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THE PREDICTION OF DYNAMIC STRAIN IN RING TYPE COMPRESSOR VALVES USING EXPERIMENTALLY DETERMINED STRAIN MODES

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INTRODUCTION

The design of a reciprocating compressor for use in a refrigeration or air conditioning application is based on two principal factors: (1) the performance of the machine in terms of work output per energy input, or efficiency, and (2) the performance in terms of reliability of the individual machine components. Previous researchers (1,2,3)* have extended the state of the art of compressor modeling to the point where it is possible to adapt a compressor simulation to various types of compressors and predict the operating performance in terms of efficiency quite successfully. A logical extension of the compressor simulation is its use in the second type of reliability, ie, predicting valve stress levels which can be related to valve reliability or fatigue life. However, in order to predict the valve dynamics, the natural frequencies and modes must be known. These may be determined analytically or experimentally depending on the type of valve.

Common compressor valves may generally be divided into two groups: reed valves and ring valves.

1) Reed type valves are most commonly found in fractional horsepower compressors. Since many reed valves are, in effect, cantilever beams, the most accurate method of modeling a reed valve is by using continuous system vibration theory which employs a superposition of the modes, or displacement distributions, of the valve. The predicting of strain in a reed valve is more difficult than predicting the valve displacement because the strain is directly in proportion to the second derivative of the displacement distribution

$$\epsilon(x,t) \approx \frac{d^2 y(x,t)}{d x^2}$$

The evaluation of this second derivative presents no problem if the valve is of uniform width and the theoretical modes are applied. However, when attempting to extend these modes to valves with more complex geometries, the second derivative often proves unreliable, particularly in its ability to predict high frequency strain components which are atteibuted to higher modes of vibration. This is due to the fact that the higher modes are characterized by rapid changes in curvature. Often a very small excitation of a higher mode can result in a very high strain level. Thus, it may be necessary to include several of the higher modes to predict the strain, whereas the displacement may be adequately predicted with only one mode.

2) Ring valves are usually employed in large compressors which have high flow rates. These valves are of two types, (1) those which open by flexing, and (2) those which are spring loaded and actually move as a rigid body. Simulation of ring valves which operate in bending presents a problem because the free vibration modes, which are essential to the modeling, are not easily obtainable from theory. Furthermore, the nature of the valve support, or boundary condition, is usually such that evan approximate solutions for the modes are difficult to obtain. An exact description of a ring valve displacement distribution requires two spacial coordinates, such as polar coordinates r and θ, as shown in Figure 1.

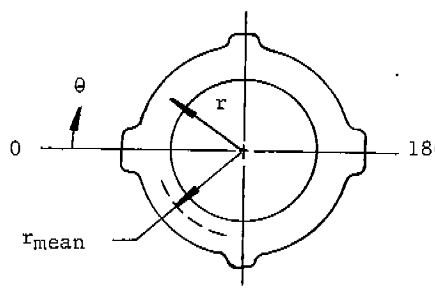

Figure 1 Ring Valve in Polar Coordinates

The calculation of the strain at any point requires a knowledge of the strain components in both the r and θ directions. The strain in the θ direction is proportional to derivatives in both the r and θ

*Numbers in brackets refer to corresponding numbers in References.

directions

$$\epsilon_\theta(r,\theta,t) \approx \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{1}{r}\frac{\partial w}{\partial r}$$

while the strain in the radial direction is

$$\epsilon_r(r,\theta,t) \approx \frac{\partial^2 w}{\partial r^2}$$

Thus it becomes apparent that a very accurate knowledge of the displacement distribution is required.

There are essentially three methods for obtaining the natural frequencies and modes when the boundary conditions do not allow an exact solution.

1) experimental measurement of the vibrating system
2) computer solution using the finite difference or finite elment technique
3) an approximate method, such as the Rayleigh - Ritz or energy method, in which a function is assumed which satisfies the geometric boundary conditions

Of the three, the last method is probably the easiest since it requires neither experimental equipment nor a computer. However, the success of such an approximate method depends largely on how well the assumed function describes the actual system. The method becomes difficult for the higher modes because many practical systems have geometries for which a functional representation of the higher displacement modes is impossible because of abrupt changes in curvature.

Although the finite element technique has been used to calculate displacement modes and natural frequencies of complex systems, an inherent problem is the number of elements which must be used so that the "grid" is fine enough to allow a reliable numerical differentiation. This introduces the problem of computer size and execution time. However, recent advances by Friley and Hamilton (7) in this area suggest that this may become the method to be used in the future.

The experimental approach of resonant testing is a proven method for obtaining the displacement modes and natural frequencies of systems such as beam or plates (4). Its usefulness is limited only by the ability of the exciting force to drive the system with amplitudes sufficient for reliable measurements. In this paper it will be shown that resonant testing can be utilized to obtain the strain information classically obtained by differentiating the displacement mode. This information will be referred to as the "strain-mode".

The specific objectives of this paper are thus:

1) To extend the present mathematical modeling techniques to include a reciprocating compressor having ring valves which open by flexing, so that the valve dynamics can be predicted with sufficient accuracy to calculate the dynamic strain.

2) To develop a method for experimentally obtaining the strain modes which are used to predict the dynamic valve strain.

