An Experiment Comparing Fortran Programming Times with the Software Physics Hypothesis

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Introduction

Recent discoveries in the area of Algorithm Structure or Software Physics [1-25] have produced a number of hypotheses. One of these relates the number of elementary mental discriminations required to implement an algorithm to measurable properties of that algorithm, and the results of one set of experiments confirming this relationship have been published [16]. That publication, while significant, made no claim to finality, suggesting instead that further experiments were warranted. This paper will present the results of a second set of experiments, having the advantages of being conducted in a single implementation language, Fortran, from problem specifications readily available in computer textbooks.

Section I of this report presents the timing hypothesis, and the elementary equations upon which it rests. Section II presents the details of the experiment and the results which were obtained, and Section III contains an analysis of the data.
SECTION I - TIMING HYPOTHESIS

Measurable properties of any implementation of any algorithm include:

\( n_1 \) = The count of distinct operators
\( n_2 \) = The count of distinct operands
  (variables or constants)
\( N_1 \) = Total uses of operators
\( N_2 \) = Total uses of operands

The vocabulary, \( n \), is given by:

\[ n = n_1 + n_2 \]  \hspace{1cm} (1)

and the length, \( N \), is:

\[ N = N_1 + N_2 \]  \hspace{1cm} (2)

From these properties, it is possible to obtain the volume, \( V \), in bits, as:

\[ V = N \log_2 n \]  \hspace{1cm} (3)

and the implementation level, \( L \), where \( L \leq 1 \), as:

\[ L = \frac{n_1^*}{n_1} \cdot \frac{n_2}{N_2} \]  \hspace{1cm} (4)

where \( n_1^* \), the minimum possible number of operators, will equal 2 for most algorithms. (One for the name of a function, plus one for a grouping symbol operator). It has been shown [4] that the product \( L \times V \) is invariant under translation from one language to another, and that for programs without impurities [3,6,8]:

\[ N = n_1 \log_2 n_1 + n_2 \log_2 n_2 \]  \hspace{1cm} (5)

From this point, the following nine steps yield the timing equation:

1. A program consists of \( N \) selections from \( n \) elements.
2. A binary search of \( n \) elements requires \( \log_2 n \) comparisons.

3. A program is generated by making \( N \log_2 n \) comparisons.

4. Therefore, the volume, \( V \), is a count of the number of comparisons required.

5. The number of elementary mental discriminations required to complete one comparison measures the difficulty of the task.

6. The level, \( L \), is the reciprocal of the difficulty.

7. Therefore, \( E \), the count of elementary mental discriminations required to generate a program, is given by:

\[
E = \frac{V}{L} \tag{6}
\]

8. \( S \), the speed with which the brain makes elementary mental discriminations can be obtained from psychology [26] as:

\[
5 \leq S \leq 20 \text{ discriminations per second.}
\]

9. Therefore, the time to generate a preconceived program, by a concentrating programmer, fluent in a language, is:

\[
\hat{T} = \frac{V}{S L} \tag{7}
\]

Equation 7 may be expressed in more basic terms by substituting for \( V \) from equation 3, and for \( L \) from equation 4, with \( n_1^x = 2 \), giving:

\[
\hat{T} = \frac{n_1 n_2 N \log_2 n}{2 S n_2} \tag{8}
\]

The effect of possible impurities [5] may be eliminated from equation 8 by substituting for \( N \) from equation 5. Letting \( S = 60 \times 18 = 1080 \) will then give, for time in minutes:

\[
\hat{T} = \frac{n_1 n_2 (n_1 \log_2 n_1 + n_2 \log_2 n_2) \log_2 n}{2160 n_2} \tag{9}
\]

Each of the variables on the right hand side of equation 9 can be readily measured (or counted) in any computer program, and the experiment described in the next section was designed to compare results from that equation with observed programming times.
SECTION II.  EXPERIMENTAL PROCEDURE

Eleven problems were arbitrarily selected from two published sources. In selecting candidates for the experiment, problems were sought which were stated in a non-procedural form. Further, the problem statement had to be complete. That is, in the course of solving a particular problem, specific laws of physics, mathematics, etc. would not have to be derived. The problems finally selected were taken from Knuth [27], and from Maurer and Williams [28], and cover a wide range of topics including character manipulation, list processing, simulation experiments, and mathematical analysis. The source of each problem statement is cited in Table 1.

On each of eleven days, one of these problems was implemented by the senior author. In order to maintain a consistent level of performance all work was conducted in a quiet room, free from distractions, during the same period of the day. The time required to fully implement the problem was obtained. This total time included the number of minutes spent reading the statement of the problem, preparing flowcharts and writing preliminary versions of the code, writing the final version of the code, desk checking, and the time spent working to correct errors in the program. Time to keypunch was not included.

