1974

Computer Simulation of Multicylinder Compressor

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INTRODUCTION

Relatively little information had been available in the published literature on the mathematical modeling of reciprocating compressors until 1972[1] when an extensive, detailed and valuable conference on the subject was held for the first time. Although the reciprocating compressor may seem simple and at best appears to be a "special case" of the internal combustion engine, which evidently has been investigated extensively, its analytical simulation is not at all a simple matter. Often, numerous assumptions, which are thought to be of second order, are made in the analysis in order to analyze first order effects. Such is the case, for example, with the unsteady flow pattern in the suction cavity, discharge cavity and discharge tube. The valve dynamics analysis which includes valve displacement, valve velocity and valve stress and strain is another example where simplifications are often made and an experimental information is incorporated in order to avoid a very complicated and extensive analysis. It appears at present that no compressor simulation can be completed without being supplemented by some experimental correlation. Two pieces of information which are commonly determined experimentally are the effective flow area and the effective force area. The effective flow area is a major factor in the determination of the mass flow rate and volumetric efficiency, while the valve dynamics analysis is greatly influenced by the effective force area of the suction and discharge valves. The success or failure of a computer simulation should be judged by comparing the simulation with available experimental data.

Typical experimental data which may be made available at a compressor manufacturing plant, may include suction and discharge pressures, speed (RPM), an estimate of the cylinder wall temperature, an estimate of the suction temperature, volumetric efficiency, mass flow rate and the cycle-average torque. The measurement of the instantaneous torque requires delicate instruments and cannot be achieved easily. The input to the computer simulation will include suction and discharge pressures, speed, cylinder wall temperature and suction temperature, while the predicted results may include the volumetric efficiency, mass flow rate and the required torque.

SIMULATED PHENOMENON

The present computer simulation is an extension to and a modification of a previous analytical simulation[2] and thus includes some of the assumptions made previously. Most notable among the assumptions is the utilization of an ideal-gas relationship in formulating the rate form of the first law of thermodynamics with temperature, rather than internal energy, as a dependent variable. The above substitution causes our analysis to be mixed ideal gas-real gas analysis, in that the temperature is computed as stated above and in reference 2, however the pressure is computed from the refrigerant equation of state using the temperature which was obtained from the solution of the first law of thermodynamics. We have extended the previous model by incorporating into it a single degree of freedom valve analysis, a torque analysis for an n-cylinder reciprocating compressor assuming a constant nominal rotational speed and a provision for R-22 properties in addition to the existing R-12 properties.

ANALYTICAL PRESENTATION

The thermal-fluid dynamics model has been explained earlier[2] and will not be repeated here. For the sake of completeness, however, we restate the rate form of the first law of thermodynamics which has been incorporated into the model:

$$\frac{\dot{T}}{T} = \frac{\dot{Q}}{C_v T W} - \frac{R}{JC_v}\frac{\dot{V}}{V} + k\frac{T_s}{T}\frac{\dot{W}_s}{W} - k\frac{T_D}{T}\frac{\dot{W}_D}{W} - \frac{\dot{W}}{W} \quad (1)$$

where T is the gas temperature, Q is the heat flux, W is the gas mass, V is the gas volume, $k = C_p/C_v$ is the ratio between constant pressure and constant volume specific heats, R is the universal gas constant, J = 778 ft-lb_f/Btu, the subscripts D and s designate discharge and suction respectively and a dot above a symbol designates the time rate form of a variable, e.g., $\dot{Q} = dQ/dt$.

The valve dynamics analysis considers the single degree of freedom displacement and velocity analysis of ring valves. Of the two ring valves, the discharge valve is spring-activated (see Figure 1) and the suction valve is inertia-activated. For the discharge valve, the equation of motion for a single degree of freedom spring-mass system neglecting the damping effect is

$$\frac{d^2y(t)}{dt^2} + \frac{k}{m}y(t) - \frac{F(t)}{m} = 0 \quad (2)$$

where y(t) is the valve displacement as a function of time t, k is the spring constant, m is the sum of the mass of the valve and half of the mass of the spring, and F(t) is the differential pressure force acting on the valve as a function of time.

