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Christoph M. Hoffmann
Purdue University, cmh@cs.purdue.edu

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Christoph M. Hoffmann
Computer Science Department
Purdue University
West Lafayette, Indiana 47907

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Christoph M. Hoffmann

Computer Science Department
Purdue University

Abstract

Algorithms are developed which construct from a given LL(1) grammar a recursive descent parser with as much recursion resolved by iteration as is possible without introducing auxiliary memory. Unlike other proposed methods in the literature designed to arrive at parsers of this kind, the algorithms do not require extensions of the notational formalism nor alter the grammar in any way.

The algorithms constructing the parsers operate in $O(k \cdot s)$ steps, where s is the size of the grammar, i.e. the sum of the lengths of all productions, and k is a grammar - dependent constant. A speedup of the algorithm is possible which improves the bound to $O(s)$ for all LL(1) grammars, and constructs smaller parsers with some auxiliary memory in form of parameters to some of the routines.

Keywords and phrases: compilers, top down parsers, recursive descent, complexity.

1. Introduction

Among the various proposed parsing techniques, the method of recursive descent has been particularly attractive from a practical point of view. A parser using this method is, in our experience, very readable and hence readily modified to allow flexible extensions handling semantic processing [12], as well as giving good error diagnostics and recover gracefully [6, 14]. Because of its advantages, the method has found acceptance both commercially, e.g. [3], as well as academically, [13], and serves as a basis for research into compiler compilers [10, 11].

A recursive descent parser is usually obtained from an LL(1) grammar G† by a simple transcription of the productions of G, using the LL(1) lookahead sets. Parsers obtained in this way are frequently unnecessarily recursive, as noted in [11], because of the technique of 'left factoring' (see e.g. [16]) applied to make G LL(1). This inherent problem has been attacked in the past by hand modifications of the LL(1) grammar [9, 11], thereby running risk of having to backtrack, or by ad-hoc modifications of the parser itself.

We present algorithms which construct recursive descent parsers directly from an LL(1) grammar avoiding recursion wherever this can be done without introducing auxiliary variables. The resulting parsers are as good as the ones obtained by hand using the methods of [9, 11], without any need to modify G. Assuming a machine environment in which recursion is not unduely expensive, the resulting parsers are competitive in speed with table driven techniques such as LL(1) parsers.

The constructor algorithms are shown to operate in O(k·s) steps,

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† Note, however, the results of [1] which show that recursive descent parsing is more powerful than LL(1) parsing.

where s is the size of the grammar (i.e. the sum of the lengths of all productions), and k is a grammar dependent constant. Ways of reducing k to 1 by a modification of the algorithms are indicated. As derived in [8], the lookahead sets for checking that G is LL(1) can be constructed within the same bound, hence the construction of our parsers is faster than, for example, the SLR(1) algorithms of [5].

2. Preliminaries

We use the standard notation for grammars as, for example, in [2]. A context free grammar G is a quadruple (N, Σ, P, S), where N is the set of nonterminal symbols, Σ the set of terminal symbols, P the set of productions, where each production is an element of $N \times (N \cup \Sigma)^*$ and written $A \to \alpha$. S is a distinguished element of N, the start symbol.

Symbols in N are denoted by A,B,C,... symbols in Σ by a,b,c..., strings in $(N \cup \Sigma)^*$ by $\alpha, \beta, \gamma \ldots$, and strings in Σ^* by u,w,x... The empty string is denoted by ε. The size of G is the sum of the lengths of all productions of G.

The relation $\Rightarrow$ ("leftmost derives") is defined on $(N \cup \Sigma)^* \times (N \cup \Sigma)^*$ as follows: $wA\beta \Rightarrow w\alpha\beta$ if $A \to \alpha$ is a production in P and w is in Σ^*. The relations $\overset{+}{\Rightarrow}$ and $\overset{*}{\Rightarrow}$ are the transitive and the reflexive transitive closure of $\Rightarrow$, respectively.

A string w in Σ^* is in $L_A(G)$, the language derived by A in G, iff $A \overset{*}{\Rightarrow} w$, and the language derived by G is the language derived by S in G and denoted L(G).

