO PIPE

THE PROCESS MAY BE CONSIDERED AS AN ISENTROPIC CONTRACTION TO A THROAT SECTION (IF ANY) FOLLOWED BY AN ADIABATIC SUDDEN EXPANSION TO PIPE CONDITIONS.







_ DENOTES UNKNOWN VARIABLES

IN THIS INSTANCE THE ENTROPY OF THE FLUID AT SECTION P IS NOT KNOWN. THE INCOMING CHARACTERISTIC C_{IN} HAS TO BE GUESSED. A SOLUTION TO THE BOUNDARY EQUATIONS IS EFFECTED AND THEN AN ITERATIVE PROCEDURE IS USED TO OBTAIN COMPATIBLE ENTROPY, PRESSURE AND VELOCITY VALUES.

ASSUMING STEADY ADIABATIC FLOW BETWEEN THE REGION R, t AND P THE ENERGY EQUATION BECOMES

$$A_R^2 = A_t^2 + \frac{k-1}{2}U_t^2 = A_p^2 + \frac{k-1}{2}U_p^2$$

NOW $C_{IN} = A_p - \frac{k-1}{2}U_p$ AND $C_{OUT} = A_p + \frac{k-1}{2}U_p$

HENCE

$$A_R^2 = A_p^2 + \frac{k-1}{2}U_p^2 = \left[\frac{C_{IN} + C_{OUT}}{2}\right]^2 + \frac{k-1}{2}\left[\frac{C_{OUT} - C_{IN}}{k-1}\right]^2$$

THIS EQUATION CAN BE SOLVED FOR C_{OUT} FOR GIVEN VALUES OF C_{IN} AND A_R.

$$\text{i.e.} \quad \frac{C_{OUT}}{A_R} = \frac{C_{IN}}{A_R}\left(\frac{3-k}{k-1}\right) + \frac{2}{k+1}\left[(k-1)(k+1) - 2(k-1)\left(\frac{C_{IN}}{A_R}\right)^2\right]^{1/2}$$

SINCE C_{OUT} CAN BE DETERMINED DIRECTLY FROM C_{IN} THE PROBLEM IS TO CALCULATE THE CORRECT ENTROPY VALUE A_{ap} FROM WHICH C_{IN} IS OBTAINED.

THE CONTINUITY EQUATION IS $\rho_p U_p = \phi \rho_t U_t$ WHERE $\phi = F_t/F_p$ THE MOMENTUM EQUATION BETWEEN t AND P MAY BE WRITTEN

$$(P_t - P_p)F_p = \rho_p U_p^2 F_p - \rho_t U_t^2 F_t$$

NOW $\frac{P_p}{P_R} = \left(\frac{A_p}{A_R}\cdot\frac{A_{aR}}{A_{ap}}\right)^{2k/k-1} = \frac{P_p}{P_t}\cdot\frac{P_t}{P_R}$

SINCE $A^2 = kP/\rho$

$\frac{P_p}{P_t} = \frac{\rho_p}{\rho_t}\left(\frac{A_p}{A_t}\right)^2$

FROM THE CONTINUITY EQUN.

$\frac{\rho_p}{\rho_t} = \phi\frac{U_t}{U_p}$ AND HENCE

$\frac{P_p}{P_t} = \phi\frac{U_t}{U_p}\left(\frac{A_p}{A_t}\right)^2$

FOR ISENTROPIC FLOW TO t

$A_{at} = A_{aR}$ AND HENCE

$\frac{P_t}{P_R} = \left(\frac{A_t}{A_R}\right)^{2k/k-1}$

SUBSTITUTION FOR $\frac{P_p}{P_t}$ AND $\frac{P_t}{P_R}$ AND SOLVING FOR A_{ap} GIVES

$$A_{ap} = \frac{A_{aR}\left(\frac{A_p}{A_t}\right)}{\left[\phi\;\frac{U_t}{U_p}\left(\frac{A_p}{A_t}\right)^2\right]^{(k-1)/2k}}$$

A_{ap} IS THUS A FUNCTION OF A_t AND U_t SINCE ϕ AND A_{aR} ARE KNOWN, WHILST U_p AND A_p ARE ESTIMATED FROM A GUESSED VALUE OF C_{IN}.