For a number of reasons, including availability and fluency, all of the algorithms were implemented in Fortran. In the course of solving a problem the correctness of the implementation was checked by executing a sufficiently complex test case for which a correct answer was known. In some cases the solution to a problem was written as a subroutine and testing required that a main routine be written. In such a case only the preparation of the subroutine was considered for the experiment.
In addition, several implementations made use of subroutines previously written. Such routines were also not included. The complete text of each of the eleven programs is included in Appendix A.

After each program was completed, a careful count was made to determine values of $n_1$, $n_2$, $N_1$, and $N_2$. In obtaining these values all read, write, declarative statements and comments were ignored. The results are shown in Table 1.
TABLE 1. EXPERIMENTAL DATA

<table>
<thead>
<tr>
<th>No.</th>
<th>Program Specifications</th>
<th>Software Parameters</th>
<th>Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ref.*</td>
<td>Page</td>
<td>Problem</td>
</tr>
<tr>
<td>G1</td>
<td>K</td>
<td>158</td>
<td>21</td>
</tr>
<tr>
<td>G2</td>
<td>K</td>
<td>159</td>
<td>23</td>
</tr>
<tr>
<td>G3</td>
<td>K</td>
<td>196</td>
<td>7</td>
</tr>
<tr>
<td>G4</td>
<td>K</td>
<td>377</td>
<td>17</td>
</tr>
<tr>
<td>G5</td>
<td>K</td>
<td>158</td>
<td>22</td>
</tr>
<tr>
<td>G6</td>
<td>K</td>
<td>154</td>
<td>10</td>
</tr>
<tr>
<td>G7</td>
<td>M</td>
<td>32</td>
<td>3.2.21</td>
</tr>
<tr>
<td>G8</td>
<td>M</td>
<td>32</td>
<td>3.2.23</td>
</tr>
<tr>
<td>G9</td>
<td>M</td>
<td>88</td>
<td>8.3.2</td>
</tr>
<tr>
<td>G10</td>
<td>M</td>
<td>89</td>
<td>8.3.4</td>
</tr>
<tr>
<td>G11</td>
<td>M</td>
<td>27</td>
<td>3.2.4</td>
</tr>
</tbody>
</table>

*K = Knuth [27], M = Maurer and Williams [28].
SECTION III - ANALYSIS OF THE DATA

The programming time predicted by theory was obtained for each program by applying equation 9 to the data in Table I. This result, T, can be compared with the observed value, T, in Table 2. In addition, a count of the number of statements in each program was obtained, and the programs were ordered according to these values.

The average of the calculated values, 34 minutes, is fortuitously close to the observed value, 35 minutes. The coefficient of correlation is 0.934, only slightly smaller than the value of 0.952 reported in an earlier experiment [16]. In further agreement with that experiment, the correlation between length and observed times, 0.887, is lower than between observed and calculated times.

In conclusion, it may again be observed that one more set of experimental data do not contradict the simple hypothesis. As a result, further carefully controlled experiments by others would appear to be warranted.
<table>
<thead>
<tr>
<th>Program Number</th>
<th>Statement Count</th>
<th>Programming Time-Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>G7</td>
<td>7</td>
<td>T observed: 5, T Equ. 9: 4.6</td>
</tr>
<tr>
<td>G11</td>
<td>8</td>
<td>T observed: 5, T Equ. 9: 5.4</td>
</tr>
<tr>
<td>G5</td>
<td>11</td>
<td>T observed: 21, T Equ. 9: 2.5</td>
</tr>
<tr>
<td>G6</td>
<td>15</td>
<td>T observed: 30, T Equ. 9: 6.8</td>
</tr>
<tr>
<td>G3</td>
<td>18</td>
<td>T observed: 16, T Equ. 9: 15.6</td>
</tr>
<tr>
<td>G1</td>
<td>18</td>
<td>T observed: 19, T Equ. 9: 14.6</td>
</tr>
<tr>
<td>G8</td>
<td>18</td>
<td>T observed: 24, T Equ. 9: 22.9</td>
</tr>
<tr>
<td>G4</td>
<td>32</td>
<td>T observed: 39, T Equ. 9: 43.6</td>
</tr>
<tr>
<td>G2</td>
<td>36</td>
<td>T observed: 92, T Equ. 9: 81.5</td>
</tr>
<tr>
<td>G9</td>
<td>38</td>
<td>T observed: 43, T Equ. 9: 49.2</td>
</tr>
<tr>
<td>G10</td>
<td>59</td>
<td>T observed: 91, T Equ. 9: 128.5</td>
</tr>
</tbody>
</table>

Means: 35.0, 34.1
REFERENCES:


Additional References:


Final listings of the eleven experimental programs.
TEST PROGRAM 1

DIMENSION MAGIC(23,23)
DATA MAGIC /529*0/

N=1
IR=1
IC=23/2 + 1

100 MAGIC(IR,IC)=N
   IF (N.EQ.529) GO TO 900
   N=N+1
   JR=IR-1
   IF (JR.LT.1) JR=23
   JC=IC-1
   IF (JC.LT.1) JC=23
   IF (MAGIC(JR,JC).EQ.0) GO TO 200
   JR=IR+1
   IF (JR.GT.23) JR=1
   JC=IC