3) To verify the predicted valve displacement and strain by comparing the predicted strain time histories with experimental measurements made on the compressor being simulated.

MATHEMATICAL MODELING TECHNIQUE

The basic method for the computer simulation of a reciprocating compressor has been discussed by other researchers (1,2) and need not be repeated here. Since the emphasis of this work is on the valve strain, the equations for the valve displacement will be reviewed and then extended to valve strain.

The dynamic motion of a two dimensional valve, using the modal expansion technique can be expressed in polar coordinates as:

$$w(r,\theta,t) = \sum_{n=1}^{\infty} \phi_n(r,\theta)\, q_n(t) \tag{1}$$

where $\phi_n(r,\theta)$ are the displacement modes

and $q_n(t)$ are the participation factors obtained by the solution to the differential equation:

$$\ddot{q}_n(t) + 2\zeta\omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{F_n(t)}{M_n} \tag{2}$$

Note that while the ring valve displacement at any point requires only one equation, which is a function of the two spatial coordinates r and θ, the calculation of the strain at any point requires a knowledge of the strain in both the r and θ directions. The strain in the tangential direction is a dependent on derivatives in both the r and θ directions (5):

$$\epsilon_\theta(r,\theta,t) = -c\left[\frac{1}{r^2}\frac{\partial w^2}{\partial \theta^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right] \tag{3}$$

substituting equation (1) into equation (3)

$$\epsilon_\theta(r,\theta,t) = -c\sum_{n=1}^{\infty} q_n(t)\left[\frac{1}{r^2}\frac{\partial^2\phi_n}{\partial\theta^2} + \frac{1}{r}\frac{\partial\phi_n}{\partial r}\right] \tag{4}$$

Note that the tangential strain is thus a function of the same participation factors, $q_n(t)$, obtained by the solution to Equation 2, and the term in brackets which will be designated as the tangential strain mode.

Similarly, the strain in the radial direction is

$$\epsilon_r(r,\theta,t) = -c\sum_{n=1}^{\infty} q_n(t)\frac{\partial^2\phi_n}{\partial r^2} \tag{5}$$

where the term $\frac{\partial^2\phi_n}{\partial r^2}$ is the radial strain mode. An experimental method for obtaining these strain modes is given in the following sections.

STRAIN MODE CONCEPT

A physical explanation may be useful in clarifying the strain mode concept. Consider the ring valve of Figure 1 which is excited at the n^{th} natural frequency. If the valve were transversed with a displacement probe the resulting measurements could be plotted to give the well known displacement mode. Furthermore, if strain gages were located at a number of points to measure the strain in the tangential direction, the resulting strains could be plotted as a function of their r and θ location on the plate. That plot would yield the tangential strain mode which is analagous to the more familiar displacement mode. If the displacement mode had been normalized to give a maximum value of unity, the strain gage data would have to be normalized in such a manner so that the two types of modes would be compatable in the modeling. However, the normalizing would not change the character of the curve, only the amplitude. The mathematics which relate the strain mode data to the displacement mode measurements will now be developed. The displacement of a plate vibrating at its nth natural frequency is

$$w(r,\theta,t) = A\phi_n(r,\theta)\sin\omega_n t \tag{6}$$

where A = the amplitude of displacement

$\phi_n(r,\theta)$ = the mode corresponding to the n^{th} natural frequency

ω_n = the n^{th} natural frequency

Note that the amplitude A is dependent on the initial conditions which started the valve in motion, or the magnitude of the forcing function.

At this frequency, the associated tangential strain is

$$\epsilon_{n\theta}(r,\theta,t) = -Ac\left[\frac{1}{r^2}\frac{\partial^2\phi_n}{\partial\theta^2} + \frac{1}{r}\frac{\partial\phi_n}{\partial r}\right]\sin\omega_n t \tag{7}$$

which can be written as

$$\epsilon_{n\theta}(r,\theta,t) = Ac\,\phi''_{n\theta}(r,\theta)\sin\omega_n t \tag{8}$$

where

$$\phi''_{n\theta}(r,\theta) = -\left[\frac{1}{r^2}\frac{\partial^2\phi_n}{\partial\theta^2} + \frac{1}{r}\frac{\partial\phi_n}{\partial r}\right] \tag{8a}$$

The displacement in Equation 7 has a maximum amplitude when

$$\sin\omega_n t = 1 \quad \text{at} \quad t = t_p$$

Thus the maximum tangential strain at any point r, θ at t = t_p is

$$\epsilon_{n\theta}(r,\theta,t_p) = -Ac\left[\frac{1}{r^2}\frac{\partial^2\phi_n}{\partial\theta^2} + \frac{1}{r}\frac{\partial\phi}{\partial r}\right]$$

Solving for the term in brackets:

$$\frac{1}{r^2}\frac{\partial^2\phi_n}{\partial\theta^2} + \frac{1}{r}\frac{\partial\phi_n}{\partial r} = -\frac{\epsilon_{n\theta}(r,\theta,t_P)}{Ac} \tag{9}$$

