1976

An Algorithmic Approach to the Detection and Prevention of Plagiarism

Karl J. Ottenstein

Report Number:
76-200

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Karl J. Ottenstein
Computer Sciences Department
Purdue University
West Lafayette, Indiana 47907

CSD-TR 200
August 1976

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The significant problem of detecting (nearly) identical student homework papers is non-trivial since a grader for a large class cannot remember all previously graded papers while examining the current one. This problem can be reduced by quantifying papers in such a way that equivalent ones are given equal values. Here we discuss one possible quantification which works well when applied to student computer programs.

The desired quantification is a function which maps the "homework space" into some value space. The ideal function, f, would impose a partitioning on the set of papers in the sense that if x and y are homework papers and the P_i are partitions of the homework space with $x \in P_i$ and $y \in P_j$ then i=j iff f(x)=f(y). If f(x)=f(y) and x and y are unique, then one of x and y is a plagiarized version of the other. In other words, when all partitions have but one element, no cheating has occured. This ideal function is unobtainable for several reasons: it is possible for identical work to be performed independently, the semantic equivalence of two items cannot always be shown deterministically, and there is a subjective area between plagiarism and paraphrasing.

Our task, then, is to find a good approximation to this function. The approximation should at least map all potentially equivalent homework papers into the same partition. It may not guarantee accuracy in that two papers being in the same partition will not imply that they are <u>necessarily</u> plagiarized. If P_1, P_2, ...P_n are the ideal partitions, our approximation should create Q_1, Q_2, ...Q_m where each Q_i is either some P_j or the union of several P_j's. That is, the partitions are merely cruder.

The constant functions satisfy our requirements for an approximation since only one partition will be created; but, they do not simplify our initial problem since all elements must be individually inspected for cheating. A function which maps a homework paper into the integer representing its length in characters will invariably create numerous partitions, but they will not be the desired Q_i: the replacement of one token by a synonym of a different length will place plagiarized assignments in separate partitions. A length function based on the number of tokens would eliminate this problem, but will still group together totally unrelated assignments simply because they have the same length. A function which takes into account some measure of the information content of a homework paper should give us more accurate partitions.

Any meaningful language can have its symbols classified into three sets:

- operators
- operands
- "syntactic sugar": symbols used only for readability

The information content of an element of a language, then, depends on the operators and operands, some function of which should lead to a good approximation to our ideal partitioning. This is simply a more formal description of the approach employed by [Bulut 1973].

In his study of student FORTRAN programs, Bulut counted the basic software science [Halstead 1972, 1977] parameters:

- η_1 - the number of unique operators
- η_2 - the number of unique operands
- N_1 - the total number of occurences of operators
- N_2 - the total number of occurences of operands

He noted that "the probability of using η_1 and η_2 symbols exactly N_1 and N_2 times in two different...[expressions] is very slim." Plagiarized copies were found by hand checking programs with identical η_1, η_2, N_1, and N_2 values. Bulut observed that, as with the length function above, the results of this method are not affected by changes to operand names since such changes will not modify η_2 or N_2.

A program to count these four parameters for FORTRAN modules was written [Ottenstein 1976] and used to confirm Bulut's work. Table 1 shows the partitioning imposed on 47 student programs from CS 210 at Purdue University by the "software science method". In the formalism developed here, we consider this method a mapping of programs into 4-tuples, $(\eta_1,\eta_2,N_1,N_2) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} \times \mathbf{N}$, where $\mathbf{N}$ denotes the set of natural numbers. Two partitions (A and B) have two programs in them; the rest have one. One program in partition B is a copy of the other, with slightly different comments and margining. The other pair is not as immediately detectable as being plagiarized because one author apparently changed all of the variable names and label numbers. Other programs with close correspondence of the parameters were compared, but without positive results. Thus it seems that a good partitioning was obtained. Copies of the programs in partition A are included in Appendix A with the parameter counts.

The size of a program (in tokens) is given as column N in Table 1. Since $N=N_1+N_2$, the partitions created by the length function mentioned above are supersets of those created by the software science method. Here, the length function creates 10 partitions of size greater than one, while the software science method seems to have given us the ideal partitioning. So at least in this case, the additional information provided by η_1 and η_2 is well worth the small effort required to obtain it.

