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# Approximating the Number of Blocks Accessed in Data Base Organizations

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APPROXIMATING THE NUMBER OF BLOCKS ACCESSED
IN DATA BASE ORGANIZATIONS

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When data records are grouped into blocks in secondary storage,
it is frequently necessary to estimate the number of blocks $X_D$ accessed
for a given query.  In a recent paper [1], Cardenas gave the expression

$$X_D = m(1 - (1 - 1/m)^k),\tag{1}$$

assuming that there are n records divided into m blocks and that the
k records satisfying the query are distributed uniformly among the m
blocks.  The derivation of the expression was left to the reader as an
exercise.

Let us take a closer look at the expression.  $(1 - 1/m)$ gives the
probability that a particular block does not contain a particular record.
If k records are selected independently, then the probability that a
particular block not being "hit" is given by $(1 - 1/m)^k$.  Hence $1 - (1 - 1/m)^k$
gives the probability that a particular block is "hit", and the expression
follows.

The assumption that the k records are selected independently implies
selection with replacement.  Since a record may be selected more than once,
the k records may not be distinct.  This is not valid in the case of a query
access which retrieves all k distinct records at one time.  In fact, Rothnie
and Lozano showed that the result of eq. (1) gives the lowerbound of the
expected number of blocks accessed [2].  A more accurate analysis based
on selection without replacement was given by Severance, but the precision
problem makes the expression obtained computationally intractable
(Appendix D in [3]).  A similar approach by Siler results in a rather complicated
recursive formula which can be computed (Appendix B in [4]).  Using a different

approach, a simple closed form was obtained by Yao in a different context
[5]. The resulting expression was used in several applications [5,6,7] to
estimate the expected number of data blocks accessed. Comparing to the
Cardenas' approximation, it is shown that this refinement is significant
when the blocking factor n/m is small. For large blocking factors (e.g.,
n/m $\geq$ 10), the error involved in Cardenas' approximation is practically
negligible.

Theorem (Yao). Given n records grouped into m blocks (1 < m $\leq$ n), each
contains n/m records. If k records (k $\leq$ n - n/m) are randomly selected
from the n records, the expected number of blocks hit (blocks with
at least one record selected) is given by

$$m \cdot \left[ 1 - \prod_{i=1}^{k} \frac{nd - i + 1}{n - i + 1} \right] \quad \text{where} \quad d = 1 - \frac{1}{m} \qquad (2)$$

Proof: Let X be a random variable representing the number of blocks
hit and let $I_j$ be a random variable where

$$I_j = \begin{cases} 1 & \text{when at least one record in the j-th} \\ & \text{block is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

The j-th block has p = n/m records and there are n - p records
not in the j-th block. The probability that no records are selected
from the j-th block is

$$\frac{C_k^{n-p}}{C_k^n} \quad \text{or} \quad \frac{C_k^{nd}}{C_k^n} \quad \text{where } d = 1 - \frac{1}{m} \quad .$$

It follows that the expectation of $I_j$ is

$$E(I_j) = 1 - \frac{C_k^{nd}}{C_k^n} \quad .$$

Hence the expected number of blocks hit is

$$E(X) = \sum_{j=1}^{m} E(I_j)$$

$$= m \cdot \left( 1 - \frac{C_k^{nd}}{C_k^n} \right)$$

Using the identity $C_y^x = \frac{x!}{y!(x-y)!}$ , we have

$$E(X) = m \cdot \left( 1 - \frac{(nd)!\,(n-k)!}{n!\,(nd-k)!} \right)$$

$$= m \cdot \left( 1 - \prod_{i=1}^{k} \frac{nd - i + 1}{n - i + 1} \right)$$

Q.E.D.

The following corollary of the theorem is obvious:

Corollary. If $k > n - n/m$ or $m = 1$, then all m blocks are hit.