The amplitude A is found by normalizing the displacement. At t = t_p Equation 6 becomes

$$w(r,\theta,t_p) = A\phi_n(r,\theta) \tag{10}$$

Assume that the displacement $w(r,\theta,t_p)$ can be measured at various points along the vibrating valve. If the measured values are plotted, the resulting curve is the displacement mode at the n^{th} natural frequency. Furthermore, if each of the measured values are divided by the maximum value of $w(r,\theta,t_p)$ the resulting curve is said to be a normalized displacement mode, which has a maximum value of 1. Although normalizing to give $\phi_n(r,\theta)$ a maximum value of unity is an arbitrary choice, it is the most commonly used. Suppose that the maximum value $w(r,\theta,t_p)$ occurs at some point on the ring valve $r=r_o, \theta=\theta_o$. Equation 10 becomes

$$w(r_o,\theta_o,t_p) = A\phi_n(r_o,\theta_o) \tag{11}$$

but, by normalization

$$\phi_n(r_o,\theta_o) = 1$$

Thus the amplitude A in Equation 11 is

$$A = w(r_o,\theta_o,t_p)$$

Equation 9 becomes

$$\frac{1}{r^2}\frac{\partial^2\phi_n}{\partial\theta^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} = \frac{-\epsilon_{n\theta}(r,\theta,t_p)}{c\,w(r_o,\theta_o,t_p)} \tag{12}$$

Equating 8a and 12, the nth tangential strain mode is

$$\phi''_{n\theta} = \frac{\epsilon_{n\theta}(r,\theta,t_p)}{c\,w(r_o,\theta_o,t_p)} \tag{13}$$

Since $\epsilon_{n\theta}(r,\theta,t_p)$ can be measured with a strain gage, $w(r_o,\theta_o,t_p)$ can be measured with a displacement transducer, and the sign is determined by the phasing between the strain and the displacement, the strain mode can be found experimentally. Similarly, the strain mode component in the radial direction is found to be

$$\phi''_{nr} = \frac{\epsilon_{n\theta}(r,\theta,t_P)}{c\,w(r_o,\theta_o,t_p)}$$

However, while the displacement mode can be determined by traversing the displacement transducer around the valve, Equations 13 and 13a indicate that in order to obtain more complete knowledge of the entire strain mode, a number of strain gages must be located at various positions about the valve.

EXPERIMENTAL PROCEDURE AND MEASUREMENT OF STRAIN MODES

The method used to excite the natural frequencies was that described by Lowery and Cohen (4), and consisted of an electromagnet driven by an oscillator and power amplifier. A fixture was built which held the valve and simulated the boundary conditions. A Bently proximity tranducer of .125" tip diameter was held above the valve to measure the dynamic displacement and valve frequency.

The dynamic strain was measured with a Tectronix 53-A oscilloscope and type Q plug - in unit which makes it possible to measure dynamic strain in the magnetic field induced by the electromagnet. This experimental equipment is shown in Figure 2.

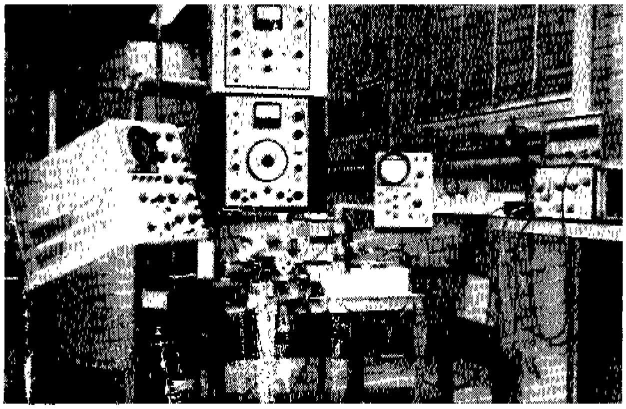

Figure 2. Equipment for Strain Mode Measurement

It should be realized that the strain mode, as well as the displacement mode, has an algegraic sign. This is indicated by Equations 12, 13, and 13a. The sign was determined experimentally by triggering the strain signal with the Bently output signal. Using a beam as an example, when the beam is curved upward, any point on the top at the beam is in compression and the strain signal will be negative. Thus if the displacement signal has a positive slope while the strain has a negative slope, the strain mode has a negative sign at that point.

First Symmetric Mode For Valve In Two Point Support

The valve being modeled is shown in its position in the compressor in Figure 3. Note that the valve rests on two supports, and that it has two other tabs which impact a stop to limit the maximum excursion.

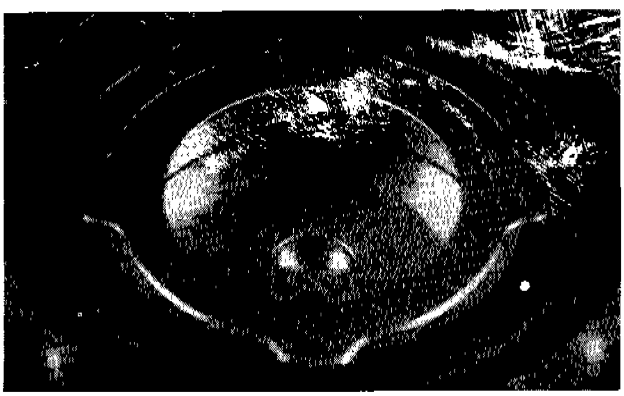

Figure 3. Suction Valve on Support Ring

The fixture was originally set to support the valve on only the two end tabs. Strain gages (.031" gage length) were located on six standard thickness valves: one gage at 0, 30, 45, 60, 75 and 90°. Each gage was located so as to measure the tangential strain at $r = r_{mean}$. The Bently distance detector was located at 90o (the point of maximum amplitude found by the earlier displacement mode measurements). The radial strain was found to be neglegible in this particular valve.

