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Transfer Matrix Approach to the Estimation of the Fundamental Acoustical Properties of Noise Control Materials

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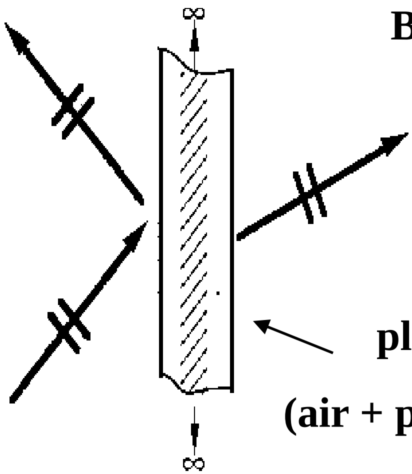
Motivation

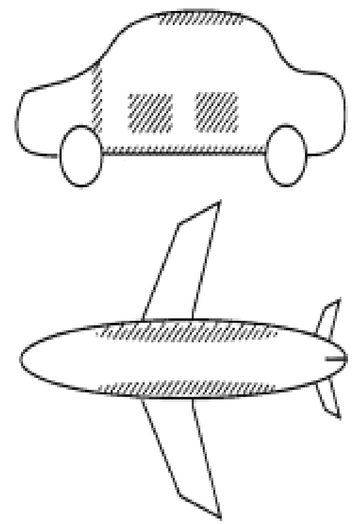
- **Analytical capabilities** **- available by J. S. Bolton and J. F. Allard et al.**



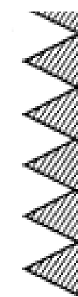
plane infinite layers

(air + panel + porous layers)

- **Practical treatments**



- FEM Predictions require complex density and sound speed

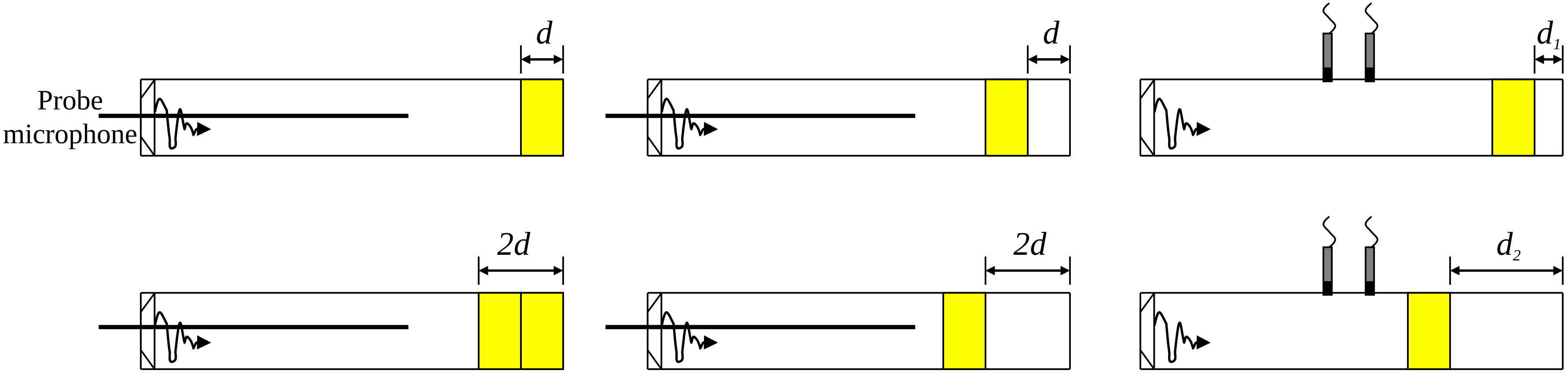


Objective

Develop procedure for measuring fundamental acoustical properties of limp or rigid porous materials quickly and easily

Background I

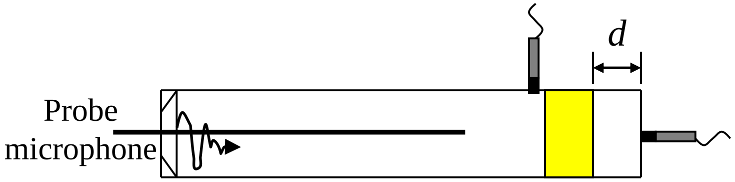
- Two thickness method (Ferrero and Sacerdote, 1951).
- Two cavity method (Yaniv, 1973).
- Arbitrary cavity, two microphone method (Utsuno et al., 1989).