Bulut called the chances of two student programs having equal 4-tuples "slim". We can get a more quantitative probability estimate by observing that η_1, η_2, N_1, and N_2 all appear to have somewhat normal distributions, in agreement with our intuition. (Appendix B gives the histograms for the four parameters.) In our particular sample, we have:

	mean	median	mode	s.d.	min	max
η_1	17.00	17	15	2.07	13	24
η_2	35.38	35	35	3.93	27	45
N_1	145.77	139	135	19.30	116	195
N_2	111.36	106	101	17.26	84	154

Assuming this normal distribution, there is clearly a greater likelihood of finding a pair of independently written programs with equal parameter values near the means as there is of finding such a pair with values on the tails. Thus, we can be more confident of a partition's accuracy as its individual parameter values approach the tails of their distribution curves.

Since the four parameters are not mutually independent, we use a multivariate normal density function, g, determined by the means vector, $m=(17.00,\ 35.38,\ 145.77,\ 111.36)$, and the covariance matrix $[C]$, $C_{ij}=E(X_i-m_i)(X_j-m_j)$ with $X=(\eta_1,\ \eta_2,\ N_1,\ N_2)$, to get a feel for the closeness of a 4-tuple to the means vector. The expression $g(X)/g(m)$ is 1 at $X=m$ and approaches 0 as we move away from the mean. Evaluated at the 4-tuple X for partitions A and B, this expression results in 0.45 and 0.018, respectively. This indicates that the programs in partition B are very probably plagiarized (the partition is accurate), while those in A are less probably so. Visual inspection of the programs is clearly warranted in any case, but one would be particularly suspicious of those in partition B. Since the accuracy of the partitions varies according to the location of the 4-tuples in the distribution space, it would seem advantageous to find a partitioning function whose range has a constant distribution. The existence of such a function is not known at present, although one would expect that if such a function were found, it would not be particularly accurate. In general, meaningful measurements of human behaviour produce uneven distributions.

Many alterations made by students to copied programs will be transparent to this method. Cosmetic transformations such as the reordering time independent statements, recommenting, reformatting of text, and renaming variables and labels will have no effect at all on η_1, η_2, N_1 or N_2. Most non-cosmetic alterations fall into one of six well-defined impurity classes[1], all of which are detectable by a slightly more sophisticated counter. Unfortunately, a student who cheated on only <u>part</u> of a program will not be detected.

Since the parameter counting routine was developed for other purposes, its $300 or so developement cost is not significant here: its running cost is about five cents (5¢) per 100 line student program on a CDC 6500. (This would be less were it not that the routine was written in ANSI-FORTRAN for portability and self-analysis.) Thus, this method of detecting plagiarism is both inexpensive and rapid. The preventive element mentioned in the title is simply the deterent created by making it difficult to cheat successfully.

It seems that this method can not only be applied to programs in other computer languages, but to <u>any</u> assignment which requires the submission of written material. Of course, programs are the only practical item for measurement since they are already in machine-readable form, but software science has been applied with some success to English [Kulm 1975, Halstead 1977] and one might hypothesize that similar results can be obtained there.

ACKNOWLEDGEMENTS

Special notes of thanks are due Dwight Andrews of Purdue University for collecting copies of his students' programs expressly for this study and to Professor Halstead, also at Purdue, for his encouragement and insights into software science.

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[1] The impurity classes are [Bulut 1974]:
(1) self-cancelling operations
(2) ambiguous usage of an operand
(3) synonymous usages of operands
(4) common subexpressions
(5) unnecessary replacements
(6) unfactored expressions

