

# Effect of Fractal Dimension of Fine Aggregates on the Concrete Chloride Resistance

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## ABSTRACT

The relationship between fractal dimension of fine aggregates and the chloride resistance of concrete was investigated in this study. Both concrete and mortar specimens were cast. Concrete specimens were in the same mix design as the mortar specimens except for the coarse aggregates. The specimens were divided into different groups based on the gradation of the fine aggregates. The chloride resistances of concrete specimens were tested by using the rapid chloride migration method. The results indicate that high volume fractal dimensions of fine aggregates have positive impacts on the chloride resistance of concrete. The simplified calculating formulas were proposed.

## 1. INTRODUCTION

Aggregate takes up 60%–90% of the total volume of concrete. Proper selection of aggregate particle size distribution affects the main properties of concrete: workability, strength, permeability, chloride resistance, and even the total cost of hardened concrete. Therefore, aggregate design is an essential part of concrete mix design. The fineness module ( $M_x$ ) is widely used to describe the aggregate characters. However, it is rough in describing the characters of the fine aggregates. Fractal dimension of the aggregate is recently adopted in aggregates researches.

The fractal theory, which focused on the irregular geometric items, was established by Mandelbrot and so called “fractal geometry” (Mandelbrot, 1984). It was then developed quickly and extended to use in the researches on the other irregular or unstable problems in nature, such as the studies of soil particles and geotechnical materials (Lange, Jennings, & Shah, 1993; Saouma & Barton, 1994; Xie, 1997). Current researches reported that fractal value of a certain object actually describes its complexity quantitatively. It is the potential to connect the macro-performance with the microstructures by using this parameter.

Concrete material is well known for its anisotropy. It could be attributed to the complexity of its microstructures, which is a kind of fractal system. The aggregates were composed of large numbers of particles in different sizes. The distribution of the particles is considered as a generalized fractal problem in mathematics (AASHTO, 1989). Current design standard only provide the size limitations on aggregates (ACI Committee 318 (ACI 318), 2005).

In this study, the mass and volume fractal dimensions of the fine aggregates were calculated. By taking concrete and mortar durability experiments, the effects of fractal dimensions of fine aggregates on the chloride resistance of concrete were investigated. The fractal dimension is suggested to be a design index in concrete mixtures.

## 2. FRACTAL DIMENSIONS

The particle size (sieve size) generates a distribution which could be described using a fractal method. In addition, the mass distribution (sieving ratio) and the volume architectures can both also be described by using fractal method as presented below.

### 2.1 Volume fractal dimension

For the particles of size  $x$ , the distribution function could be written as Eq. (1):

$$F(x) = \frac{N(x)}{N} \quad (1)$$

where  $N(x)$  is the number of fine aggregates whose size are less than or equal to  $x$ .

Since the size of fine aggregates could be classified as “point problem,” the fractal dimension  $N(x)$  could be calculated as Eq. (2), according to Xie (1997):

$$N(x) = N_0 \left( \frac{x}{x_{\max}} \right)^{0-D} \quad (2)$$

where  $x_{\max}$  represents the maximum size of the fine aggregate particles,  $N_0$  is a constant, and  $D$  is the size fractal dimension value.

Classifying the piled particles of the fine aggregates as a “volume problem,” the volume fractal character could be used to describe the void filling capacity of those particles (Xu, 1991). The fractal dimension  $V(x)$  could be calculated as Eq. (3), comparing to Eq. (2), according to Xie (1997):

$$V(x) = V_0 \left( \frac{x}{x_{\max}} \right)^{3-D_v} \quad (3)$$

where  $V(x)$  is the fractal volume and  $D_v$  is the volume fractal dimension.

$$dV(x) = dV_0 \left( \frac{x}{x_{\max}} \right)^{3-D_v} \quad (4)$$

$$dV_0 = \frac{M \cdot dP(x)}{\rho} \quad (5)$$

Based on the simultaneous Eqs (3–5), the fractal volume could be calculated using Eq. (6):

$$V(x) = \frac{3-D}{6-D-D_v} \cdot \frac{M}{\rho} \cdot \frac{x_{\min}^{6-D-D_v} - x_{\max}^{6-D-D_v}}{x_{\min}^{3-D} - x_{\max}^{3-D}} \cdot x_{\max}^{D_v-3} \quad (6)$$

In addition, when numerous particles with different sizes are piled together, there is void among them. The void volume of the particles pile could be calculated using Eq (7):

$$\begin{aligned} V_{\text{void}} &= \frac{\frac{M}{\rho} - V}{\frac{M}{\rho}} \\ &= 1 - \frac{3-D}{6-D-D_v} \cdot \frac{x_{\min}^{6-D-D_v} - x_{\max}^{6-D-D_v}}{x_{\min}^{3-D} - x_{\max}^{3-D}} \cdot x_{\max}^{D_v-3} \\ &= 1 - \frac{\gamma}{\rho_0} \end{aligned} \quad (7)$$

where  $\gamma$  is the compacted density and  $\rho_0$  is the gross density of the particles pile.