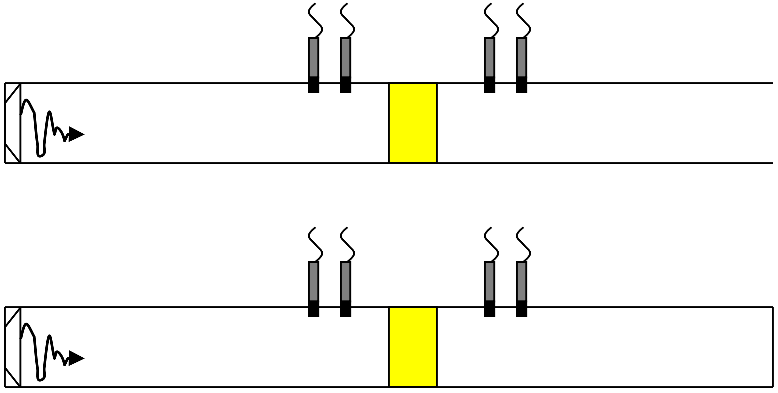


General problem : <u>inaccurate</u> for highly dissipative materials.

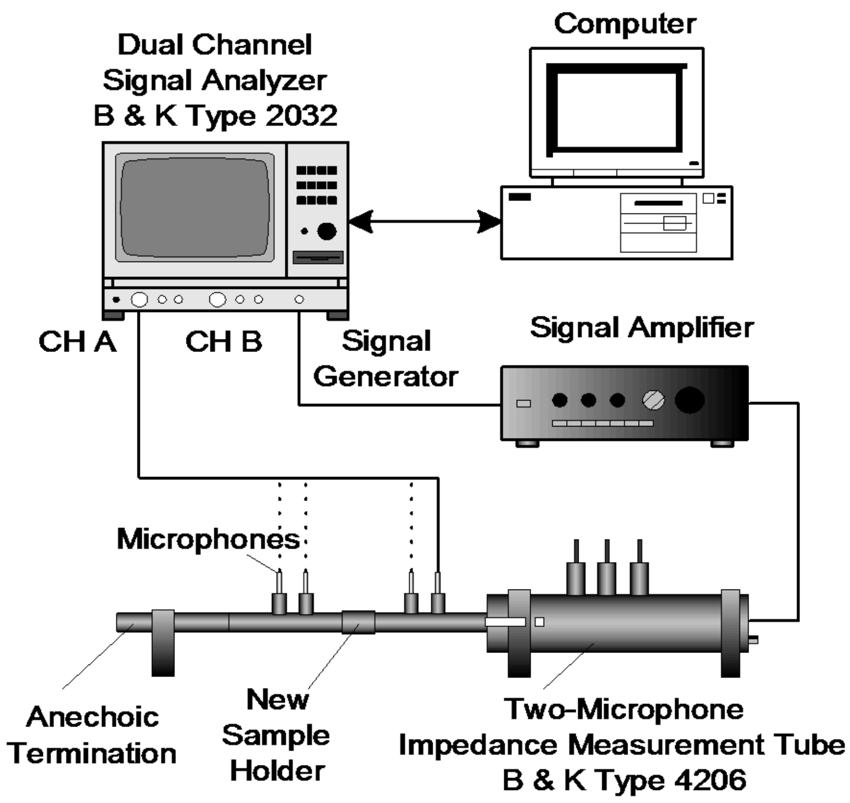
Background II

- **Transfer function method (Champoux and Stinson, 1991)**
  - Champoux and Stinson: pure tone only (time consuming)
- **Two load method (Munjal and Doige, 1990) can be used for highly dissipative materials**
  - Munjal (two separate measurements)





