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INTRODUCTION

This paper looks at the basic steps in compressor operation with examples showing their relation to the language of thermodynamics textbooks.

The various types of compressors: centrifugal, axial, rotary, reciprocating, helical screw, and others, differ considerably in construction and means of compression, nevertheless, they all have the same common threefold task to perform upon the gas between suction and discharge flanges.

1. Suction - Containing the gas within the compressor.
2. Compression - Raising the gas pressure to discharge pressure.
3. Discharge - Moving the compressed gas into the discharge line.

To understand these three steps in detail, let's consider a single acting frictionless piston and cylinder operating with the following conditions:

Cylinder capacity at inlet conditions: 1 pound of dry air.

Inlet pressure: $p_1 = 14.7$ lbs./in.^{2}

Discharge pressure: $p_2 = 29.4$ lbs./in.^{2}

Inlet temperature: T_1 (40°F.) 500° Rankine

From this data we can calculate:

Inlet Volume:

$$V_1 = \frac{\text{wt.} \times R \times T}{P_1} \quad \frac{1 \times 53.3 \times 500}{144 \times 14.7}$$

$$= 12.6 \text{ ft.}^3$$

Further, we'll initially assume no heat is added to, or removed from the cylinder during these three steps - this is called adiabatic compression, and would result in the following temperature and volume.

Discharge Temperature (Adiabatic):

$$T \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = 500 \left(\frac{29.4}{14.7}\right)^{.286} = 610^\circ R$$

Discharge Volume:

$$V_2 = \frac{w \times R \times T}{P_1} = \frac{1 \times 53.3 \times 610}{144 \times 29.4}$$

$$= 7.68 \text{ ft.}^3$$

One Complete Cycle Using the Mechanical Approach

Here the events during one compressor cycle would be in terms of how many foot pounds of work the crankshaft must provide for each one of these steps in a complete cycle. This approach will differ from the textbook, heat balance approach for calculating work, but the net result must be the same in each case.

a. Crankshaft work during suction stroke.
Because there is no difference in pressure between the top and bottom sides of the piston, there is no crankshaft work during the suction stroke.

b. Compression work.
At the beginning of compression the pressure on both the top and bottom of the piston is the same. In our calculations, absolute air pressure values are used to measure the force resisting upward piston movement, therefore, we must also take into account the opposite force provided by atmospheric pressure in the crankcase. Thus, the total work to compress the air is equal to the sum of crankshaft work plus the work resulting from atmospheric pressure on the bottom area of the piston. Since we assumed the piston is frictionless and no heat is added to, or removed from, the confined air being compressed, between work and internal energy is:

$$\text{Work Expressed In } \frac{\text{BTU}}{\text{lb. of air}} = C_v \times \Delta T = \Delta u$$

Where:

Δu = Change in internal energy, BTU per lb.

ΔT = Change in air temperature, °F or °R

C_v = Specific heat at constant volume ($.172 \frac{\text{BTU}}{^\circ F}$ for each lb. of dry air)

We'll emphasize here that internal energy is strictly a function of temperature and it is unrelated to other gas properties. Thus:

1. Total work of compression = .172 (610 - 500)

= 18.9 BTU

2. Work supplied by atmospheric air in the crank-case.

$$\text{Work (BTU)} = \text{PSIA} \times \text{Piston Area} \times \text{Piston Travel} \times \frac{144 \frac{\text{Sq. In.}}{\text{Sq. Ft.}}}{778 \frac{\text{Ft. Lb.}}{\text{BTU}}}$$

$$= \text{PSIA} \times (\text{Initial ft.}^3 - \text{Final ft.}^3)$$

$$= 14.7 \times (12.6 - 7.68) \times \frac{144}{778}$$

= 13.4 BTU

3. Thus, work supplied by crankshaft, to compress the air:

Total Compression Work	(18.9)
- Work Provided by Atmos. Air	(13.4)
Crankshaft Work	(5.5 BTU)

c. Work to push the air out of the cylinder.

1. Total work = PSIA x (Initial - Final volume) x $\times \frac{144}{778}$

$$= 29.4 \times (7.68 - 0) \times \frac{144}{778}$$

= 41.8 BTU

2. Work supplied by atmospheric air in the crank case.

$$\text{Work (BTU)} = 14.7 \times (7.68 - 0) \times \frac{144}{778}$$

= 20.9 BTU

3. Work supplied by crankshaft to push the air out of the cylinder.

41.8 - 20.9 = 20.9

d. Summing up the work supplied by the crankshaft for one complete revolution.