The solution of Equation (2) by Laplace transform method yields the velocity, $\dot{y}(t)$, and displacement, y(t), characteristics of a single-degree of freedom spring-activated ring valve:

$$y(t) = \frac{\dot{y}(0)}{a} \sin at + y(0) \cos at + \frac{1}{am} \int_0^t F(\tau) \sin \left[a(t-\tau) \right] d\tau \tag{3}$$

and

$$\dot{y}(t) = \dot{y}(0) \cos at - a\, y(0) \sin at + \frac{1}{m} \int_0^t F(\tau) \cos \left[a(t-\tau) \right] d\tau \tag{4}$$

where $a^2 = k/m$, $y(0) = y(t)$ at t=0 and $\dot{y}(0) = \dot{y}(t)$ at t=0. Equations (3) and (4) can be written for each time step at which the numerical solution is sought if we assume that F(t) remains constant within the small time step under considerations, i.e., $F(t) \approx \tilde{F}$ = constant. This assumption is realistic since the numerical solution progresses in small steps (approximately 0.5 crank-angle degrees). Integration of Equations (3) and (4) with the above assumption yields

$$y(\Delta t) = \frac{\tilde{F}}{ma^2} + \frac{\dot{y}(0)}{a} \sin (a\Delta t) + \left[y(0) - \frac{\tilde{F}}{ma^2} \right] \cos (a\Delta t) \tag{5}$$

$$\dot{y}(\Delta t) = \dot{y}(0) \cos(a\Delta t) - a \left[y(0) - \frac{\tilde{F}}{ma^2} \right] \sin (a\Delta t) \tag{6}$$

In Equations (5) and (6) $\dot{y}(0)$ and y(0) are the valve velocity and the valve displacement respectively at the beginning of the time interval Δt and are in turn equal to $\dot{y}(\Delta t)$ and $y(\Delta t)$ of the preceding time step.

The formulation of the valve displacement and velocity as given in Equations (5) and (6) allows us to evaluate y(t) and $\dot{y}(t)$ directly without resorting to any iterative procedure. The only assumptions made are the assumptions involved in the basic equation of motion (2) and that the force $\tilde{F}$ which is known at the beginning of the time interval remains constant throughout the time interval.

The suction valve which is located on the piston is an inertia-activated ring valve. The equation of motion for a simple rectilinear motion of a mass in a force field is:

$$\frac{d^2y(t)}{dt^2} = F(t)/m \tag{7}$$

where m is the mass of the valve, and F(t) is the sum of the differential pressure force acting on the valve and the inertia force of the valve as a function of time. Integration of Equation (7) gives the velocity and displacement characteristics of the suction valve:

$$\dot{y}(t) = \frac{1}{m} \int_0^t F(\tau) d\tau + \dot{y}(0) \tag{8}$$

$$y(t) = y(0) + \dot{y}(0)t + \frac{1}{m} \int_0^t \int_0^{t_1} F(\tau) d\tau dt_1 \tag{9}$$

Similar to the formulation of the discharge valve we may integrate Equations (8) and (9) assuming $F(t) \approx \tilde{F}$ = constant within the small time interval Δt:

$$\dot{y}(\Delta t) = \dot{y}(0) + \frac{\tilde{F}}{m} \Delta t \tag{10}$$

$$y(\Delta t) = y(0) + \dot{y}(0)\Delta t + \frac{\tilde{F}}{2m} (\Delta t)^2 \tag{11}$$

To determine the torque requirement of a multi-cylinder compressor, we restrict our attention to a single cylinder first and proceed in a manner similar to that reported by Gatecliff[3]. By reference to Figures 2, 3 and 4, a force and moment balance on the piston, the connecting rod and the crankshaft gives,

$$F_{3px} + F_{WF} - P = M_p A_p \tag{12}$$

$$F_{WP} - F_{3py} = 0 \tag{13}$$

$$F_{23x} - F_{3px} = M_{cr} A_{cgx} \tag{14}$$

$$F_{23y} + F_{3py} = M_{cr} A_{cgy} \tag{15}$$

$$F_{3py}(M-L)\cos\phi + F_{3px}(M-L)\sin\phi + F_{23y} M\cos\phi - F_{23x} M\sin\phi + T_{23} - T_{3p} = I_{cr} A_3 \tag{16}$$

$$F_{23x} R\sin\theta - F_{23y} R\cos\theta + T_r - T_b - T_{23} = 0 \tag{17}$$

where F_{3px} is the force exerted on the piston in the x-direction by the connecting rod; P is the force exerted on the piston due to differential gas pressure; F_{WP} is the force exerted on the piston normal to the cylinder wall; F_{WF} is the force due to friction exerted on the piston parallel to the cylinder wall; T_{3p} is the torque due to friction exerted on piston by the connecting rod; A_p is the acceleration of the piston in the x-direction; M_p