200 IR=JR
   IC=JC
   GO TO 100

PRINT MAGIC SQUARE

900 DO 920 IR=1,23,1
   920 WRITE (6,1000) (MAGIC(IR,IC), IC=1,23,1)
   STOP

1000 FORMAT(23I4)
END
INTEGER DIGIT(10)
DATA DIGIT /1H0, 1H1, 1H2, 1H3, 1H4, 1H5, 1H6, 1H7, 1H8, 1H9/
DIMENSION MAT(24, 24)
DATA MAT /576*1/
INTEGER WHITE(3, 5)
DATA WHITE /1H, 1H, 1H+/,
$ 1H, 1H, 1H+,
$ 1H, 1H, 1H+,
$ 1H, 1H, 1H+,
$ 1H, 1H, 1H+,
$ 1H, 1H, 1H+,
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$ 1H, 1H, 1H+,
DEBUG PRINTOUT

DO 300 IR=1,24,1
300 WRITE (6,90000) (MAT(IR,IC), IC=1,24,1)

MOVE TO OUTPUT ARRAY

N=0
DO 380 IR=1,23,1
    JR=(IR-1)*3
    DO 370 IC=1,23,1
        JC=(IC-1)*5
    370 OUT(IR+JC+J)=WHITE(I,J)
    GO TO 380
380 OUT(IR+JC+J)=EDGE(I,J,M0DE)
    GO TO 370

BLACK SQUARE

IF (MAT(IR,IC).NE.1) GO TO 325
DO 310 I=1,3,1
    DO 310 J=1,5,1
310 OUT(I,J)=1H+
    GO TO 370

EDGE SQUARE

IF (MAT(IR,IC).EQ.0) GO TO 350
    MODE=1
    IF (MAT(IR,IC+1).EQ.0) MODE=MODE+1
    IF (MAT(IR+1,IC).EQ.0) MODE=MODE+2
    IF (MODE.EQ.1 .AND. MAT(IR+1,IC+1).EQ.0) MODE=5
    DO 330 I=1,3,1
    DO 330 J=1,5,1
330 OUT(I,J+1)=EDGE(I,J,M0DE)
    GO TO 370

WHITE SQUARE

350 OUT(I,J)=WHITE(I,J)

NUMBERED...

IF (.NOT.
    (MAT(IR+1,IC).EQ.0 .AND. MAT(IR-1,IC).NE.0 .OR.
    MAT(IR,IC-1).NE.0 .AND. MAT(IR,IC+1).EQ.0)) GO TO 370
    N=N+1
    NT=N/10
    NU=N-NT*10
    OUT(I,J)=DIGIT(NT+1)
    OUT(I,J+1)=DIGIT(NU+1)
370 CONTINUE
380 CONTINUE

OUTPUT PUZZLE

WRITE (6,3000)
DO 500 I=1,120,1
500 WRITE (6,2000) (OUT(I,J), J=1,120,1)
STOP

1000 FORMAT(23I1)
2000 FORMAT(IX,120A1)
3000 FORMAT(1H1,/,1H1)
9000 FORMAT(24I3)
END

*EOR
100001111111111111111111
001000111111111111111111
000010111111111111111111
TEST PROGRAM 3

INTEGER C, CO, PI, PO
INTEGER INPUT(8), OUTPUT(8), DIGIT(10)
DATA OUTPUT /8*1H/
DATA DIGIT /1H0, 1H1, 1H2, 1H3, 1H4, 1H5, 1H6, 1H7, 1H8, 1H9/
INTEGER GETCH
LOGICAL NUMERIC
NUMERIC(I)=I.GE.1H0 .AND. I.LE.1H9

READ (5, 1000) INPUT
1000 FORMAT(8A10)
WRITE (6, 2000) INPUT
2000 FORMAT(#0INPUT IS #,8A10)

PI=1
PO=1
CO=GETCH(INPUT, PI)
PI=PI+1

100 KOUNT=1
200 C=GETCH(INPUT, PI)
PI=PI+1

IF (C.EQ.1H) GO TO 200
IF (C.NE.CO) GO TO 300
KOUNT=KOUNT+1
GO TO 200

300 IF (KOUNT.EQ.1 .AND. .NOT.NUMERIC(CO)) GO TO 400
CALL PUTCH (DIGIT(KOUNT), OUTPUT, PO)
PO=PO+1

400 CALL PUTCH (CO, OUTPUT, PO)
PO=PO+1
CO=C

IF (CO.NE.1H.) GO TO 100
CALL PUTCH (CO, OUTPUT, PO)
WRITE (6, 3000) OUTPUT
3000 FORMAT(#0OUTPUT IS #,8A10)
STOP
END

THE ROUTINES GETCH AND PUTCH WERE NOT WRITTEN AS PART OF THIS EXPERIMENT.

