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APPROXIMATE CALCULATION METHOD FOR THE RUN OF CHANGES OF PULSATION PRESSURE IN RECIPROCATING COMPRESSORS PIPELINES

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INTRODUCTION

In reciprocating compressors, lately produced, the pulsation pressure values at the outlet are decidedly greater than ones acknowledged as permissible even when resonant effects don't appear in connected installation. Therefore the adequate pulsation pressure dampers are required. The knowledge of the run concerning maximum values of pressure pulsation Δp_a /peak-to-peak/ along the road of pipeline is the basis for evaluating the effect of decreasing pressure pulsation in a pipeline by a damper. That run permits to define the maximum pressure fluctuation rate in the installation and to state whether its value is larger or smaller than the value acknoledged as permissible.

Analytical solution for the procedure of pulsating gas stream flow in branching pipeline systems is confronted with great difficulties and is practically abridged to more simple cases. The most effective way for solution of these problems is the application of digital computers, particulary electric models based on analogy principles. Many investigators were engaged in this problem, e.g.: R.S. Benson /1,2/, J. Bráblik /3/, Y.N. Chen /4/, G. Damewood /9/, S.S. Grover /5/, W.A. Kozłow /6/, P. Kuhlmann/7/, W. Nimitz /9,10/, J.M. Sharp and E.N. Henderson /11/, A.S. Üçer /1/, D. Woolaatt /2/ and others.

EQUATIONS OF THE PULSATING GAS STREAM FLOW IN A PIPELINE

Installation of the gas under compression is a wave-guide, which can be modelled as a system of series connected tubes of various diameters. The calculation of the run of changes in a pressure pulsation in this installation requires the solution of the partial differential equations which describe pulsating flow of gas with equivalant boundary conditions taken into account. The equations describing pulsating flow of gas in a straight pipeline of constant cross-section are considered below. These equations are derived by means of various simplifying assumptions on the base of equation of movement, equation of continuity and equation of state. Assuming that the gas density is constant and equal to its mean value with respect to time and linearizing non-linear resistance expression, this equation can be presented in the following form:

$$\left.\begin{aligned} -\frac{\partial v}{\partial t} &= \alpha \frac{\partial p}{\partial x} + \gamma v \\ -\frac{\partial p}{\partial t} &= \beta \frac{\partial v}{\partial x} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots /1/$$

where: $\alpha = \frac{A}{\rho_o}$, $\beta = \frac{\rho_o a^2}{A}$, $\gamma = \frac{\lambda w_o}{2D}$;

symbol p denotes here variable pressure of gas, t - time, D - internal diameter of pipeline, a - sonic gas velocity, λ - friction losses factor, w_o - mean analytical gas velocity, A - cross-sectional area of the tube, ρ_o - mean gas density, v - variable volumetric velocity.

Laplace's transformation of equations /1/ under zero initial conditions, i.e. in the case of $v_o/x/\equiv 0$ and $p_o/x/\equiv 0$, gives:

$$\left.\begin{aligned} \frac{d^2\bar{p}}{dx^2} - \varrho^2\bar{p} &= 0 \\ \frac{d^2\bar{v}}{dx^2} - \varrho^2\bar{v} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots/2/$$

General solution of the system of equations /2/ is:

$$\left.\begin{aligned} \bar{p}/x,s/ &= Z_f\left[B_1/s/\ e^{-\varrho x} - B_2/s/\ e^{\varrho x}\right] \\ \bar{v}/x,s/ &= B_1/s/\ e^{-\varrho x} + B_2\ e^{\varrho x} \end{aligned}\right\} \ldots/3/$$

where:

$$\varrho = \sqrt{\frac{s/s+\chi/}{\mathcal{L}\beta}} = \frac{1}{a}\sqrt{/s+\delta/^2 - \delta^2} \quad \ldots\ldots\ldots/4/$$

$$Z_f = \sqrt{\frac{\beta/s+\chi/}{\mathcal{L}s}} = \frac{\sigma/s+2\delta/}{\sqrt{/s+\delta/^2 - \delta^2}} \quad \ldots\ldots\ldots/5/$$

s is the parameter of Laplace's transformation, symbols a, σ and δ are defined:

$$a = \sqrt{\mathcal{L}\beta}, \quad \sigma = \sqrt{\frac{\beta}{\mathcal{L}}}, \quad \delta = \frac{\chi}{2}.$$

<u>SOLUTION FOR THE PIPELINE OF CONSTANT CROSS-SECTION</u>

Let us consider straight pipeline segment of the constant cross-section A and the length l coupled with compressor which causes periodical in time excitations described by the function $\psi/t/$. The second end of the tube is ended in the arbitrary closing which characterizes the impedance Z_l. Let us assume additionally that the piston travel of the compressor is neglectible small in comparison with the length of pipeline and that the impedance at the beginning of the tube is Z_1 /Fig. 1/.

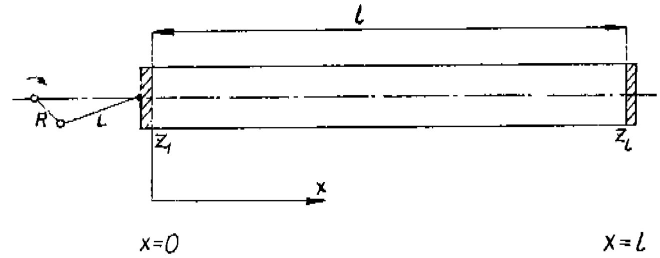


Fig. 1. Pipeline of constant cross-section coupled with compressor

For the piping system shown on Fig. 1 boundary conditions can be written in the form:

$$x = 0 \qquad\qquad x = l$$

$$\psi/s/ = \bar{v}_1/s/ + \frac{\bar{p}_1/s/}{Z_1/s/}\ ; \quad \bar{p}_l/s/ = Z_l/s/\ \bar{v}_l/s/ \quad \ldots\ldots\ldots/6/$$

where: $\bar{v}_1/s/ = \bar{v}/0,s/$; $\bar{p}_1/s/ = \bar{p}/0,s/$; $\bar{v}_l/s/ = \bar{v}/l,s/$; $\bar{p}_l/s/ = \bar{p}/l,s/$.

Coeffitients of reflection at the beginning of the tube and at the end of the tube, called here respectively primary q_1 and secondary q_l, are defined as follows:

$$q_1 = \frac{Z_1 - Z_f}{Z_1 + Z_f}\ ; \quad q_l = \frac{Z_l - Z_f}{Z_l + Z_f} \quad \ldots\ldots\ldots/7/$$

For the reason of the complete closing of the beginning of the tube by the piston-head impedance $Z_1 = \infty$ then $q_1 = 1$. Taking into consideration boundary conditions /6/ and introducing relations /4/, /5/ and /7/ we can determine constants B_1 and B_2 in the solution /3/ and for the case $\delta = 0$ we can obtain following transforms of the solutions:

$$\bar{p}/x,s/ = \sigma\,\bar{v}_1/s/\sum_{k=0}^{\infty}\left(q_l^k\, e^{-\frac{2kl+x}{a}s} + q_l^{k+1}\, e^{-\frac{2/k+1/l-x}{a}s}\right)$$

$$\bar{v}/x,s/ = \tilde{v}_1/s/\sum_{k=0}^{\infty}\Big(q_1^k\, e^{-\frac{2kl+x}{a}s} - \ldots\ldots + q_1^{k+1}\, e^{-\frac{2/k+1/l-x}{a}s}\Big) \qquad \ldots./8/$$

Let us now perform Laplace's retransformation $\mathcal{L}^{-1}$ for transforms /8/, using, term after term, according to the theorem of translation and taken into account previonsly neglected frictional term in the form of function $e^{-\delta t}$. It means that the exponential type of damping of the wave amplitude along the wave way without change of the shape of time runs is accepted. For $0 \leqslant x \leqslant l$ and $t > 0$ we obtain thus the solution in the following form:

$$p/x,t/ = \sigma \sum_{k=0}^{\infty}\Big[q_1^k\, v_1\big(t-\frac{2kl+x}{a}\big)1\big(t-\frac{2kl+x}{a}\big)\cdot e^{-\delta\frac{2kl+x}{a}} + q_1^{k+1}\, v_1\big(t-\frac{2/k+1/l-x}{a}\big)\cdot 1\big(t-\frac{2/k+1/l-x}{a}\big)\, e^{-\delta\frac{2/k+1/l-x}{a}}\Big]$$

$$v/x,t/ = \sum_{k=0}^{\infty}\Big[q_1^k\, v_1\big(t-\frac{2kl+x}{a}\big)1\big(t-\frac{2kl+x}{a}\big)\cdot e^{-\delta\frac{2kl+x}{a}} - q_1^{k+1}\, v_1\big(t-\frac{2/k+1/l-x}{a}\big)\cdot 1\big(t-\frac{2/k+1/l-x}{a}\big)\, e^{-\delta\frac{2/k+1/l-x}{a}}\Big] \qquad \ldots./9/$$

where: $1(t-\ldots.)$ means unit step function.

