A Simple Numerical Solution for Compressor Valves with One Degree of Freedom

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An approximate numerical method for calculating the lift and pressure drop for self actuating compressor valves with one degree of freedom is described. This method is in some ways more efficient than those usually used and lends itself to rapid calculation by digital computer. Results of this method are compared with measured results and those calculated by a conventional method and agreement is shown to be good.

INTRODUCTION:

As is well known, the dynamics of the moving elements in a compressor valve play a vital role in determining the horsepower, capacity and reliability of a compressor. In a machine that is built for a limited range of applications, the best valve design can be determined during development testing, but in a machine that is built for a wide range of operating pressures and gas composition, this is impossible.

Nowadays computers are used for sizing compressors for a given application, but it is usually impractical to include a full calculation of the valve action in a sizing program. For that reason, several sets of guide rules using the parameters that control valve performance have been set up and used for valve selection. While they have been extremely useful, these methods do not always predict unsatisfactory valve operation.

The calculation method described here allows an evaluation of the valve element movement that is considerably more complete than that provided by parametric methods and yet is not as complicated or lengthy as the many excellent research and development programs previously written for calculating valve element dynamics.

NOMENCLATURE:

- a — Dimensionless cyl. Volume= $\dfrac{\text{Cyl. Volume}}{\text{Swept Volume}}$
- a_s — Spring force at zero lift
- A_{eq} — Valve equivalent flow area at full lift
- A_1 to A_5 - Constants (see Table II)
- b_s — Spring constant
- B_1 to B_5 - Constants (see Table II)
- C — Constant (see Table II)
- CN — Constant (see Table II)
- C_D — Pressure force on element/pressure drop across valve
- d — Damping factor
- E — Dimensionless natural frequency of element = 60 Nat'l Frequency/Compr. RPM
- f — Natural frequency of valve element
- $f(y)$ — The equivalent flow area of the valve/ The equivalent flow area at full lift
- G — Variable defined by equation(5) Table I
- H — Variable defined by equation(6) Table I
- m — Mass of gas in cylinder
- M — Mass of element
- n_v — Isentropic volume exponent of gas
- N — Compressor speed
- p — Passage pressure
- Δp — Pressure drop across valve
- p_c — Pressure in cylinder
- p_{FL} — Pressure drop to fully open valve
- p_O — Pressure drop to start element opening
- P_O — Dimensionless form of p_O=$P_O/(p_{FL} - p_O)$
- P_s — Dimensionless pressure = $\pm p/(p_{FL} - p_O)$ (+ for suction valve; - for discharge valve)
- R — Gas constant
- r — Dimensionless cylinder pressure = Cylinder Pressure/Passage Pressure
- SN — Constant (see Table II)
- t — Time from start of step
- T — Gas temperature
- V — Fraction of critical damping= $d/4\pi Mf$
- V_c — Cylinder volume
- V_{sw} — Swept Volume
- y — Lift of valve element
- y_{FL} — Full lift
- Y — Dimensionless lift = Lift/Full Lift
- θ — Crank angle
- ρ — Gas density

Subscripts:

- $_1$ — Before step
- $_2$ — After step

OUTLINE OF THE NUMERICAL METHOD:

The equations used for the numerical solution for the valve element movement and pressure drop are given in Table I and their derivation is outlined in a later section. The solution procedes by calculating the conditions at the end of a time step, indicated by suffix 2, from those at the beginning of the step, indicated by suffix 1. The calculation procedes as shown in the following list.

1. Calculate the A, B and C constants (Table II).
2. Set the initial values of θ, r, y, and $dY/d\theta$.
3. Calculate a_1, $(da/d\theta)_1$, and a_2 from the compressor geometry.
4. Calculate $(dy/d\theta)$, from equation (3) (Table I).
5. Calculate G and H from equation (5) and (6) (Table I).
6. Calculate Y_2, $(dY/d\theta)_2$ and r_2 from equations (1), (2), and (4) (Table 1).
7. Allow for the valve reaching a stop (seat or guard).
8. Repeat for the next time step starting with step 3.

Thus, the calculation loop involves only the calculation of the cylinder volume (a) and rate of change of volume $(da/d\theta)$, the evaluation of six algebraic expressions given by equations (1) to (6) and checks for impact on the stops.