INTEGER FUNCTION GETCH(IWORD, IPOS)

THE FUNCTION GETCH RETURNS A 6 BIT CHARACTER FROM A CHARACTER STRING OF POSSIBLY SEVERAL WORDS IN LENGTH. THE VALUE RETURNED IS LEFT JUSTIFIED, BLANK FILLED. CHARACTERS ARE NUMBERED LEFT TO RIGHT, 1, 2, . . . .

AUTHOR: KEVIN KOLIS (23 JAN 75)

DIMENSION IWORD(1)
SUBROUTINE PUTC (CHAR, STRING, POS)

SUBROUTINE PUTC PLACES A GIVEN CHARACTER INTO A STRING AT THE SPECIFIED POSITION. CHARACTERS ARE NUMBERED LEFT TO RIGHT, 1, 2, ..., *

AUTHOR: RONALD GORDON (24 JAN 78)

INTEGER CHAR, STRING(10), POS

IW=(POS-1)/10+1
IC=(1-POS+IW*10)*6
M=SHIFT(77, 77, 77, 77, 77, 77, 77, 77, 77, IC)
STRING(IW)=OR (AND (STRING(IW), M), AND (SHIFT(CHAR, IC), COMPL(M)))
RETURN

*END

SUBROUTINE PUTC (CHAR, STRING, POS)
INTEGER PICK, CARD
LOGICAL DEBUG
INTEGER POP
INTEGER DECK(52)
INTEGER KOUNT(53)
DATA KOUNT /53*0/
INTEGER PILE(13,5)
COMMON PILE

DEBUG=.TRUE.
PC=RANF(13,0)

K=1
DO 100 I=1,4,1
DO 100 J=1,13,1
DECK(K)=I*100 + J
100 K=K+1

DO 900 N=1,500,1
DO 150 I=1,13,1
150 PILE(I,1)=1

CALL SHUFFLE (DECK,52)

K=1
DO 200 I=1,4,1
DO 200 J=1,13,1
CALL PUSH (DECK(K),J)
200 K=K+1

IF (DEBUG) WRITE (6,1000) PILE
1000 FORMAT(16,5(/,1X,13I5)/)

L=53
PICK=1

300 CARD=POP(PICK)
IF (CARD.EQ.0) GO TO 400
L=L-1
IF (DEBUG) WRITE (6,2000) CARD, PICK
2000 FORMAT(14,I2,PICKED FROM#,I3)
PICK=MOD(CARD,100)
GO TO 300

400 KOUNT(L)=KOUNT(L)+1
K=L-1
IF (DEBUG) WRITE (6,3000) K
3000 FORMAT(16,I3,2 CARDS LEFT)

900 CONTINUE

WRITE (6,5000)
DO 920 I=0,52,1
PC=FLOAT(KOUNT(I+1))/500.0 * 100.0
920 WRITE (6,4000) I, KOUNT(I+1), PC
4000 FORMAT(17,I2,F11.2)
5000 FORMAT(1X,1CARDS LEFT N TIMES PERCENT)
STOP
END
SUBROUTINE PUSH (ITEM, PICK)
INTEGER PICK
INTEGER PILE(13,5)
COMMON PILE
PILE(PICK,1)=PILE(PICK,1)+1
PILE(PICK, PILE(PICK,1) ) = ITEM
RETURN
END
INTEGER FUNCTION POP (PICK)
INTEGER PICK
INTEGER PILE(13,5)
COMMON PILE
IF (PILE(PICK,1).EQ.1) GO TO 100
POP=PILE(PICK, PILE(PICK,1) )
PILE(PICK,1)=PILE(PICK,1)-1
RETURN
C 100 POP=0
RETURN
END
C
C SUBROUTINE SHUFFLE WAS NOT WRITTEN AS PART OF THIS EXPERIMENT.

C SUBROUTINE SHUFFLE (LIST, N)
DIMENSION LIST(N)
C
C THIS ROUTINE WILL RANDOMLY SHUFFLE A LIST OF ITEMS.

C REF: KNUTH, VOL. 2, P. 125, ALGORITHM P.
J=N
100 U=RANF(0,1)
K=FLOAT (J)*U+1
KEEP=LIST(K)
LIST(K)=LIST(J)
LIST(J)=KEEP
J=J-1
IF (J.GT.1) GO TO 100
RETURN
END
TEST PROGRAM 5

CREATE CIRCULAR LIST

KILL N-1 MEN

COUNT M MEN

KILL CURRENT MAN

KILL LAST MAN

KILL (L1) = N
WRITE (6,1000) (KILL(I), I=1,N+1)
1000 FORMAT (40I3)
STOP
END

*EOR
0, 4
TEST PROGRAM 6

THE MAIN PROGRAM WAS NOT WRITTEN AS PART OF THIS EXPERIMENT.