Provided the measured value of  $V_{\text{void}}$ ,  $x_{\max}$  and  $x_{\min}$  of a certain pile of fine aggregates, the volume fractal dimension  $D_v$  could be calculated using Eq. (7).

### 3. RELATIONSHIP BETWEEN FRACTAL DIMENSIONS AND MORTAR/CONCRETE DURABILITY PERFORMANCE

#### 3.1 Fine aggregates gradation

Seven series of fine aggregate particle size distribution were designed in this study. All the fine aggregates were oven dried and sieved to different sizes (see in

Table 1) using the automatic sieve shaker. Then, the particles of different size were mixed following the designed distribution in Table 1. The maximum size of fine aggregates is 4.75 mm. The void volumes of each aggregate set were tested according to the standard (JGJ/T 70-2009, 2009). The volume fractal dimensions were calculated using Eq. (7) and presented in Table 2. Figure 1 shows a decreasing linear relationship between the volume fractal dimension values  $D_v$  and void volumes. An linear relationship between the percentage of voids and  $D_v$  could be obtained as Eq. (8):

$$V_{\text{void}} \% = -31.61 \times D_v + 113.58 \quad (8)$$

It may be explained that with the increase of volume fractal dimension, the filling capacity of particles also increases, which results in reduction of void.

Table 1. Fine aggregate particle size distribution.

Sieve size (mm)	Accumulated sieve ratio (%)						
	I	II	III	IV	V	VI	VII
4.75	4	3	3	2	2	1	1
2.36	45	41	38	34	31	26	22
1.18	69	65	61	57	53	46	41
0.60	83	80	77	73	70	63	58
0.30	92	83	87	85	83	77	73
0.15	97	96	95	93	92	89	87
<0.15	100	100	100	100	100	100	100

Table 2. Fractal dimensions of fine aggregates.

Groups	$\rho_0$ (g/cm <sup>3</sup> )	$\gamma$ (g/cm <sup>3</sup> )	Void (%)	$D_v$
I	2.00	1.6393	18.03	2.6458
II	2.50	1.6667	33.33	2.5775
III	2.38	1.7544	26.32	2.7183
IV	2.33	1.7241	25.86	2.7429
V	2.70	1.8868	30.19	2.7040
VI	2.50	1.8519	25.93	2.7825
VII	2.33	1.7241	25.86	2.7980

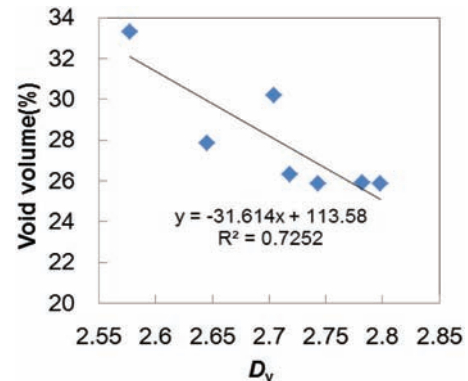


Figure 1. Relationship between  $D_v$  and void%.

### 3.2 Relationship between fractal dimensions and chloride resistance of mortar specimens

Seven series in total of seven mortar specimens were tested for chloride resistant test. The mixture information of mortar specimens was presented in Table 3.

**Table 3.** Mixture design of the mortar specimens (kg/m<sup>3</sup>).

Cement	Fly ash	Slag	Sand	Water	Water reducer
313	102	137	629	176	12

All the specimens were kept in tap water under room temperature (average of 25°C) for 28 days. Then, the specimens were put into an artificial accelerating testing chamber and subjected to wet-dry cycles for 60 days. Mangat and Gurusamy (1987) found a significant difference between the diffusion rates of chloride through the casting face and the bottom face (during casting) of the prism. To eliminate the casting effect, all the faces of the specimens for chloride resistant test were coated with epoxy except the top surface. The top surface is vertical to the casting direction. The specimens were under salt water spraying for 3 h (wet cycle) out of 48 h and, for the remaining 45 h, they were left in the ambient laboratory conditions at a temperature of 45°C (dry cycle). After 60 days' cycling, the powder samples were drilled out from each specimen along the ingress direction. Three holes were drilled in dry condition using a 14-mm diameter rotary impact drill for each specimen. Powdered samples were taken at every 5 mm depth. The chloride concentration profile of each powder sample was measured by using the potentiometric titration method. Previous research (Gao, 2008) indicates that diffusing dominate the chloride's moving inside concrete away from the surface more than 7 mm. In this study, the apparent chloride diffusion coefficient  $D_a$  (square centimeter per second) was obtained by Fick's second law without the test data at the depth of 5 mm:

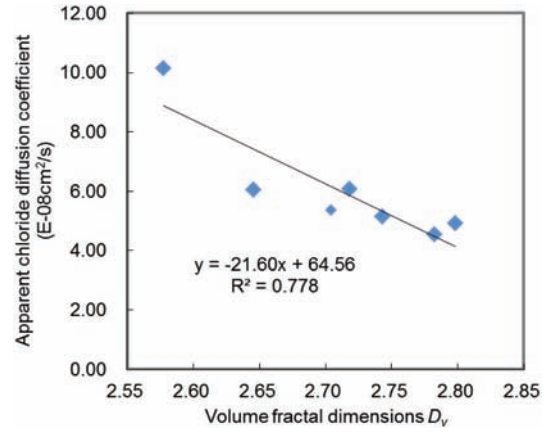
$$C(x,t) = C_s \times \left( 1 - \operatorname{erf} \left( \frac{x}{2\sqrt{D_a \cdot t}} \right) \right) \quad (9)$$

where  $C(x,t)$  is the chloride concentration at depth  $x$  and age  $t$ ;  $C_s$  is the surface chloride concentration.

The results are presented in Table 4 and Figure 2.

**Table 4.** Apparent chloride diffusion coefficient ( $\times 10^{-12}$  cm<sup>2</sup>/s).

I	II	III	IV	V	VI	VII
6.03	11.30	6.08	5.13	5.37	4.55	4.91



**Figure 2.** Relationship between  $D_v$  and apparent chloride diffusion coefficient.

$$D_a = -21.6 \times D_v + 64.56 \quad (10)$$

It should be noticed that all the specimens were cast in same mixture design except for the fine aggregate gradation. Therefore, the difference of chloride resistant ability of the mortar specimens could be only attributed to the difference of the fine aggregate distribution. The results indicate mortar specimens with high  $D_v$  have high chloride resistance. The difference of fine aggregate causes approximately 250% difference in the chloride resistant ability of mortar specimens. It may be explained that higher volume fractal dimension leads to higher compacting ability of the particles. It might complicate the transferring paths of the chloride ions. This might result in the reduction in the apparent chloride diffusion coefficients.

### 3.3 Relationship between fractal dimensions and chloride resistance in concrete specimens

For concrete specimens, coarse aggregates take about half of the total volume. The chloride moving route in concrete is probably changed due to the existing of coarse aggregate. Therefore, the effect of fractal dimensions of fine aggregates on the chloride resistance of concrete is investigated in this study.

Four series in total of 12 concrete specimens were tested, and each series has three specimens. The fine aggregate distributions of each series are presented in Table 5, and the mixture is presented in Table 6. Only the fine aggregate distribution is the variable parameter for all specimens. The mass and volume fractal dimensions were calculated using Eq. (7) and presented in Table 7. The specimens were prepared in the same way as the mortar specimens. After a 28-day curing period, the rapid chloride migration test method (CCES01-2004, 2005) was used to measure the chloride penetration depths of concrete specimens. The chloride diffusion coefficient  $D_{RCM}$

values were calculated based on the measured chloride ingress depth (CCES01-2004, 2005):

$$D_{RCM} = 2.872 \times 10^{-6} \frac{Th(x_d - \alpha\sqrt{x_d})}{t} \quad (11)$$

$$\alpha = 3.338 \times 10^{-3} \sqrt{Th}$$

where  $T$  is the average temperature of the anticathode cell (K),  $h$  is the height of the specimen (mm),  $t$  is the testing duration (s), and  $x_d$  is the average ingress depth (mm).