Given a context free grammar G = (N, Σ, P, S), we define for each production $A \to \alpha$ in P a lookahead set $H(A \to \alpha)$ as follows:

$$H(A \to \alpha) = \{ a \text{ in } \Sigma \mid S \overset{*}{\Rightarrow} uA\gamma \text{ ; } A\gamma \Rightarrow \alpha\gamma \overset{+}{\Rightarrow} ax; \quad u, ax \text{ in } \Sigma^* \}.$$

Definition A context free grammar G is LL(1) if, for every A in N, $A \to \alpha$ and $A \to \beta$ in P, $H(A \to \alpha) \cap H(A \to \beta) = \emptyset$.

The definition is equivalent to other definitions in the literatur, e.g. [2, 17].

Given an LL(1) grammar G, there is a recursive descent parser for L(G) obtained, loosely speaking, as follows: For every non-terminal A in N create a recursive procedure A. The body of A contains, for every production $A \to \alpha$ in P, a statement of the form

if next input symbol is in $H(A \to \alpha)$ then begin

$\langle \alpha \rangle$; goto exit end;

where $\langle \alpha \rangle$ is a series of statements calling procedures for non-terminals occurring in α and scanning the input for terminals in α. An example may clarify.

Example Let N = {S, E, E1, T, T1, F},

Σ = {+, *, (,), i, $},

and let G_1 be the grammar (N, Σ, P, S), where P consists of the productions

S → E $	E1 → ε	T1 → ε
E → T E1	T → F T1	F → i
E1 → + T E1	T1 → * F T1	F → (E)

G_1 is LL(1). The recursive descent parser for $L(G_1)$ obtained as described above is as follows.

```
procedure S; begin
     E; if nextch = '$' then scan else error;  end;
procedure E; begin
     T; E1; end;
procedure E1; begin
     if nextch in {'+'} then begin
          scan; T; E1; goto exit; end;
     if nextch in {'*', ')', '$'} then goto exit;
     error;
     exit: end E1;
  .
  .
  .
procedure F; begin
     if nextch in {'i'} then begin
          scan; goto exit; end;
     if nextch in {'('} then begin
          scan; E;
          if nextch = ')' then scan else error;
          goto exit; end;
     error;
     exit: end F;
```

Note that the parser parses the string i+i+i$ in L(G) by a recursion in E1. Evidently, an iteration is sufficient, and is an example of a situation in which recursion should be removed. It can also be seen, that the procedures E1 and T1 should be eliminated eventually.

Theorem The parser obtained as described parses L(G) deterministically if G is LL(1).

The theorem follows quite easily from the properties of LL(1) grammars.

3. Constructor Algorithms

In this section we develop the constructor algorithms for fast recursive descent parsing and establish their correctness. A relation ϱ defined on $N \times N$ is used to determine which nonterminals A permit iteration in the parse of $L_A(G)$ without auxiliary memory. Note that ϱ, as defined below, is analogous to $\hat{\varrho}$ used in [8] to compute the lookahead sets for LL(1) grammars.

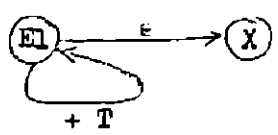
Definition The relation ϱ is defined for pairs of nonterminals by $A \varrho B$ iff there is a production $A \to \alpha B$ in G (α possibly empty).

Let ϱ^+ and ϱ^* denote the transitive and the reflexive transitive closure of ϱ, respectively.

Definition The nonterminal A in N is quasi-regular, if $A \varrho^+ A$.

Given a nonterminal A, define a directed graph D(A) as follows: Let R(A) be the subset of all those nonterminals B for which $A \varrho^* B$ and $B \varrho^* A$. Then the node set of D(A) is $R(A) \cup \{X\}$, where X is a symbol not in $N \cup \Sigma$. Furthermore, for every B in R(A), if $B \to \alpha C$ is a production of B and C is in R(A), then there is a directed edge in D(A) from B to C labelled α. For all other productions $B \to \beta$ of B, there is a directed edge from B to X in D(A) labelled β.