is the piston mass; F_{3py} is the force exerted on the piston in the y-direction by the connecting rod; ϕ is the angle between the connecting rod and the x-direction; F_{23x} is the force exerted on the connecting rod in the x-direction by the crankshaft; F_{23y} is the force exerted on the connecting rod in the y-direction by the crankshaft; T_{23} is the torque due to the friction exerted by the crankshaft on the connecting rod; M is the distance from the crankshaft bearing center to the connecting rod center of gravity; L is the length between the connecting rod bearing centers; A_{cgx} is the acceleration of the connecting rod center of gravity in the x-direction; A_{cgy} is the acceleration of the connecting rod center of gravity in the y-direction; M_{cr} is the connecting rod mass; I_{cr} is the connecting rod polar moment of inertia about its center of gravity; A_3 is the angular acceleration of the connecting rod; T_b is the torque due to the friction exerted on the crankshaft by the crankshaft bearings; T_r is the torque required by the whole assembly; R is the crankshaft throw and θ is the crank angle. In the above force and torque analysis, the crankshaft speed has been assumed constant and the effect of cyclic variation of angular velocity has been omitted. This effect is not considered to be important in multi-cylinder compressors. The following kinematic and torque-force relations exist among the various variables (see Figure 5).

$$\phi = \text{Arc sin } (\frac{R}{L} \sin\theta) \tag{18}$$

$$W_3 = \frac{R \cos \theta}{L \cos \phi} W_2 \tag{19}$$

$$A_3 = (W_3^2 - W_2^2) \tan\phi \tag{20}$$

$$A_{cgy} = - \frac{(L - M)}{L} R W_2^2 \sin\theta \tag{21}$$

$$A_{cgx} = M (A_3\sin\phi + W_3^2\cos\phi) - RW_2^2\cos\theta \tag{22}$$

$$A_p = L (A_3\sin\phi + W_3^2\cos\phi) - RW_2^2\cos\theta \tag{23}$$

$$F_{WF} = S_{WP}M_{WP}F_{WP} \tag{24}$$

$$T_{3p} = S_{3p} \sqrt{F_{3px}^2 + F_{3py}^2} M_{3p}R_{3p} \tag{25}$$

$$T_{23} = \sqrt{F_{23x}^2 + F_{23y}^2} M_{23}R_{23} \tag{26}$$

$$T_b = \sqrt{F_{23x}^2 + F_{23y}^2} M_{cs}R_{cs} \tag{27}$$

$$S_{WP} = \frac{-V_p}{|V_p|} \frac{F_{3py}}{|F_{3py}|} \tag{28}$$

$$S_{3p} = \frac{W_3}{|W_3|} \tag{29}$$

where W_2 is the angular velocity of the crankshaft; W_3 is the angular velocity of the connecting rod; M_{WP} is the coefficient of friction between the piston and the cylinder; M_{3p} is the coefficient of friction between the connecting rod and the wrist pin; M_{23} is the coefficient of friction between the connecting rod and the crankshaft; M_{CS} is the coefficient of friction between the crankshaft and the main bearings; R_{3p} is the radius of the wrist pin bearings; R_{23} is the radius of the throw bearing; R_{cs} is the radius of the main bearing and V_p is the velocity of the piston.

When Equations (13), and (18) through (29) are substituted into Equations (12), and (14) through (16), the four resulting equations are solved iteratively for the four unknowns, F_{23x}, F_{23y}, F_{3px} and F_{3py}. Substituting F_{23x}, F_{23y}, T_{23} and T_b into Equation (17) yields the required torque T_r. The friction coefficients in Equations (26) and (27) are calculated by the following two equations:[4]

$$(\frac{R}{C})^2 \frac{\mu N}{P} (\frac{L}{D})^2 = \frac{(1-\varepsilon^2)^2}{\pi\varepsilon} \left[\frac{1}{\pi^2(1-\varepsilon^2) + 16 \varepsilon^2}\right]^{1/2} \tag{30}$$

$$f (\frac{R}{C}) (\frac{L}{D})^2 = \frac{2\pi}{(1-\varepsilon^2)^{1/2}} \left\{\frac{(1-\varepsilon^2)^2}{\pi\varepsilon} \times \left[\frac{1}{\pi^2(1-\varepsilon^2) + 16 \varepsilon^2}\right]^{1/2}\right\} \tag{31}$$