	Module	η_1	η_2	N_1	N_2	N
	35	16	34	116	84	200
	14	16	27	121	94	215
	8	15	29	125	93	218
	17	15	33	125	93	218
	1	19	33	131	93	224
	26	15	34	132	96	228
	22	19	32	135	93	228
	31	16	32	132	98	230
(A)	27	18	32	135	95	230
(A)	41	18	32	135	95	230
	45	16	35	133	99	232
	44	17	36	131	101	232
	47	20	33	134	98	232
	11	18	32	129	105	234
	2	17	37	136	101	237
	16	15	36	137	101	238
	39	15	35	138	101	239
	36	17	28	139	103	242
	32	18	36	139	103	242
	40	19	37	141	101	242
	20	17	35	139	105	244
	28	15	37	139	107	246
	5	16	35	138	108	246
	12	16	35	135	111	246
	21	16	36	135	111	246
	13	18	35	142	104	246
	24	15	36	135	113	248
	43	18	34	146	106	252
	6	15	33	143	111	254
	18	15	42	144	112	256
	9	16	35	139	117	256
	4	21	35	150	106	256
	38	16	29	141	117	258
	10	19	35	149	109	258
	33	16	40	143	121	264
	46	18	37	153	120	273
	3	18	41	163	121	284
	19	13	39	161	127	288
	30	20	40	162	126	288
	23	15	36	158	133	291
	29	19	31	180	120	300
	15	14	41	170	142	312
	34	17	39	179	142	321
	7	24	34	182	143	325
	42	15	40	195	147	342
(B)	25	19	45	193	154	347
(B)	37	19	45	193	154	347

Table 1:
47 student program parameter values as partitioned by the software science method (left) and the length function (right).

APPENDIX A

Source Listings of Programs in Partition B

```
      N=1
      IQ1=0
      IF1=0
      IP1=0
      IQ2=0
      IF2=0
      IP2=0
C     HAVE REFERENCED THE COUNTERS
      READ 111, KQTS, KFTS, KPTS
  111 FORMAT (I2, X, I2, X, I2)
      READ 120, MAX
  120 FORMAT (I2)
C     HAVE READ QUANTITIES ON HAND AND HEADER NUMBER
   10 READ 100, NUM, ICODE, IOQTS, IOFTS, IOPTS
  100 FORMAT (I1, X, I4, X, I2, X, I2, X, I2)
C     HAVE READ A DATA CARD.  THE THREE SUCCEEDING IF STATEMENTS
C     CHECK ORDER QUANTITIES AGAINST QUATITIES ON HAND...
C     'INSUFFICIENT QUANTITY' RECEIPT PRINTED IF APPLICABLE
      IF (IOQTS.LE.KQTS) GO TO 20
      PRINT 200, NUM, ICODE
      PRINT 205
      PRINT 300
      GO TO 44
   20 IF (IOFTS.LE.KFTS) GO TO 30
      PRINT 200, NUM, ICODE
      PRINT 205
      PRINT 300
      GO TO 44
   30 IF (IOPTS.LE.KPTS) GO TO 40
      PRINT 200, NUM, ICODE
      PRINT 205
      PRINT 300
      GO TO 44
C     IF ORDER CAN BE FILLED, COSTS ARE COMPUTED