Transfer Matrix Method

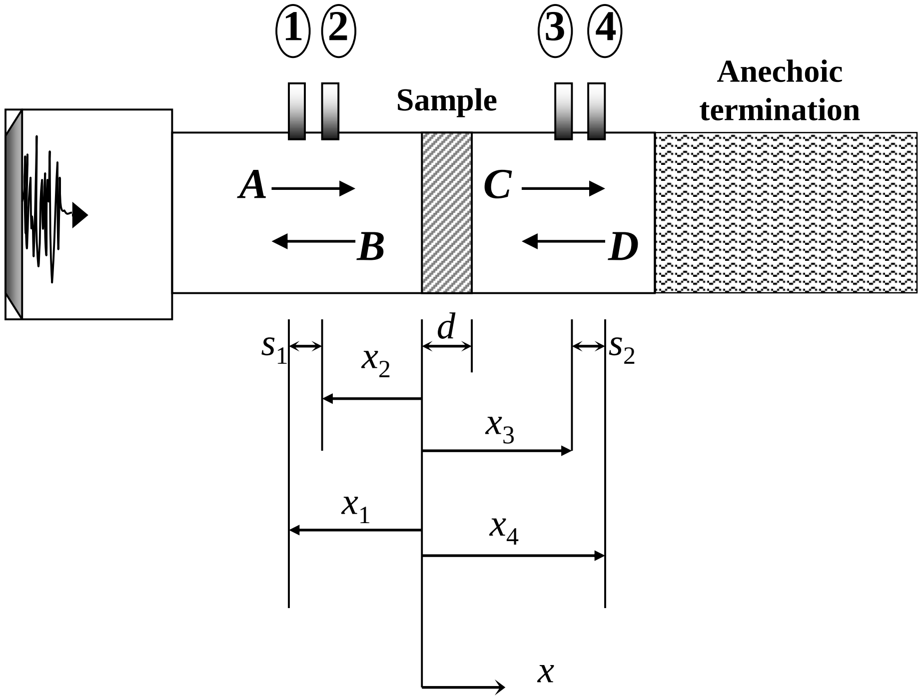


- Four microphone method based on modified E 1050 standing wave tube.

- Sound transmission through homogeneous porous materials is symmetric and reciprocal: number of measurements is reduced compared to two-load method.

- Knowledge of tube termination impedance is not required.

Experimental procedure



1. Measure complex pressures:

$P_1, P_2, P_3, P_4.$

2. Perform plane wave decomposition to obtain: *A, B, C, D*.

3. Use *A, B, C, D* to determine

$$\begin{bmatrix} P \\ V \end{bmatrix}_{x=0} \text{ and } \begin{bmatrix} P \\ V \end{bmatrix}_{x=d}.$$

4. Solve for transfer matrix elements and other desired quantities.

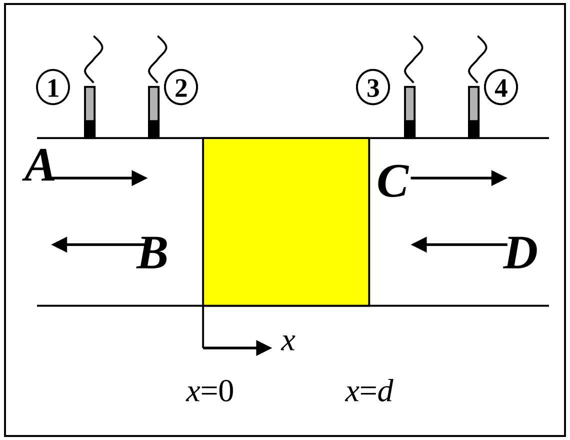
Sound field representation using wave decomposition

$$A = \frac{j(P_1 e^{jkx_2} - P_2 e^{jkx_1})}{2\sin k(x_1 - x_2)}$$

$$B = \frac{j(P_2 e^{-jkx_1} - P_1 e^{-jkx_2})}{2\sin k(x_1 - x_2)}$$

$$C = \frac{j(P_3 e^{jkx_4} - P_4 e^{jkx_3})}{2\sin k(x_3 - x_4)}$$

$$D = \frac{j(P_4 e^{-jkx_3} - P_3 e^{-jkx_4})}{2\sin k(x_3 - x_4)}$$



$$P\big|_{x=0} = A + B$$

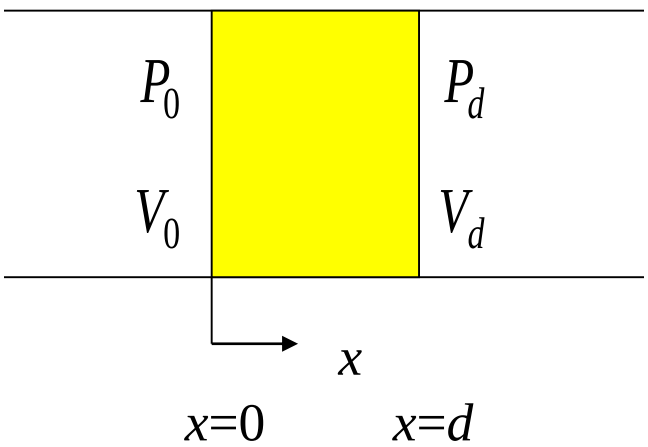
$$V\big|_{x=0} = \frac{A - B}{\rho_0 c}$$

$$P\big|_{x=d} = Ce^{-jkd} + De^{+jkd}$$

$$V\big|_{x=d} = \frac{Ce^{-jkd} - De^{+jkd}}{\rho_0 c}$$

Porous layer transfer matrix I

- **In plane wave case for limp or rigid materials**



$$\begin{bmatrix} P \\ V \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix}_{x=d}$$

subject to constraints:

$T_{11} = T_{22}$ **(symmetry)**

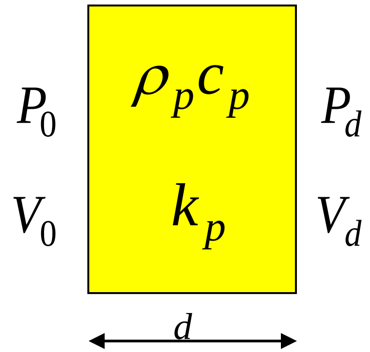
$T_{11}T_{22} - T_{12}T_{21} = 1$ **(reciprocity)**

∴ When P and V known on both sides sample, two equations in two unknowns and all transfer matrix elements can be determined.