Work during:	Suction	(0)
	Compression	(5.5)
	+ Discharge	(20.9)
	Net crankshaft work	(26.4 BTU)

Now the Textbook Way!

Consider the same problem with the thermodynamic textbook heat balance approach where changes in enthalpy (i.e., the sum of internal and potential energy) are calculated. Calculations can be considerably simpler than the previous example by omitting the effect of crankcase pressure, providing we treat the suction stroke as work done by, rather than on the air. Starting with air in the inlet line, we'll review the same fundamental steps of compressor operation.

a. Total energy or enthalpy (h) in the intake line

h = internal energy and potential energy

$$h \text{ (BTU)} = C_v T + pv \times \frac{144}{778}$$

$$= .172 \times 500 + \frac{14.7 \times 144}{778} \times 12.6$$

= 120.3 BTU

b. Work done by the air during suction stroke. The minus sign will show that work was done by, rather than on, the air.

$$\text{Work (BTU)} = p_1 v_1 \times \frac{144}{778}$$

$$= \text{PSIA (Initial volume - Final volume} \times \frac{144}{778}$$

$$= 14.7 \times (0 - 12.6) \times \frac{144}{778}$$

= -34.3 BTU

c. Work done on the air during compression

Work (BTU) = .172 (610 - 500)

= 18.9 BTU

d. Work done on the air during discharge

$$\text{Work (BTU)} = p_2 v_2 \times \frac{144}{778}$$

$$= 29.4 \times 7.68 \times \frac{144}{778}$$

= 41.8 BTU

e. Total energy or enthalpy in the discharge line

h at inlet + the net work on the air = h at discharge

120.3 - 34.3 + 18.9 + 41.8 = 146.7 BTU

Check:

$$h = .172 \times 610 + \frac{144}{778} (29.4 \times 7.68) = 146.7 \text{ BTU}$$

And: Change in Enthalpy:

146.7 - 120.3 = 26.4 BTU

Therefore, the compressor has increased the air's total energy by 26.4 BTU which is equal to the net work done by the crankshaft in our first approach. This is obvious, for when there is no friction or heat flow into, or out of the air, all the work done by the crankshaft must be absorbed by the air. Accordingly when we summarize and simplify the work of suction, compression, and discharge the following equation for compressor work results:

$$\text{Work (BTU)} = p_1V_1 \times \frac{k}{k-1}\left[\frac{(P_2)}{(P_1)}^{\frac{k-1}{k}} - 1\right] \times \frac{144}{778}$$

The math involved in transforming the three steps of the compressor process into this equation, for adiabatic compression is readily available in most thermodynamic texts. Our object here is primarily to show the meaning of the three factors which form the equation.

Side Issues, Friction and Cooling

Most shop men take a dim view of explanations limited to frictionless devices such as we have assumed. Thus, we'll show how the basic process changes with the inevitably present heat of friction. Friction simply results in the addition of heat to the air being compressed, thus, we'll visualize its effects by stopping the piston at the completion of its intake stroke and add heat through the cylinder walls. In accordance with Charles' Law, the absolute pressure of the confined air will rise in direct proportion to the rise in absolute temperature. Hence, if we add enough heat the air pressure within the cylinder will reach discharge line pressure even though no work of compression has been done by the piston. We can then restart the compressor and complete the cycle by discharging the air which is already at discharge line pressure. At first glance, it appears that the addition of heat would result in less total compressor work since no piston work was required to compress the air. However, recalling that total horsepower is the sum of suction, compression and discharge work, we find:

Total Compressor Work
= Suction Work + Compression Work* + Discharge Work

$$= \left[P_1 \text{ (Initial Vol. - Final Vol.)} + 0 + P_2 \text{ (Initial Vol. - Final Vol.)}\right] \times \frac{144}{778}$$

$$= \left[14.7\ (0 - 12.6) + 0 + 29.4\ (12.6 - 10)\right] \times \frac{144}{778}$$

$= 34.3$ BTU

* In this case, the compressor did not provide the work.

Recall that for our example where no friction or external heat was added to the compressor, the compressor work for one cycle was only 26.4 BTU. Thus, we have shown that although friction or external heat reduces the compression work, the net result for a complete cycle is increased compressor horsepower.