C     AND A RECEIPT PRINTED
   40 KQTS=KQTS-IOQTS
      KFTS=KFTS-IOFTS
      KPTS=KPTS-IOPTS
      QCOST=6.05*FLOAT(IOQTS)
      FCOST=4.15*FLOAT(IOFTS)
      PCOST=2.25*FLOAT(IOPTS)
      TOT=QCOST+FCOST+PCOST
      IF(N.EQ.1) GO TO 66
      PRINT 200, NUM, ICODE
      GO TO 77
   66 PRINT 201, NUM, ICODE
   77 PRINT 210
      PRINT 220, IOQTS, QCOST
      PRINT 230, IOFTS, FCOST
      PRINT 240, IOPTS, PCOST
      PRINT 250, TOT
      PRINT 300
C     AFTER THE RECEIPT IS PRINTED, THE COSTS FOR EACH STORE ARE
C     UPDATED TO BE RECALLED AS A SUMMARY WHEN ALL CARDS ARE READ.
C     SUMMARY VARIABLES HAVE APPROPIATE SUFFIXES,
C     1 FOR STORE NUMBER 1 AND 2 FOR STORE NUMBER 2.
      IF(NUM.EQ.1) GO TO 33
      IQ2=IQ2+IOQTS
```

```
      IF2=IF2+IØFTS
      IP2=IP2+IØPTS
      QCØS2=6.05*FLØAT(IQ2)
      FCØS2=4.15*FLØAT(IF2)
      PCØS2=2.25*FLØAT(IP2)
      GTØT2=QCØS2+FCØS2+PCØS2
      GØ TØ 44
   33 IQ1=IQ1+IØQTS
      IF1=IF1+IØFTS
      IP1=IP1+IØPTS
      QCØS1=6.05*FLØAT(IQ1)
      FCØS1=4.15*FLØAT(IF1)
      PCØS1=2.25*FLØAT(IP1)
      GTØT1=QCØS1+FCØS1+PCØS1
   44 N=N+1
C     THE NEXT STEP CHECKS THE CARD CØUNT AGAINST THE HEADER
      IF(N.LE.MAX) GØ TØ 10
      PRINT 260
      PRINT 210
      PRINT 220, IQ1, QCØS1
      PRINT 230, IF1, FCØS1
      PRINT 240, IP1, PCØS1
      PRINT 270, GTØT1
      PRINT 300
      PRINT 280
      PRINT 210
      PRINT 220, IQ2, QCØS2
      PRINT 230, IF2, FCØS2
      PRINT 240, IP2, PCØS2
      PRINT 270, GTØT2
  200 FØRMAT ('0',15X,'STØRE ', I1, 3X,'ØRDER CØDE ', I4)
  201 FØRMAT ('1',15X,'STØRE ', I1, 3X,'ØRDER CØDE ', I4)
  205 FØRMAT ('0', '**** ØRDER NØT FILLED,'
     $   ' INSUFFICIENT STØCK ØN HAND ****')
  210 FØRMAT ('0',14X,'ITEM',9X,'PRICE',5X,'CØST')
  220 FØRMAT ('0',12X, I2,' QUART(S)     $6.05    $',F6.2)
  230 FØRMAT (' ',12X, I2,' FIFTH(S)     $4.15    $',F6.2)
  240 FØRMAT (' ',12X, I2,' PINT(S)      $2.25    $',F6.2)
  250 FØRMAT ('0',27X,'TØTAL  $',F7.2)
  260 FØRMAT ('1',17X,'STØRE 1    TØTAL BILL')
  270 FØRMAT ('0',21X,'GRAND TØTAL  $',F7.2)
  280 FØRMAT ('0',17X,'STØRE 2    TØTAL BILL')
  300 FØRMAT ( ' ', )
      STØP
      END
```