Example The graph D(E1) of E1 in G_1 is



Definition The graph D(A) of A in N is the deriving graph of A.

Intuitively, the deriving graph of A describes the body of the procedure A which parses $L_A(G)$. Note that D(A) is always connected.

Lemma 3.1 D(A) is acyclic if and only if A is not quasi-regular.

Proof If A is quasi-regular, then $A \varrho^+ A$, hence D(A) must contain a cycle. The rest is immediate.

In order to demonstrate the connection between traversals of deriving graphs and derivation trees in G, we decompose derivation trees into 'cuts'. Given a derivation tree $\mathcal{T}$ of w in L(G) with an interior node A, an A-cut of $\mathcal{T}$ is a tree defined as follows:

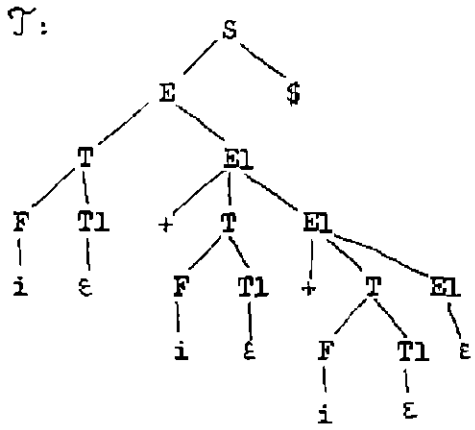
(1) The node A is the root of the A-cut, and all immediate descendants of A in $\mathcal{T}$ are immediate descendants of the root of the A-cut.

(2) If B is a node in the A-cut $\mathcal{S}'$ and is the rightmost descendant of a node C in $\mathcal{S}$, then the immediate descendents of the corresponding node B in $\mathcal{T}$ are descendants of B in $\mathcal{S}'$ provided that B $\not\in^{*}$ A.

Definition An A-cut $\mathcal{S}'$ of a derivation tree $\mathcal{T}$ is maximal if, whenever the root A of $\mathcal{S}'$ corresponds to a node A in $\mathcal{T}$ which is the rightmost descendant of another node B in $\mathcal{T}$, then A is not in R(B).

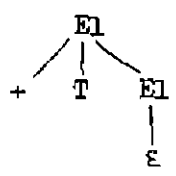
Example Consider the derivation tree $\mathcal{T}$ of i+i+i$ in G_1:

$\mathcal{T}$:

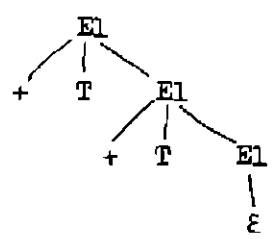


Then $\mathcal{T}_1$ is an E1-cut of $\mathcal{T}$ but not maximal, and $\mathcal{T}_2$ is another E1-cut of $\mathcal{T}$ and is maximal:

$\mathcal{T}_1$:



$\mathcal{T}_2$:



The following lemma should be obvious.

Lemma 3.2 Every derivation tree $\mathcal{T}$ in G has a unique decomposition into maximal cuts.

Proof By induction on the structure of $\mathcal{T}$.

A <u>simple traversal</u> of the deriving graph D(A) is a string in $\{N\{N \cup \Sigma\}^*\}^*\{X\}$ defining a path from A to X in D(A), that is, a string of alternating node and edge labels spelling out a path in D(A), with possible repetition, leading from A to X. A simple traversal t describes a tree as follows:

(1) The tree $\mathcal{T}_0$ consisting of the root node A alone is described by the prefix A of t.

(2) If the tree $\mathcal{T}_{i+1}$ is described by the prefix t_iB of t, and $t_{i+1} = t_iB\alpha C$ is a prefix of t where the edge label $\alpha = B_1B_2\ldots B_k$, then the tree $\mathcal{T}_{i+1}$ described by t_{i+1} is obtained from $\mathcal{T}_i$ by making B_1, B_2, .. B_k, C the immediate descendants of the rightmost occurrence of B in $\mathcal{T}_i$ (as leaf) provided C is not X; otherwise, only B_1, ..., B_k descend from B.