where R is the shaft radius; R_b is the bearing radius; $C = R_b - R$ is the radial clearnce; D = 2R is the shaft diameter; L is the axial length of the bearing; e is the eccentricity; $\varepsilon = e/c$ is the eccentricity ratio; μ is the oil viscosity (lb-sec/in^2); N is the speed (R.P.S.); W is the load; P = W/DL is the normal unit load (psi); and f is the coefficient of friction. The value of T_r calculated from Equation (17) is the torque required for a single cylinder. To calculate the torque required by an n-cylinder compressor, assuming a constant nominal speed, the values of $T_r(\theta)$ should be added at $2\pi/n$ phase angles.

NUMERICAL SOLUTION

A predictor-corrector scheme was used to obtain a simultaneous solution of the mass flow rate, the heat flow rate and the energy equation. The volume of the gas in the cylinder was determined directly from geometric considerations and the pressure was determined from the fluid equation of state. Knowledge of the pressure and the effective force area at each time step permits the evaluation of the forces which act on the valves and on the piston. These in turn can be used in evaluating the valve displacement and velocity and the torque required by the compressor as explained earlier. The solution was started by assuming initial conditions for the temperature, pressure and mass flow rate and was pursued until the computed values at the end of the cycle agreed to within the desired accuracy with the assumed initial conditions.

RESULTS AND DISCUSSIONS

The success or failure of the model can be judged by comparing its predictions with available experimental data covering a wide range of operating conditions. Some of the experimental data which were supplied to us by Mr. H. Grant of the Thermo King Corporation, Minneapolis, Minnesota for compressor model no. 426 are listed in Table I together with the model prediction.

Table I indicates that the simulation prediction of the volumetric efficiency (and hence the mass flow rate) compares favorably with a wide range of experimental conditions which include a three-fold variation in compression ratio, two different refrigerants, two different speeds and three different suction temperatures. The agreement between the predicted and measured average torque is not as good as the agreement between the volumetric efficiencies. Two possible reasons for this discrepancy are the omission of the oil pump torque from the simulation (this is merely an additive factor which will close the gap between the experimental data and the predictions since the simulation mostly underpredicts the torque), and the assumption of a constant nominal speed.

In a parametric study we have found that the value of the suction flow coefficient has a marked effect on the simulation prediction and should be determined very carefully. The results which are reported in Table I were made with a constant suction flow coefficient.

The cyclic variations of the valve displacement, valve velocity, torque of a single cylinder compressor, torque of a multicylinder compressor and cylinder pressure are displayed in Figures 6, 7, and 8. Figure 6 shows the suction valve and discharge valve displacement superimposed on the cylinder pressure. Figure 7 shows the suction valve and discharge valve velocity superimposed on the cylinder pressure and Figure 8 shows the torque of a single cylinder and the torque of a multicylinder compressor superimposed on the cylinder pressure.

The predicted cylinder pressure compared favorably with an oscilloscope display of the cylinder pressure. However, no other experimental data were available for comparison with the simulation prediction. The torque curve in Figure 8 displays four peaks in accord with the number of cylinders. Although a negative torque is exhibited for a single cylinder, the torque is always positive for the multicylinder compressor.

CONCLUSIONS

The computer simulation of the multicylinder compressor as presented herein is capable of adequately predicting the gas pressure in the cylinder, the gas temperature, mass flow rate, volumetric efficiency, torque, valve displacement and valve velocity. Undoubtedly there is room for improving the simulation by a more rigorous analysis of the valve dynamics, torque, energy equation, and transient flow effects; however, a valuable insight into the compressor behavior may be gained by using the present version of the simulation.

ACKNOWLEDGMENT

The experimental data were furnished by Mr. H. Grant of Thermo King Corporation, Minneapolis, Minnesota. His cooperation is greatfully acknowledged.

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3. Gatecliff, G. W., "A Digital Simulation of a Reciprocating Hermetic Compressor Including Comparison With Experiments", PhD Thesis, The University of Michigan, 1969.