PURDUE SOFTWARE SCIENCE FORTRAN ANALYZER VERSION 1.0

STATISTICS FOR THIS MODULE:

OPERATOR	FREQUENCY
E.O.S.	40
() OR DO	9
IF	6
*	9
+	13
-	3
=	29
.LE.	4
.EQ.	2
GOTO 20	1
GOTO 44	4
GOTO 30	1
GOTO 40	1
GOTO 66	1
GOTO 77	1
GOTO 33	1
GOTO 10	1
FLOAT	9

ETA1= 18
N1= 135

OPERAND	FREQUENCY
TOT	1
N	5
KFTS	3
QCOST	2
QCOS1	2
QCOS2	2
PCOST	2
PCOS1	2
PCOS2	2
IOQTS	5
NUM	1
GTOT1	1
GTOT2	1
IOPTS	5
IOFTS	5
IP1	4
IP2	4
FCOST	2
FCOS1	2
FCOS2	2
IF1	4
IF2	4
KQTS	3
IQ1	4
IQ2	4
MAX	1
KPTS	3
1.00E+00	4
0	6
6.05E+00	3
4.15E+00	3
2.25E+00	3

ETA2= 32
N2= 95

```
C                                  PRØGRAM 2 CS 210
C
C        FØLLØWING CALCULATIØNS ARE FØR D.T.W.D. PERTAINING TØ WEEKLY SALES
         N=1
         IQTS=0
         IFIF=0
         IPTS=0
         IQT=0
         IFI=0
         IPT=0
         READ333,LQTS,LFTS,LPTS
     333 FØRMAT(I2,X,I2,X,I2)
         READ330,NMAX
     330 FØRMAT(I2)
      10 READ335,NUMST,IØRCØD,IQTB,IFIB,IPTB
     335 FØRMAT(I1,X,I4,X,I2,X,I2,X,I2)
C        THIS DETERMINES WHETHER ØR NØT THE ØRDER CAN BE FILLED.
         IF(IQTB.LE.LQTS)GØTØ11
C        IF THE ØRDER CANNØT BE FILLED, THIS INFØRMATIØN WILL BE PRINTED.
         PRINT100,NUMST,IØRCØD
         PRINT110
         GØTØ55
      11 IF(IFIB.LE.LFTS)GØTØ12
         PRINT100,NUMST,IØRCØD
         PRINT110
         GØTØ55
      12 IF(IPTB.LE.LPTS)GØTØ20
         PRINT100,NUMST,IØRCØD
         PRINT110
         GØTØ55
      20 LQTS=LQTS-IQTB
         LFTS=LFTS-IFIB
         LPTS=LPTS-IPTB
C        FØLLØWING DETERMINES ALL CØST INFØRMATIØN IF ØRDER CAN BE FILLED.
         QCØST=6.05*FLØAT(IQTB)
         FCØST=4.15*FLØAT(IFIB)
         PCØST=2.25*FLØAT(IPTB)
         TØT=QCØST+FCØST+PCØST
         IF(N.EQ.1)GØTØ77
         PRINT100,NUMST,IØRCØD
         GØTØ22
      77 PRINT101,NUMST,IØRCØD
C        THIS PRINTS ØUT STØRE ØRDERS.
      22 PRINT111
         PRINT112,IQTB,QCØST
         PRINT113,IFIB,FCØST
         PRINT114,IPTB,FCØST
         PRINT115,TØT
         PRINT119
         IF(NUMST.EQ.1)GØTØ25
         IQT=IQT+IQTB
         IFI=IFI+IFIB
         IPT=IPT+IPTB
         QCØS=6.05*FLØAT(IQT)
         FCØS=4.15*FLØAT(IFI)
         PCØS=2.25*FLØAT(IPT)
         GTØT=QCØS+FCØS+FCØS
         GØTØ55
```

```
   25 IQTS=IQTS+IQTB
      IFIF=IFIF+IFIB
      IPTS=IPTS+IPTB
      QCØSQ=6.05*FLØAT(IQTS)
      FCØSQ=4.15*FLØAT(IFIF)
      PCØSQ=2.25*FLØAT(IPTS)
      GTØTQ=QCØSQ+FCØSQ+PCØSQ
   55 N=N+1
      IF(N.LE.NMAX)GØTØ10
C     THIS PRINTS ØUT THE TØTAL BILL.
      PRINT116
      PRINT111
      PRINT112,IQTS,QCØSQ
      PRINT113,IFIF,FCØSQ
      PRINT114,IPTS,PCØSQ
      PRINT117,GTØTQ
      PRINT119
      PRINT118
      PRINT111
      PRINT112,IQT,QCØS
      PRINT113,IFI,FCØS
      PRINT114,IPT,PCØS
      PRINT117,GTØT
  100 FØRMAT('0',15X,'STØRE',X,I1,3X,'ØRDER CØDE',X,I4)
  101 FØRMAT('0',15X,'STØRE',X,I1,3X,'ØRDER CØDE',X,I4)
  110 FØRMAT('0','**** ØRDER NØT FILLED. '
     1   'INSUFFICIENT STØCK ØN HAND ****')
  111 FØRMAT('0',14X,'ITEM',9X,'PRICE',6X,'CØST')
  112 FØRMAT('0',12X,I2,X,'QUART(S)     $6.05     $',F6.2)
  113 FØRMAT(' ',12X,I2,X,'FIFTH(S)     $4.15     $',F6.2)
  114 FØRMAT(' ',12X,I2,X,'PINT(S)      $2.25     $',F6.2)
  115 FØRMAT('0',27X,'TØTAL   $',F7.2)
  116 FØRMAT('1',15X,'STØRE 1      TØTAL BILL')
  117 FØRMAT('0',21X,'GRAND TØTAL   $',F7.2)
  118 FØRMAT('0',15X,'STØRE 2    TØTAL BILL')
  119 FØRMAT(' ')
      STØP
      END
```