To obtain the necessary data for the first symmetric strain mode in the two point boundary condition each of the six valves had to be located in the fixture. In order that the boundary conditions be the same for each test, a fixture applying a constant force at the boundary was used. The test was conducted by locating a valve in the test stand, exciting the first natural frequency, and recording the strain and amplitude. This procedure was carried out for each of the six valves, and was then repeated to check the repeatability of the measurements. The resulting strain mode (designated as Φ''_{21} for two point suspension, first mode) is shown in Figure 4 and indicates that the repeatability of the measurements was satisfactory. Since the maximum value of Φ''_{21} was expected at 90° rather than 75°, the measurements at 60, 75, and 90° were each repeated two additional times. The results verified the first two runs.

A functional form of the displacement mode from the displacement measurements was hypothesized as:

$$\Phi(r,\theta) = \frac{1}{2}\left[1 - \cos 2\theta + 1.54(r_0 - r)\cos 2\theta\right]$$

An energy method was used to calculate the natural frequency, which came within 4% of the measured value. For comparison, the strain mode was calculated from this function and evaluated at $r = r_{mean}$. This theoretical strain mode is shown with the experimental data, and although the maximum values are comparable, the predicted strain mode is unsatisfactory on a point to point basis.

First Symmetric Mode For Valve In Four Point Support

When the valve was at its maximum opening in the compressor, the tabs in contact with the stop, the boundary conditions changed from a two "point" support to a four "point" support. This required changing the equations of motion with appropriate initial conditions entered into the computer simulation. These equations are discussed in detail in Reference (5). Experimentally, it was necessary to build a fixture which simulated the valve with this four point boundary condition. Furthermore, in order to excite the valve to sufficient amplitudes so that the displacement and strain measurement could be made, a scaling law was applied, Reference (6), which allowed the measurements to be made on a thinned version of the standard valve. The valve used for this set of experiments was ground from the standard .027" to .0166".

The measured strain mode Ψ''_{41}, is shown in Figure 5. Of particular interest is the fact that the maximum value of the strain mode for the valve with this boundary condition is 6.5 or approximately 4.5 times larger than the maximum value of Φ''_{21}. This indicates that even a small excitation of this mode can produce high strain levels.

Second Symmetric Mode For Valve In Four Point Support

Strain mode information for the second symmetric mode in the four point support was obtained using a .0133" thick valve. The results at three points are shown in Figure 6. The scatter of the points reflects the difficulty encountered in exciting this mode to amplitudes large enough to make accurate measurements. However, the magnitude of the strain modes at those points verifies that the higher mode strain contribution can be very large even if the mode is only slightly excited.

PREDICTION OF DYNAMIC STRAIN

Having obtained valve mode, natural frequency, and the strain mode information, the compressor simulation will now be utilized to predict the valve strain. Figure 7 is a normalized comparison of the predicted tangential strain at 75° with the experimentally measured strain. The line labeled t_c indicates that point at which the valve impacts the stop. The strain after that point is known as "overshoot" strain. The important point to note is that this prediction is based on a single mode approximation both before the valve contacts the stop and after the valve contacts the stop.

Figure 8 shows the effect of adding the second mode when the valve is in contact with the stop. Since the predicted strain at contact is 18% greater than the experimental value at contact, the total predicted strain is larger than the experimental. This discrepancy in the predicted strain before stop contact could be caused by two factors (1) the participation of higher modes not included in the modeling, (2) an error in the measurement of the strain mode Φ_{21}'' .

Figure 9 shows the strain contribution from the first two overshoot modes and the experimental overshoot strain to emphasize the importance of including the second overshoot mode. In analyzing the frequency content of the measured overshoot strain it is clear that modes even higher than the second symmetric mode have been excited.

Strain predictions were also made at other points on the valve with similar success.

SUMMARY AND CONCLUSIONS

A method was presented for experimentally obtaining the strain modes which were then used to predict dynamic valve strain. For valves of complex geometries, this technique is presently thought to be the best method for obtaining strain mode information for the higher modes of vibration.

The strain modes were utilized in a compressor simulation to predict the valve strain. Comparisons of the predicted and measured strain indicate that it may be desirable to include the first two modes before the valve contacts the stop, and at least the first two modes when the valve is in contact with the stop. The resulting simulated strain values are more than adequate in predicting trends which can be used by the designer to evaluate the effects of various parameter changes on the valve strain.

For the designer who does not have a compressor simulation at his disposal, a knowledge of the strain mode may be useful in determining the location of the maximum strain on the valve. Since the bench testing method becomes routine once the technique has been developed, it is possible to study the strain mode fields of more than one valve and use this information in arriving at the best configuration.

ACKNOWLEDGEMENTS

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LIST OF SYMBOLS

A Amplitude of displacement of vibrating plate - natural vibration.

c Perpendicular distance from neutral axis to point where strain is measured.

F_n Generalized force (Equation 2).

M_n Generalized mass (Equation 2).

q_n Participation factor, generalized coordinate.

r Radial coordinate in radial direction.

r_{mean} Mean radius of valve (Figure 1).

w Plate or valve deflection.

y Beam or valve deflection.

