1974

Prediction of the Oil Free Screw Compressor Performance Using Digital

M. Fujiwara  
*Hitachi Ldt.*

H. Mori  
*Hitachi Ldt.*

T. Suwama  
*Hitachi Ldt.*

Follow this and additional works at: [https://docs.lib.purdue.edu/icec](https://docs.lib.purdue.edu/icec)
INTRODUCTION
The screw compressor was invented by Lysholm in 1934 and was mainly developed by SRM (Svenska Rotor Maskiner AB, Sweden). Since the late 1950s, it has received practical applications for industrial use. Two meshing rotors of the screw compressors rotate at constant speeds without crank and piston mechanism. Therefore, in comparison with the reciprocating compressor it may be compact, light and free from mechanical vibration. Moreover the maintenance is easier, as no valves are necessitated at either suction or discharge. Nowadays, most compressor manufacturers produce screw compressors and their products are increasing year by year, because of the above described merits. However, very few theoretical investigations have been carried out in the screw compressor performance. The effects of the geometrical dimensions of rotors and the operating conditions on the performance have been investigated mostly by the experiments.

We have developed a computer program for predicting the oil free screw compressor performance, mainly assuming that the internal leakage power loss would be most predominant. And applying this program to an air compressor, we have calculated the effects of following factors on the performance; (a) clearances, (b) blow hole, (c) built-in pressure ratio, (d) operating pressure ratio and (e) speed.

COMPRESSION PROCESS
Though the screw compressor is rotating type, the process of compression is quite different from that of turbocompressors and the gas is compressed by the decrease in volume. The screw compressor have a couple of rotors with parallel axis, enclosed by casing. Each rotor has a small number of lobes on it, which are cut out in helix and mesh each other in the casing as shown in Fig.1. The casing and the grooves on the rotors make the working chambers which correspond to the cylinder and piston of reciprocating compressors. A meshing part of each lobe acts as a piston.

The oil free screw compressors have timing gears to prevent the contact between the rotors. In order to rotate the meshing rotors smoothly in operation, it is required to have minimum clearances between lobe tips and casing wall, and between meshing rotors. Therefore there are gas leakages through these clearances and this results in the decrease of the volumetric efficiency and the increase of power loss. Besides these clearances, so-called blow hole is formed between two rotors, which is particular to the screw compressors. This is a triangular vent through which gas leakage also occurs. Considering these internal leakages, we calculate step by step the change of state of the working gas in the control volume and obtain the volumetric efficiencies and the indicated adiabatic efficiencies.

THEORETICAL BASIS AND THE CALCULATING METHOD
In deriving the thermodynamic equation describing the change of state in the control volume, we assume as follows:
1. Gas properties are uniform throughout the control volume at any instant of time.
2. Working substance is perfect gas.
3. Heat transfer to the casing and rotor walls is neglected.
4. Gas enthalpy is equal before and after the leakage path.

The model of the compression process is illustrated in Fig.2. The volume change due to the rotation of the rotors and the mass change due to the internal leakage during a step in time causes the changes of the temperature, the pressure and the internal energy.

Let P, T, G, V and U represent the pressure, the temperature, the mass, the volume and the internal energy at the time t respectively, then following equations are derived with the previous assumptions.

\[
\frac{dP}{P} = \frac{dG_i}{G} + \frac{dT}{T} - \frac{dV}{V} \tag{1}
\]

energy equation
\[
dU = -PdV + C_pT_i dG_i - C_p T dG \tag{2}
\]

the law of conservation of mass
\[
dG = dG_i - dG \tag{3}
\]

where \(dP\), \(dT\), \(dG\), \(dV\) and \(dU\) are the small variations of \(P\), \(T\), \(G\), \(V\) and \(U\) during time interval \(dt\) respectively, \(dG_i\) and \(dG\) are masses of the gas flowing into and out of the control volume respectively, \(C_p\) is the specific heat of gas at constant pressure and \(T_i\) is the temperature of the gas flowing into the volume.

\[
dU = (dG_i - dG) C_p T_i + G C_v dT \tag{4}
\]

where \(C_v\) is the specific heat of gas at constant volume.
From Eqs. (2) and (4), we get
\[
\frac{dT}{T} = -(K-1) \frac{dV}{V} + (K \zeta_i - 1) \frac{dG_i}{G} - (K-1) \frac{dG_e}{G} \tag{5}
\]

where \( K = C_p / C_v \) and \( \zeta_i = T_i / T \).
This equation gives the relationship to calculate step by step the changes of the temperature.

From Eqs. (1), (3) and (5), we get the following relationship to calculate step by step the changes of the pressure.
\[
\frac{dP}{P} = K \left( \frac{dV}{V} + \zeta_i \frac{dG_i}{G} - \frac{dG_e}{G} \right) \tag{6}
\]

The control volume \( V \) in Eqs. (5) and (6) is presented as a function of the rotor turning angle, if the profile and the dimensions of the rotors are given.

Mass of leakage gas is calculated by the following standard formula for steady flow through the converging nozzle.
\[
dG = \left\{ \begin{array}{ll}
\frac{C A_P}{V} \sqrt{\frac{2 K_B}{(K-1) R} \left( \frac{2}{R^2} - \frac{K_{th}}{K} \right) dt}, & 0 \leq r \leq \left( \frac{2}{K_{th}} \right) \frac{K_{th}}{K}
\\
\frac{C A_P}{V} \sqrt{\frac{K_B}{R} \left( \frac{2}{K-1} \right) dt}, & 0 \leq r \leq \left( \frac{2}{K_{th}} \right) \frac{K_{th}}{K}
\end{array} \right. \tag{7}
\]

where
- \( C \): flow coefficient
- \( A \): leakage area
- \( P_i \): the upstream pressure
- \( T_i \): the upstream temperature
- \( g \): acceleration due to gravity
- \( r \): pressure ratio ( \( r \leq 1 \) )
- \( R \): gas constant

Leakage areas except blow hole are given from the products of sealing line lengths and mean clearance, and the leakage areas are given as a function of the rotor turning angle.