4. Raimondi, A. A., Private communication, 1973.

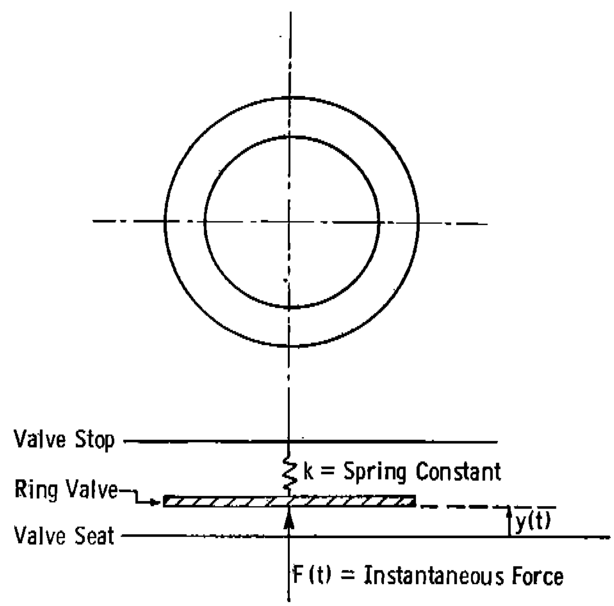

Fig. 1 — Schematic diagram of the ring valve with forces acting on it

TABLE I

COMPARISON BETWEEN THE COMPRESSOR SIMULATION PREDICTION AND THERMO KING EXPERIMENTAL DATA

Compression Ratio	Refrigerant	Speed (RPM)	Suction Temp. °F	Mass Flow Rate (lbm/hr)	Vol. Efficiency %	Torque ft-lb
2.91	R-22	1800	65	(1930)	82 (83.8)	41.7 (42.4)
3.58	R-22	2200	65	(2685)	78 (78.4)	55.9 (55.9)
4.25	R-22	1800	65	(1860)	76 (80.7)	52.2 (52)
4.82	R-12	2200	110	1364 (1380)	77 (77.5)	33.2 (28)
5.99	R-12	2200	100	884 (912)	71 (73.2)	24.8 (20.4)
9.69	R-12	2200	100	457 (472)	66 (68.4)	16.8 (13.3)

Note: The values in parenthesis are the simulation predictions.