DIMENSION MATRIX (9,8)
READ (5,1000) MATRIX
WRITE (6,1001) MATRIX
CALL SADDLE (MATRIX, I, J)
PRINT, I, J
STOP
1000 FORMAT(9(I1,I1))
1001 FORMAT(9(T2))
END

SUBROUTINE SADDLE (MAT, I, J)
DIMENSION MAT(9,8)
DIMENSION KEI.PR(9), KEEP(8)

C COMPUTE MIN = ROW
DO 160 IR=1,9,1
MIN=MAT(IR,1)
DO 150 IC=2,9,1
IF (MIN.GT.MAT(IR,IC)) MIN=MAT(IR,IC)
150 CONTINUE
160 KEEP(IR)=MIN
C COMPUTE MAX = COL
DO 260 IC=1,9,1
MAX=MAT(1,IC)
DO 250 IH=2,9,1
IF (MAX.LT.MAT(IH,IC)) MAX=MAT(IH,IC)
250 CONTINUE
260 KEEP(IC)=MAX
C LOOK FOR MATCH
DO 370 I=1,9,1
DO 370 J=1,8,1
IF (KEEP(I).EQ.KEEP(J)) RETURN
370 CONTINUE
I=0
J=0
RETURN
END

*EOR
1 4 2 2 4 3 0 1 1
2 4 6 1 5 5 2 2 2
2 5 6 9 6 4 0 3 2
3 5 6 7 7 3 3 4 3
3 5 6 5 8 2 0 5 5
3 5 1 1 9 1 4 4 5
4 5 9 2 8 0 0 7 5
4 6 5 3 7 1 5 8 6
INTEGER SUMDIV

SEARCH FOR SOME FRIENDLY NUMBERS

DO 100 N=1000,1500,1
   M=SUMDIV(N)
   IF (SUMDIV(M).NE.N) GO TO 100
   WRITE (6,1000) N, M
100 CONTINUE
STOP

1000 FORMAT(I5,' and ',I5,' are friendly.')
END

INTEGER FUNCTION SUMDIV(N)

CALCULATE SUM OF DIVISORS OF N

SUMDIV=1
DO 100 I=2,N-1,1
   IF (N/I+I.NE.N) GO TO 100
   SUMDIV=SUMDIV+I
100 CONTINUE
RETURN
END
C
C
C

LOGICAL PRIME
COMMON LIST(100)

LIST(1)=2
LIST(2)=3
N=3
DO 110 I=3,100,1
100 N=N+1
IF (.NOT. PRIME(N)) GO TO 100
110 LIST(I)=N
C
WRITE (6,1000) LIST
C
N=1
DO 500 L=1,10
250 DO 300 I=0,L-1,1
IF (.NOT. PRIME(N+I)) GO TO 300
N=N+I+1
GO TO 250
300 CONTINUE
WRITE (6,2000) L, N
500 CONTINUE
STOP
C
1000 FORMAT(1X,20I6)
2000 FORMAT('#SEQUENCE OF #I2, # BEGINS AT #I4)
END
C
LOGICAL FUNCTION PRIME (N)
COMMON LIST(100)
C
PRIME=.TRUE.,
LIM=SQR( FLOAT(N ) ) + 0.5
DO 100 I=1,100,1
IF (LIST(I).GT.LIM) RETURN
IF (N/LIST(I).GT.LIM) RETURN
IF (N/LIST(I)*LIST(I).NE.N) GO TO 100
PRIME=.FALSE.
RETURN
100 CONTINUE
STOP 1
END
TEST PROGRAM 9

INTEGER GT
INTEGER WAIT, TOTAL
LOGICAL DEBUG
DO 100 GT=10,90,10
TOTAL=0
DEBUG=.TRUE.
DO 110 I=1,3,1
CALL SIMUL (GT, WAIT, DEBUG)
TOTAL=TOTAL+WAIT
WRITE (6,1000) GT, WAIT
DEBUG=.FALSE.

110 CONTINUE
AVG=FLOAT(TOTAL)/3.0
WRITE (6,2000) AVG
100 CONTINUE
STOP

1000 FORMAT(I3,# SECOND GREEN LIGHT. WAIT =#.I6)
2000 FORMAT(# AVERAGE WAIT =#,F7.1,/) END

SUBROUTINE SIMUL (GT, WAIT, DEBUG)
INTEGER TIME, Q1, Q2, WAIT1, WAIT2, GT, RANDOM
LOGICAL DEBUG
INTEGER WAIT
INTEGER ON, OFF