Porous layer transfer matrix II

- **From porous material theory**

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos k_p d & j\rho_p c_p \sin k_p d \\ j \sin k_p d / \rho_p c_p & \cos k_p d \end{bmatrix}$$



$\therefore$ **when matrix elements known**

- **Complex wave number:**

$$k_p = \frac{1}{d} \cos^{-1} T_{11}$$

- **Complex characteristic impedance:**

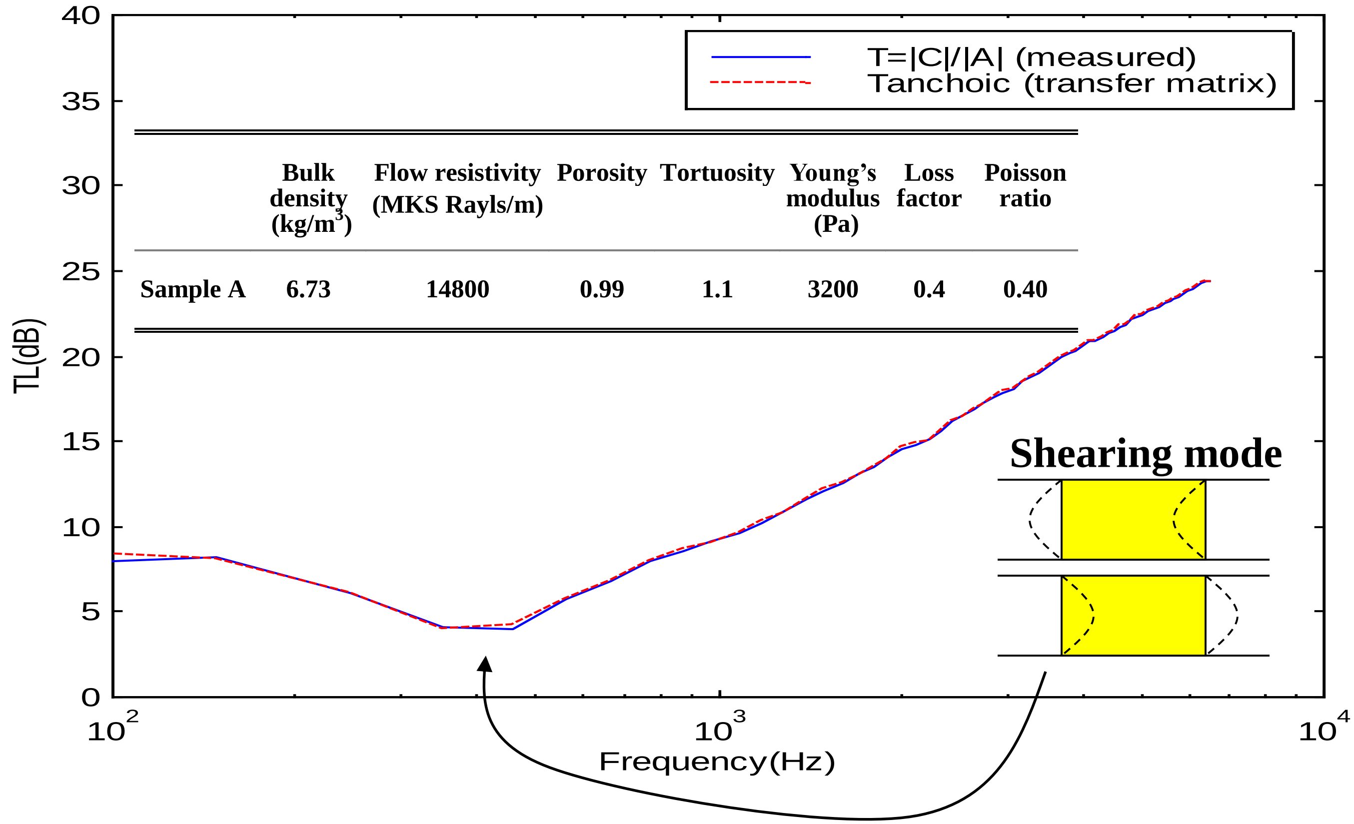
$$\rho_p c_p = \sqrt{\frac{T_{12}}{T_{21}}}$$