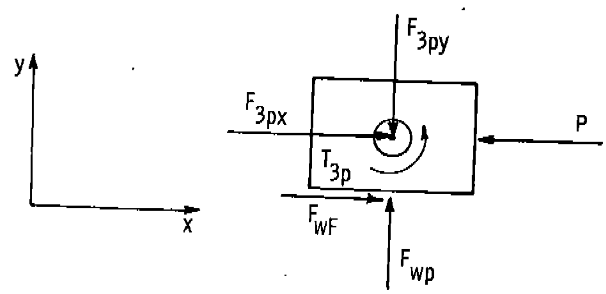

Fig. 2 —Free body diagram of piston

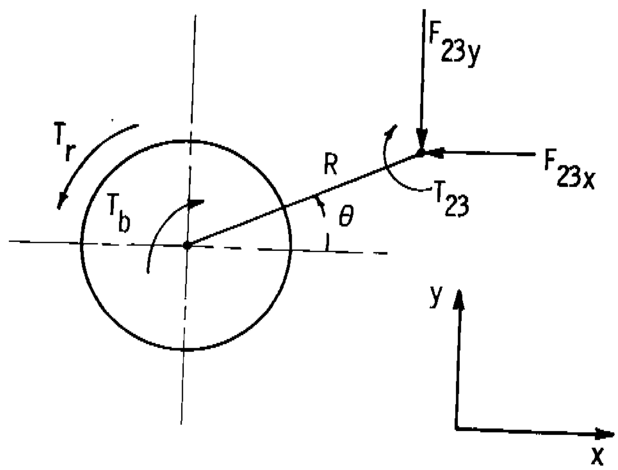

Fig. 4 —Free body diagram of crankshaft

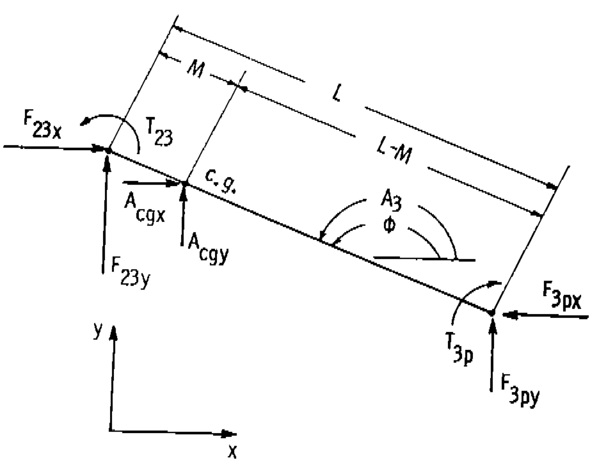

Fig. 3 —Free body diagram of connecting rod

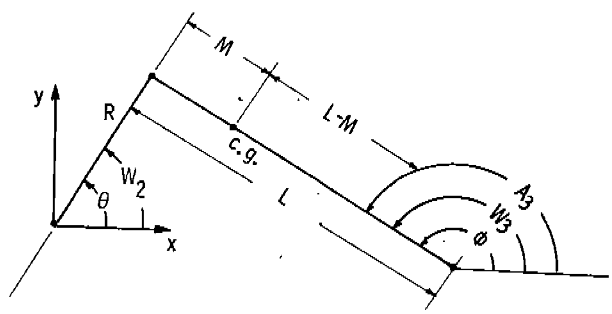