Q1=0
Q2=0
WAIT1=0
WAIT2=0
TIME=0
LIGHT=1
ON=0
OFF=0
IF (DEBUG) WRITE (6,3000) GT

ADD TO QUEUES

100 Q1=Q1+RANDOM(5,15)
Q2=Q2+RANDOM(6,24)
IF (DEBUG) WRITE (6,2000) TIME, LIGHT, Q1, WAIT1, Q2, WAIT2

REMOVE FROM QUEUES IF GREEN

C
IF (LIGHT.EQ.1) GO TO 200
Q2=Q2-MIND(Q2,20)
GO TO 250

200 Q1=Q1-MIND(Q1,36)

ACCUMULATE WAITING TIME

C
250 CONTINUE
WAIT1=WAIT1+Q1*10
WAIT2=WAIT2+Q2*10
IF (DEBUG) WRITE (6,2000) TIME, LIGHT, Q1, WAIT1, Q2, WAIT2

CHANGE LIGHT

C
TIME=TIME+10
IF (LIGHT.EQ.0) GO TO 300
ON=ON+10
IF (ON.EQ.GT) LIGHT=0
GO TO 400

OFF=OFF+10
IF (OFF+ON.NE.100) GO TO 400
LIGHT=1
ON=0
OFF=0

C
400 IF (TIME.LT.300) GO TO 100
IF (DEBUG) WRITE (6,1000) Q1, WAIT1, Q2, WAIT2
WAIT=WAIT1+WAIT2
RETURN

C
1000 FORMAT(*6CARS LEFT IN Q1 =*,I3,* WAITING TIME =*,I5,/,
+ * CARS LEFT IN Q2 =*,I3,* WAITING TIME =*,I5)
2000 FORMAT(6I10)
3000 FORMAT(*1SIMULATION OF*,I3,* SECOND GREEN LIGHT*,//
+ * TIME LIGHT Q1 WAIT1 Q2 WAIT2*)
END

INTEGER FUNCTION RANDOM (I, J)
X=J-I
RANDOM=X*RANF(0,0)
RANDOM=RANDOM+I
RETURN
END
C C
DO 100 I=1,2,1
WRITE (6,1000)
CALL SIMUL (250)
100 CONTINUE
STOP
1000 FORMAT(#1BEGIN SIMULATION#)
END

SUBROUTINE SIMUL (LIMIT)
C
C PERFORM SIMULATION WITH LIMIT PEOPLE
C
INTEGER TIME, RANDOM, CO, WAIT, TOTAL
INTEGER Q(3,600), QL(3)
COMMON Q
C
DO 100 I=1,3
100 QL(I)=0
KOUNT=0
TIME=0
TOTAL=0
NA=0

C 175 IF (TIME.NE.NA) GO TO 200
KOUNT=KOUNT+1
C
MIN=1
IF (QL(MIN)).GT.QL(2)) MIN=2
IF (QL(MIN)).GT.QL(3)) MIN=3
CO=RANDOM(100,350)
QL(MIN)=QL(MIN)+1
Q(MIN,QL(MIN))=KOUNT*100000000 + CO
NA=TIME+RANDOM(0,160)
IF (KOUNT.EQ.LIMIT) NA=-1
WRITE (6,1000) TIME, KOUNT, MIN, CO, QL(MIN), NA
GO TO 175
C
200 TIME=TIME+1
DO 250 I=1,3,1
IF (QL(I).EQ.0) GO TO 250
C
DO 210 J=1,QL(I),1
210 Q(I,J)=Q(I,J)+1000
C
MAN=Q(I,1)/100000000
WAIT=(Q(I,1)-MAN*100000000)/1000
CO=MOD(Q(I,1),1000) - 1
Q(I,1)=MAN*100000000 + WAIT*1000 + CO
IF (CO.GT.0) GO TO 250
C
QL(I)=QL(I)-1
DO 220 J=1,QL(I),1
220 Q(I,J)=Q(I,J)+1
WRITE (6,3000) TIME, MAN, I, WAIT, QL(I)
CALL MAXHAIT (0, WAIT, MAN)
250 CALL LINELEN (0, I, QL(I))
IF (NAG.EQ.0) GO TO 175
IF (QL(I) + QL(2) + QL(3).GT.0) GO TO 175
CALL LINELEN (I, I, J)
CALL MAXWAIT (I, I, J)
RETURN

+ * Q LENGTH=", I3," NEXT ARRIVAL=", I5")
+ * Q LENGTH=", I3)
END

SUBROUTINE MAXWAIT (MODE, L, N)
C GATHERS STATS ON WAITING LINES
C DIMENSION MAX(10), NUM(10)
INTEGER AVG, TOTAL
DATA MAX, NUM, TOTAL, KNT /22*0/
C IF (MODE.NE.0) GO TO 200
TOTAL=TOTAL+N
KNT=KNT+1
DO 100 I=1,10,1
IF (MAX(I).GT.L) GO TO 100
MAX(I)=L
NUM(I)=N
RETURN
100 CONTINUE
RETURN
C 200 WRITE (6,1000)
DO 300 I=1,10,1
WRITE (6,2000) MAX(I), NUM(I)
MAX(I)=0
300 NUM(I)=0
AVG=FLOAT(TOTAL)/FLOAT(KNT)+0.5
WRITE (6,3000) AVG
TOTAL=0
KNT=0
RETURN
1000 FORMAT(*0LONGEST WAITS WERE:*)
3000 FORMAT(* AVERAGE WAIT=", I5)
END