THE CONTINUITY, MOMENTUM AND EQUATION OF STATE MAY BE COMBINED TO GIVE:-

$$A_t^2 = \phi\frac{U_t}{U_p}\left[A_p^2 + kU_p(U_p - U_t)\right]$$

THIS EQUATION MAY BE COMBINED WITH THE ENERGY EQUATION TO GIVE :-

$$\left[k\phi - \frac{k-1}{2}\right]\left(\frac{U_t}{A_R}\right)^2 - \phi\left[\frac{1 + \frac{k-1}{2}\left(\frac{U_p}{A_R}\right)^2}{\frac{U_p}{A_R}}\right]\left(\frac{U_t}{A_R}\right) + 1 = 0$$

WHICH MAY BE SOLVED FOR (U_t/A_R)

FOR SONIC FLOW AT t THE SOLUTION IS TRIVIAL SINCE $U_t = A_t$ AND ONE HAS

$$\frac{U_t}{A_R} = \frac{A_t}{A_R} = \left(\frac{2}{k+1}\right)^{1/2}$$

CHART A - BOUNDARY CONDITION AT PIPE/VOLUME/JUNCTION

THE VALUE OF AREA RATIO $\phi = \phi_{CRIT}$ FOR WHICH A GIVEN FLOW WILL BE CHOKED MAY BE OBTAINED FROM :-

$$\frac{1}{\phi_{CRIT}} = \left[\frac{1 + \frac{k+1}{2}\left(\frac{U_P}{A_R}\right)^2}{\frac{U_P}{A_R}\left(\frac{2}{k+1}\right)^{1/2}} - k \right]$$

A CHECK MAY BE CARRIED OUT TO DETERMINE WHETHER THE FLOW IS SONIC OR SUBSONIC AT t

I.E. $\phi \leqslant \phi_{CRIT}$ THE FLOW IS SONIC AT t

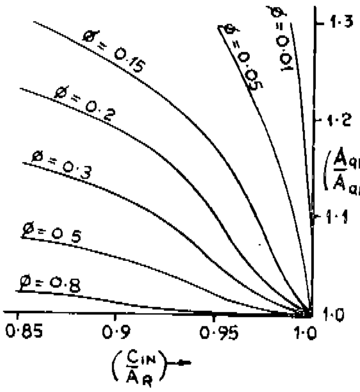
$\phi > \phi_{CRIT}$ THE FLOW IS SUBSONIC AT t

INFLOW SOLUTION AND ENTROPY CORRECTION PROCEDURE:-

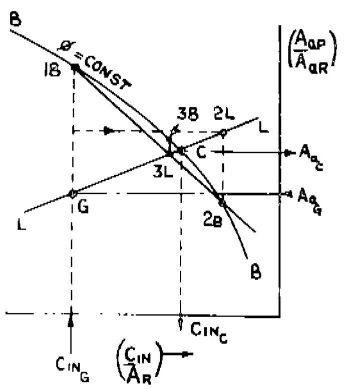
(1) GUESS A VALUE FOR A_{aP_G} (VALUE FROM PREVIOUS TIME).
(2) CALCULATE C_{IN} FROM CHARACTERISTIC LINE
(3) CALCULATE CORRESPONDING VALUE OF C_{OUT}
(4) CALCULATE VALUES OF U_P, A_P, AND ϕ_{CRIT}
(5) FOR SONIC OR SUBSONIC FLOW DETERMINE (U_t/A_R) AND A_t
(6) CALCULATE CORRESPONDING VALUE OF A_{aP}
(7) DOES $(A_{aP})_{CALC} = (A_{aP})_G$ IF YES SOLUTION COMPLETE
(8) IF NO, CORRECT C_{IN} BY USING THE FACT THAT

$$C_{IN_C} = \left(\frac{A_W}{A_{aW}}\right)(A_{a_C} - A_{a_G}) + C_{IN_G}$$

I.E. THE CORRECT SOLUTION LIES ON A LINE OF SLOPE OF (A_{aW}/A_W) IN THE C_{IN}, A_a PLANE WHERE THIS LINE INTERSECTS THE APPROPRIATE ϕ = CONST. CURVE IN THE SAME PLANE