$\epsilon_{n\theta}$ Strain of n^{th} mode in θ, tangential, direction.

θ Polar coordinate in tangential direction.

ϕ_n The n^{th} displacement mode of plate or valve.

$\phi''_{n\theta}$ The n^{th} strain mode in the tangential direction.

ϕ''_{nr} The n^{th} strain mode in the radial direction.

ω_n The n^{th} natural frequency of the plate or valve.

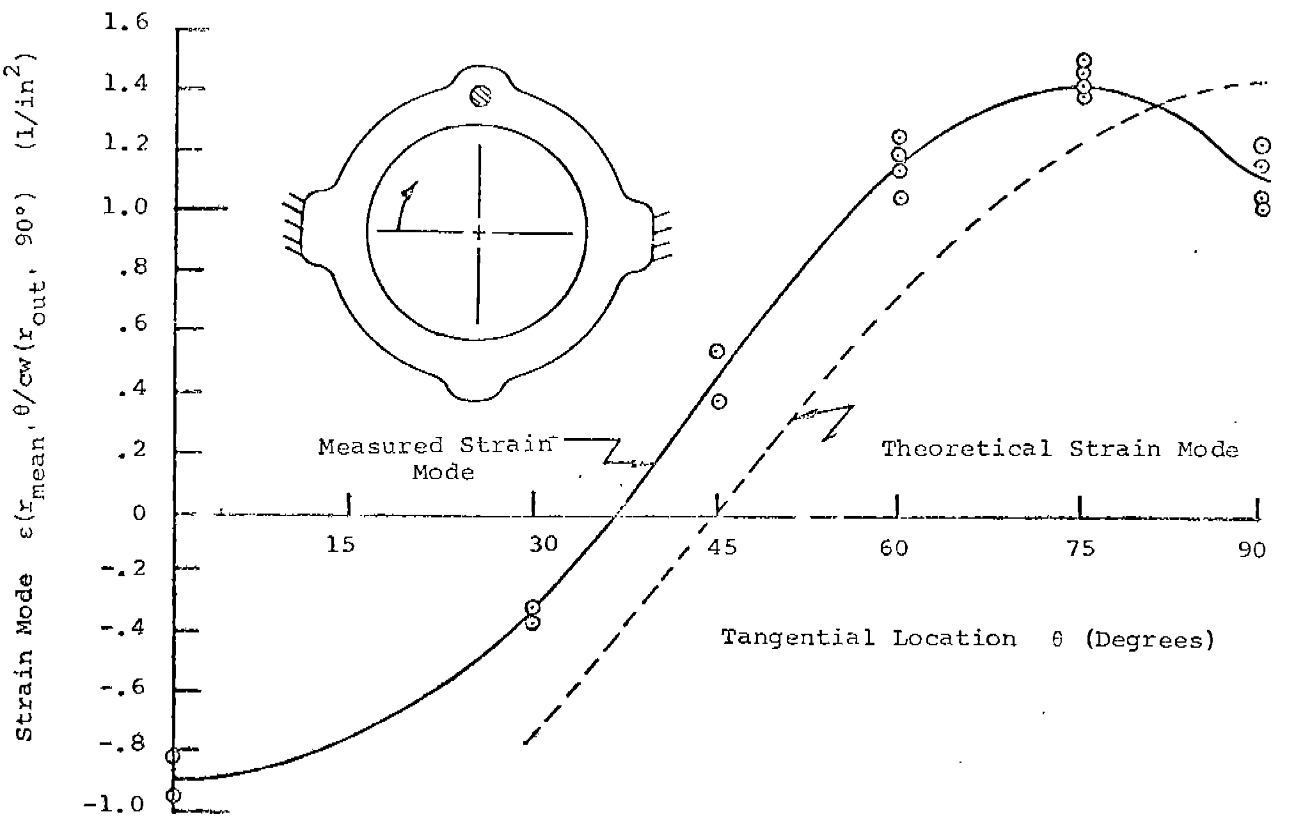

Figure 4. Measured Strain Mode ϕ_{21}'' For Suction Valve in Two Point Support Boundary