STATISTICS FOR THIS MODULE:

OPERATOR	FREQUENCY
E.O.S	40
() OR DO	9
IF	6
*	9
+	13
-	3
=	29
.LE.	4
.EQ.	2
GOTO 11	1
GOTO 55	4
GOTO 12	1
GOTO 20	1
GOTO 77	1
GOTO 22	1
GOTO 25	1
GOTO 10	1
FLOAT	9

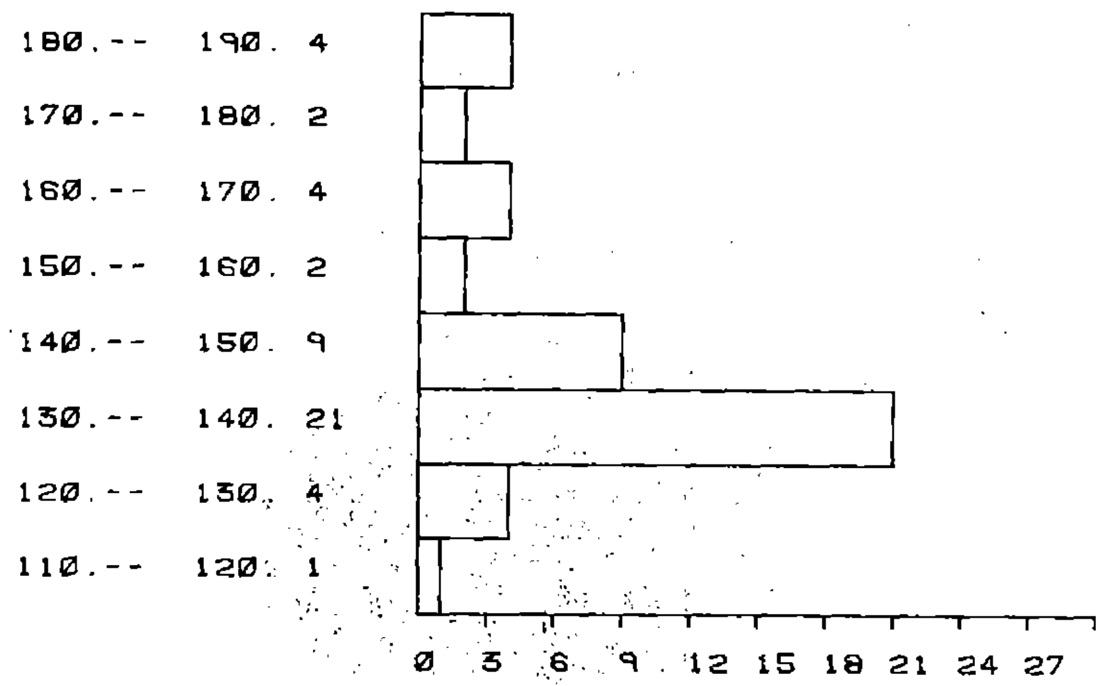
ETA1= 18
N1= 135

OPERAND	FREQUENCY
TOT	1
N	5
QCOSQ	2
QCOS1	2
PCOSQ	2
PCOS1	2
GTOT	1
NMAX	1
IOTE	5
GTOTO	1
FCOS	2
IOTS	4
IPTE	5
QCOS	2
IFIC	5
IPTS	4
IFIF	4
IPT	4
LQTS	3
FCOSQ	2
FCOST	2
NUMST	1
LPTS	3
PCOS	2
IFI	4
LFTS	3
IQT	4
1.00E+00	4
0	6
6.05E+00	3
4.15E+00	3
2.25E+00	3

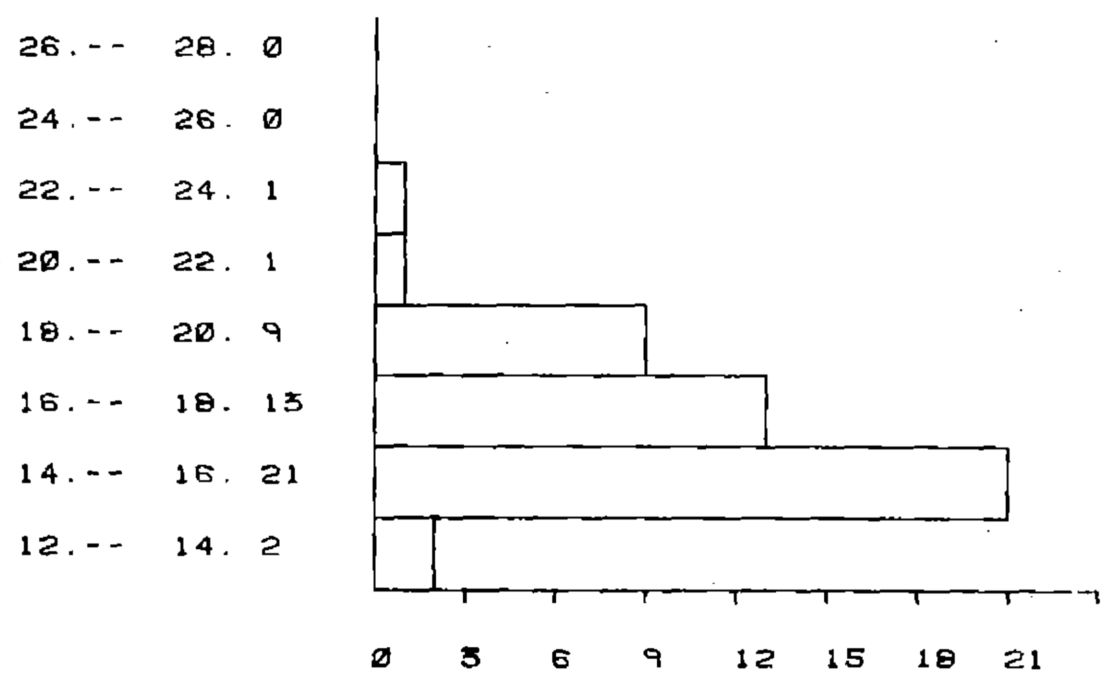
ETA2= 32
N2= 95

APPENDIX B

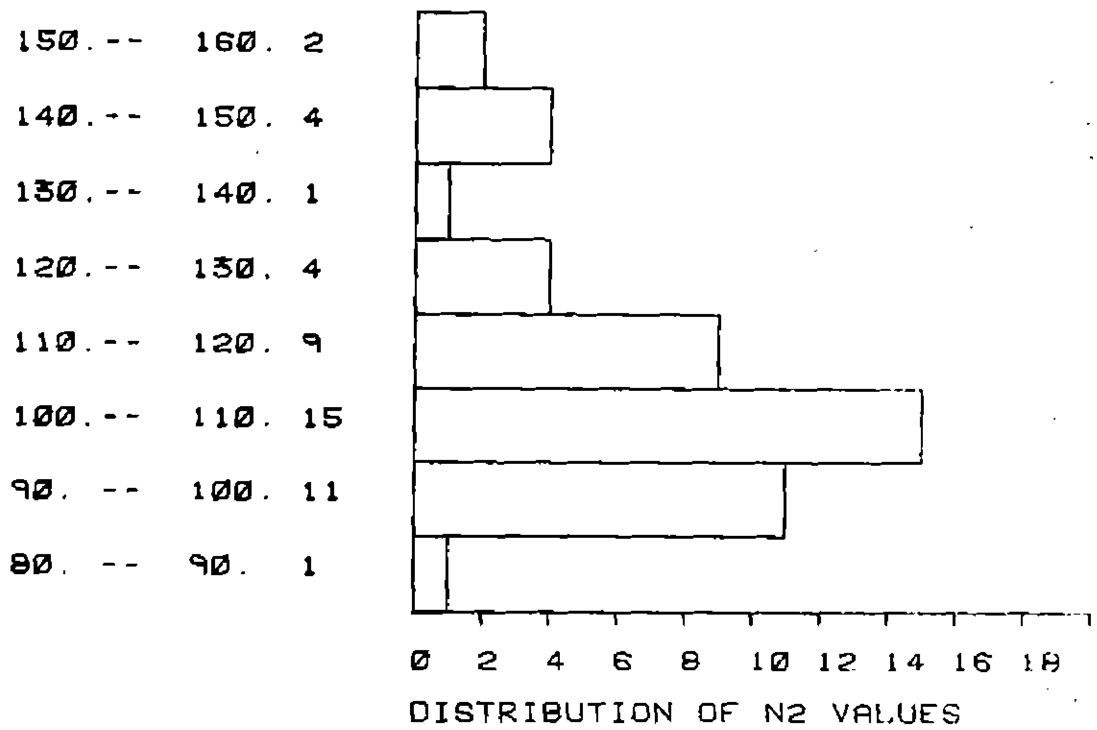
Histograms for η_1, η_2, N_1, and N_2
for the Observed Sample