SUBROUTINE LINELEN (MODE, Q, L)
C GATHERS LINE LENGTH STATS
C INTEGER Q, AVG
INTEGER LEN(3), MAX(3), KNT(3)
DATA LEN, MAX, KNT /9*0/
C IF (MODE.NE.0) GO TO 100
LEN(Q)=LEN(Q)+L
KNT(Q)=KNT(Q)+1
IF (MAX(Q).LT.L) MAX(Q)=L
RETURN
C 100 WRITE (6,2000)
DO 200 I=1,3,1
AVG = FLOAT(LEN(I)) / FLOAT(KNT(I)) + 0.5
WRITE (6, 1000) I, MAX(I), AVG
LEN(I) = 0
MAX(I) = 0
200 KNT(I) = 0
RETURN
1000 FORMAT (# Q=##,I1,## MAX=##,I3,## AVERAGE=##,I3)
2000 FORMAT (#0LINE LENGTH STATISTICS##)
END
TEST PROGRAM 11

INTEGER SUNDIG3

THIS PROGRAM HUNTS FOR SOME NUMBERS SUCH THAT THE SUM OF
THE CUBES OF THE DIGITS OF THE NUMBER EQUALS THE NUMBER.

DO 100 I=2,500,1
   IF (I .NE. SUNDIG3(I)) GO TO 100
   WRITE (6,1000) I, I
100 CONTINUE
STOP

1000 FORMAT(80,THE SUM OF THE CUBES OF THE DIGITS OF I4,* EQUALS I4)
END

INTEGER FUNCTION SUNDIG3 (N)

COMPUTE THE SUM OF THE CUBES OF THE DIGITS OF N

INTEGER WORKER
   SUNDIG3=0
   WORKER=IABS(N)
100 IF (WORKER .EQ. 0) RETURN
   SUNDIG3=SUNDIG3 + MOD(WORKER,10) ** 3
   WORKER=WORKER/10
   GO TO 100
END
APPENDIX B
SOFTWARE PARAMETER DATA FOR PROGRAMS
G1 THROUGH G11.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \text{OPERATOR} )</th>
<th>( f_{1,i} )</th>
<th>( \text{OPERAND} )</th>
<th>( f_{2,i} )</th>
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\[ n_2 = 11, \quad N_2 = 51 \]
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\( n_2 = 24, \quad N_2 = 197 \)
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\( n_1 = 14, N_1 = 29 \)

\( n_2 = 10, N_2 = 21 \)
An experiment comparing Fortran programming times with the software physics hypothesis

by R. D. GORDON and M. H. HALSTEAD
Purdue University
Lafayette, Indiana

ABSTRACT
Recent discoveries in the area of Algorithm Structure or Software Physics have produced a number of hypotheses. One of these relates the number of elementary mental discriminations required to implement an algorithm to measurable properties of that algorithm, and the results of one set of experiments confirming this relationship have been published. That publication, while significant, made no claim to finality, suggesting instead that further experiments were warranted. This paper will present the results of a second set of experiments, having the advantages of being conducted in a single implementation language, Fortran, from problem specifications readily available in computer textbooks.

The first section of this paper presents the timing hypothesis, and the elementary equations upon which it rests. The second section presents the details of the experiment and the results which were obtained, and the third section contains an analysis of the data.

TIMING HYPOTHESIS

Measurable properties of any implementation of any algorithm include:

\[ \eta_1 = \text{The count of distinct operators} \]
\[ \eta_2 = \text{The count of distinct operands (variables or constants)} \]
\[ N_1 = \text{Total uses of operators} \]
\[ N_2 = \text{Total uses of operands} \]

The vocabulary, \( \eta \), is given by:

\[ \eta = \eta_1 + \eta_2 \]  
and the length, \( N \), is:

\[ N = N_1 + N_2 \]  

From these properties, it is possible to obtain the volume, \( V \), in bits, as:

\[ V = N \log_2 \eta \]  
and the implementation level, \( L \), where \( L \leq 1 \), as:

\[ L = \frac{\eta_1^* \eta_2}{\eta_1 N_2} \]

where \( \eta_1^* \), the minimum possible number of operators, will equal 2 for most algorithms. (One for the name of a function, plus one for a grouping symbol operator). It has been shown that the product \( L \times V \) is invariant under translation from one language to another, and that for programs without impurities:

\[ N = \eta_1 \log_2 \eta_1 + \eta_2 \log_2 \eta_2 \]  

From this point, the following nine steps yield the timing equation:

1. A program consists of \( N \) selections from \( \eta \) elements.
2. A binary search of \( \eta \) elements requires \( \log_2 \eta \) comparisons.
3. A program is generated by making \( N \log_2 \eta \) comparisons.
4. Therefore, the volume, \( V \), is a count of the number of comparisons required.
5. The number of elementary mental discriminations required to complete one comparison measures the difficulty of the task.
6. The level, \( L \), is the reciprocal of the difficulty.
7. Therefore, \( E \), the count of elementary mental discriminations required to generate a program, is given by:

\[ E = \frac{V}{L} \]  

8. \( S \), the speed with which the brain makes elementary mental discriminations can be obtained from psychology as:

\[ 5 \leq S \leq 20 \text{ discriminations per second.} \]

