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# On the Simulation of Anechoic Pipes in Helmholtz Resonator Models of Compressor Discharge Systems

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## INTRODUCTION

The purpose of this paper is to report an addition to the work of reference [1], where a two cylinder discharge system was modeled using a Helmholtz resonator approach. For that particular system it turned out that the effect of the discharge pipe itself on the gas oscillations in the Helmholtz resonator system was negligible due to the special system dynamics. However, in more general applications, the discharge pipe effect has to be included (this problem is of lesser importance for air compressors, which usually discharge into a tank whose acoustic characteristic can easily be modeled).

As discharge pipe in a refrigeration system we will define that portion of piping that comes after discharge chambers, plenum and muffler components and attaches to the condenser. Two phase flow may already exist in portions of the discharge pipe and will occur of course eventually at some point in the condenser. It is felt that the existence of such two phase flow makes the discharge pipe and condenser system "acoustically dead", that is, we may model many discharge pipes as anechoic. This means that we assume that pressure pulses propagating in the discharge line will not be reflected.

If the discharge system is modeled in terms of its sinusoidal response (the impedance approach of reference [2]), then the anechoic termination does not cause any mathematical difficulty. The impedance of the anechoic discharge pipe is simply [2]

$$Z = \frac{\rho c}{A}$$

However, if we operate in the time domain as with the Helmholtz resonator approach, where we actually solve a family of differential equations, a knowledge of the anechoic impedance is of little help. We rather need an expression that relates instantaneous volume velocity at the pipe entrance to the instantaneous acoustic pressure at that entrance.

## BRIEF REVIEW OF THE HELMHOLTZ RESONATOR MODEL

The following is a brief review of the

modeling of a typical discharge system that consists of a combination of volumes, with relatively short connecting passages, as reported in reference [1].

It is assumed that one has two kinds of flow moving from the discharge valves to the lines. One is a mean flow which by itself may increase pressures in the discharge or suction cavities because of the restrictive action of the connecting passages. The other kind is an oscillatory flow where one has to picture the fluid particles accelerating and decelerating inside the discharge system. The pressures of these two types of flow can be added if it may be assumed that the two cases described are equivalent to an acoustic pressure oscillation superimposed on mean flow pressures.

Rather than operating with the acoustic wave equation directly, it is of advantage from a practical viewpoint (in light of the normally rather irregular shape discharge cavities, plenum chambers and connecting passages of the typical compressor) to use the Helmholtz resonator approximation. In this approximation the gas is considered to have inertia only in the connecting passages and to be inertialess in the cavities and plenum because of the relative differences of acceleration level. On the other hand, the gas is considered to be compressible in the cavities and plenum, but incompressible in the connecting passages because of the relative differences in volume. One may, therefore, picture the acoustic model as consisting of incompressible plugs of gas in the connecting passages, that oscillate like pistons on springs that are provided by the elasticity of the compressible gas in the cavities and plenums.

The difficulty is, therefore, to attach to the discrete Helmholtz resonator model, which is described in form of ordinary differential equations, the continuous discharge pipe, which is described by a partial differential equation. Thus, the following is aimed toward reducing the partial differential equation description of an anechoic pipe to an ordinary differential equation form. For this purpose we investigate first the response of anechoic pipes

to unit volume velocity impulses and use then a convolution integral description to allow for general inputs at the discharge pipe entrance.

#### PROPAGATION OF AN UNIT VELOCITY IMPULSE IN AN ANECHOIC PIPE

An anechoic pipe is in the applied mechanics sense a semi-infinite pipe. The acoustic displacements are given by the well known one dimensional wave equation

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2} \quad (1)$$

with the initial conditions

$$p(x, 0) = 0 \quad (2)$$

$$\frac{\partial f}{\partial t}(x, 0) = 0 \quad (3)$$

and boundary conditions

$$\frac{\partial f}{\partial x}(0, t) = \frac{1}{A} \delta(t - \tau) \quad (4)$$

$$p(\infty, t) = 0 \quad (5)$$

The first condition defines the unit volume velocity impulse, applied at  $t = \tau$  and located mathematically by  $\delta(t - \tau)$ , the dirac-delta function. The second condition defines the anechoic termination. The acoustic pressure is related to the acoustic displacement by