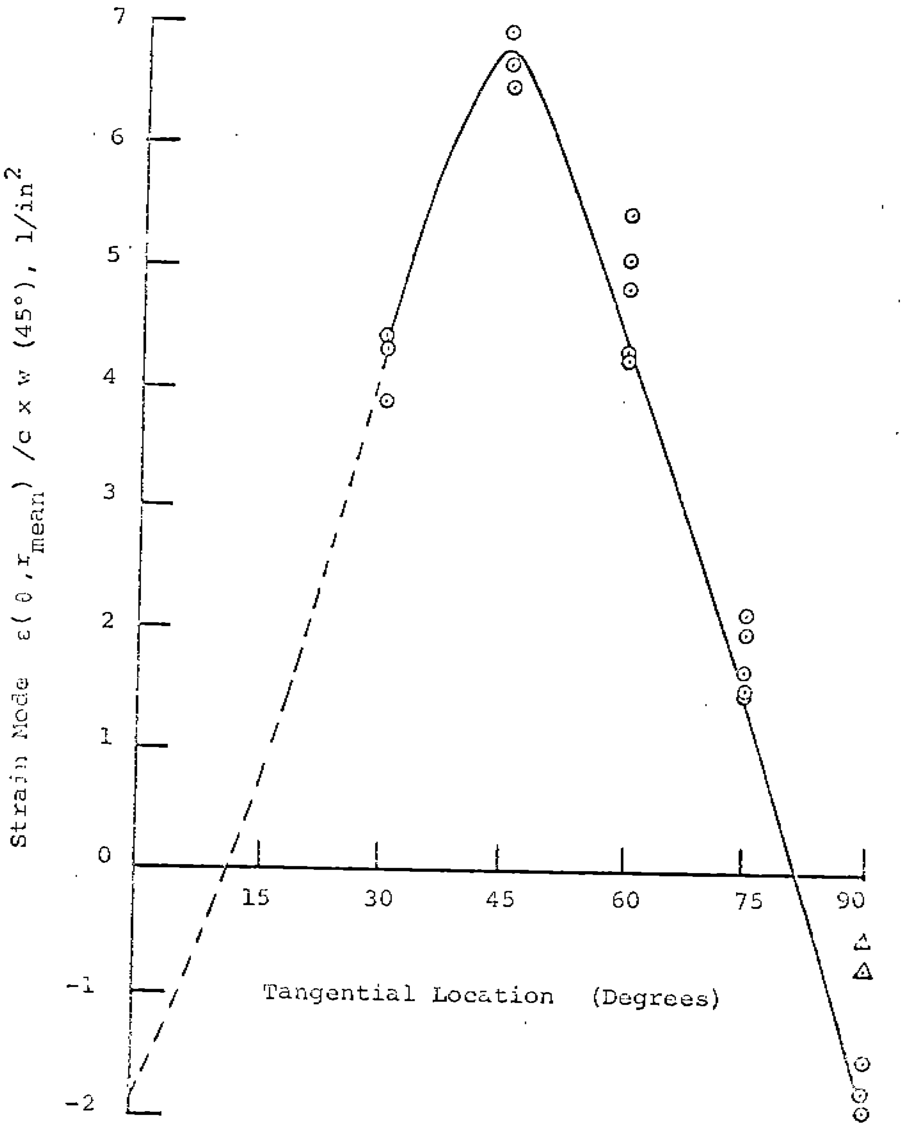

Figure 5. Measured Strain Mode ψ_{41}'' For Suction Valve in Four Point Support Boundary

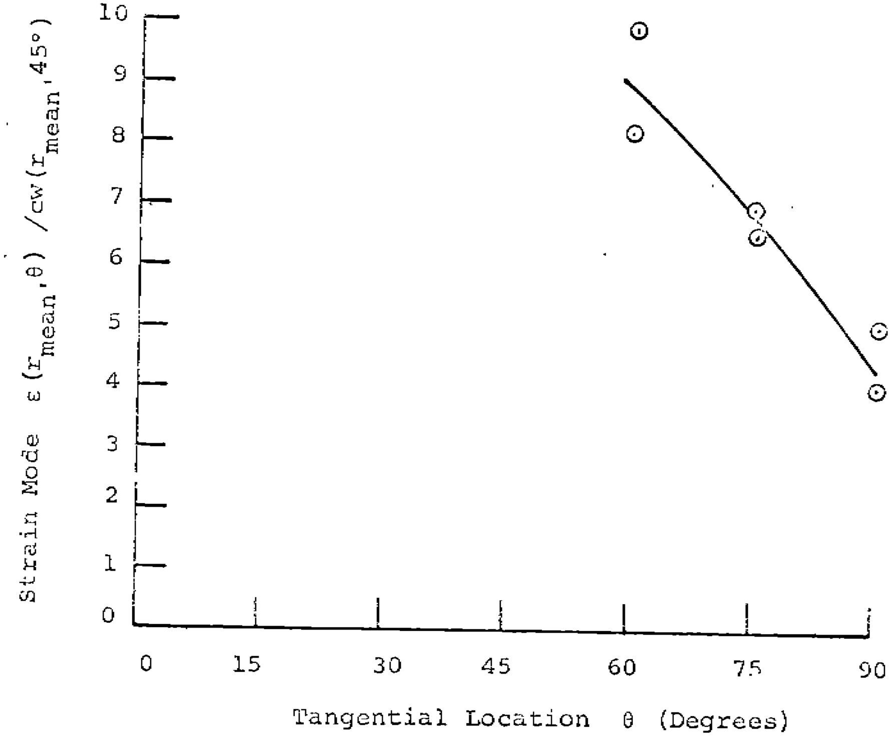

Figure 6. Measured Strain Mode ψ_{42}'' for Suction Valve in Four Point Support Boundary