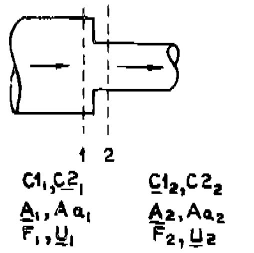


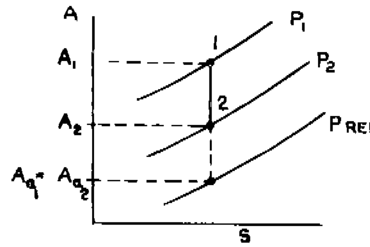
INFLOW SOLUTION CURVES



ITERATION TECHNIQUE

(9) POINT G CORRESPONDS TO A_{a_G}, C_{IN_G}
(10) POINT 1B CORRESPONDS TO C_{IN_G}, AND SYSTEM OF BOUNDARY EQUATIONS.
(11) USING A MEAN ENTROPY VALUE BETWEEN POINTS G AND 1B FIND POINT 2L ON THE CORRECTION LINE LL.
(12) USE CORRESPONDING VALUE OF C_{IN} AND DETERMINE POINT 2B ON BOUNDARY CURVE.
(13) FROM THE INTERSECTION OF LINE THROUGH POINTS 1B, 2B, [iB, (i+1)B] AND LL DETERMINE POINT 3L [(i+2)L]
(14) REPEAT STEPS 12 & 13 UNTIL THE ENTROPY DIFFERENCE BETWEEN POINTS iB AND iL IS ACCEPTABLY SMALL





— UNKNOWN VARIABLE

A SUDDEN CONTRACTION MAY BE ANALYSED BY ASSUMING ISENTROPIC FLOW BETWEEN SECTIONS 1 AND 2 AND IGNORING VENA CONTRACTA EFFECTS.

THE USE OF THE CONTINUITY, CONSERVATION OF ENERGY AND CHARACTERISTIC EQUATIONS ALLOWS A SOLUTION TO BE EFFECTED.

(1) FROM PATH LINE CALCULATION DETERMINE A_{a_1}
(2) $A_{a_1} = A_{a_2}$ (ISENTROPIC FLOW)
(3) FROM CHARACTERISTIC CALCULATIONS DETERMINE $C1_1$ AND $C2_2$
(4) SINCE

$$A_1^2 + \frac{k-1}{2} U_1^2 = A_2^2 + \frac{k-1}{2} U_2^2$$

$$A_1^{\frac{2}{k-1}} U_1 F_1 = A_2^{\frac{2}{k-1}} U_2 F_2$$

$$C1_1 = A_1 + \frac{k-1}{2} U_1 \quad \text{AND} \quad C2_2 = A_2 - \frac{k-1}{2} U_2$$

I.E. WE HAVE A SYSTEM OF FOUR EQUATIONS IN FOUR UNKNOWNS A_1, A_2, U_1, U_2. U_1 AND U_2 MAY BE ELIMINATED YIELDING TWO EQUATIONS IN A_1 AND A_2

$$A_1^{\frac{2}{k-1}}(C1_1 - A_1)F_1 = A_2^{\frac{2}{k-1}}(A_2 - C2_2)F_2$$

$$A_1^2 + \frac{2}{k-1}(C1_1 - A_1)^2 = A_2^2 + \frac{2}{k-1}(C2_2 - A_2)^2$$

THESE EQUATIONS MAY BE SOLVED USING THE NEWTON - RAPHSON TECHNIQUE

ONCE A_1 AND A_2 HAVE BEEN FOUND U_1 AND U_2 ARE EASILY CALCULATED.

$$U_1 = \frac{2}{k-1}(C1_1 - A_1) \quad \text{AND} \quad U_2 = \frac{2}{k-1}(A_2 - C2_2)$$

THE SOLUTION IS CHECKED TO ENSURE SUBSONIC FLOW (OR IN THE LIMIT SONIC FLOW AT PLANE 2)

I.E. $U_1/A_1 < 1.0$ AND $U_2/A_2 \leqslant 1.0$

IF SUBSONIC FLOW CONDITIONS ARE SATISFIED THEN

$$C1_2 = A_2 + \frac{k-1}{2} U_2 \quad \text{AND} \quad C2_1 = A_1 - \frac{k-1}{2} U_1$$

FOR SONIC FLOW AT PLANE 2, A_1/A_2 IS OBTAINED FROM

$$\phi = \frac{F_2}{F_1} = \left(\frac{A_1}{A_2}\right)^{\frac{2}{k-1}} \left[\frac{k+1}{k-1} - \frac{2}{k-1}\left(\frac{A_1}{A_2}\right)^2\right]^{1/2}$$

SINCE FOR SONIC FLOW AT PLANE 2

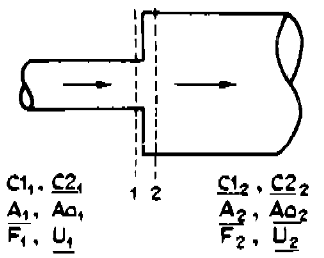
$$\frac{U_1}{A_2} = \frac{\phi}{(A_1/A_2)^{2/k-1}} \quad \text{AND}$$