It is interesting to note that eq. (1) is independent of the number of records n. We observe that the approximation of eq. (1) is good if $k \ll n$ and $m \ll n$. Intuitively, if n is large and each block contains many records, the selection with and without replacement makes little difference, especially when a small number of records are selected. In many actual applications it is true that $k \ll n$, otherwise sequential processing of the records can be considered. Using Siler's example, the results from Cardenas, Siler, and Yao are compared in Figure 1. The Siler's result for 300 records grouped into 20 blocks with a blocking factor of 15 matches Yao's result. The errors of Cardenas' expression are plotted in Figure 2.
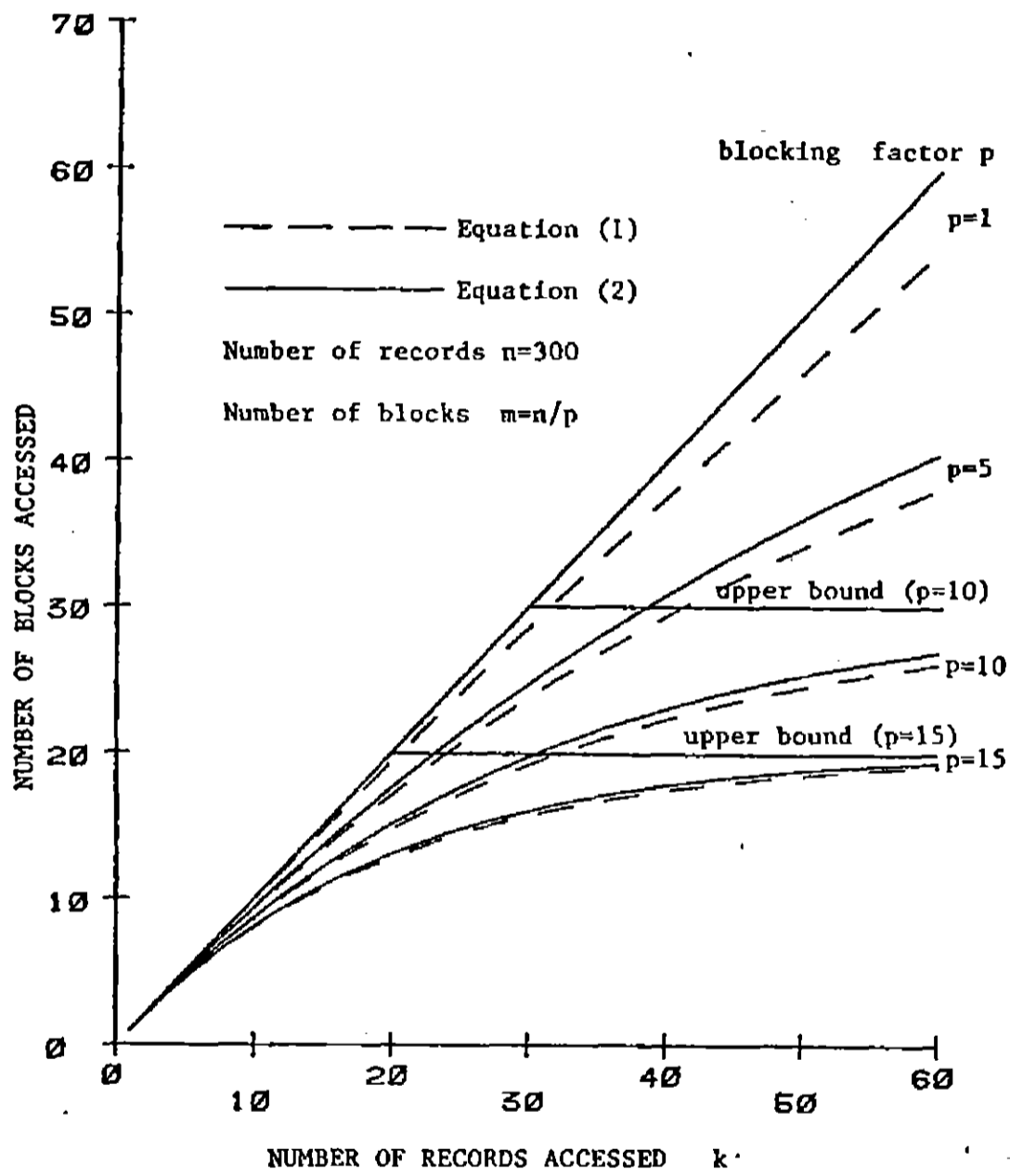
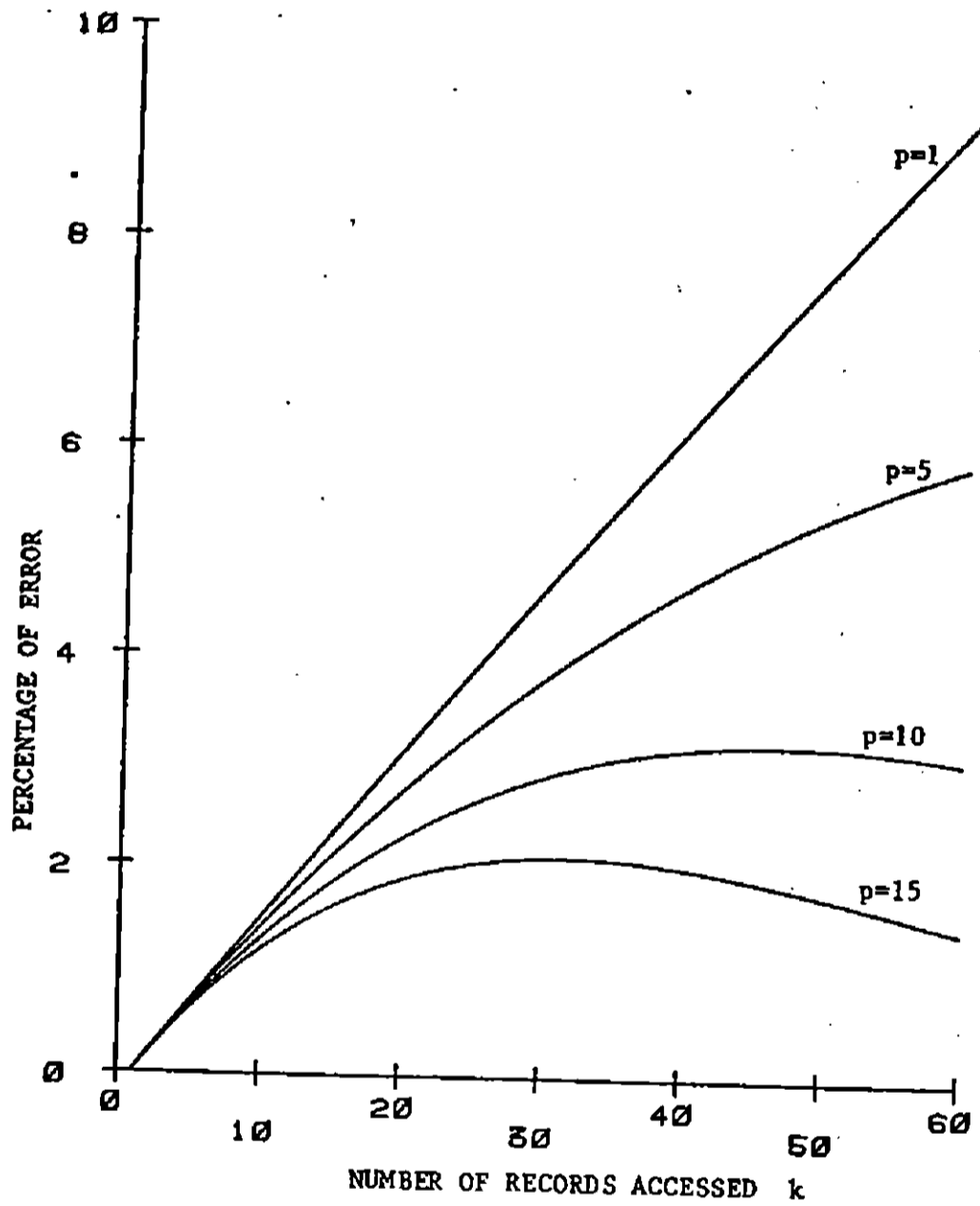Figure 1. Comparison of expected block accesses.

Figure 2. Errors of equation (1) approximation.

## References

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