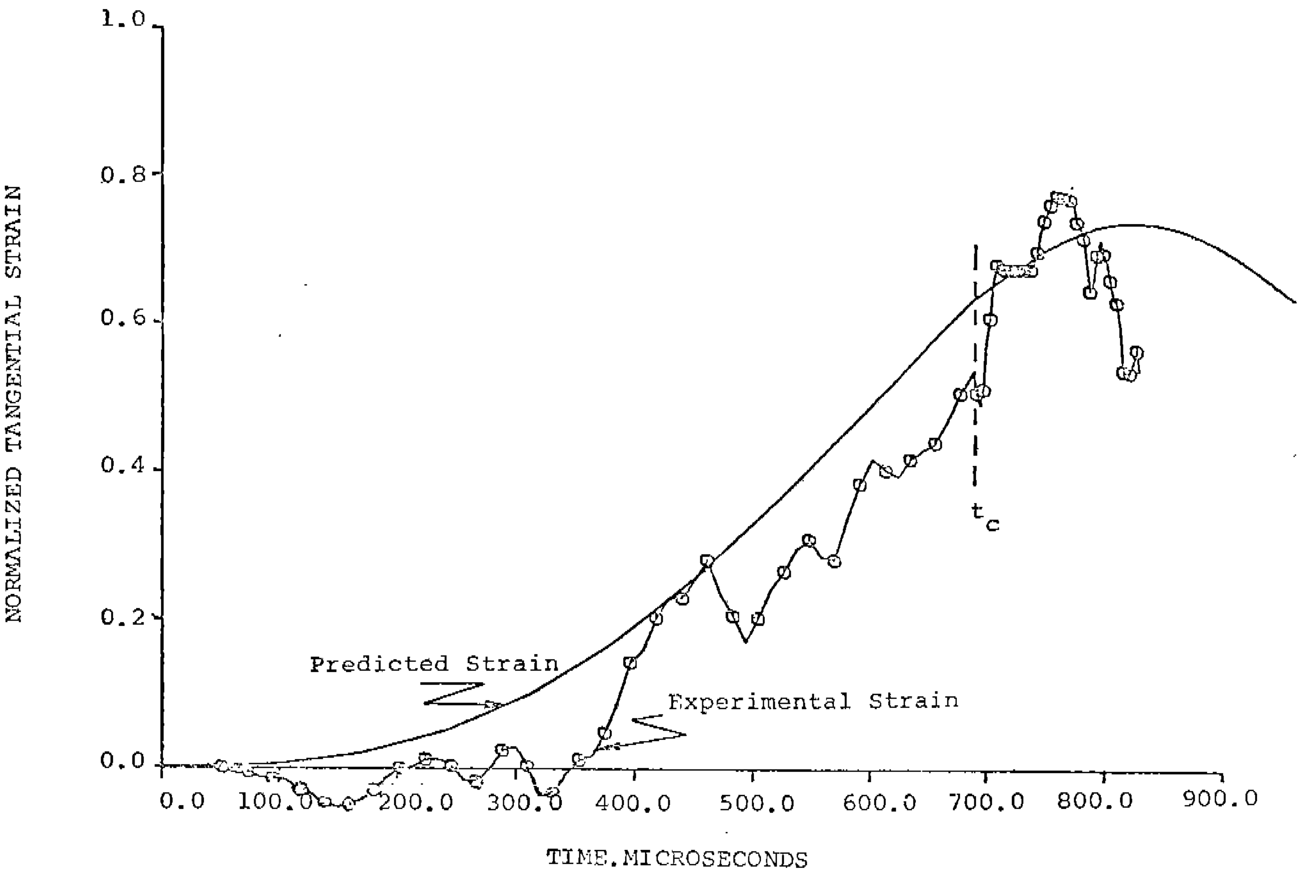

Figure 7. Total Strain Prediction Using Single Mode Approximation

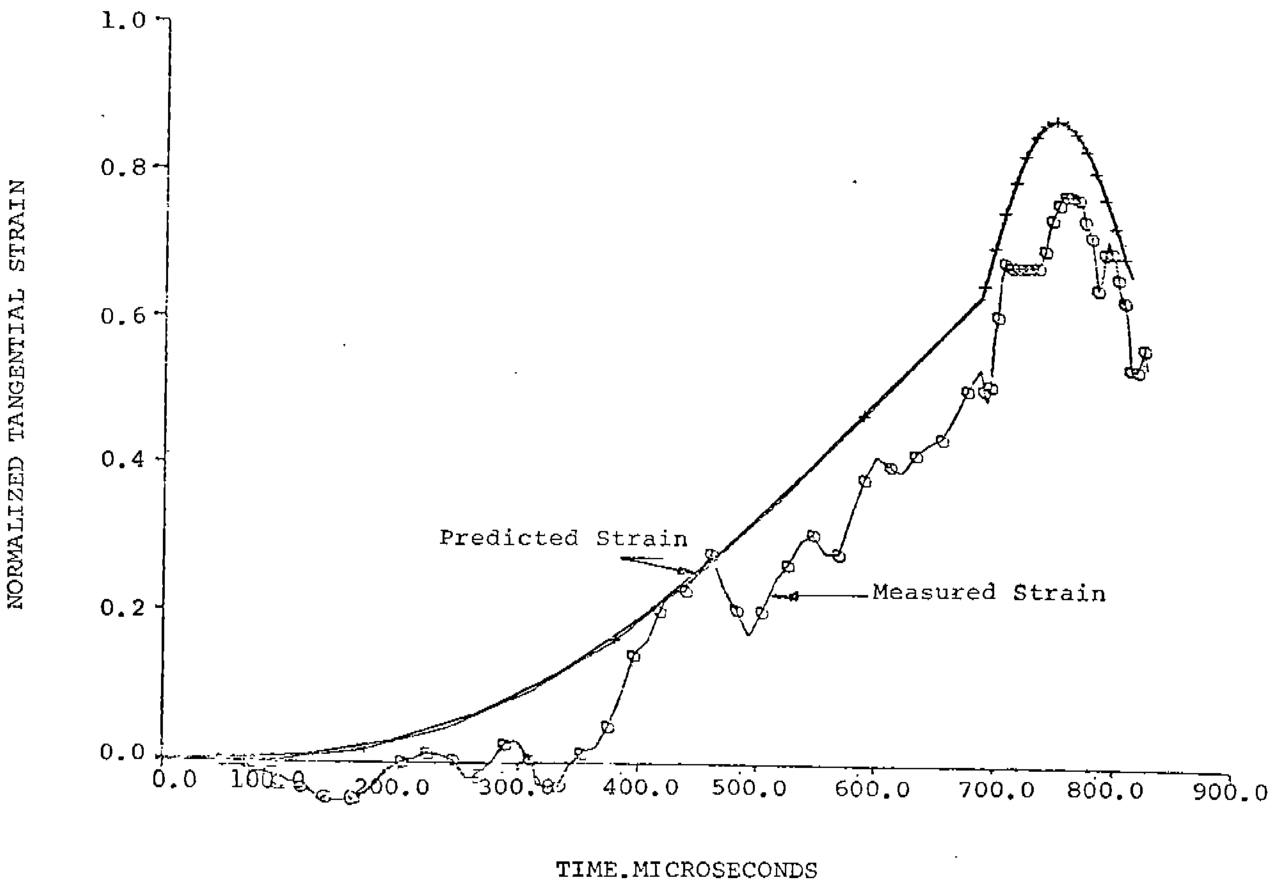