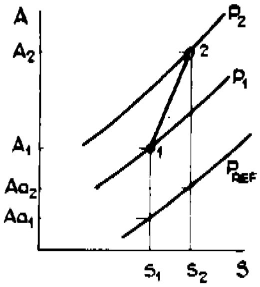
$$C1_1 = A_1 + \frac{k-1}{2} U_1 \;;\; C2_1 = A_1 - \frac{k-1}{2} U_1$$

$$C2_1 = C1_1 \left[\frac{\frac{A_1}{A_2} - \frac{k-1}{2}\frac{U_1}{A_2}}{\frac{A_1}{A_2} + \frac{k-1}{2}\frac{U_1}{A_2}}\right]$$

$$C1_2 = \frac{k+1}{3-k} C2_2$$

CHART B-PIPE/VOLUME INFLOW SOLUTION SUDDEN CONTRACTION





(—) DENOTES UNKNOWN VARIABLE

FLOW IN A SUDDEN EXPANSION MAY BE ANALYSED USING THE CONTINUITY, ENERGY AND MOMENTUM EQUATIONS BETWEEN SECTIONS 1 AND 2. IN THE MOMENTUM EQUATION THE PRESSURE IMMEDIATELY DOWNSTREAM OF THE PLANE OF ENLARGEMENT IS ASSUMED TO BE EQUAL TO P_1 HENCE :-

CONTINUITY
$$\frac{A_1^{\frac{2}{k-1}} U_1 F_1}{(Ao_1)^{2k/k-1}} = \frac{A_2^{\frac{2}{k-1}} U_2 F_2}{(Ao_2)^{2k/k-1}}$$

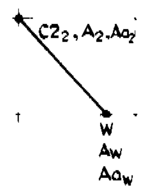
ENERGY
$$A_1^2 + \frac{k-1}{2} U_1^2 = A_2^2 + \frac{k-1}{2} U_2^2$$

MOMENTUM
$$F_2 \cdot \left[\left(\frac{A_2}{Ao_2}\right)^{\frac{2k}{k-1}} - \left(\frac{A_1}{Ao_1}\right)^{\frac{2k}{k-1}}\right] = k\left[\left(\frac{A_1}{Ao_1}\right)^{\frac{2k}{k-1}} \frac{U_1^2 F_1}{A_1^2} - \left(\frac{A_2}{Ao_2}\right)^{\frac{2k}{k-1}} \frac{U_2^2 F_2}{A_2^2}\right]$$

CHARACTERISTICS
$$C1_1 = A_1 + \frac{k-1}{2} U_1$$
$$C2_2 = A_2 - \frac{k-1}{2} U_2 = C2_{2GUESS} + \left(\frac{A_2}{Ao_2}\right)\left[Ao_2 - Ao_{2GUESS}\right]$$

N.B. THE LATTER VERSION OF THE ENTROPY CORRECTION EQUATION IS FOUND TO GIVE ACCEPTABLE RESULTS COMPARED WITH THE ACTUAL CORRECTION EQUATION

$$C2_2 = C2_{2GUESS} + \left(\frac{A_W}{Ao_W}\right)\left[Ao_2 - Ao_{2GUESS}\right]$$



THE SYSTEM OF EQUATIONS MAY BE REDUCED AND SIMPLIFIED BY WORKING IN TERMS OF "STARRED" VARIABLES WHERE :-

$$A_1^* = \frac{A_1}{Ao_1} \; ; \; U_1^* = \frac{U_1}{Ao_1} \; ; \; A_2^* = \frac{A_2}{Ao_2} \; ; \; U_2^* = \frac{U_2}{Ao_2} \; ; \; C2^*_{2GUESS} = \frac{C2_{2GUESS}}{Ao_{2GUESS}}$$
$$C1_1^* = \frac{C1_1}{Ao_1} \; ; \; C2_1^* = \frac{C2_1}{Ao_1} \; ; \; C1_2^* = \frac{C1_2}{Ao_2} \; ; \; C2_2^* = \frac{C2_2}{Ao_2}$$