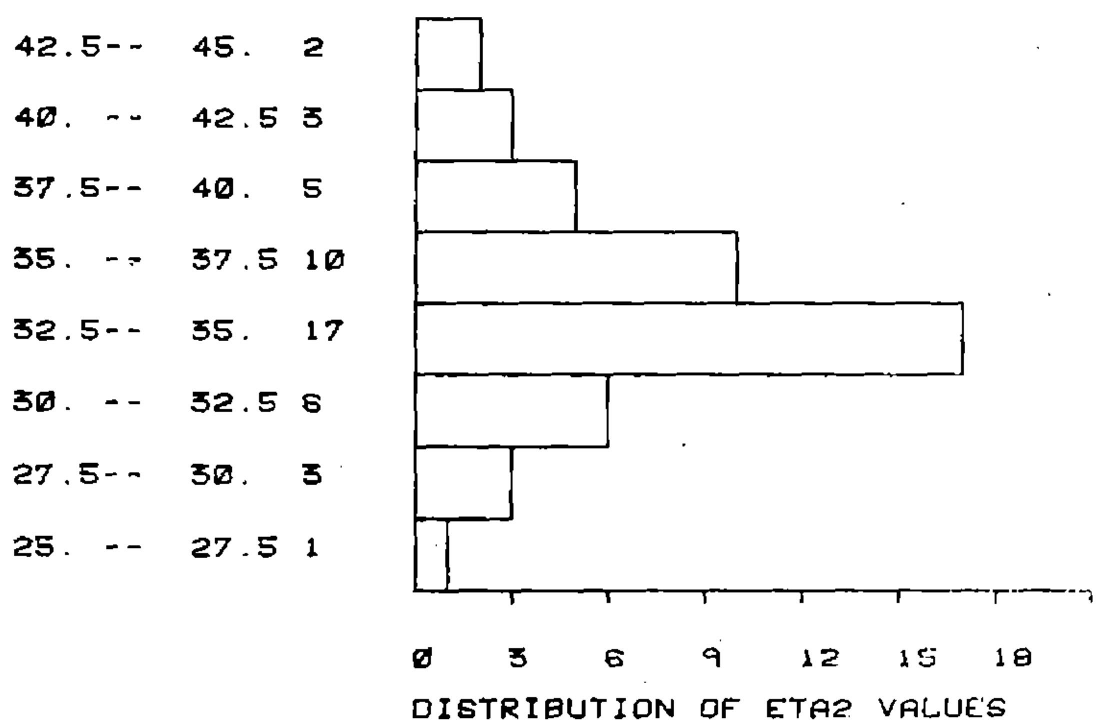


DISTRIBUTION OF N1 VALUES



DISTRIBUTION OF ETA1 VALUES


150. -- 160. 2
140. -- 150. 4
130. -- 140. 1
120. -- 130. 4
110. -- 120. 9
100. -- 110. 15
90. -- 100. 11
80. -- 90. 1
0 2 4 6 8 10 12 14 16 18
DISTRIBUTION OF N2 VALUES


42.5-- 45. 2
40. -- 42.5 3
37.5-- 40. 5
35. -- 37.5 10
32.5-- 35. 17
30. -- 32.5 6
27.5-- 30. 3
25. -- 27.5 1
0 3 6 9 12 15 18
DISTRIBUTION OF ETA2 VALUES

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