<u>Lemma 3.3</u> For any simple traversal t of D(A) there exists a derivation tree $\mathcal{T}$ of some sentence w in L(G) and an A-cut $\mathcal{S}$ of $\mathcal{T}$ such that t describes $\mathcal{S}$. Conversely, given an A-cut $\mathcal{S}$ in a derivation tree $\mathcal{T}$, there is a simple traversal of D(A) describing the A-cut.

<u>Proof</u> Let $\mathcal{S}'$ be the tree described by t. Note, that by construction of D(A), $\mathcal{S}'$ can be embedded into a derivation tree $\mathcal{T}$. If $\mathcal{S}'$ is not an A-cut of $\mathcal{T}$, then this must be because the rightmost leaf of $\mathcal{S}'$ has descendants in the A-cut. But the last edge of the traversal must lead to X in D(A), hence the rightmost leaf of

$\mathcal{S}'$ is either a terminal or a nonterminal B such that not B ρ^* A. Hence $\mathcal{S}'$ is an A-cut.

The converse part is shown by induction on the size of the A-cut and is straight-forward.

Thus, there is a close connection between parts of derivation trees and traversals of deriving graphs. By a suitable notion of substitution we will be able to relate any derivation tree to certain traversals. Let $t_A = t_1 \alpha t_2$ be a simple traversal of D(A) and assume that α is an edge label occurring in D(A) where $\alpha = \alpha_1 B \alpha_2$. Let t_B be a simple traversal of D(B). A <u>substitution</u> of t_B into t_A is the string $t_1 \alpha_1 [t_B] \alpha_2 t_2$ and is a <u>traversal</u> of D(A). Note, that t_B is substituted for only one occurrence of B as part of an edge label in t_A. A substitution corresponds, in an obvious manner, to attaching the tree $\mathcal{S}_B$ described by t_B with its root to the leaf B in the tree described by t_A in the corresponding position. Substitution is extended, permitting the substitution of traversals into traversals. Formalizing these notions is routine and is left to the reader.

If a traversal $\bar{t}$ does not contain edge labels containing nonterminals, then $\bar{t}$ is a <u>complete traversal</u>. No substitution is possible into complete traversals. Complete traversals correspond to trees the leaves of which are all labelled by terminals.

Theorem 3.4 Assume that $L_A(G)$ is not empty. Then the following is true:

(1) Given a complete traversal t of $D(A)$, there exists a derivation tree $\mathcal{T}$ of w in $L_A(G)$ such that w is obtained from t by deleting all node labels and brackets.

(2) Given a derivation tree $\mathcal{T}$ of w in $L_A(G)$, there is a complete traversal t of D(A) such that the string obtained from t by deleting all node labels and brackets is w.

Proof (1): Given a complete traversal t of D(A), consider the tree $\mathcal{S}$ it describes. It is easily verified that $\mathcal{S}$ is a derivation tree for $L_A(G)$. Since t is complete, all occurring edge labels are terminal strings, hence w can be obtained from t as described.

(2): This is proved by induction on the structure of $\mathcal{T}$ in its decomposition into maximal cuts (Lemma 3.2).

Basis $\mathcal{T}$ is an A-cut. By Lemma 3.3 there is a simple traversal t of D(A) describing $\mathcal{T}$, and, since $\mathcal{T}$ is a derivation tree, t is complete. The rest follows easily.

Induction Step Assume $\mathcal{T}$ is a derivation tree of w in $L_A(G)$. Decompose $\mathcal{T}$ into the maximal A-cut $\mathcal{S}_A$ with its root corresponding to the root of $\mathcal{T}$, and the trees $\mathcal{T}_{B_1}$, $\mathcal{T}_{B_2}$, ..., $\mathcal{T}_{B_k}$ attached to $\mathcal{S}_A$ at the nonterminal leaves B_1, ..., B_k. By Lemma 3.3, the simple traversal t_A describes $\mathcal{S}_A$. Also, each tree $\mathcal{T}_{B_i}$ is a derivation tree for some string u_i in $L_{B_i}(G)$ (which therefore cannot be empty), hence, by induction hypothesis, there are complete traversals t_{B_i} of of $D(B_i)$ satisfying the theorem.