$$p(x, t) = -\rho c^2 \frac{\partial f}{\partial x}(x, t) \quad (6)$$

Differentiating Eq. (1) with respect to  $x$  and substituting Eq. (5) gives

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \quad (7)$$

Let us apply the Laplace transform to Eq. (7), with respect to time. We get

$$\frac{s^2}{c^2} p(x, s) = \frac{\partial^2 p(x, s)}{\partial x^2} \quad (8)$$

The solution of this equation is

$$p(x, s) = \bar{A} e^{-\frac{x}{c}s} + \bar{B} e^{\frac{x}{c}s} \quad (9)$$

Because of boundary condition (5),

$$\bar{B} = 0 \quad (10)$$

and

$$p(x, s) = \bar{A} e^{-\frac{x}{c}s} \quad (11)$$

To evaluate  $\bar{A}$ , we make use of the relationship

$$\frac{\partial p}{\partial x} = -s^2 \frac{\partial f}{\partial x} \quad (12)$$

which we have obtained from Eqs. (1) and (5). Taking the Laplace transformation of Eq. (12) gives

$$\frac{\partial p(x, s)}{\partial x} = -s^2 g f(x, s) \quad (13)$$

and substituting Eq. (12) results in

$$-\bar{A} \frac{x}{c} e^{-\frac{x}{c}s} = -s^2 g f(x, s) \quad (14)$$

At  $x = 0$ , we get

$$\bar{A} = s c g f(0, s) \quad (15)$$

By transforming boundary condition (4) we get

$$f(0, s) = \frac{1}{A} \frac{e^{-s\tau}}{s} \quad (16)$$

and thus

$$\bar{A} = \frac{c g}{A} e^{-s\tau} \quad (17)$$

and Eq. (11) becomes

$$p(x, s) = \frac{c g}{A} e^{-(\frac{x}{c} + \tau)s} \quad (18)$$

Inverting this expression gives

$$p(x, t) = \frac{c g}{A} \delta(t - \frac{x}{c} - \tau) \quad (19)$$

We recognize this expression as a dynamic Green's function and write it

$$G(x, t; 0, \tau) = \frac{c g}{A} \delta(t - \frac{x}{c} - \tau) \quad (20)$$

where  $G(x, t; 0, \tau)$  is the acoustic pressure at pipe location  $x$  and at time  $t$  due to a unit volume velocity impulse at  $x = 0$  (pipe entrance) and time  $\tau$ .

#### RESPONSE OF PIPE TO INSTANTANEOUS VOLUME VELOCITIES OR PRESSURES AT THE PIPE ENTRANCE

Instantaneous pressure, in terms of the dynamic Green's function and the instantaneous volume velocity at the pipe entrance  $Q(0, t)$ , is given by the convolution integral

$$p(x, t) = \int_0^t Q(0, \tau) G(x, t; 0, \tau) \quad (21)$$

Substituting Eq. (20) gives

$$p(x, t) = \frac{c g}{A} Q(0, t - \frac{x}{c}) \quad (22)$$

This means for instance, that the pressure at a time  $t$ , and a pipe position  $x$ , is equal to  $c g / A$  multiplied by the instantaneous volume velocity  $Q(0, t)$  at time  $t = t - \frac{x}{c}$ . Thus, for instance for the case of a rectangular volume velocity pulse, the propagating pressure is a rectangular pulse too, propagating with velocity  $c$ .

Of special interest is the case at  $x = 0$

$$p(0, t) = \frac{c g}{A} Q(0, t) \quad (23)$$

For instance, if  $p(0, t)$  is calculated first, we get

$$Q(0, t) = \frac{A}{c g} p(0, t) \quad (24)$$

and

$$p(x, t) = p(0, t - \frac{x}{c}) \quad (25)$$

Of primary interest for the Helmholtz resonator model application is the acoustic displacement at the pipe entrance. It is given by either

$$\frac{\partial f}{\partial t}(0, t) = \frac{1}{A} Q(0, t) \quad (26)$$

or

$$\frac{\partial f}{\partial t}(0, t) = \frac{1}{\rho c} p(0, t) \quad (27)$$

INCLUSION OF ANECHOIC PIPE IN SYSTEM EQUATIONS

Let us now consider as example the system of reference (18), with the anechoic discharge pipe attached. Considering the effective pressure forces acting on each gas plug as shown in Fig. 1, we get

$$4 \rho_1 S \frac{df_1}{dt} = (p_{dc1}^e - p_{pl}^e) S_1 \quad (28)$$

$$L_2 \rho_2 S \frac{df_2}{dt} = (p_{dc2}^e - p_{pl}^e) S_2 \quad (29)$$

where  $p_{dc1}^e = p_{dc1} + p_{dc1}^a$  (30)

$$p_{dc2}^e = p_{dc2} + p_{dc2}^a \quad (31)$$

and where  $p_{pl}^e = p_{pl} + p_{pl}^a$  (32)

$$p_{dc1}^a = - \frac{c_1 \rho_1 S_1}{V_{dc1}} f_1 \quad (33)$$

$$p_{dc2}^a = - \frac{c_2 \rho_2 S_2}{V_{dc2}} f_2 \quad (34)$$

$$p_{pl}^a = \frac{c_1 \rho_1}{V_{pl}} (S_1 f_1 + S_2 f_2 - A f(0,t)) \quad (35)$$

The acoustic displacement  $f(0,t)$  is given by Eq. (27) :

$$\frac{df(0,t)}{dt} = \frac{1}{c_1 \rho_1} p_{pl}^a \quad (36)$$

since

$$p(0,t) = p_{pl}^a \quad (37)$$

Thus, instead of two differential equations as given in reference [1], we have now three:

$$4 \rho_1 S \frac{df_1}{dt} + c_1^2 \rho_1 S_1^2 \left( \frac{1}{V_{dc1}} + \frac{1}{V_{pl}} \right) f_1 + \frac{c_1^2 \rho_1 S_2^2}{V_{pl}} f_2 - \frac{c_1^2 \rho_1 A}{V_{pl}} f(0,t) = (p_{dc1} - p_{pl}) S_1 \quad (38)$$

$$L_2 \rho_2 S \frac{df_2}{dt} + c_2^2 \rho_2 S_2^2 \left( \frac{1}{V_{dc2}} + \frac{1}{V_{pl}} \right) f_2 + \frac{c_2^2 \rho_2 S_1^2}{V_{pl}} f_1 - \frac{c_2^2 \rho_2 A}{V_{pl}} f(0,t) = (p_{dc2} - p_{pl}) S_2 \quad (39)$$

$$\frac{df(0,t)}{dt} + \frac{c_1 A}{V_{pl}} f(0,t) - \frac{c_1 S_1}{V_{pl}} f_1 - \frac{c_1 S_2}{V_{pl}} f_2 = 0 \quad (40)$$

Pressures in the discharge pipe at location  $x$  and time  $t$  are given by Eq. (25). Mean flow pressures are calculated in the usual manner.

SUMMARY AND CONCLUSION

The Helmholtz resonator approach to compressor discharge system analysis was extended to allow addition of an anechoic discharge pipe. It is felt that the approach with this added capability is general enough to allow its application to a large variety of compressor discharge systems.

In the applied mechanics sense the paper has proven that instantaneous nonperiodic acoustic displacement at the anechoic pipe entrance is related to instantaneous nonperiodic pressure at the pipe entrance in the same way as sinusoidal acoustic displacement is related to sinusoidal pressure by the impedance. This was not obvious at the beginning of this study and applies only to the anechoic pipe case.

REFERENCES

1. Soedel, W., Padilla Navas, E., Kotalik, B.D., "On Helmholtz Resonator Effects in the Discharge System of A Two-Cylinder Compressor", Journal of Sound and Vibration, Vol. 30, No. 3, 1973, pp. 263-277.
2. Elson, W., and Soedel, W., "Simulation of the Interaction of Compressor Valves with Acoustic Back Pressures in Long Discharge Lines", to be published in Journal of Sound and Vibration.

LIST OF SYMBOLS

- $Z$  = impedance
- $\rho$  = mass density [lb sec<sup>2</sup>/in<sup>4</sup>]
- $c$  = speed of sound [in/sec]
- $A$  = crosssectional area of pipe [in<sup>2</sup>]
- $f$  = instantaneous nonperiodic acoustic displacement [in]
- $t$  = time [sec]
- $x$  = coordinate [in]
- $p$  = instantaneous nonperiodic acoustic pressure [psi]
- $s$  = Laplace transformation variable
- $G$  = Green's function [psi]
- $Q$  = volume velocity [in<sup>3</sup>/sec]
- $L$  = neck length [in]
- $S$  = neck crosssection [in<sup>2</sup>]
- $e$  = superscript to indicate effective pressures
- $a$  = superscript to indicate acoustic pressures
- $1, 2$  = subscripts to indicate necks
- $dc_1, dc_2, pl$  = subscripts to indicate discharge chambers 1 and 2 and discharge plenum
- $V$  = volume [in<sup>3</sup>]

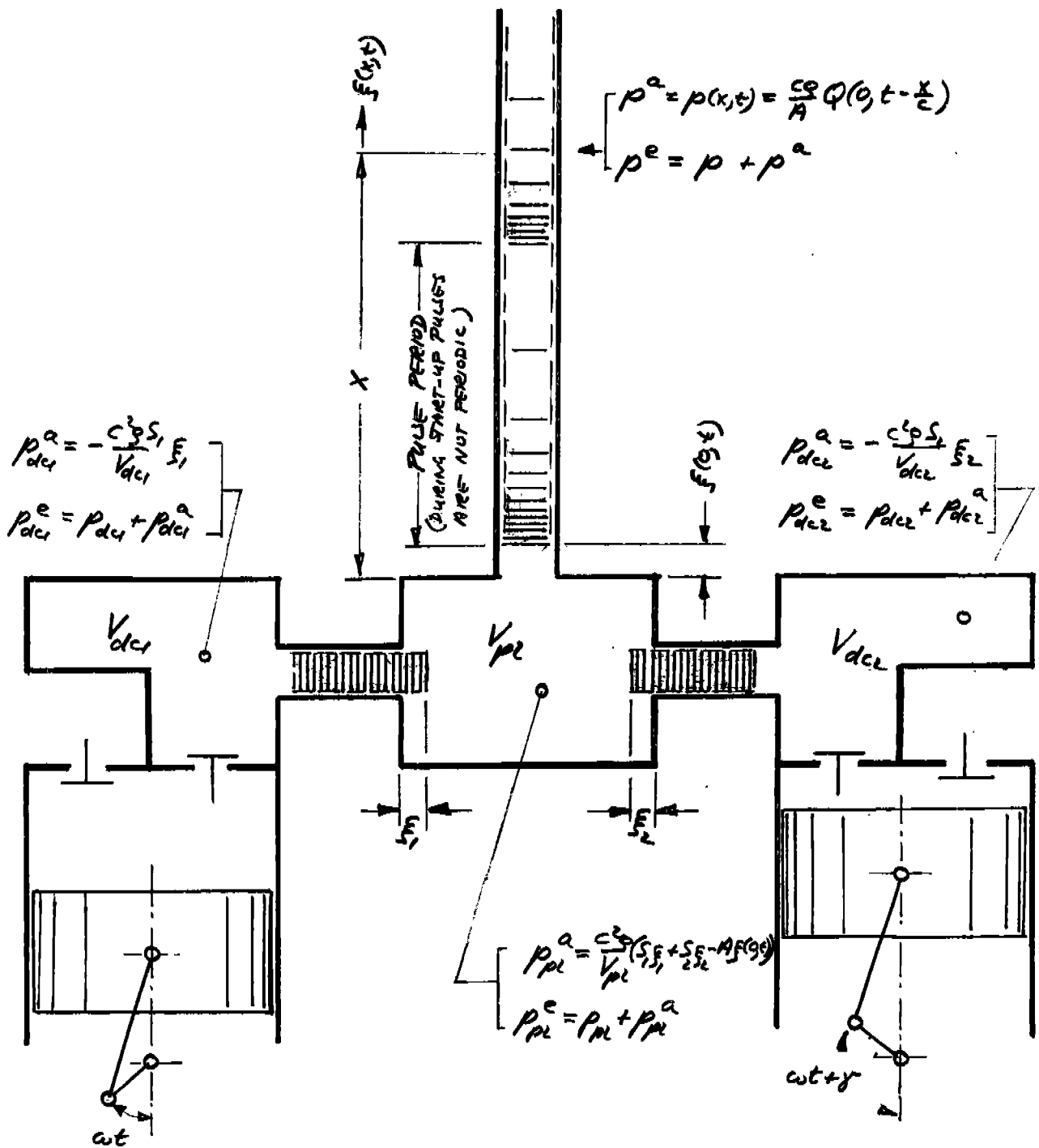


Fig. 1 Discharge System With Anechoic Pipe Termination