Figure 8. Total Strain Prediction Using First Two Symmetric Overshoot Modes

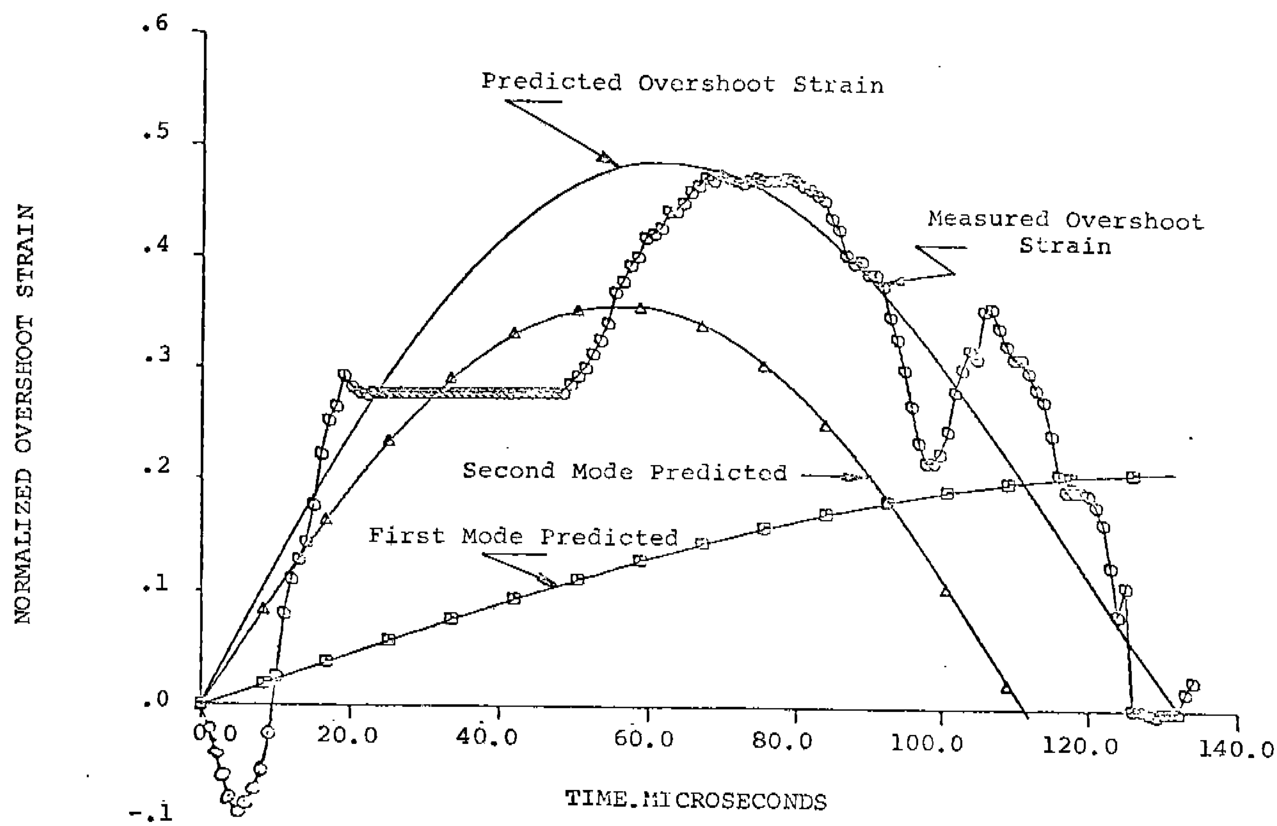

Figure 9. Overshoot Strain Prediction