The string obtained from t_A by deleting all node labels and brackets is the frontier of $\mathcal{S}_A$ and is $x_1B_1x_2B_2\ldots B_kx_{k+1}$, x_i in Σ^*. Hence $w = x_1u_1\ldots u_kx_{k+1}$. Substituting t_{B_i} for B_i in t_A we obtain the desired complete traversal of D(A) satisfying the teorem.

■

The graphs D(A) are converted into parsing routines as follows: Corresponding to D(A) is a recursive procedure A which predicts a simple traversal of D(A) edge by edge. Associated with each edge label α, $\alpha = B_1B_2\ldots B_k$, is a sequence of computations χ_i where χ_i is a call of procedure B_i if B_i is in N, otherwise χ_i checks if the next input symbol is B_i if B_i is in Σ.

Assuming that the grammar is LL(1), the simple traversal can be predicted deterministically from a one symbol lookahead [16]. We sketch the algorithm generating procedure A below. In it, code α refers to the code generated for the edge label α. Note that if the first symbol of α is terminal, the code corresponding to it can be generated to merely advance the input pointer since the presence of the nonterminal in the input has already been checked by interrogating the lookahead set $H(B\rightarrow\ldots)$. If the parse fails at any point, the routine error is called which is a standard procedure supplied which issues an appropriate message.

Algorithm 1 (Construct Procedure A)

Input: LL(1) grammar G, nonterminal A.

Output: Procedure A parsing $L_A(G)$ using iteration where possible.

1. [Initialization]

   Initialize list $\mathcal{L}$ to contain A unmarked.
   Emit: "procedure A; begin"

2. [Check if all done]

   If all elements of $\mathcal{L}$ are marked, goto Step 6;
   otherwise, take the next unmarked B in $\mathcal{L}$ and mark it,
   emit: "LB:".

3. [Process B]

   For every production $B \rightarrow \beta$ in G perform Step 4. Thereafter, goto Step 5.

4. [Process $B \rightarrow \beta$]

   4.1 Emit: "if nextch in $H(B \rightarrow \beta)$ then begin".

   4.2 If $\beta = \alpha C$, where C is in N and $C \rho^{*} A$, then
   code α,
   emit: "goto LC;"
   add C to $\mathcal{L}$ unmarked unless C is already in $\mathcal{L}$;
   otherwise,
   code β.

   4.3 Emit: "end else".

5. [End of processing B]

   Emit: "error; goto exit;". Goto Step 2.

6. [Procedure A coded]

   Emit: "exit: end A;".
   Stop.

The driving algorithm calls Algorithm 1 to construct procedures, beginning with procedure S, for only those nonterminals A for which there is a call on A. In addition, some simple optimizations are performed to make the code neater, e.g. the lookahead set is not checked for single production nonterminals, a procedure body is substituted as open subroutine if only one call to it is made in the parser, etc. The resulting code parsing $L(G_1)$ is given in the example below.

Example The parser generated from grammar G_1 is as follows:

```
procedure S;  begin
              E; if nextch = '$' then scan else error;
              end S;

procedure E;  begin
              T;
              LE1:  if nextch in {'+'} then begin
                              scan;  T;  goto LE1  end
                    else if nextch in {'*', ')', '$'} then
                    else error;
              exit: end E;

procedure T;  begin
              F;
              LT1:  if nextch in {'*'} then begin
                              scan;  F;  goto LT1  end
                    else if nextch in {'+', ')', '$'} then
                    else error;
              exit:  end T;
```

```
procedure F;  begin
              if nextch in {'i'} then scan
              else if nextch in {'('} then begin
                      scan;  E;
                      if nextch = ')' then scan else error
                      end
              else error;
              end F;
```

Theorem 3.5 When calling S, the parser generated by the algorithms described above parses L(G) deterministically provided that G is LL(1).

