6-2015

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Rui Cao

Ray W. Herrick Laboratories, cao101@purdue.edu

J. Stuart Bolton

Ray W. Herrick Laboratories, bolton@purdue.edu

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Cao, Rui and Bolton, J. Stuart, "Improved Model for Coupled Structural-Acoustic Modes of Tires" (2015). *Publications of the Ray W. Herrick Laboratories*. Paper 114.
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IMPROVED MODEL FOR COUPLED STRUCTURAL-ACOUSTIC MODES OF TIRES

Rui Cao, J. Stuart Bolton, Ray W. Herrick Laboratories,
School of Mechanical Engineering, Purdue University

I. Introduction

I. Introduction

Objective:

1. Build a model coupling the tire structure and air cavity
2. Identify tire structural vibration
3. Study sound characteristics in interior air cavity
4. Investigate spinning influence

II. Literature Review

Structure-borne sound on a smooth tyre

— Kropp

Effects of Coriolis acceleration on the free and forced in-plane vibrations of rotating rings on elastic foundation

— Huang & Soedel

A wave model of a circular tyre. Part 1: belt modelling

— Pinnington

Effects of rotation on the dynamics of a circular cylindrical shell with application to tire vibration

— Kim and Bolton

A coupled tire structure/acoustic cavity model

— Molisani, Burdisso & Tsihlas

The Influence of Tyre Air Cavities on Vehicle Acoustics

— Fernandez

The wave number decomposition approach to the analysis of tire vibration

— Bolton, Song, Kim & Kang

III. Analytical Model

Review of previous models

* Cao & Bolton, NoiseCon 2013

* Cao & Bolton, NoiseCon 2014

III. Analytical Model

Fully coupled circular cavity model

1. The tire rotates about a fixed axle
2. The wheel rim is rigid
3. Tire sidewall is represented by springs in radial and tangential directions
4. Ring structure includes flexural and longitudinal waves

III. Analytical Model

Rotating ring structure

Assume harmonic solutions for displacements:

$$w = \alpha e^{-jk_\theta \theta} e^{j\omega t} \quad u = \beta e^{-jk_\theta \theta} e^{j\omega t}$$

Substitution into rotating ring EOMs and write solutions in matrix form:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Where M_{11} , M_{12} , M_{21} and M_{22} are expressions of structure-related constants and variables k_θ and ω . For example:

$$M_{11} = j \left(\frac{Eh^3}{12R^4} k_\theta^3 + \frac{Eh}{R^2} k_\theta + 2 \frac{\sigma_\theta h}{R^2} k_\theta + 2\rho h \Omega \omega \right)$$

$$M_{12} = \frac{Eh^3}{12R^4} k_\theta^2 + \frac{Eh}{R^2} k_\theta^2 + \frac{\sigma_\theta h}{R^2} k_\theta^2 + \frac{p_0}{R} + k_u + 2\rho h \Omega k_\theta \omega - \rho h \omega^2$$

III. Analytical Model

Circular air cavity

Velocity of the flowing air is expressed as

$$v_{\theta} = \frac{v_0}{R} r$$

By using velocity potential ψ , the wave equation in the circular air cavity is

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} = \frac{1}{c_0^2} \left(\frac{\partial^2 \psi}{\partial t^2} + 2 \frac{v_0}{R} \frac{\partial^2 \psi}{\partial t \partial \theta} + \frac{v_0^2}{R^2} \frac{\partial^2 \psi}{\partial \theta^2} \right)$$

Harmonic solution of pressure is assumed in circumferential direction while Bessel function is assumed in radial direction:

$$\psi(r, \theta, t) = g_m(\kappa_m r) e^{-jm\theta} e^{j\omega t}$$

$$g_m(\kappa_m r) = A_m J_m(\kappa_m r) + B_m Y_m(\kappa_m r)$$

$$p = -\rho_0 \left[\frac{\partial \psi}{\partial t} + \vec{v}_{flow} \cdot \text{grad} \psi \right]$$

III. Analytical Model

Coupling relations

III. Analytical Model

Solving the coupled system

Substituting sound pressure as distributed load in radial direction into the characteristic equations of the ring structure and express p as function of α and β by using the boundary conditions:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} - FL & M_{22} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Where the fluid loading term FL can be expressed as

$$FL = \frac{p}{e^{-jm\theta}} = (-j\omega\rho_0 + jmv\rho_0)\alpha C_{A/\alpha}(J_m(\kappa_m r) + C_{B/A}Y_m(\kappa_m r))e^{j\omega t}$$

By supplying the mode number m , which is equivalent to wavenumber, we have

$$f(\omega) = M_{11}M_{22} - M_{12}(M_{21} - FL) = 0$$

The values of ω that satisfy this equations are the natural frequencies of the coupled model.

Tire mobility measurement set up

IV. Testing

Tire mobility measurement set up

Table of model parameters

Ring density	$\rho = 1200 \text{ kg/m}^3$
Air density	$\rho_0 = 1.24 \text{ kg/m}^3$
Outer radius	$r_1 = 0.3 \text{ m}$
Inner radius	$r_2 = 0.2 \text{ m}$
Ring thickness	$h = 0.008 \text{ m}$
Tire inflation pressure	$p_0 = 20600 \text{ Pa}$
Radial stiffness	$k_w = 1 \times 10^5 \text{ N/m}$
Tangential stiffness	$k_u = 1 \times 10^5 \text{ N/m}$
Young's modulus	$E = 4.8 \times 10^8 \text{ Pa}$

Tire measured

Goodyear 225/55 R17

Dispersion relation (static case)

Dispersion relation (static case)

Dispersion relation (static case)

V. Results

Dispersion relation (static case)

Dispersion relation (static case)

Dispersion relation (no fluid loading)

Acoustical waves disappear

Fluid loading has minor impact on structural features

Dispersion relation (rotating case)

Natural frequencies split into two at each mode of all waves

Dispersion relation (experimental)

Dispersion relation (experimental)

Dispersion relation (experimental)

V. Results

Dispersion relation (experimental)

Dispersion relation (experimental)

V. Results

Phase speed (static)

Phase speed (rotating)

V. Results

Radial pressure distribution in cavity

Mode number is 2, at the natural frequencies of each wave

1st structural wave

1st acoustical wave

V. Results

Radial pressure distribution in cavity

Mode number is 2, at the natural frequencies of each wave

1st structural wave

1st acoustical wave

V. Results

Pressure distribution in cavity (static)

Mode number is 2, at the natural frequencies of each wave

2nd structural wave

2nd acoustical wave

V. Results

Pressure distribution in cavity (static)

Mode number is 2, at the natural frequencies of each wave

2nd structural wave

2nd acoustical wave

VI. Conclusion

- ❑ The ring model allows for motions in radial and circumferential directions, which are associated with flexural waves and longitudinal waves, respectively
- ❑ The air cavity acts as a fluid loading on the ring structure
- ❑ Rotation of tire causes frequency split phenomenon
- ❑ Acoustical wave in tire radial directions exist – “depth modes” detectable in tire surface vibration
- ❑ In circular air cavity, phase speed of circumferential acoustical wave varies with radius due to planar nature of waves

Thank you

Question?