The case of valves with elements of different geometries can readily be solved by calculating y_2 and $(dY/d\theta)_2$ separately for each type of element and then using the total equivalent area in the calculation of r_2.

ASSUMPTIONS AND INACCURACIES:

The main assumptions made in the calculation procedure described here are:

1. The valve elements have a single degree of freedom with a natural frequency independent of lift.
2. Unsteady flow effects in the cylinder or piping are neglected.
3. The gas temperature in the cylinder is not calculated correctly.
4. The valve parameters can be described as shown in the following section (Fig. 1).
5. To simplify the calculation, some errors are introduced into the numerical integration scheme. This is, of course, true of any numerical integration.
6. Damping is of the viscous type with the damping force proportional to the velocity.
7. The gas behaves as an incompressible fluid for flows through the valve.
8. The gas behaves as a perfect gas for change of state in the cylinder.

SPECIFICATION OF VALVE PARAMETERS:

We have found it conveneint to define the valve geometry with reference to results of steady flow tests as plotted in Fig. 1A. The quantity p_o will be negative as shown for a valve with 'free lift' (i.e. one for which the spring does not start to operate until the valve is partially open). p_o will be positive for a valve with 'preload' in which there is a positive spring force even with the valve closed. For this analysis, we assume that the spring force is a linear function of lift (Fig. 1B) and it is then easy to calculate the relationship between equivalent area and lift (Fig. 1C) represented by f(y).

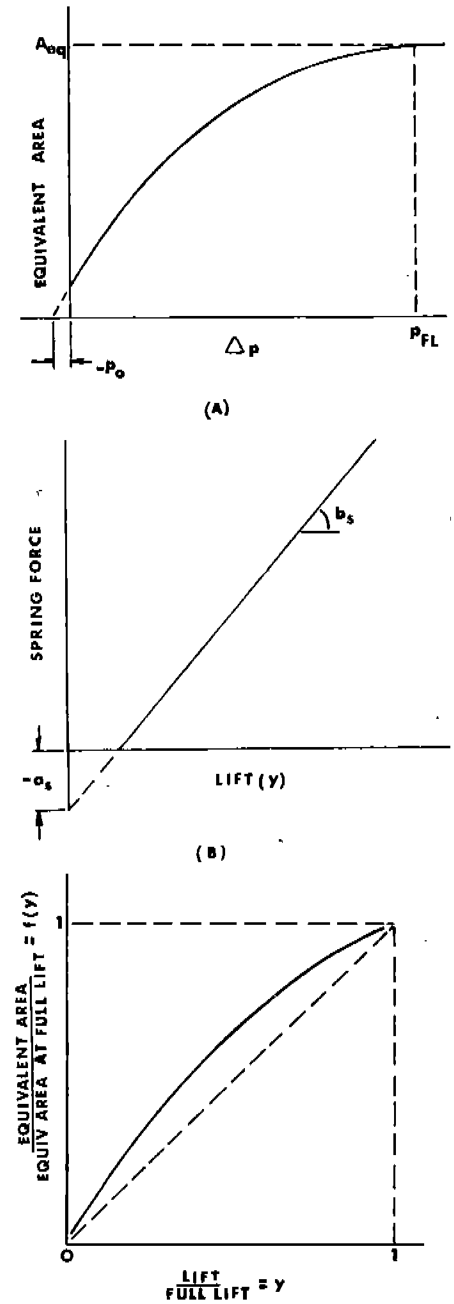


FIG 1

Also of importance are the natural frequency of the valve element which may be calculated for a simple geometry or obtained experimentally, the damping and coefficient of restitution for impact on the stops.

DERIVATION OF EQUATIONS FOR VALVE ELEMENT MOVEMENT:

To describe the techniques involved, the method will first be described for a valve with no damping, preload or free lift.

1. No damping, preload or free lift

   For this case the spring force is $b_s y$ and when the valve is at full lift, but not pressing on the stop, the spring force ($b_s y_{FL}$) equals the pressure force ($C_D P_{FL}$).

$$b_s = \frac{C_D\, P_{FL}}{y_{FL}}$$

   The equation of motion is:

$$M \frac{d_2 y}{dt^2} = C_D\, \Delta p - \frac{C_D\, P_{FL}}{y_{FL}}\, y$$

   We will seek a solution of this equation that applies to a time step Δt and will assume that, during this step, the pressure drop Δp is given by:

$$\Delta p = \Delta p_1 + \left(\frac{d\Delta p}{dt}\right) t$$
$$= p(1-r_1) - p\left(\frac{dr}{dt}\right)_1 t$$

   where suffix 1 refers to conditions at the start of the step.

   The equation of motion then becomes:

$$M(d_2 y/dt^2) = C_D(p(1-r_1) - p(dr/dt)_1 t) - C_D\, P_{FL}\, y/y_{FL}$$

   The solution to which is:

$$y = (p/P_{FL})(1-r_1)y_{FL} - (p/P_{FL})(dr/dt)_1 y_{FL}\, t$$
$$+ \cos[\sqrt{((C_D\, P_{FL})/(My_{FL}))}t + \phi]$$

   where α, ϕ are arbitrary constants.

   This solution represents simple harmonic motion about the equilibrium position defined by the varying pressure drop.

   Solving for α and ϕ using the conditions that $y = y_1$ and $dy/dt = (dy/dt)_1$ at $t = 0$, converting from t to θ and to nondimensional variables gives equations (1) and (2) (Table I). This involves only simple algebraic manipulation and will not be given here.

   In contrast to the conventional methods, this numerical solution is exact except for the assumption regarding the pressure drop across the valve. The time step should be chosen such that this approximation is reasonable.

2. With damping and preload or free lift.

   In this section it is assumed that the spring force is given by $a_s + b_s y$ and that the damping force is $-d(dy/dt)$. The case for a valve with free lift at low lift when the spring force is zero is considered in the next section.

   Equating pressure and spring force with the valve in equilibrium at zero and full lift gives:

$$C_D P_o = a_s$$

   and

$$C_D P_{FL} = a_s + b_s y_{FL}$$

   $\therefore$ The spring force is

$$C_D\{P_o + (P_{FL} - P_o)y/y_{FL}\}$$

   and the equation of motion becomes:

$$M(d_2 y/dt^2) = C_D\{p(1-r_1) - p(dr/dt)_1\}t - C_D\{P_o + (P_{FL} - P_o)y/y_{FL}\} - d(dy/dt)$$

   the solution to which is:

$$y = \frac{pd(dr/dt)_1 y^2_{FL}}{(P_{FL} - po)^2\, CD} + \frac{p(1-r_1) - po}{P_{FL} - po}\, y_{FL} - \frac{p(dr/dt)_1 y_{FL}}{P_{FL} - po}\, t$$
$$+ \alpha\, e^{(-dt/2M)} \cos(\sqrt{(}\frac{C_D\, (P_{FL} - po)}{y_{FL}} - \frac{d^2}{4M^2}\, t) + \phi)$$

   Solving for the lift and velocity at the end of a time step in the same way as used for the more simple case described in 1. gives the same form of solution, equations (1) and (2) (Table I), but with A and B constants modified as shown in Table II.

3. With no spring force or damping

   In a valve with free lift, there is a period when the only force acting on the strip is the pressure force. The equation of motion is then:

$$M(d_2 y/dt^2) = C_D\, \Delta p$$

   The solution to which is:

$$y = (C_D/M)\, \Delta p t^2/2 + \alpha t + \beta$$

   where α, β are arbitrary constants.

   Solving for Y_2 and $(dy/d\theta)_2$ as in the previous sections again yields a solu- of the form of equations (1) and (2) (Table I) with the A and B constants as defined in the third column of Table II. The surprising result that the dimension-less natural frequency is an important parameter in this case is explained by the use of the pressure drop to fully open the valve (P_{FL}) or ($P_{FL} - P_o$) in the definition of the dimensionless pressure P_s.

DERIVATION OF EQUATIONS FOR VALVE PRESSURE DROP

To illustrate the method, the case of in-flow through a suction valve with change of state in the cylinder assumed to be iso-thermal will be considered.

The equation of state for the perfect gas gives:

$$d(p_c V_c)/dt = RT(dm/dt)$$

and assuming incompressible flow through the valve:

$$dm/dt = A_{eq}\, f(y)\sqrt{(2\rho\Delta p)}$$

Introducing the dimensionless variables r and a and eliminating dm/dt from the above equation gives:

$$d(ar)/d\theta = cf(Y)\sqrt{(1-r)} \qquad \text{(a)}$$

where the constant C is defined in Table II.

The modified Euler's method will be used to obtain a solution for the step from θ_1 to θ_2 and Y will be assumed to be constant with a value of 1/2 ($Y_1 + Y_2$) during the step. Thus:

$$(a_2 r_2 - a_1 r_1)/\Delta\theta = cf(Y_1/2 + Y_2/2)\sqrt{(1-(r_1+r_2)/2)}$$

which may be solved for r_2 giving equation (4) on Table I.

If it is assumed, more realistically that the change of state in the cylinder during the time step is isentropic instead of isothermal, the above equation (a) becomes:

$$d(ra^{n_v})/d\theta = n_v\, c\, f(Y) a^{n_v - 1}\, r^{(n_v-1)/n_v} \sqrt{(1-r)} \qquad \text{(a1)}$$

To solve this by the above method it is assumed that the average value of "r" during the step is $r_1 + 1/2\, (dr/d\theta)_1 \Delta\theta$ and this is used to calculate $r \uparrow (n_v-1)/n_v$. The solution is then as given in Table I.

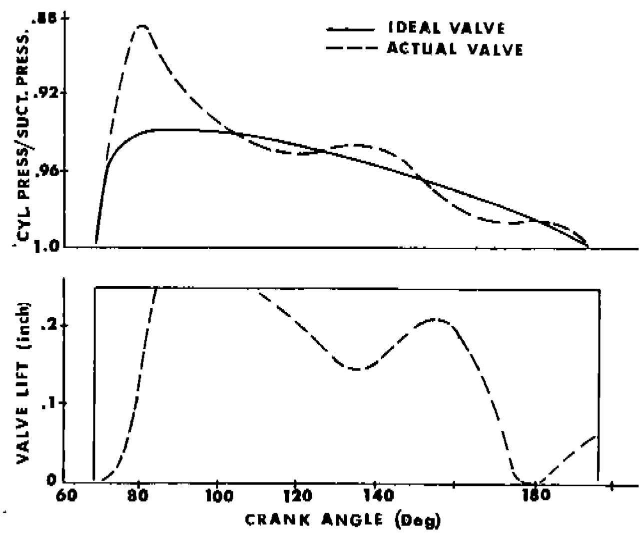
Equation (3) on Table I for ($dr/d\theta$) can be obtained from equation (a) above.

TYPICAL RESULTS:

Comparison of Actual and Ideal Valve

Fig. 2 shows the calculated pressure drop across a typical valve and its lift as a function of crank angle superimposed on the diagrams obtained assuming that the valve has no inertia or spring and so opens and closes instantaneously as the pressure drop changes sign. Note that due to the compressibility of the gas, the pressure drop is not zero at the dead center even with the ideal valve. The magnitude of this effect depends on the adequacy of the valve flow area.

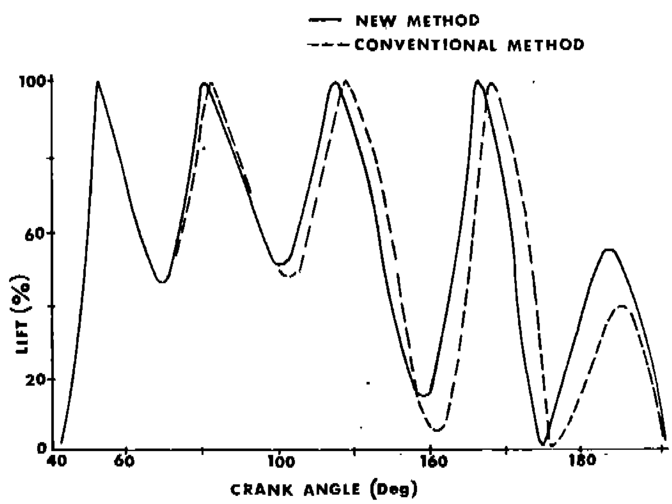
The dynamics of the actual valve have two important effects. First, the finite opening rate causes the pressure drop across the valve to increase to almost twice the value it would have for an ideal valve.



COMPARISON OF IDEAL AND ACTUAL VALVE

FIG 2

The pressure drop peak could have a significant influence on the compressor horsepower. Second, the valve flutters while closing. This is unlikely to affect the horsepower of the compressor significantly, but could in some cases influence the impact velocity of the element on the seat and may change the crank angle at which the valve closes. Thus, flutter can affect valve reliability and compressor capacity.



COMPARISON OF NEW AND COVENTIONAL METHODS

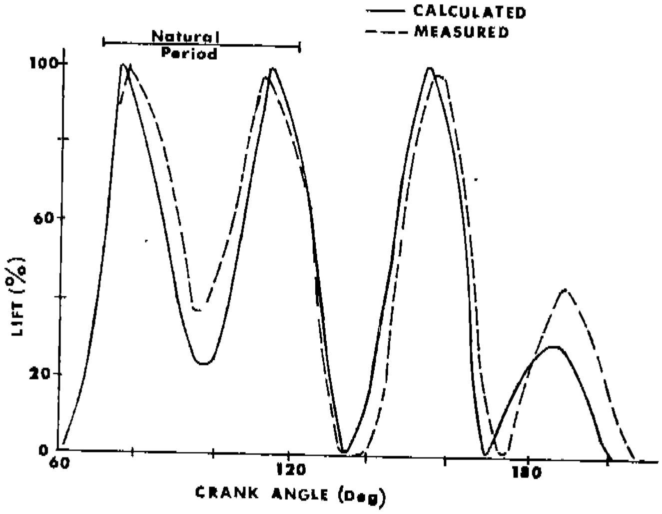
FIG 3

Comparison of this and conventional calculation methods.

Fig. 3 compares the valve lift diagram calculated by the method described here and by a more conventional method using the Runge Kutta integration method, the correct compressible flow equation for flow through the valve and the energy equation to calculate the cylinder temperature. The example chosen is one with a large amount of flutter of the valve element that serves as a severe test of either method and the agreement obtained is considered to be very satisfactory.

Comparison with measured valve lift diagram

The calculated and measured valve lift diagrams for suction valve are compared on Fig. 4. A case of a valve with an unnecessarily high flow area that flutters has again been chosen to provide a severe test of the calculation method. The curve of equivalent area against pressure drop used was that measured under steady flow conditions, and the coefficients of restitution and damping were chosen to give the best agreement with measured diagrams. The values used being 2% of critical damping and a coefficient of restitution of 0.2, both of which seem reasonable.



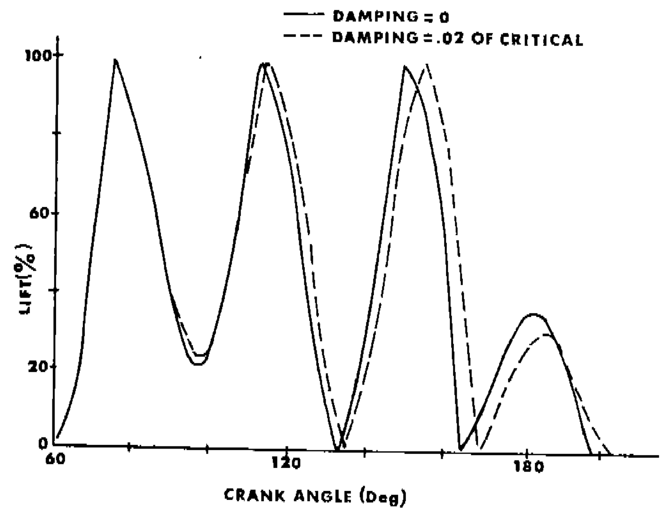
COMPARISON OF MEASURED AND CALCULATED DIAGRAMS

FIG 4

The effect of damping is not great for this valve design (Fig. 5) and the constant A_1 to A_4, B_1 to B_4, CN and SN (Table II) are simplified significantly if damping is neglected (V=O). This may be attractive in some cases although it must be remembered that the constants are only calculated once for each calculation and the saving in calculation time introduced by this simplification is very small.

The Flutter Frequency

It is of interest to note that the valve element does not oscilate at its natural frequency. As shown on Fig. 4, the actual period is shorter than that given by the natural frequency. There are three reasons for this. First, the period is increased very slightly by the damping as given by the factor $\sqrt{(1 - V^2)}$ in the definition of CN and SN (Table II). Second, the interaction of the valve lift and the cylinder pressure tends to reduce the period. If the valve tends to open, the pressure drop tends to fall which acts to reduce the tendency of the valve to open. This effect is analogous to an increase of the spring rate and, hence, increases the effective frequency. Third, the situation in the example shown (Fig. 4) is distorted by the valve element hitting the stop either at full or zero lift on each flutter. This decreases the apparent period.



COMPARISON OF LIFT DIAGRAMS WITH AND WITHOUT DAMPING

FIG 5

CONCLUSIONS:

The method described here for calculating the lift and pressure drop diagrams for compressor valves is simple enough to allow it to be used for compressor valve selection and performance prediction and is probably as accurate as required for any purpose other than detailed research investigation. It is, however, limited to cases where the fundamental vibration mode of the valve element is dominant.

The main limitations to the method are the calculation of the cylinder pressure and the assumption of constant passage pressure. It may or may not be possible to improve the calculation in these areas without compromising the simplicity that is the methods main attraction.

TABLE I

$$Y_2 = A_1(dr/d\theta)_1 + A_2(1-r_1) + A_3Y_1 + A_4(dy/d\theta)_1 + A_5 \qquad (1)$$

$$(dY/d\theta)_2 = B_1(dr/d\theta)_1 + B_2(1-r_1) + B_3Y_1 + B_4(dY/d\theta)_1 + B_5 \qquad (2)$$

Isothermal Change of Cylinder Conditions	Isentropic Change of Cylinder Conditions
$(dr/d\theta)_1 = (c/a_1)\sqrt{(1-r_1)}f(Y) - (r_1/a_1)(da/d\theta)_1$ (3a)	$= n_v r^{(n_v-1)/n_v}(c/a_1)\sqrt{(1-r_1)}f(Y) - n_v(r_1/a_1)(da/d\theta)_1$ (3b)

$$r_2 = H \pm G/2 \mp \sqrt{\{G^2/4 \pm GH \pm G(r_1-2)\}} \qquad (4)$$

Use top sign for flow out of cylinder

Isothermal	Isentropic
$G = \{C\Delta\theta f(Y_1/2 + Y_2/2)\ /a_2\}^2/2$ (5a)	$G = \{n_v C(a_1/2 + a_2/2)^{(n_v-1)}\{r_1 + 0.5(dr/d\theta)_1\Delta\theta\}^{(n_v-1)/n_v}(\Delta\theta/a_2^{n_v})f(Y_1/2 + Y_2/2)\}^2\ /2$ (5b)
$H = (a_1/a_2)r_1$ (6a)	$H = (a_1/a_2)^{n_v}\ r_1$ (6b)

The A, B and C constants are defined in Table II

Equations for Calculations of Valve Movement and Pressure Drop

A. VALVE MOVEMENT:

	Without Damping & Preload or Free Lift	With Damping & Preload or Free Lift	During Free Lift
A_1	$Ps(\frac{SN}{E} - \Delta\theta)$	$Ps(\frac{2V}{E}(1-CN) - \Delta\theta + \frac{(1-2V^2)}{E} SN)$	$-E^2 Ps \frac{\Delta\theta^3}{6}$
A_2	$Ps(1-CN)$	$Ps(1-CN-V\,SN)$	$E^2 Ps \frac{\Delta\theta^2}{2}$
A_3	CN	$CN + V\,SN$	1
A_4	$\frac{SN}{E}$	$\frac{SN}{E}$	$\Delta\theta$
A_5	0	$Po(CN-1+V\,SN)$	0
B_1	$Ps(CN-1)$	$Ps(CN-1+V\,SN)$	$-E^2 Ps \frac{\Delta\theta^2}{2}$
B_2	$PsE\ SN$	$PsE\ SN$	$E^2 Ps \Delta\theta$
B_3	$-E\ SN$	$-E\ SN$	0
B_4	CN	$CN-V\,SN$	1
B_5	0	$-Po\ E\ SN$	0
CN	$COS(E\Delta\theta)$	$e^{-V\Delta\theta} COS(E\sqrt{1-V^2}\,\Delta\theta)$	
SN	$SIN(E\Delta\theta)$	$e^{-V\Delta\theta} SIN(E\sqrt{1-V^2}\,\Delta\theta)/\sqrt{1-V^2}$	

B. PRESSURE DROP

$$C = (30/(\pi N V_{sw}))\sqrt{(2RT)}A_{eq}$$

CONSTANTS FOR CALCULATION OF VALVE MOVEMENT

AND PRESSURE DROP

TABLE II