After noting this effect of friction, or the addition of heat, on compressor horsepower, we can turn this knowledge to our advantage by removing heat from the gas being compressed. This is commonly done with fins, cooling water jackets, or spray injection. Using the conditions of our original example, we'll cool the air. When the air just reaches discharge line pressure, we'll stop the piston and cool the cylinder unitl the air temperature reaches inlet air temperature. Again, Charles' Law enters the picture with a resulting drop in pressure to:

$$29.4 \times \frac{500}{610} \text{ or } 24.1 \text{ PSIA}$$

Now we'll restart the compressor for the two steps required to complete the cycle:

a. Compress the air from 24.1 back up to 29.4 PSIA

$$\text{Resulting New } T = T \frac{(p_2)}{(p_1)}^{\frac{k}{k-1}} = 500 \frac{(29.4)}{(24.1)}^{.286}$$

$$= 530^\circ$$

Additional work of compression

$$= C_v\ (T_2 - T_1) \text{ BTU}$$

Or $.172\ (530 - 500) = 5.16$ BTU

$$\text{Then } v = 12.6 \times \frac{530}{500} \times \frac{14.7}{29.4}$$

$$= 6.67 \text{ ft.}^3$$

This is the volume of compressed air which must be pushed into the discharge line.

b. Work of discharging the compressed air

$$P_2v_2 = P_2 \text{ (Initial Vol. - Final Vol.)} \times \frac{144}{778}$$

$$P_2v_2 = 29.4 \times (6.67 - 0) \times \frac{144}{778}$$

$$= 36.3 \text{ BTU}$$

Summarizing the work in this example of cooled compression:

Total Work
= Suction Work + Compression Work + Discharge Work

$$= -Pv + C_v\left[(T_2 - T_1) + T^1{}_2 - T_1)\right] + Pv$$

$$= -34.3 + .172\left[(610 - 500) + (530 - 500)\right] + 36.3$$

$$= 26.1$$

Thus, even though cooling necessitates additional compression work, the net result is lower compressor horsepower.

Up to this point we have shown how the three basic factors in compression tie into thermodynamic terms to product the adiabatic compressor equation.

Further, we have proven with numerical examples that which the shop man knew from the start -- friction increases, and cooling decreases power requirements. Before showing how the adiabatic equation is rewritten to reflect the effects of friction and cooling, we must know several other textbook terms.

That Word Called Entropy

Many textbooks define entropy as a measure of the unavailability of some portion of a gas's total energy. Pressurized plant air which has been cooled to room temperature has no more total energy than the atmosphere air in the room. Obviously, then we need further information to complete the picture:

1. The percentage of the total energy which can be used in a machine or process, and --

2. The percentage which is unavailable.

Pressurized plant air is in a position to expand and produce work hence, the net effect of a plant compressor and aftercooler is to increase the available portion of the air's energy, and not to increase its total energy, (i.e. enthalpy).

This difference in available energy can be seen on a Mollier chart for air as a difference in entropy. For the pressurized plant air illustration this difference in availability is numerically equal to:

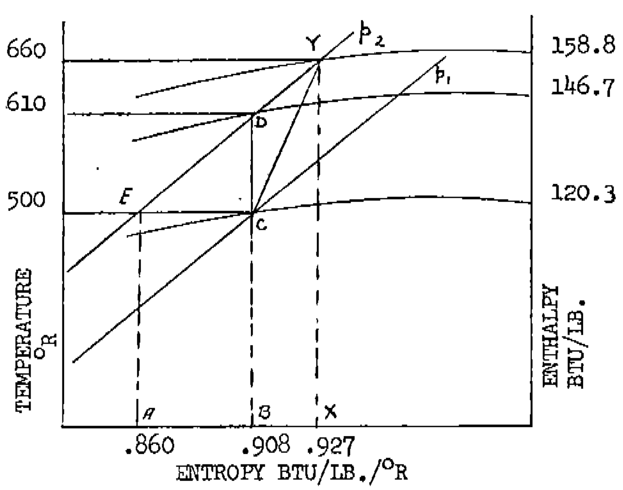
$T \times S$ = difference in availability $\frac{BTU}{lb.}$ where:

T = room and plant air temperature ^{o}R

$S = S_{@\ Atm.} - S_{@\ Plant\ Air\ Pressure}$

= difference in entropy

To gain additional familiarity with the term entropy let's calculate the approximate work of adiabatic compression with entropy values rather than the mechanical and enthalpy methods used previously. The air properties in our original example would show up on a Mollier chart for air as illustrated below.