THE CONTINUITY, ENERGY, MOMENTUM, CHARACTERISTIC AND ENTROPY CORRECTION EQUATIONS BECOME :-

$$\frac{A_1^{*\frac{2}{k-1}} U_1^* F_1}{Ao_1} = \frac{A_2^{*\frac{2}{k-1}} U_2^* F_2}{Ao_2}$$

$$Ao_1^2 \left[A_1^{*2} + \frac{k-1}{2} U_1^{*2}\right] = Ao_2^2 \left[A_2^{*2} + \frac{k-1}{2} U_2^{*2}\right]$$

$$\left[A_2^{*\frac{2k}{k-1}} - A_1^{*\frac{2k}{k-1}}\right] F_2 = kA_1^{*\frac{2}{k-1}} U_1^{*2} F_1 - kA_2^{*\frac{2}{k-1}} U_2^{*2} F_2$$

$$U_1^* = \frac{2}{k-1}\left[C1_1^* - A_1^*\right]$$

$$U_2^* = \frac{2}{k-1}\left[A_2^* - C2^*_{2GUESS}\right] \frac{Ao_{2GUESS}}{Ao_2}$$

FROM THE LATTER EQUATION

$$Ao_2^2 = \frac{1}{U_2^{*2}}\left[\left(\frac{2}{k-1}\right)\left(A_2^* - C2^*_{2GUESS}\right) Ao_{2GUESS}\right]^2$$

USING THE ENERGY EQUATION AND SUBSTITUTING FOR U_1^* AND Ao_2 YIELDS

$$\pi_1 = \frac{A_2^{*2}}{U_2^{*2}} = \frac{Ao_1^2\left[A_1^{*2} + \left(\frac{2}{k-1}\right)\left(C1_1^* - A_1^*\right)^2\right] - \left(\frac{2}{k-1}\right)\left[\left(A_2^* - C2^*_{2GUESS}\right) Ao_{2GUESS}\right]^2}{\left[\left(\frac{2}{k-1}\right)\left(A_2^* - C2^*_{2GUESS}\right) Ao_{2GUESS}\right]^2}$$

MULTIPLYING THIS EQUATION BY THE EXPRESSION FOR $Ao_2\, U_2^*$ GIVES

$$\pi_2 = \frac{Ao_2 A_2^{*2}}{U_2^*} = \frac{Ao_1^2\left[A_1^{*2} + \left(\frac{2}{k-1}\right)\left(C1_1^* - A_1^*\right)^2\right] - \left(\frac{2}{k-1}\right)\left[\left(A_2^* - C2^*_{2GUESS}\right) Ao_{2GUESS}\right]^2}{\left(\frac{2}{k-1}\right)\left[A_2^* - C2^*_{2GUESS}\right] Ao_{2GUESS}}$$

THE MOMENTUM EQUATION CAN BE COMBINED WITH THE EQUATIONS FOR U_1^* AND π_1 TO YIELD

$$A_2^{*\left(\frac{2k}{k-1}\right)}\left[1 + \frac{k}{\pi_1}\right] F_2 - A_1^{*\left(\frac{2}{k-1}\right)}\left[A_1^{*2} F_2 + k\left(\frac{2}{k-1}\right)^2 \left(C1_1^* - A_1^*\right)^2 F_1\right] = 0$$

THE CONTINUITY EQUATION MAY BE COMBINED WITH THE EQUATIONS FOR U_1^* AND π_2 TO YIELD

$$\frac{A_1^{*\left(\frac{2}{k-1}\right)}\left(\frac{2}{k-1}\right)\left[C1_1^* - A_1^*\right] F_1}{Ao_1} - \frac{A_2^{*\left(\frac{2k}{k-1}\right)} F_2}{\pi_2} = 0$$

THE LAST TWO EQUATIONS MAY BE SOLVED ITERATIVELY FOR A_1^* AND A_2^*

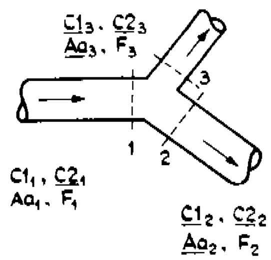
HENCE U_1^* AND U_2^* CAN BE OBTAINED

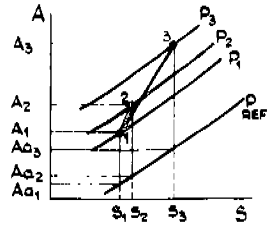
Ao_2 IS THEN DETERMINED AND THE OUTGOING CHARACTERISTICS ARE EVALUATED.

$$C2_1 = \left[A_1^* - \frac{k-1}{2} U_1^*\right] Ao_1 \; ; \; C1_2 = \left[A_2^* + \frac{k-1}{2} U_2^*\right] Ao_2$$

CHECKS ARE REQUIRED TO ENSURE THAT FOR A CONTRACTION U_1^* AND U_2^* ARE +VE AND THAT SUPERSONIC FLOW HAS NOT DEVELOPED.

CHART C - SUDDEN EXPANSION BOUNDARY SOLUTION.





(—) UNKNOWN VARIABLE

THE CONTINUITY AND ENERGY EQUATIONS FOR ADIABATIC FLOW IN "STARRED" VARIABLES MAY BE WRITTEN :-

$$\frac{A_1^{*\left(\frac{2}{k-1}\right)} U_1^* F_1}{Aa_1} = \frac{A_2^{*\left(\frac{2}{k-1}\right)} U_2^* F_2}{Aa_2} + \frac{A_3^{*\left(\frac{2}{k-1}\right)} U_3^* F_3}{Aa_3}$$

$$Aa_1^2\left[A_1^{*2} + \frac{k-1}{2} U_1^{*2}\right] = Aa_2^2\left[A_2^{*2} + \frac{k-1}{2} U_2^{*2}\right] = Aa_3^2\left[A_3^{*2} + \frac{k-1}{2} U_3^{*2}\right]$$

EMPIRICAL FORMS OF THE MOMENTUM EQUATIONS MAY BE EMPLOYED USING EXPERIMENTALLY DETERMINED COEFFICIENTS. i.e.

$$\left.\begin{aligned} P_1 - P_2 &= \theta_1 \rho_2 U_2^2 - \theta_2 \rho_1 U_1^2 \\ P_1 - P_{\cdot} &= \theta_3 \rho_3 U_3^2 - \theta_4 \rho_1 U_1^2 \end{aligned}\right\} \quad \theta_1, \theta_2, \theta_3, \theta_4 \text{ ARE EMPIRICAL COEFFICIENTS}$$

THESE MOMENTUM EQUATIONS MAY ALSO BE WRITTEN IN "STARRED" VARIABLES.

$$A_1^{*\left(\frac{2k}{k-1}\right)} - A_2^{*\left(\frac{2k}{k-1}\right)} = k\theta_1 A_2^{*\left(\frac{2}{k-1}\right)} U_2^{*2} - k\theta_2 A_1^{*\left(\frac{2}{k-1}\right)} U_1^{*2}$$

$$A_1^{*\left(\frac{2k}{k-1}\right)} - A_3^{*\left(\frac{2k}{k-1}\right)} = k\theta_3 A_3^{*\left(\frac{2}{k-1}\right)} U_3^{*2} - k\theta_4 A_1^{*\left(\frac{2}{k-1}\right)} U_1^{*2}$$

ENTROPY CORRECTION EQUATIONS MAY BE WRITTEN FOR PIPES 2 AND 3

$$U_2^* = \left(\frac{2}{k-1}\right)\left[A_2^* - C2_{2G}^*\right]\frac{Aa_{2G}}{Aa_2} \quad \text{OR} \quad Aa_2^2 = \frac{1}{U_2^{*2}}\left[\left(\frac{2}{k-1}\right)\left(A_2^* - C2_{2G}^*\right)Aa_{2G}\right]^2$$

$$U_3^* = \left(\frac{2}{k-1}\right)\left[A_3^* - C2_{3G}^*\right]\frac{Aa_{3G}}{Aa_3} \quad \text{OR} \quad Aa_3^2 = \frac{1}{U_3^{*2}}\left[\left(\frac{2}{k-1}\right)\left(A_3^* - C2_{3G}^*\right)Aa_{3G}\right]^2$$

THE DEFINITION OF $C1_1$ YIELDS $C1_1^* = A_1^* + \frac{k-1}{2} U_1^*$
THE SYSTEM OF EIGHT EQUATIONS MAY BE SOLVED FOR EIGHT UNKNOWNS ($U_1^*, A_1^*, U_2^*, A_2^*, Aa_2, U_3^*, A_3^*, Aa_3$). THE SYSTEM MAY BE REDUCED TO THREE EQUATIONS AS FOLLOWS :-

$$\pi_1 = \frac{A_2^*}{U_2^{*2}} = \frac{Aa_1^2\left[A_1^{*2} + \frac{2}{k-1}\left(C1_1^* - A_1^*\right)^2\right] - \frac{2}{k-1}\left[\left(A_2^* - C2_{2G}^*\right)Aa_{2G}\right]^2}{\left[\frac{2}{k-1}\left(A_2^* - C2_{2G}^*\right)Aa_{2G}\right]^2}$$

$$\pi_2 = \frac{Aa_2 A_2^{*2}}{U_2^*} = \frac{Aa_1^2\left[A_1^{*2} + \frac{2}{k-1}\left(C1_1^* - A_1^*\right)^2\right] - \frac{2}{k-1}\left[\left(A_2^* - C2_{2G}^*\right)Aa_{2G}\right]^2}{\left[\frac{2}{k-1}\left(A_2^* - C2_{2G}^*\right)Aa_{2G}\right]}$$