Fig. 5 —Kinematic conventions for slider-crank mechanism

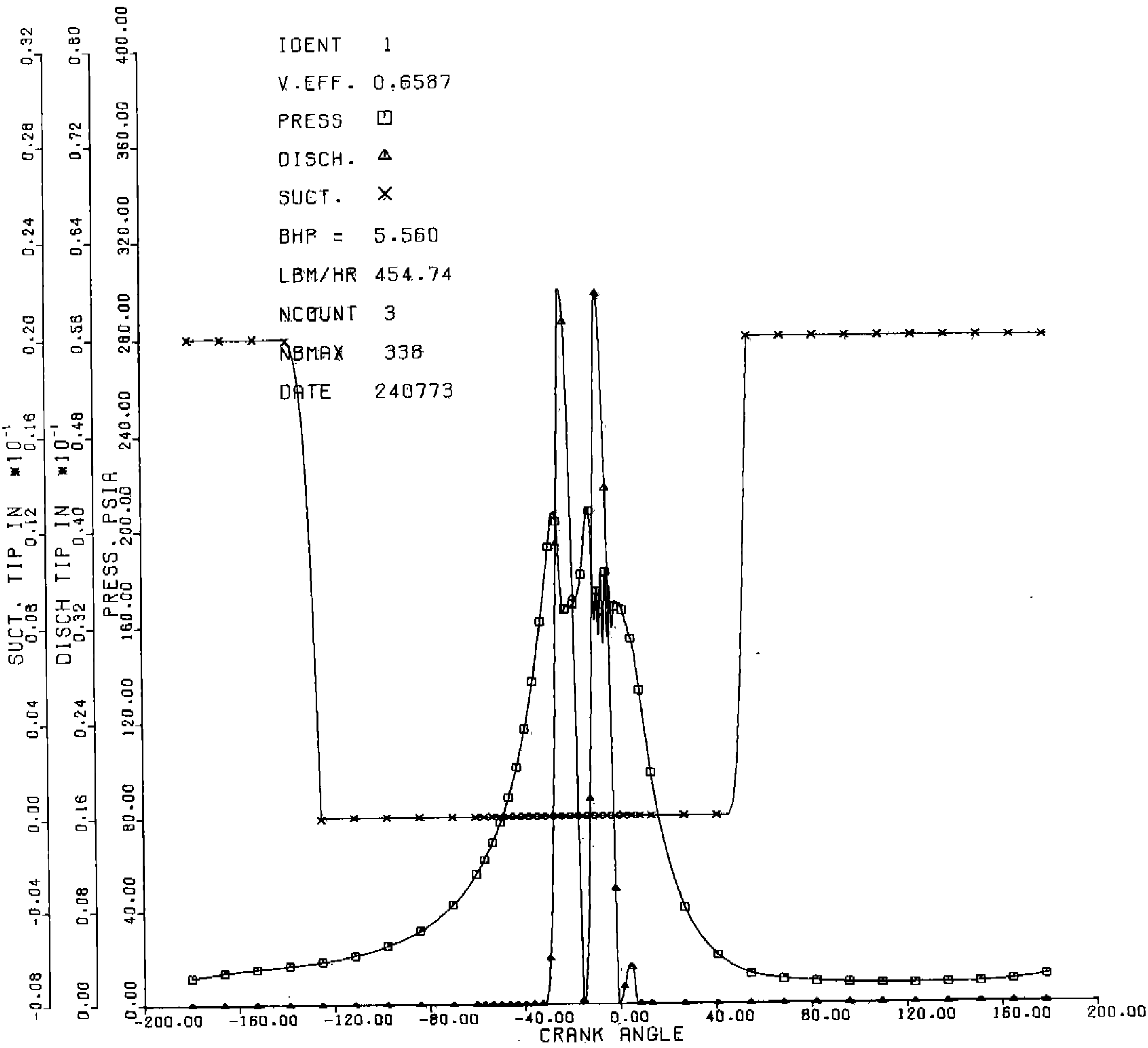

Fig. 6 - Variation of the Suction and Discharge Valve Displacements and Gas Pressure

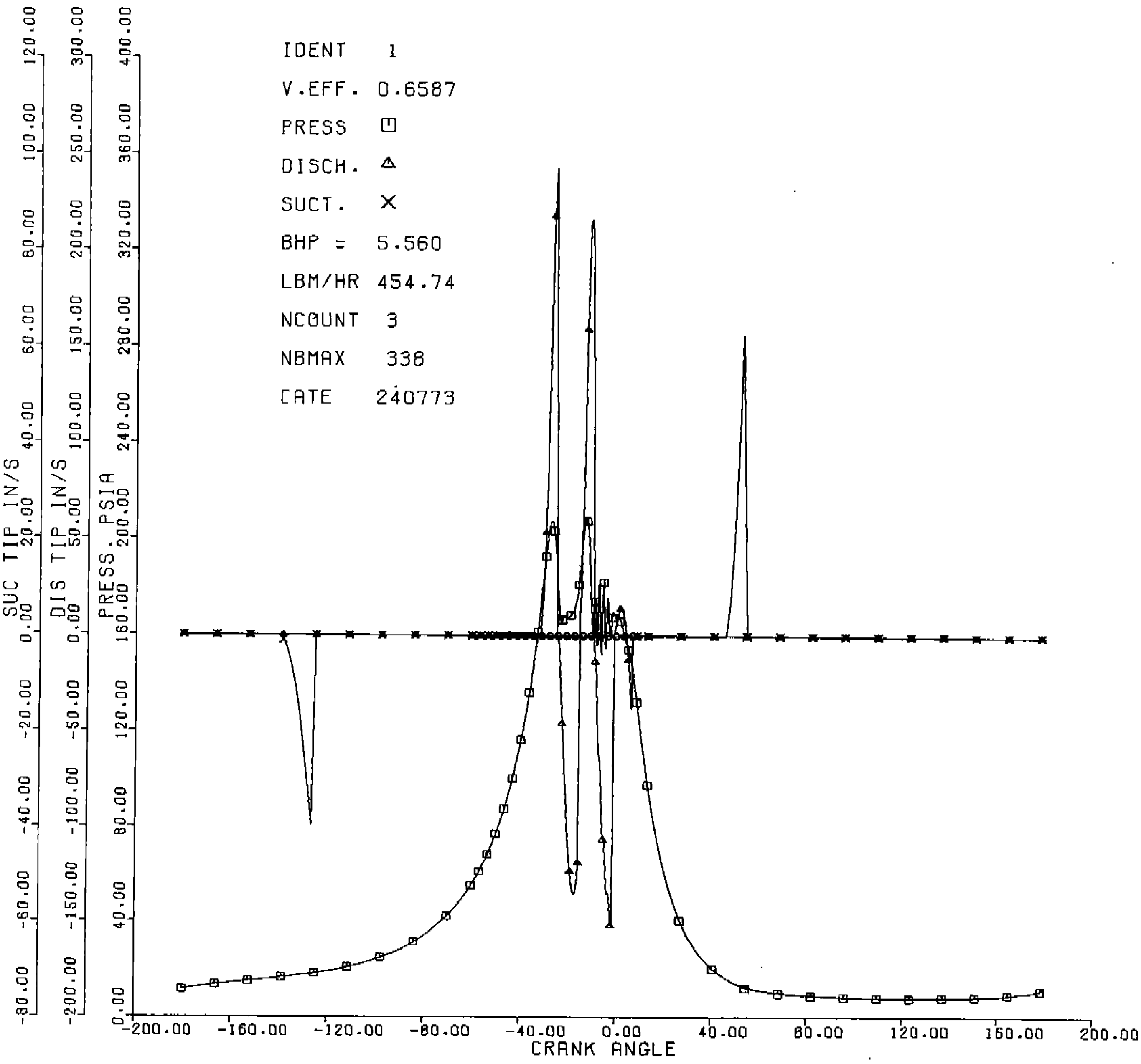

Fig. 7 - Variation of the Suction and Discharge Valve Velocities and Gas Pressure

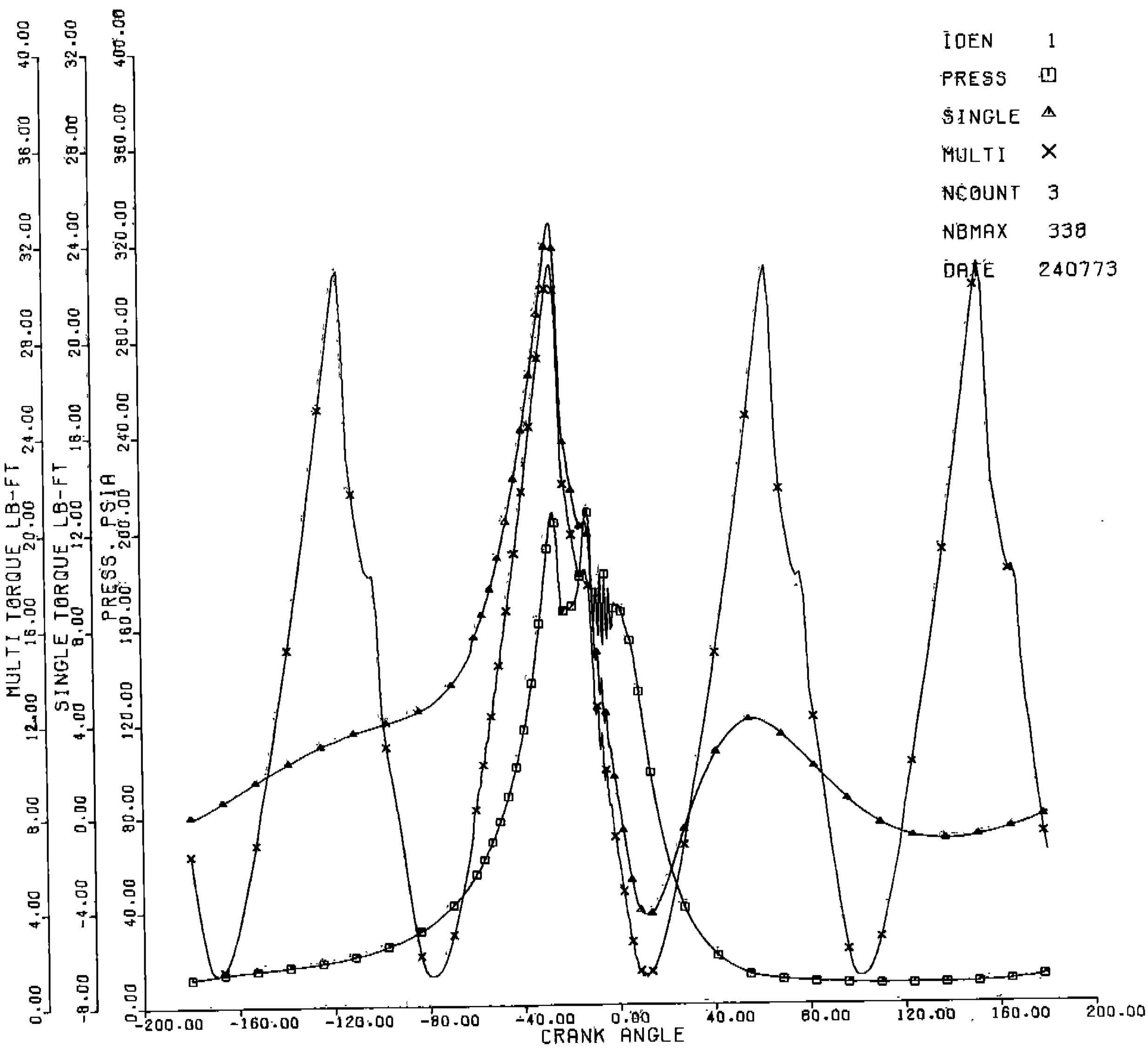

Fig. 8 - Variation of the Torque Requirements of a Single Cylinder and of a Multi-Cylinder Compressor