CD - Adiabatic Compression Path

DE - Cooling Compressed Air To Initial Temperature

AEDB - Area Representing Adiabatic Work

CY - Typical Polytropic Compression Path

BCYX - Area Representing Added Work For Typical Polytropic Compression

The area enclosed by ABDE represents the work of one complete cycle, however, its mechanical analogy is not as simple to visualize as the typical indicator card with its pressure and volume relationships. In our example of adiabatic compression the work to get the air from 14.7 PSIA and 500°F to 29.4 PSIA and 610°F was 26.4 $\frac{BTU}{lb.}$ These two conditions are represented by points C and D on the sketch. Now if we remove all the heat of compressor work in an aftercooler, we will arrive at point E, hence the area ABDE will represent the same amount of work as $h_2 - h_1$. If we wish to become more sophisticated about this, our calculations will have to adjust slightly by the fact that real gases deviate slightly from straight line relationships such as D to E. In textbooks, the heat transfer in an entropy change is accurately defined as: $dQ = Tds$. For air, we would be reasonably accurate with:

$$Q = \frac{(T_D + T_E)}{(2)} \times (S_B - S_A)$$

Where $\frac{(T_D + T_E)}{(2)}$ is the average temperature between D and E

$$Q = 555 \times (.908 - .860)$$

$$= 26.4 \frac{BTU}{lb.}$$

Available Energy

Available energy is the term which ties the picture together and solves the mystery about high pressure air having no more total energy than low pressure air at the same temperature. From our practical point of view what is important about any gas is how much work can be extracted from the gas. The equation which tells how much available energy can be extracted from a gas between two temperature levels is:

$$a_2 - a_1 = (h_2 - h_1) - T_1 (S_2 - S_1)$$

Where $a_2 - a_1$ = difference in available energy $\frac{BTU}{lb.}$

The air which we compressed to 29.4 PSIA in our example has no more total energy than the room air but, it can expand in an air tool to atmospheric pressure and in doing so, will produce work by dropping its temperature 110°F (i.e., adiabatic temperature change between 29.4 PSIA and 14.7 PSIA). Let's determine the difference in available energy between the plant air and room air, both at a temperature T_1 and accordingly $h_1 = h_2$.

$$a_2 - a_1 = (h_2 - h_1) - 500\ (.860 - .908)$$

$$a = 0 + 23.8 \text{ BTU per pound}$$

Thus, the available energy has increased even though the total energy is no greater than that of the surrounding room air at atmospheric pressure. This increase in available energy is numerically equal to the work which would be required to compress the air in our example at a constant temperature from suction to discharge. Note, the work of suction, compression at constant temperature, and discharge is:

$$\text{Work (in BTU)} = P_1V_1 \times \ln_e \frac{P_2}{P_1} \times \frac{1}{778}$$

Polytropic Efficiency

In actual practice, compressors do not follow the adiabatic temperature rise and horsepower relationships because of friction and cooling. Their actual performance is called polytropic compression, a term which takes into account the extent of friction and cooling during the cycle. Among all the types of compressor efficiencies used, polytropic efficiency is the only index which accurately reflects compressor performance.

In pointing out the importance of available energy gain, rather than gain in total energy we didn't say what practical process would be used to get the available energy down to our lower temperature.

Friction in uncooled polytropic compression increases the adiabatic horsepower by an amount equal to $\frac{(T_2 + T_1)}{(2)} \times (S_2 - S_1)$.

For this extra work, our available energy equation tells us that of this amount, $T_1\ (S_2 - S_1)$ is available. Comparing these two terms, we note that some of this additional work above adiabatic is recoverable. But this increase in available energy is only available in the form of heat whereas we want to use this air to perform work. It should be recalled that most plants throw away heat through the inter and after cooler because it is not the form of energy which can be used to perform mechanical work. With this in mind, we can now define polytropic head as the gain in that portion of available energy which can produce mechanical work. Polytropic efficiency then is polytropic head divided by the heat input (work) per pound needed to obtain this polytropic head. In equation form:

$$N_{polytropic} = \frac{(h_2 - h_1) - \frac{(T_2 + T_1)}{(2)}(S_2 - S_1)}{(h_2 - h_1)}$$

Let's alter our example of adiabatic compression to one of polytropic compression with 660°R (200°F) discharge temperature. In our example using the textbook way, we have found that the actual compressor work, per pound of air, is equal to the final enthalpy minus the initial enthalpy or $(h_2 - h_1)$. These properties as well as entropy values may be found on a Mollier chart for air. On different charts, their values may not be the same because varying basic reference temperatures are frequently used. This is unimportant because we are interested in differences in enthalpy and entropy rather than their absolute values. For this example of polytropic compression:

T_1 = 500°R	T_2 = 660°
P_1 = 14.7 PSIA	P_2 = 29.4
h_1 = 120.3 BTU per pound	h_2 = 158.8
S_1 = .908 BTU per pound per degree	S_2 = .927

polytropic efficiency

$$= \frac{(158.8 - 119.7) - \frac{(660 + 500)}{2}(.927 - .908)}{(158.8 - 120.3)}$$

$$= 71 - 1/2\%$$

This example and the explanation of polytropic efficiency is mainly to illustrate the meaning of the term. In ordinary plant practice the engineer knows what the compressor discharge temperature is and from this can find polytropic efficiency through the relationship between polytropic and adiabatic performance. To alter our basic adiabatic relationships for polytropic compression, we find the polytropic equivalent of $\frac{k-1}{k}$ and designate this as $\frac{n-1}{n}$:

$$\frac{k-1}{k} \times \frac{1}{N_{polytropic}} = \frac{n-1}{n}$$

Thus for polytropic compression, we can use all the adiabatic equations simply by replacing $\frac{k-1}{k}$ or $\frac{k}{k-1}$ with $\frac{n-1}{n}$ or $\frac{n}{n-1}$.

Having now developed the equation for common polytropic compression, we'll check our example of polytropic compression. We assumed a 660°R discharge temperature and from entropy values calculated 71 - 1/2% polytropic efficiency.

Substituting $\frac{n-1}{n}$ for $\frac{k-1}{k}$ in the adiabatic temperature rise equation results in:

Polytropic Discharge Temperature

$$= T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k} \times \frac{1}{N_{polytropic}}}$$

$$= 500 \left(\frac{29.4}{14.7}\right)^{\frac{1.4-1}{1.4} \times \frac{1}{.715}} = 660^{\circ}R$$

$\text{Work}_{\text{polytropic}}$ (BTU)

$$= \frac{p \times v \times \frac{k}{k-1} \times N_p \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k} \times \frac{1}{N_p}} - 1 \right] \times \frac{144}{778}}{N_p}$$

$$= \frac{144}{778} \times 14.7 \times 12.6 \times \frac{1.4}{.4} \times .715 \left[\left(\frac{29.4}{14.7}\right)^{\frac{1.4-1}{1.4 \times .715}} - 1 \right]$$

= 38.5 BTU per pound

The resulting work of 38.5 BTU per pound thus agrees with Mollier charts showing enthalpy changes 38.5 BTU per pound with a temperature change from 500°R to 660°R.

Conclusion

A compressor designer reading this outline would no doubt wish to amplify many of the general statements and include additional side issues which enter into the picture. Conversely a plant engineer may believe that day to day shop problems do not necessitate a working knowledge of entropy, enthalpy, et cetera.

The writer recognizes validity in both of these positions. However, it is hoped that a fundamental knowledge of what occurs between compressor suction and discharge flanges will be a useful starting point for the shop man when problems of compressor selection or operation arise.

References

1. Walter Kidde Co, Mollier Chart for Air, April 1953.

2. U. S. Naval Institute, "Energy Analysis of Naval Machinery".

3. A. J. Stepanoff, "Turbo Blowers", Wiley & Sons, 1955.

NOMENCLATURE

P	= Absolute pressure	: lbs/ft^2
p	= Absolute pressure	: lbs/in^2
T	= Absolute temperature	: °Rankine
V	= Volume	: cu. ft.
v	= Volume	: cu. ft. per lb.
R	= The gas constant	: ft. per °F
C_v	= Specific heat at constant volume	: BTU per lb. per degree, .172 for dry air
C_p	= Specific heat at constant pressure	: BTU per lb. per degree, .241 for dry air
$\frac{1}{778}$	= Conversion factor	: BTU per ft. lb.
k	= Ratio for specific heats	: C_p/C_v, 1.4 for dry air
U	= Internal energy or the measure of gas's kinetic energy	: BTU per lb.
$\frac{PV}{778}$	= Potential energy of a gas	: BTU per lb.
h	= Enthalpy or total energy	: BTU per lb.
S	= Entropy	: BTU per lb. per °F
a	= Available energy	: BTU per lb.
N_p	= Polytropic Efficiency	: %