Proof Theorem 3.4 in conjunction with the results of [16].

4. Complexity of the Algorithms

The results of [7, 8] have established that the lookahead sets $H(A \to \alpha)$ may be constructed for an LL(1) grammar G in $O(p \cdot s)$ steps, where p is the number of terminals and s is the size of G. Ultimately these results depend on the sparseness of relations on $N \times N$ which are similar to the relation $\lessdot$ used in this paper. This is usually the case for grammars. Also in [8], there is a discussion of how to lower the bound to O(s) in certain machine environments.

On closer inspection of [8] it can be seen that ϱ^* may be computed within the same bound, because it is a subset of $\hat{\varrho}$ used in [8], and therefore at least as sparse. Hence we can assume that both the lookahead sets and the relation ϱ^* can be computed within this bound. Once they are available, the parser may be constructed within the bounds derived below.

Proposition 4.1 Given nonterminal A, the sets $H(B \to \beta)$ for all B in R(A), and the relation ϱ^*, Algorithm 1 constructs procedure A in $O(s_A)$ steps, where s_A is the sum of the lengths of all productions of the nonterminals in R(A).

Proof Evident.

Let $|R(A)|$ denote the number of elements in R(A). We define an equivalence relation δ on N×N by A δ B iff A ϱ^* B and B ϱ^* A. Note that the R(A) are the equivalence classes of δ.

Theorem 4.2 Given the LL(1) grammar G, the lookahead sets $H(A \to \alpha)$ for G and the relation ϱ^*, the parser for G can be constructed as described above in $O(k \cdot s)$ steps, where s is the size of G, and k is the cardinality of the largest equivalence class of δ, i.e.

$$k = \max_{A \in N} (|R(A)|).$$

Proof Let $R(A) = \{A, A_1, \ldots, A_{m-1}\}$. The procedures A, A_1, .., A_{m-1} can then be constructed in $O(m \cdot s_A)$ steps, which may be estimated by $O(k \cdot s_A)$ steps because of k's definition.

Let B_1, .., B_r be a set of representatives of the eqivalence classes of δ, i.e.

$$R(B_i) \cap R(B_j) = \emptyset \qquad i,j \leq r,\ i \neq j$$

$$\bigcup_{j \leq r} R(B_j) = N$$

Then, clearly, $s_{B_1} + s_{B_2} + \ldots + s_{B_r} = s$, where s is the size of G. Hence all procedures $\underline{A}$ for every A in N can be constructed in $O(k \cdot s)$ steps. ■

The constant k could be fairly large for certain grammars, as seen by considering the grammars G_n, $n > 1$, which are as follows.

The nonterminals of G_n are $\{S, A_1, .., A_n\}$, the terminals are $\{a_1, .., a_n, b_1, .., b_n, \$\}$ and the productions are

$$S \rightarrow A_1\ \$$$

$$A_i \rightarrow a_i\ A_{i+1}\ a_i\ A_{i+1} \mid b_i \qquad i < n$$

$$A_n \rightarrow a_n\ A_1\ \ a_n\ A_1 \mid b_n$$

Clearly, $|R(A_i)| = n-1$.

This points out a potential weakness in our algorithms. In case that the sets R(A) are large, the code generated becomes bulky. Recall that A is in R(B) iff B is in R(A). Consequently, the procedures for A and B can be made identical except for different entry points. If the set R(A) contains m elements, there will be m such procedures. Clearly a simple scheme can be devised which replaces the m procedures associated with the nonterminals in R(A) by a single one. This new procedure, called with a parameter indicating the nonterminal to be expanded, makes an initial transfer to the appropriate label, and is otherwise exactly like procedure $\underline{A}$. Thus, the size of the parser stays comparable to that of conventional recursive descent parsers. A careful implementation of this idea would result in a bound O(s) for the construction. It should be noted, however, that for the grammars of most programming languages the sets R(A) contain usually not more than one or two elements, which means that for such grammars the algorithms are quite acceptable.

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