Porous material properties

- **Complex sound speed:** $$c_p = \frac{\omega}{k_p}$$
- **Complex density:** $$\rho_p = \frac{(\rho_p c_p)}{c_p}$$
- **Phase speed:** $$c_{ph} = \frac{\omega}{\beta_p} \quad (\beta_p = \mathrm{Re}\{k_p\})$$
- **Dissipation per wavelength:**

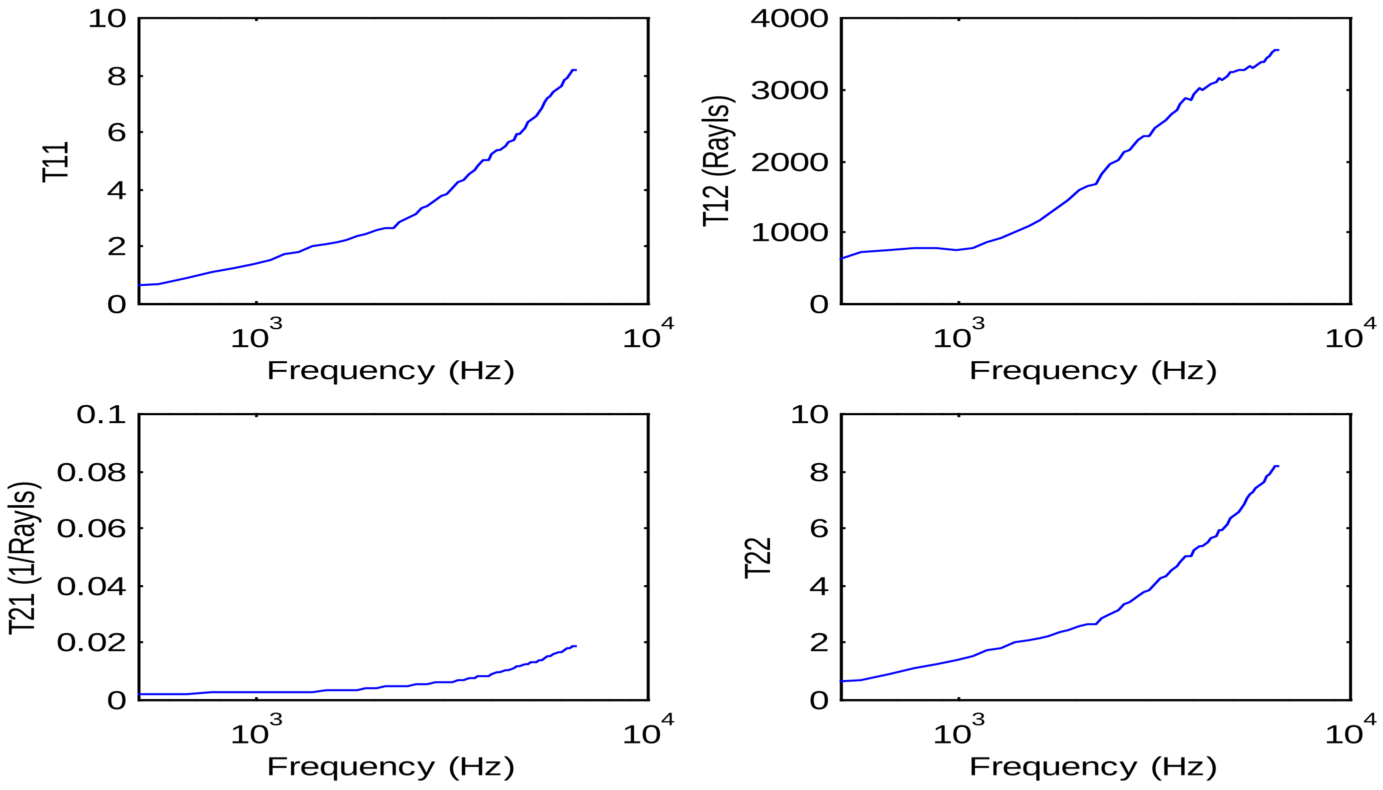
$$\alpha_p \lambda_p = 2\pi\alpha_p / \beta_p \quad (\alpha_p = \mathrm{Im}\{k_p\})$$

Transmission loss through acoustical materials

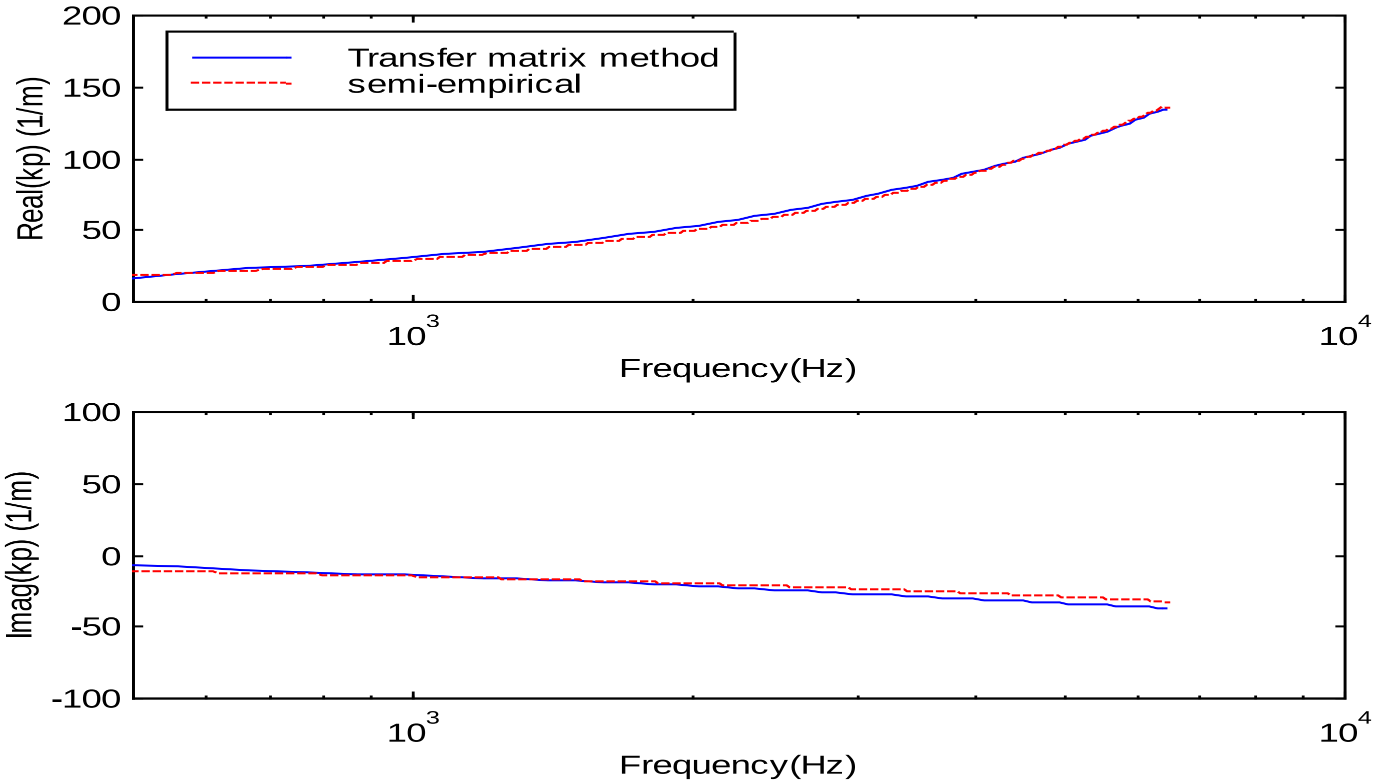


	Bulk density (kg/m^3)	Flow resistivity (MKS Rayls/m)	Porosity	Tortuosity	Young's modulus (Pa)	Loss factor	Poisson ratio
Sample A	6.73	14800	0.99	1.1	3200	0.4	0.40