**Table 5.** Fine aggregate particles distribution in concrete specimens.

Sieve size (mm)	Accumulated sieve ratio (%)			
	I	II	III	IV
4.75	3	10	10	5
2.36	11	13	25	20
1.18	20	24	26	30
0.6	41	69	43	70
0.3	70	71	89	91
0.15	98	96	90	92
<0.15	100	100	100	100

**Table 6.** Mixture design of the mortar specimens (kg/m<sup>3</sup>).

C	FA	SI	S	G	W	Admixture
313	102	137	629	1014	176	12

**Table 7.** Fractal dimensions of fine aggregates in concrete specimens.

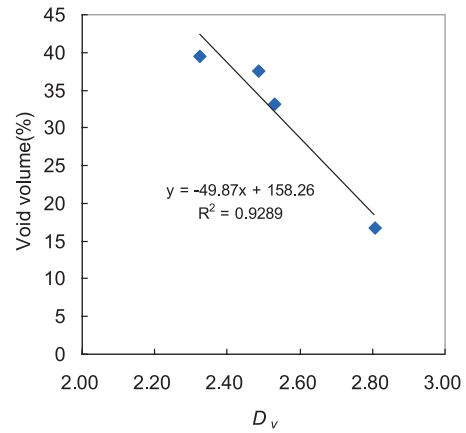
Series	$\rho_o$ (g/cm <sup>3</sup> )	$\gamma$ (g/cm <sup>3</sup> )	Void (%)	$D_v$
I	2.6667	1.6129	39.52	2.3239
II	2.1137	1.7604	16.71	2.8058
III	2.8840	1.8025	37.50	2.4871
IV	2.6736	1.7856	33.21	2.5316

The concrete testing results shows the same trend as the mortar testing results (see in Figures 3 and 4), but the effect of fine aggregate on the chloride resistance of concrete specimens is even significant. The difference of fine aggregate causes approximately 300% difference in chloride resistant ability of concrete specimens. This result indicates the importance of volume fractal dimensions of fine aggregates for the chloride resistance of concrete.

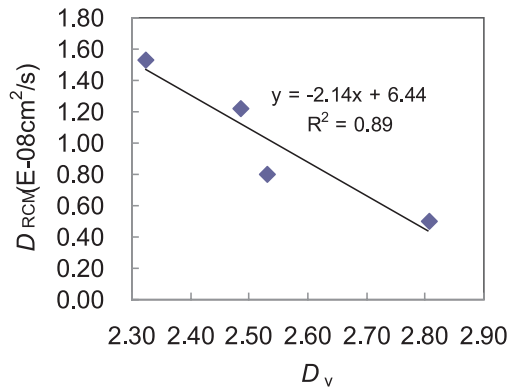
The relationship between  $V_{void}$  % or  $D_{RCM}$  and  $D_v$  in the case of concrete is written as Eqs (12) and (13) based on the testing results:

$$V_{void} \% = -49.87 D_v + 158.26 \quad (12)$$

$$D_{RCM} = -2.14 D_v + 6.44 \quad (13)$$



**Figure 3.** Relationship between  $D_v$  and void% in concrete.



**Figure 4.** Relationship between  $D_v$  and  $D_{RCM}$

#### 4. CONCLUSION

In this study, the relationship between fractal dimension of fine aggregates and chloride resistance of concrete was investigated. The testing results indicated that with the increasing of  $D_v$ , the voids of the fine aggregates piles decrease, as well as the chloride moving ability in either the mortar specimens or the concrete specimens. The equations for the relationships between the void volume, chloride resistance, and volume fractal dimension  $D_v$  in the case of mortar had been established by using fractal theory. In addition, equations for the relationship between void volume, chloride resistance, and  $D_v$  were obtained in concrete.

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#### REFERENCES

AASHTO. (1989). *Report of the joint task force on rutting*. Washington, DC: AASHTO.

- ACI Committee 318 (ACI 318). (2005). *Building code requirements for structural concrete and commentary*. Detroit, MI: American Concrete Institute.
- CCES01-2004. (2005). *Guide to durability design and construction of concrete structures*. Beijing, pp. 37–38.
- Gao, X. J. (2008). Study on chloride penetration of in-situ harbor structures (Master thesis). Zhejiang University, China.
- JGJ/T 70-2009. (2009). *Standard for test method of basic properties of construction mortar*.
- Lange, D. A., Jennings, H. M., & Shah, S. P. (1993). Relationship between fracture surface roughness and fracture behavior of cement paste and mortar. *Journal of American Ceramic Society*, 76(3), 589–597.
- Mandelbrot, B. B. (1984). Fractal character of fracture surfaces of metals of metals. *Nature*, 308, 721–723.
- Mangat, P. S., & Gurusamy, K. (1987). Chloride diffusion in steel fibre reinforced marine concrete. *Cement and Concrete Research*, 17(3), 385–396.
- Saouma, V. C., & Barton, C. C. (1994). Fractals, fracture and size effect in concrete. *Journal of Engineering and Mechanics*, 120(4), 835–854.
- Xie, H. P. (1997). *Fractal-geomechanics introduction*. Beijing, China: Science Press, pp. 168–257.
- Xu, S. F. (1991). *Study of prediction, control, prevention and cure about highway Asphalt Pavement's rutting*. Shanghai, China: Tongji University.