In order to simplify the calculation, Eqs. (5) and (6) are not applied to the suction process, but the internal leakage gas back to the suction side is considered to calculate the temperature rise of the inlet charge. The temperature of the mixed gas will be calculated by the assumptions that these gas mix at the end of the suction process and the inlet pressure drop is constant during that process. We obtain the inlet pressure and temperature at just before the compression process.

On the other hand, the pressure drop in the discharge process is ignored in this program. But when the internal compression pressure just before releasing the gas to discharge differs from the operating discharge pressure, the gas in the control volume is suddenly expanded or compressed at the beginning of the discharge process. Then, the temperature is calculated by following equation.
\[
\frac{T_d}{T_a} = \left( \frac{K-1}{K} \frac{P_d}{P_s} + \frac{1}{K} \right) T_a \tag{7}
\]

where
- \( T_d \): discharge temperature
- \( P_d \): discharge pressure
- \( P_s \): pressure just before releasing the gas to discharge
- \( T_a \): temperature just before releasing the gas to discharge

After the calculation of the change of state at suction, compression and discharge process, the volumetric efficiency \( \eta_v \) and the indicated adiabatic efficiency \( \eta_{ad} \) are obtained. They are defined as follows:
\[
\eta_v = \frac{V_s}{V_{th}} \tag{9}
\]
\[
\eta_{ad} = \frac{A_{th} \eta}{A_i} \tag{10}
\]

where
- \( V_s \): actual delivery of gas an inlet condition
- \( V_{th} \): theoretical displacement delivery of gas on inlet condition
- \( A_i \): theoretical displacement delivery of gas on inlet condition
- \( A_{th} \): isentropic compression work calculated from the P-V diagram
- \( \eta \): isentropic compression work for \( V_s \)

In this program, the leakage areas and control volume are given from subroutine programs. It is enough to replace these programs, if other relationships between the performance and the rotor wrap angle, helix angle, clearances or others are required.

RESULTS
Some results of this calculation are illustrated below for an air compressor with rotors having the symmetric circular profile. Fig. 3 shows a calculated P-V diagram for several \( \xi_s \), where \( \xi_s \) is the clearance between lobe tips and casing. \( \xi_{th} \) in the figure is the standard \( \xi_s \) value of our specifications. The theoretical curve of isentropic compression is also shown for comparison. As you see, the increase in \( \xi_s \) results in the increase of compression work.

Fig. 4 shows the effects of the clearance between meshing rotors on \( \eta_v \) and \( \eta_{ad} \). The suffix \( o \) refers to the standard value of our specifications. It shows that \( \eta_v \) and \( \eta_{ad} \) decrease linearly as the clearance increases.

Fig. 5 shows the effect of the operating pressure ratio on \( \eta_{ad} \) for several \( \xi_s \). \( \xi_s \) is the built-in pressure ratio which is defined as:
\[
\xi_s = \frac{P_i}{P_s} \tag{11}
\]
where $P_i$: internal compression pressure just before the delivery process assuming an isentropic compression.

$P_s$: inlet pressure

The maximum value of $\eta_{ad}$ exists at a certain value of $\gamma$. For the smaller $\gamma$ from this point, the slope of $\eta_{ad}$-curve is steeper than for the larger $\gamma$. This is one of the characteristics of the screw compressor.

$P_s$ is the maximum value of $\eta_{ad}$. For the smaller $\gamma$ from this point, the slope of $\eta_{ad}$-curve is steeper than for the larger $\gamma$. This is one of the characteristics of the screw compressor.

Fig. 6 shows the effects of rotating speed and blow hole on $\eta_{ad}$. The upper curve "blow hole area=0" means the efficiency for the imaginary compressor with no blow hole.

At the lower speed, the efficiency is rather low, but the higher efficiency can be attained at the higher speed. The reason is as follows: The delivery volume rate is nearly proportional to the speed, but the leakage flow rate which depends on pressure ratio is independent on the speed, and so the relative leakage flow becomes smaller as the speed increases.

It is also shown that the effect of blow hole area is much larger at the lower speed.

**SUMMARY**

We have developed a computer program for predicting the oil free screw compressor performance, assuming that the internal leakage power loss would be most predominant.

The effects of clearances, blow hole, built-in pressure ratio, operating pressure ratio, and speed on the efficiencies were calculated. Further, we can predict the performance of compressors with other rotor profiles, using this program.

---

Fig. 1 The rotors of the screw compressor

Fig. 2 Model of the compression process
Ps: suction pressure  
Vo: volume at the end of suction process

Fig. 3 The effect of clearance between rotor tips and casing walls on P - V diagram

\( \mathcal{\kappa} \): operation pressure ratio  
\( \mathcal{\kappa}_i \): built-in pressure ratio

Fig. 5 \( \eta_{adi} \) as a function of pressure ratio \( \kappa \)  
( parameter: \( \mathcal{\kappa}_i / \mathcal{\kappa}_{i0} \) )

Fig. 6 \( \eta_{adi} \) as function of speed.  
( parameter: blow hole area )