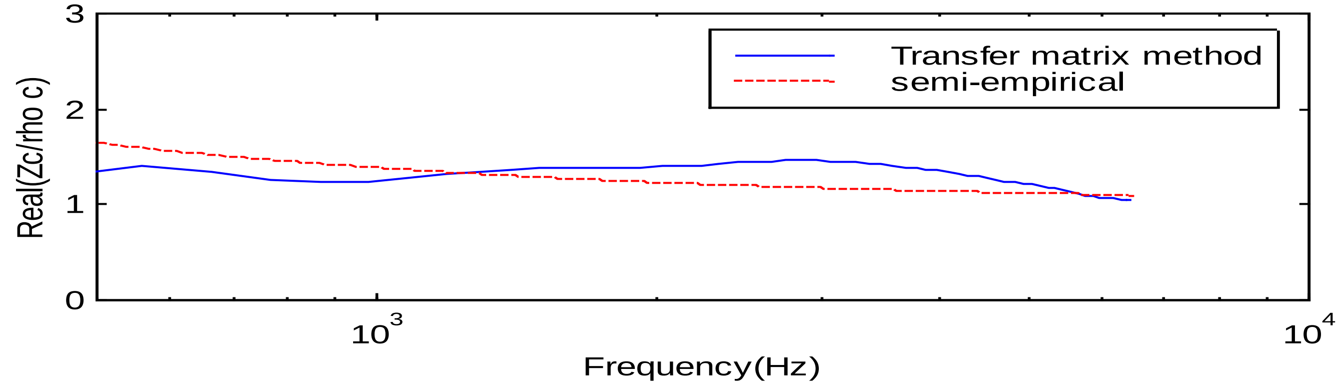
Magnitudes of transfer matrix elements

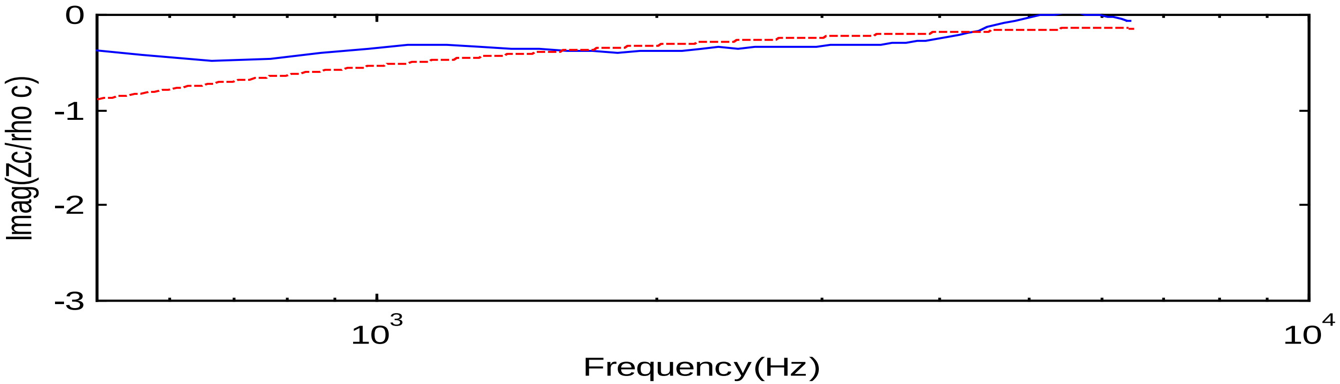


Propagation constant of lining material

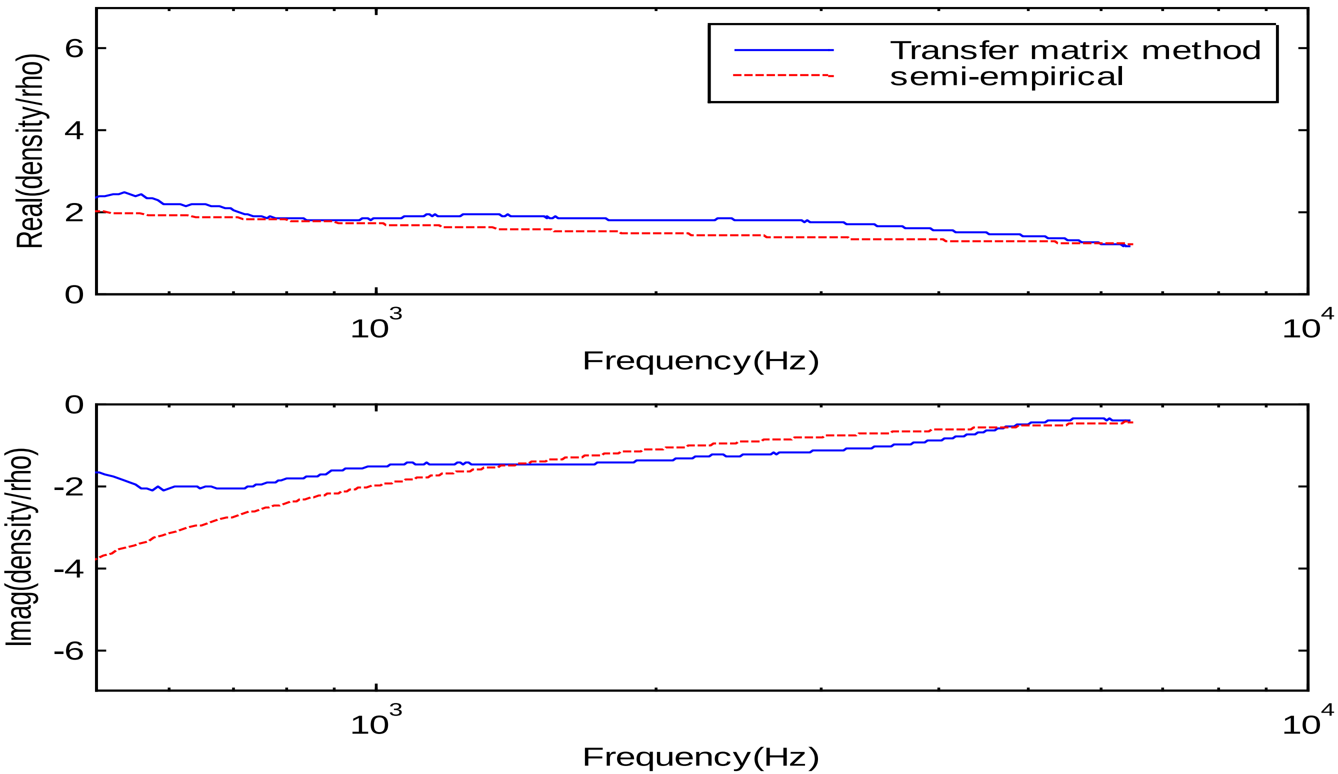


Normalized characteristic impedance of lining material

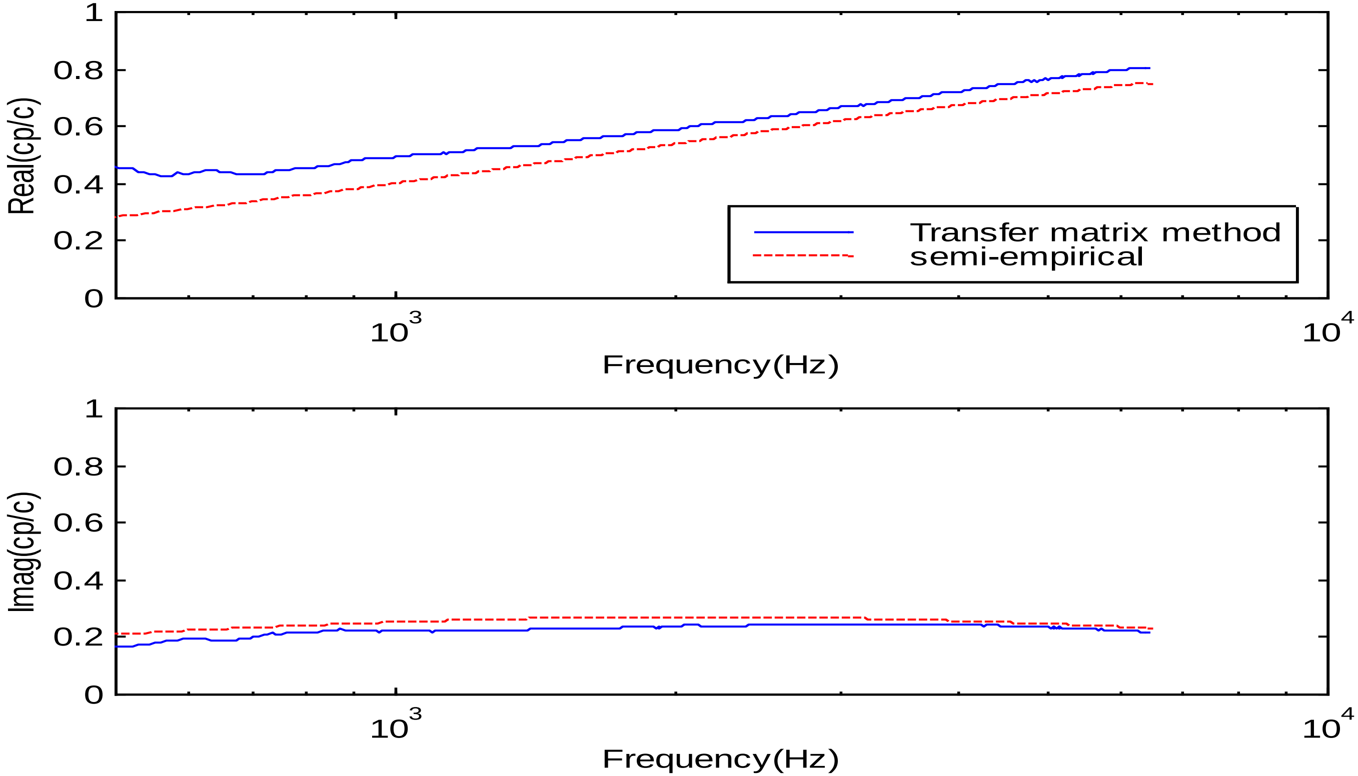




Normalized complex density in porous material



Normalized complex sound speed



Conclusions

- Quick and convenient method for determining the fundamental acoustical properties (i.e., the wave number and characteristic impedance) as well as acoustical performances (i.e., the absorption coefficients and transmission loss).
- Transfer matrix method improved by using symmetry conditions (reduces number of measurements required).
- By using this method, complex density and complex sound speed can be determined (required by FEM and BEM model).