$$\pi_3 = \frac{A_3^{*2}}{U_3^{*2}} = \frac{Aa_1^2\left[A_1^{*2} + \frac{2}{k-1}\left(C1_1^* - A_1^*\right)^2\right] - \frac{2}{k-1}\left[\left(A_3^* - C2_{3G}^*\right)Aa_{3G}\right]^2}{\left[\frac{2}{k-1}\left(A_3^* - C2_{3G}^*\right)Aa_{3G}\right]^2}$$

$$\pi_4 = \frac{Aa_3 A_3^{*2}}{U_3^*} = \frac{Aa_1^2\left[A_1^{*2} + \frac{2}{k-1}\left(C1_1^* - A_1^*\right)^2\right] - \frac{2}{k-1}\left[\left(A_3^* - C2_{3G}^*\right)Aa_{3G}\right]^2}{\left[\frac{2}{k-1}\left(A_3^* - C2_{3G}^*\right)Aa_{3G}\right]}$$

FINALLY ONE OBTAINS

$$A_1^{*\left(\frac{2}{k-1}\right)}\left[A_1^{*2} + k\theta_2\left(\frac{2}{k-1}\right)^2\left(C1_1^* - A_1^*\right)^2\right] - A_2^{*\left(\frac{2k}{k-1}\right)}\left[1 + \frac{k\theta_1}{\pi_1}\right] = 0$$

$$A_1^{*\left(\frac{2}{k-1}\right)}\left[A_1^{*2} + k\theta_4\left(\frac{2}{k-1}\right)^2\left(C1_1^* - A_1^*\right)^2\right] - A_3^{*\left(\frac{2k}{k-1}\right)}\left[1 + \frac{k\theta_3}{\pi_3}\right] = 0$$

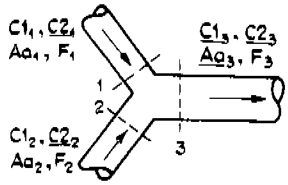
$$\frac{A_1^{*\left(\frac{2}{k-1}\right)} F_1\left(\frac{2}{k-1}\right)\left(C1_1^* - A_1^*\right)}{Aa_1} - \frac{A_2^{*\left(\frac{2k}{k-1}\right)} F_2}{\pi_2} - \frac{A_3^{*\left(\frac{2k}{k-1}\right)} F_3}{\pi_4} = 0$$

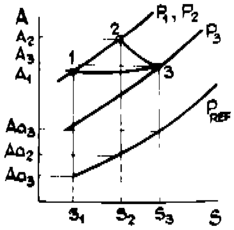
THE EQUATIONS ARE SOLVED ITERATIVELY FOR A_1^*, A_2^*, A_3^* HENCE U_1^*, U_2^*, U_3^* AND Aa_2 AND Aa_3 MAY BE EVALUATED.

CHECKS ARE REQUIRED TO ENSURE THAT U_1^*, U_2^*, AND U_3^* ARE +VE CORRESPONDING TO DIVIDING FLOW.

THE FORMULATION IS CURRENTLY LIMITED TO SUBSONIC FLOWS.

DIVIDING FLOW JUNCTION





(—) DENOTES UNKNOWN VARIABLE

A SATISFACTORY ANALYSIS FOR THE CASE OF COMBINING FLOWS IS OBTAINED BY ASSUMING THAT THE PRESSURES OF THE COMBINING FLOWS ARE EQUAL. AN EMPIRICAL FORM OF THE MOMENTUM EQUATION IS USED TO ALLOW FOR ENTROPY CHANGES. CONTINUITY, ENERGY AND ENTROPY CORRECTION EQUATIONS ARE USED AS BEFORE. i.e. USING "STARRED" VARIABLES.

$$\frac{A_1^{*\left(\frac{2}{k-1}\right)} U_1^* F_1}{Aa_1} + \frac{A_2^{*\left(\frac{2}{k-1}\right)} U_2^* F_2}{Aa_2} = \frac{A_3^{*\left(\frac{2}{k-1}\right)} U_3^* F_3}{Aa_3}$$

$$Aa_1 A_1^{*\left(\frac{2}{k-1}\right)} U_1^* F_1\left[A_1^{*2} + \frac{k-1}{2} U_1^{*2}\right] + Aa_2 A_2^{*\left(\frac{2}{k-1}\right)} U_2^* F_2\left[A_2^{*2} + \frac{k-1}{2} U_2^{*2}\right]$$
$$= Aa_3 A_3^{*\left(\frac{2}{k-1}\right)} U_3^* F_3\left[A_3^{*2} + \frac{k-1}{2} U_3^{*2}\right]$$