9. Therefore, the time to generate a preconceived program, by a concentrating programmer, fluent in a language, is:

\[ T = \frac{V}{SL} \]

Equation 7 may be expressed in more basic terms by substituting for \( V \) from equation 8, and for \( L \) from equation 4, with \( \eta_1^* = 2 \), giving:

\[ T = \frac{3N_1 N \log_2 \eta_1 N_2}{2S \eta_2} \]  

935
The effect of possible impurities may be eliminated from equation 8 by substituting for N from equation 5. Letting S = 60x18 = 1080 will then give, for time in minutes:

\[ T = \frac{\eta N_5 (\eta_1 \log_2 \eta_1 + \eta_2 \log_2 \eta_2) \log_2 \eta}{2160 \eta} \]  

(9)

Each of the variables on the right hand side of equation 9 can be readily measured (or counted) in any computer program, and the experiment described in the next section was designed to compare results from that equation with observed programming times.

EXPERIMENTAL PROCEDURE

Eleven problems were arbitrarily selected from two published sources. In selecting candidates for the experiment, problems were sought which were stated in a non-procedural form. Further, the problem statement had to be complete. That is, in the course of solving a particular problem, specific laws of physics, mathematics, etc. would not have to be derived. The problems finally selected were taken from Knuth, and from Maurer and Williams, and cover a wide range of topics including character manipulation, list processing, simulation experiments and mathematical analysis. The source of each problem statement is cited in Table I.

On each of eleven days, one of these problems was implemented by the senior author. In order to maintain a consistent level of performance all work was conducted in a quiet room, free from distractions, during the same period of the day. The time required to fully implement the problem was obtained. This total time included the number of minutes spent reading the statement of the problem, preparing flowcharts and writing preliminary versions of the code, writing the final version of the code, desk checking, and the time spent working to correct errors in the program. Time to keypunch was not included.

TABLE I—Experimental Data

<table>
<thead>
<tr>
<th>Program Specifications</th>
<th>Software Parameters</th>
<th>Implementation</th>
<th>Time-Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Ref.*</td>
<td>Page</td>
<td>Problem</td>
</tr>
<tr>
<td>G1</td>
<td>K</td>
<td>158</td>
<td>21</td>
</tr>
<tr>
<td>G2</td>
<td>K</td>
<td>159</td>
<td>23</td>
</tr>
<tr>
<td>G3</td>
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<td>277</td>
<td>17</td>
</tr>
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<td>G5</td>
<td>K</td>
<td>158</td>
<td>22</td>
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<td>G6</td>
<td>K</td>
<td>154</td>
<td>10</td>
</tr>
<tr>
<td>G7</td>
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</tr>
<tr>
<td>G8</td>
<td>M</td>
<td>32</td>
<td>3.2.23</td>
</tr>
<tr>
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<td>M</td>
<td>32</td>
<td>3.3.23</td>
</tr>
<tr>
<td>G10</td>
<td>M</td>
<td>28</td>
<td>3.3.4</td>
</tr>
<tr>
<td>G11</td>
<td>M</td>
<td>27</td>
<td>3.2.4</td>
</tr>
</tbody>
</table>

* K = Knuth, M = Maurer and Williams.

For a number of reasons, including availability and fluency, all of the algorithms were implemented in Fortran. In the course of solving a problem the correctness of the implementation was checked by executing a sufficiently complex test case for which a correct answer was known. In some cases the solution to a problem was written as a subroutine and testing required that a main routine be written. In such a case only the preparation of the subroutine was considered for the experiment. In addition, several implementations made use of subroutines previously written. Such routines were also not included.

After each program was completed, a careful count was made to determine values of \( n_1, n_2, N_1, \) and \( N_2. \) In obtaining these values all read, write, declarative statements and comments were ignored. The results are shown in Table I.

ANALYSIS OF THE DATA

The programming time predicted by theory was obtained for each program by applying equation 9 to the data in Table I. This result, \( T, \) can be compared with the observed value, \( T, \) in Table II. In addition, a count of the number of statements in each program was obtained, and the programs were ordered according to these values.

The average of the calculated values, 34 minutes, is fortuitously close to the observed value, 35 minutes. The coefficient of correlation is 0.984, only slightly smaller than the value of 0.962 reported in an earlier experiment. In further agreement with that experiment, the correlation between length and observed times, 0.887, is lower than between observed and calculated times.

In conclusion, it may again be observed that one more set of experimental data does not contradict the simple hypothesis. As a result, further carefully controlled experiments by others would appear to be warranted.

* Additional details available from the author.

TABLE II—Experimental Results

<table>
<thead>
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<th>Program Number</th>
<th>Statement Count</th>
<th>Programming Time-Minutes</th>
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<tr>
<td></td>
<td>Observed</td>
<td>T Eq. 9</td>
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<tr>
<td>G7</td>
<td>7</td>
<td>5</td>
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<tr>
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<td>5</td>
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<td>34.1</td>
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REFERENCES


Additional References: