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INTRODUCTION

Stresses in compressor valves of both the flexible reed as well as the rigid spring loaded ring plate type have been investigated in the past [1]. Primarily, efforts were directed toward the measurement and prediction of dynamic bending stresses that occur in the reeds or plates flexing under operating loads. An important stress in this context was found to be the so-called overshoot stress which occurs when the reed strikes a stop in its opening travel (Fig. 1).

Stresses which are commonly neither measured nor predicted are impact stresses. These occur locally at the point where the reed tip of Fig. 2 contacts the stop and throughout the reed when the reed impacts the valve seat at closure. The reed tip impact stress during stop contact seem to be of lesser importance, mainly because of the relatively small stop impact velocities of a well designed valve. However, seat impact is important. This has long been recognized and an empirically established, so-called setting velocity has been given in literature [2] for impacting rigid ring valves. This setting velocity v_s , has been defined as

$$v_s = \omega A \tag{1}$$

and it has been recommended that it should not exceed 4 to 8 in/sec.

Design procedures for valves with the criterium of a limiting impact velocity have also been advanced [3].

Since for typical high speed refrigeration compressors, v_s = 15 to 24 in/sec, it is felt that a more detailed examination of allowable impact velocities is necessary.

In the present paper, an attempt is made to predict impact stresses directly and formulate absolute limiting velocities. The assumption is made that impact is colinear. By this is meant that there is no appreciable velocity gradient transverse to the impact direction and the valve hits the seat squarely, all points contacting at the same time. This is a good assumption in the case of rigid ring plate valves where impact velocities are more or less uniform, except possibly at edges of ports or when the plate tumbles (Fig. 3). In case of a reed, impact velocities are approximately equal to the product of the time derivation of the first mode participation factor and the first mode [4] and are therefore not uniform. However, as a first approximation, colinear impact can be assumed.

The impact process will be investigated considering the possibility of different materials for valve plate or reed and valve seat. It will be shown that impact stress amplitudes are independent of valve plate or reed thickness. An impact stress factor will be developed and discussed. A formula for limiting velocities under ideal conditions will be given which will be a function of the stress factor and the endurance limits of the valve plate and seat material.

While the treatment includes both the flexible reed and the spring loaded rigid valve plate, both impacting the seat, it is awkward to carry both terms throughout the paper. The terminology "valve plate" will therefore in the following also mean "valve reed", and "impact velocity" will mean in case of the reed "tip velocity" or "maximum velocity".

COLINEAR IMPACT OF PLATE ON SEAT

Let the thickness of the valve plate be h and the thickness of the seat be H . The plate impacts the seat with velocity $v_h(0)$. The seat is at rest with $v_H(0)=0$. The coordinate origin is the upper edge of the plate, away from the impact point (Fig. 4). The governing equation [5] for the plate is, after impact contact has been made,

$$\frac{\partial^2 u_h}{\partial t^2} = c_h^2 \frac{\partial^2 u_h}{\partial x^2} \tag{2}$$

The governing equation of the seat is, after impact contact has been made,

$$\frac{\partial^2 u_H}{\partial t^2} = c_H^2 \frac{\partial^2 u_H}{\partial x^2} \tag{3}$$

u_h and u_H are the elastic displacement of plate and seat during impact.

The solutions to these two wave equations are

$$u_h(x,t) = f_h(x - c_h t) + g_h(x + c_h t) \tag{4}$$

$$u_H(x,t) = f_H(x - c_H t) + g_H(x + c_H t) \tag{5}$$

Stresses are given by

$$\sigma_h(x,t) = E_h \frac{\partial u_h}{\partial x} = \rho_h c_h^2 [f_h'(x - c_h t) + g_h'(x + c_h t)] \tag{6}$$

$$\sigma_H(x,t) = E_H \frac{\partial u_H}{\partial x} = \rho_H c_H^2 [f_H'(x - c_H t) + g_H'(x + c_H t)] \tag{7}$$

Velocities are given by

$$v_h(x,t) = \frac{\partial u_h}{\partial t} = c_h [-f_h'(x - c_h t) + g_h'(x + c_h t)] \tag{8}$$

$$v_H(x,t) = \frac{\partial u_H}{\partial t} = c_H [-f_H'(x - c_H t) + g_H'(x + c_H t)] \tag{9}$$

where

$$c_h = \sqrt{\frac{E_h}{\rho_h}} \tag{10}$$

$$c_H = \sqrt{\frac{E_H}{\rho_H}} \tag{11}$$

The functions f_h or f_H represent waves traveling in the positive direction with velocity c_h or c_H, while the functions g_h or g_H represent waves traveling in the opposite directions. Primes indicate derivatives with respect to the arguments of these functions. For conveniences of writing, these arguments are dropped in the following.

Initial velocity conditions are

$$c_h(-f_h' + g_h') = v_h(0) \tag{12}$$

for $t=0$ and $0 \le x \le h$,

$$-f_H' + g_H' = 0 \tag{13}$$

for $t=0$ and $h \le x \le H$, and, reflecting the fact of a zero initial stress state before impact,

$$\begin{aligned} f_H' + g_H' &= 0 \\ f_h' + g_h' &= 0 \end{aligned} \tag{14}$$

From Eqs. 12 and 13 we obtain the initial magnitudes of the traveling waves:

$$f_h' = -\frac{v_h(0)}{2c_h} \tag{15}$$

$$g_h' = \frac{v_h(0)}{2c_h} \tag{16}$$

$$f_H' = 0 \tag{17}$$

$$g_H' = 0 \tag{18}$$

Since a free boundary cannot support a normal stress,

$$f_h' + g_h' = 0 \tag{19}$$

at $x=0$, but at any time t. This gives

$$f_h' = -g_h' \tag{20}$$

and establishes the fact that at the free end wave components reflect with equal amplitude, but opposite in phase.

At the impact location during impact contact, stresses on the valve plate boundary have to equal stresses at the valve seat boundary. Also, particle velocities have to be identical. Thus, at $x=h$,

$$\rho_h c_h^2 (f_h' + g_h') = \rho_H c_H^2 (f_H' + g_H') \tag{21}$$

$$c_h(-f_h' + g_h') = c_H(-f_H' + g_H') \tag{22}$$

These equations give

$$f_H' = \frac{f_h'}{2}\left[\frac{\rho_h}{\rho_H}\left(\frac{c_h}{c_H}\right)^2 + \frac{c_h}{c_H}\right] + \frac{g_h'}{2}\left[\frac{\rho_h}{\rho_H}\left(\frac{c_h}{c_H}\right)^2 - \frac{c_h}{c_H}\right] \tag{23}$$

$$g_H' = \frac{f_h'}{2}\left[\frac{\rho_h}{\rho_H}\left(\frac{c_h}{c_H}\right)^2 - \frac{c_h}{c_H}\right] + \frac{g_h'}{2}\left[\frac{\rho_h}{\rho_H}\left(\frac{c_h}{c_H}\right)^2 + \frac{c_h}{c_H}\right] \tag{24}$$

For identical materials ($\rho_h = \rho_H$; $c_h = c_H$)

$$f_H' = f_h' \tag{25}$$

$$g_H' = g_h' \tag{26}$$

The physical interpretation of the general case is that for instance a component f_h' approaching the impact boundary will split into a transmitted wave component f_H' and a reflected component g_h' due to the change in material properties. For the case of identical materials, no reflection takes place.

The boundary condition at the other side of the valve seat cannot be clearly defined since the valve seat is part of the total cylinder head. It is however safe to assume that

$$H \gg h \tag{27}$$

and that therefore all waves traveling in positive x direction are dispersed and do not return.

Let us investigate the case when f_h' approaches the impact boundary. It will split into a reflected component g_h' and a transmitted component f_H'. The value of g_H' is zero. Thus, the reflection will be

$$g_h' = f_h' \frac{\rho_H c_H - \rho_h c_h}{\rho_H c_H + \rho_h c_h} \tag{28}$$

and the transmitted component will be

$$f_H' = f_h' \frac{2\rho_h c_h}{\rho_H c_H + \rho_h c_h} \left(\frac{c_h}{c_H}\right) \tag{29}$$

The peak impact stress amplitudes in the valve reed and the valve seat are therefore, by properly adding forward moving and reflecting wave components,

$$\sigma_{h\,max} = \sigma_{H\,max} = -\frac{\rho_h c_h}{1 + \frac{\rho_h c_h}{\rho_H c_H}} \; v(o) \tag{30}$$

The developing impact stress wave is shown in Fig. 5 for the cases of identical materials and nonidentical materials with soft seat.

Relationship (30) is plotted in Fig. 6. We recognize, taking the case of equal material for valve plate and seat as the point of departure, that in order to reduce impact stresses in a given valve plate we should make the seat more elastic and reduce the mass density of the seat material such that

$$\rho_H c_H < \rho_h c_h \tag{31}$$

and of course make $v(o)$ as small as possible. Typical values of ρc are given in Table 1. In order to explore the influence of different materials further, let us formulate an impact stress factor

$$S_{hH} = \frac{\rho_h c_h}{1 + \frac{\rho_h c_h}{\rho_H c_H}} \tag{32}$$

Such that

$$\sigma_{h\,max} = \sigma_{H\,max} = -S_{hH} \, v(o) \tag{33}$$

The valve plate is designated by h and the valve seat by H. Typical values of the impact stress factor are given in Table 2. Note that we have reciprocity

$$S_{hH} = S_{Hh} \tag{34}$$

It does therefore not matter, as far as impact stress amplitudes are concerned, if the valve plate is of steel impacting a brass seat or a brass valve plate is impacting a steel seat.

The physical interpretation of S_{hH} is that it is the amount of stress created per unit impact velocity.

Note further that the thickness of the valve plate does not enter into the stress relationship. Merely reducing the thickness of the valve plate will therefore not change the impact stresses.

PERMISSIBLE IMPACT VELOCITIES

The permissible impact velocity can be found by assuming that Sine's equation [6] describes adequately permissible endurance stresses. For colinear impact, the stress state is uniaxial and Sine's equation becomes

$$\sigma_{a/} \leq S_e - \frac{S_e}{S_{Y.S.}} \sigma_{m/} \tag{35}$$

Strictly speaking, this equation only applies to a sinusoidal change of stress in time. Our loading condition is of the nature of individual pulses. A stress indicator located at the impact location will register a stress time plot as shown in Fig. 17a if one impact occurs per cycle or as shown in Fig. 17b if more than one impact occurs per cycle. The number of impacts per cycle does not play a role in our considerations since we consider infinite fatigue life only.

The largest pulse magnitude is from here on referred to as σ_{max}:

$$\sigma_{max} = -S_{hH} \, v(o)_{max} \tag{36}$$

For stress pulses, it is current practice [7] to set

$$\sigma_{a/} = \sigma_{m/} = \left|\frac{\sigma_{max}}{2}\right| \tag{37}$$

Furthermore, as an estimate, we set

$$\frac{S_e}{S_{Y.S.}} \cong \frac{1}{2} \tag{38}$$

Thus, we get

$$|\sigma_{max}| \leq \frac{4}{3} S_e \tag{39}$$

The permissible impact velocity is therefore

$$v_{max}(o) \leq \frac{4}{3} \frac{S_e}{S_{4H}} \tag{40}$$

Typical endurance limits are listed in Table 3. These values represent severe approximations since their magnitude is very sensitive to surface finish and heat treatment. It may be more advisable to utilize direct impact endurance testing [8].

Let us consider a few examples. If a steel valve plate strikes a steel seat, with like endurance limits, let us say $S_e = 50{,}000$ psi, then both valve plate and valve seat are likely to fail when

$$v_{max}(o) \geq 930 \text{ in/sec} \tag{41}$$

Typically, impact velocities measured in well designed small high speed refrigeration compressors are $100-200$ in/sec. One has to remember, however, that the theoretical value of maximum velocity represents the ideal case of colinear impact. The allowable velocity will be lower if we deviate from the ideal situation.

If a steel valve plate of endurance limit $S_e = 50{,}000$ psi strikes an aluminum valve seat of $S_e = 20{,}000$ psi, the steel valve plate will likely fail around an impact velocity of

$$v_{max}(o) \geq 1810 \text{ in/sec} \tag{42}$$

which, at first glance, seems an improvement, however, the aluminum seat will probably fail already at velocities of

$$v_{max}(o) \geq 720 \text{ in/sec} \tag{43}$$

Taking steel on steel as our reference point, we like therefore to select a combination steel and material Y where the endurance limit of material Y is equivalent to the endurance limit of steel and the S_{4H} factor for both is less than 71.9. For instance, a titanium valve plate impacting a steel seat seems to be a promising combination. For optimum endurance limits of $S_e = 100{,}000$ psi each, we do not get likely failure for both valve plate and seat until

$$v_{max}(o) \geq 2550 \text{ in/sec} \tag{44}$$

MEASUREMENTS OF IMPACT STRESSES IN OPERATING COMPRESSOR VALVES

The duration of impact stress at the impact face of the valve plate is

$$\tau = \frac{2h}{c_h} \tag{45}$$

and is not a function of impact velocity $v(o)$. For example, for a steel valve plate of thickness $h = 0.05$ in, the impact duration is of the order of $5(10^{-7})$ sec. We are talking therefore about rather narrow stress pulses which require a high frequency response characteristic of the measuring device. Furthermore, on the side of the valve plate that is not in contact during impact, stresses are zero (a free boundary cannot support a stress normal to it). Thus, the possibility of applying a conventional straingage on this surface and using the Poisson effect to gain an indication of impact stress does not exist. It is therefore not too surprising that impact stresses of the type discussed in this paper have not yet been measured in compressor valves. Rather, impact velocity is used as a failure indicator since it can be determined by displacement transducers or simulated on the computer.

The only feasible way to measure impact stresses directly seems to be at present to imbed piezo-electric crystals in a specially prepared plastic valve seat and to correct measurements theoretically to account for the effect of a different valve seat material.

SUMMARY AND DISCUSSION

It was established that stress amplitudes in both valve plate and seat during impact are given by

$$\sigma_{h\,max} = \sigma_{H\,max} = -S_{4H}\, v(o) \tag{46}$$

Stress at the impact interface is a narrow rectangular pulse in time of duration

$$\tau = \frac{2h}{c_h} \tag{47}$$

The permissible impact velocity is given by

$$v_{max}(o) \leq \frac{4}{3} \frac{S_e}{S_{4H}} \tag{48}$$

The problem of making meaningful impact stress measurements was discussed.

In general, it has to be remembered that any condition that makes the system deviate from colinear impact will have the effect of stress concentration. Any foreign particle that lodges between plate and seat will be a stress raiser. Any edge impact will serve as stress raiser.

Any impairment in surface smoothness will lower the endurance limit. Rust, as it occurs in air compressors, will lower the endurance limit. Various surface treatments have effects that have to be investigated from case to case.

Thus, all numerical values reported in this paper are only intended as a guideline for the compressor valve designer, have to be re-examined carefully for each special situation and have to be adjusted downward for every stress raising effect.

It is suggested that a factor η be introduced that is a function of the degree of noncolinear impact and other stress raising effects such that

$$v_{max}(0) \le \eta \frac{S_e}{S_{hH}} \tag{49}$$

A design that should be examined in the future both theoretically, following the outlined approach, and experimentally (endurance limit), is the plated valve seat.

Also, more work in general should be done in the area of impact endurance testing.

NOMENCLATURE

v_s = setting velocity [in/sec]

ω = rotational speed of compressor [rad/sec]

A = amplitude of valve travel [in]

h = thickness of valve plate or reed [in], subscript

H = thickness of valve seat [in], subscript

$v(0)$ = velocity at start of impact [in/sec]

u = elastic displacement [in]

t = time [sec]

x = coordinate [in]

c = wave speed [in/sec]

f, f' = wave traveling in positive direction

g, g' = wave traveling in negative direction

E = Young's modulus [lb/in^2]

σ = stress [lb/in^2]

ρ = mass density [lb sec^2/in^4]

S_{hH} = impact stress factor [lb sec/in^3]

S_e = endurance limit [lb/in^2]

$S_{Y.S.}$ = yield strength [lb/in^2]

σ_{a} = reversing stress amplitude [lb/in^2]

σ_{m} = mean stress [lb/in^2]

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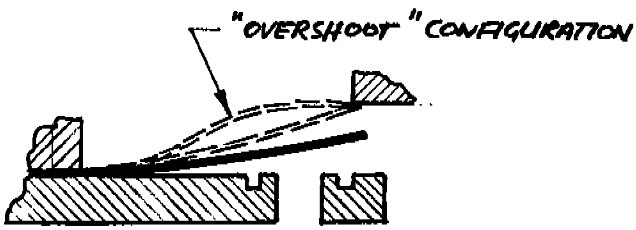

Fig. 1 Overshoot Deflection

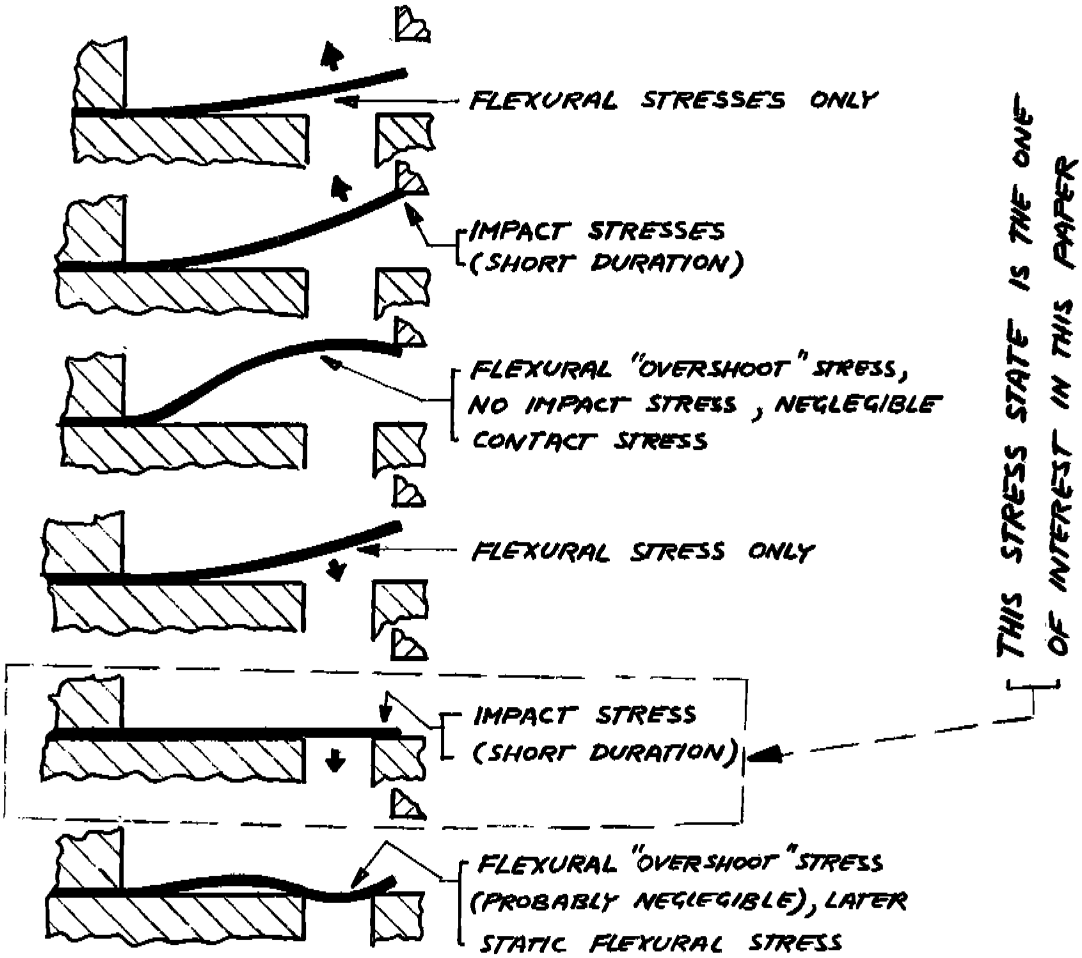

Fig. 2 Stress States

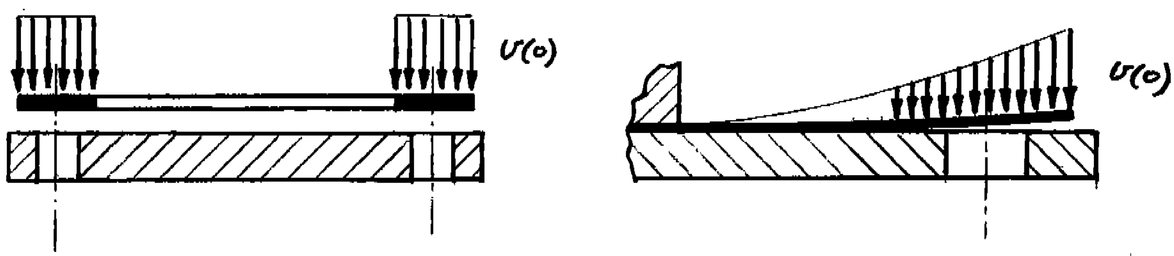

Fig. 3 Impact Velocities

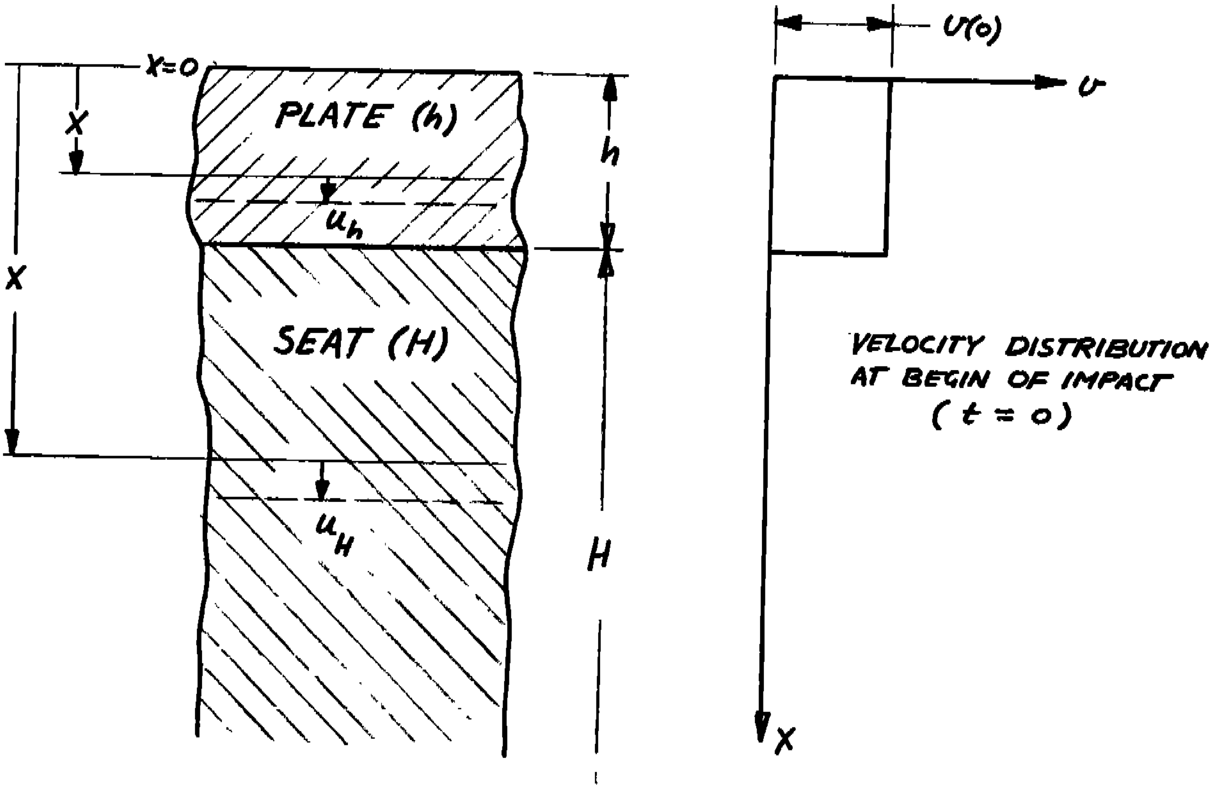

Fig. 4 Definition of Coordinates and Initial Velocity Distribution

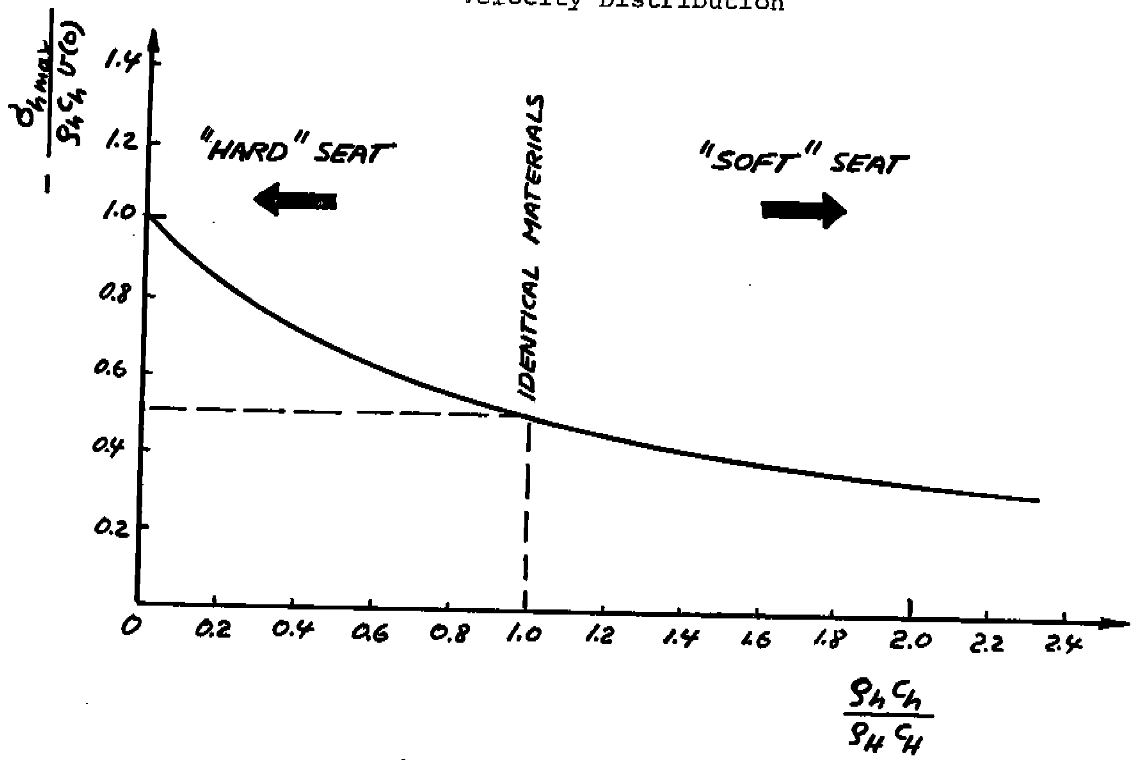

Fig. 6 Maximum Stresses

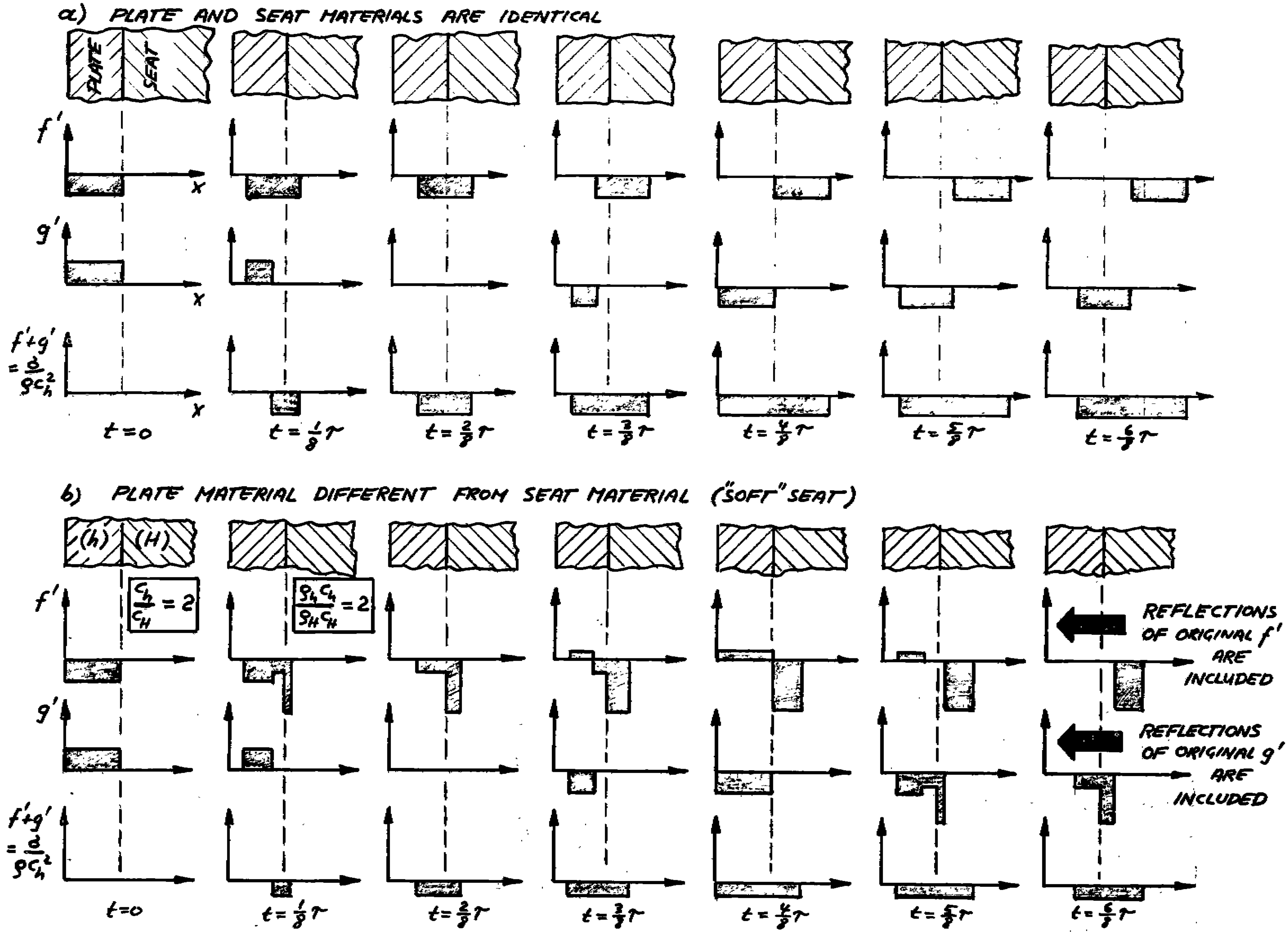

Fig. 5 Stress Wave Propagation

Material	ρ	c	ρc
Aluminum	$247(10^{-6})$	$201(10^{3})$	49.65
Brass	800	138	110.40
Bronze	763	135	103.01
Copper	833	140	116.62
Gold	1806	69	124.61
Grey Cast Iron	663	175-218	116.03 -144.53
Lead	1064	48	51.07
Magnesium	163	181	29.50
Nickel	805	196	157.78
Silver	983	103	101.25
Steel	730	197	143.81
Titanium	421	239	100.62

Table 1: Speeds of Sound and Material Densities

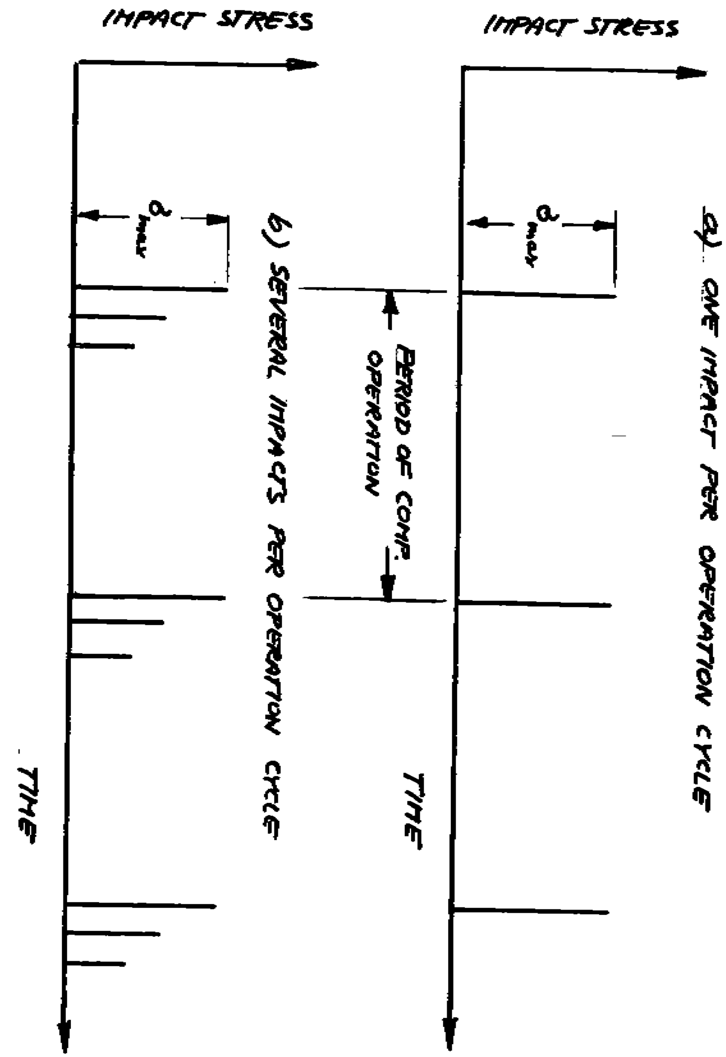

Fig. 7 Impacts Per Operation Cycle

Valve Plate (h) \ Valve Seat (H)	Aluminum	Brass	Bronze	Copper	Gold	Grey Cast Iron	Lead	Magnesium	Nickel	Silver	Steel	Titanium
Aluminum	24.8	34.3	33.6	34.9	35.5	34.7-37.0	25.2	18.6	37.7	33.3	36.9	33.2
Brass	34.3	55.2	53.1	56.6	58.5	56.6-62.6	35.0	23.2	64.9	52.7	62.5	52.7
Bronze	33.6	53.1	51.5	55.2	56.4	54.5-60.2	34.0	22.9	62.4	51.1	59.9	50.8
Copper	34.9	56.6	55.2	58.3	60.2	58.3-64.5	35.5	23.6	67.1	54.1	64.5	54.1
Gold	35.5	58.5	56.4	60.2	62.0	60.1-66.9	36.2	23.8	69.5	55.9	66.9	55.6
Grey Cast Iron	34.7-37.0	56.6-62.6	54.5-60.2	58.3-64.5	60.1-66.9	58.0-72.3	35.5-37.8	23.6-24.6	67.1-75.5	54.1-59.6	64.2-72.3	53.8-59.2
Lead	25.2	35.0	34.0	35.5	36.2	35.5-37.8	25.5	18.7	38.6	34.0	37.6	33.9
Magnesium	18.6	23.2	22.9	23.6	23.8	23.6-24.6	18.7	14.8	24.8	22.9	24.5	22.8
Nickel	37.7	64.9	62.4	67.1	69.5	67.1-75.5	38.6	24.8	78.9	61.7	75.3	61.4
Silver	33.3	52.7	51.1	54.1	55.9	54.1-59.6	34.0	22.9	61.7	50.6	59.4	50.6
Steel	36.9	62.5	59.9	64.5	66.9	64.2-72.3	37.6	24.5	75.3	59.4	71.9	59.2
Titanium	33.2	52.7	50.8	54.1	55.6	53.8-59.2	33.9	22.8	61.4	50.6	59.2	50.3

Table 2: Values of Impact Stress Factor S_{hH}

Material	$\sim S_e$
Aluminum	10,000 - 30,000
Brass	15,000
Bronze	15,000
Copper	5,000 (10 million cycles)
Gold	5,000 (10 million cycles)
Grey Cast Iron	10,000 - 20,000
Lead	1,000
Magnesium	8,000
Nickel	30,000 - 40,000
Silver	5,000
Steel	30,000 - 100,000
Titanium	30,000 - 100,000

Table 3: Approximate Endurance Strength