$P_1 = P_2$ OR $A_1^* = A_2^*$

$P_1 - P_3 = \theta_5 \rho_3 U_3^2 - \theta_6 \rho_1 U_1^2$ WHERE θ_5 AND θ_6 ARE COEFFICIENTS

$$\text{i.e.} \quad A_1^{*\left(\frac{2k}{k-1}\right)} - A_3^{*\left(\frac{2k}{k-1}\right)} = k\theta_5 A_3^{*\left(\frac{2}{k-1}\right)} U_3^{*2} - k\theta_6 A_1^{*\left(\frac{2}{k-1}\right)} U_1^{*2}$$

$$U_3^* = \left(\frac{2}{k-1}\right)\left[A_3^* - C2_{3G}^*\right]\frac{Aa_{3G}}{Aa_3}$$

$$C1_1^* = A_1^* + \frac{k-1}{2} U_1^* \quad \text{AND} \quad C1_2^* = A_2^* + \frac{k-1}{2} U_2^*$$

THE SYSTEM OF SEVEN EQUATIONS IN SEVEN UNKNOWNS ($A_1^*, U_1^*, A_2^*, U_2^*, A_3^*, U_3^*, Aa_3$) MAY BE REDUCED TO TWO EQUATIONS AS FOLLOWS :-

$$\pi_5 = \frac{U_3^{*2}}{A_3^{*2}} = \frac{2}{k-1}\left[\frac{Aa_1 A_1^{*\left(\frac{2}{k-1}\right)}\left(C1_1^* - A_1^*\right)\left[A_1^{*2} + \frac{2}{k-1}\left(C1_1^* - A_1^*\right)^2\right]F_1}{A_3^{*\left(\frac{2k}{k-1}\right)}\left(A_3^* - C2_{3G}^*\right)Aa_{3G}F_3} + \frac{Aa_2 A_1^{*\left(\frac{2}{k-1}\right)}\left(C1_2^* - A_1^*\right)\left[A_1^{*2} + \frac{2}{k-1}\left(C1_2^* - A_1^*\right)^2\right]F_2}{A_3^{*\left(\frac{2k}{k-1}\right)}\left(A_3^* - C2_{3G}^*\right)Aa_{3G}F_3} - 1\right]$$

$$\pi_6 = \frac{U_3^*}{Aa_3 A_3^{*2}} = \frac{Aa_1 A_1^{*\left(\frac{2}{k-1}\right)}\left(C1_1^* - A_1^*\right)\left[A_1^{*2} + \frac{2}{k-1}\left(C1_1^* - A_1^*\right)^2\right]F_1}{A_3^{*\left(\frac{2k}{k-1}\right)}\left(A_3^* - C2_{3G}^*\right)^2 Aa_{3G}^2 F_3} + \frac{Aa_2 A_1^{*\left(\frac{2}{k-1}\right)}\left(C1_2^* - A_1^*\right)\left[A_1^{*2} + \frac{2}{k-1}\left(C1_2^* - A_1^*\right)^2\right]F_2}{A_3^{*\left(\frac{2k}{k-1}\right)}\left(A_3^* - C2_{3G}^*\right)^2 Aa_{3G}^2 F_3} - \frac{1}{\left(A_3^* - C2_{3G}^*\right)Aa_{3G}}$$

FROM THE MOMENTUM EQUATION

$$A_1^{*\left(\frac{2}{k-1}\right)}\left[A_1^{*2} + k\theta_6\left(\frac{2}{k-1}\right)^2\left(C1_1^* - A_1^*\right)^2\right] - A_3^{*\left(\frac{2k}{k-1}\right)}\left[1 + k\theta_5\pi_5\right] = 0$$

AND FINALLY USING THE EXPRESSIONS FOR π_5 AND π_6

$$\frac{A_1^{*\left(\frac{2}{k-1}\right)}\left(\frac{2}{k-1}\right)\left(C1_1^* - A_1^*\right)F_1}{Aa_1} + \frac{A_1^{*\left(\frac{2}{k-1}\right)}\left(\frac{2}{k-1}\right)\left(C1_2^* - A_1^*\right)F_2}{Aa_2} - A_3^{*\left(\frac{2k}{k-1}\right)}\pi_6 F_3 = 0$$

COMBINING FLOW JUNCTION

CHART D - THREE BRANCH JUNCTION BOUNDARY