PIPING SYSTEM COMPOSED OF TUBES OF VARIOUS CROSS-SECTIONS

Previonsly considered pipeline of constant cross-section is elementary part of am arbitrary piping system. As mentioned above compressor installation consists of number of series connected tubes of various lengths and diameters as well as of different vessels /Fig. 2/.

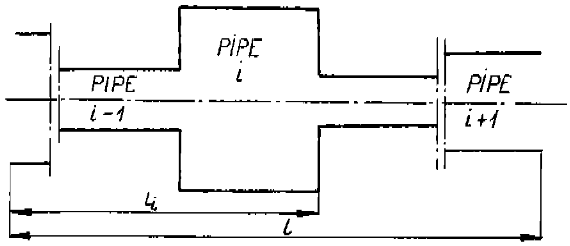


Fig. 2. Scheme of piping system composed of tubes of various cross-sections

For the **simplicity** reasons vessels in the calculation model are treated as tubes of equivalant lengths and diameters. For the each elementary tube equations /1/ and their solutions /9/ are valid. In each place of step change of the cross-section of the tube incomplete reflection and partial transfer of pressure pulsation wave takes place. Continuity condition in the place of the change of cross-section of pipeline can be written for pressure waves as follows:
in the case of waves moving from the left to right

$$p_i + q_{i,\,i+1}\, p_i = T_{i,\,i+1}\, p_i \qquad \ldots../10/$$

in the case of waves moving from the right to left

$$p_{i+1} + q_{i+1,\,i}\, p_{i+1} = T_{i+1,\,i}\, p_{i+1} \qquad \ldots/11/$$

where: $T = 1+q$ denotes coefficient of transmission.

Values of q must be chosen from the range /-1, +1/, so it is easy to notice that corresponding values of T must be greater than zero and smaller than + 2. As a conclusion the coefficient of the transmission is always positive.

EXAMPLE OF CALCULATIONS

In the purpose to examine the accordance of the results of the calculations of pressure pulsation runs obtained by the use of proposed method with practice were performed calculations and measurements on the installation of compressed air which is illustrated in Fig. 3a. The calculation model for this installation consists of four series connected tubes and is shown in Fig. 3b.

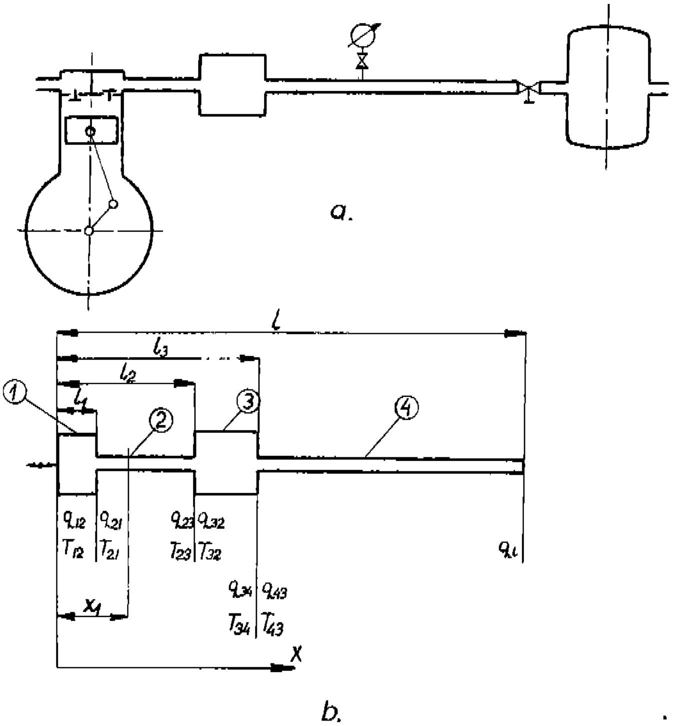


Fig. 3. Scheme /a/ and calculation model /b/ of outlet installation

The excitation function illustrated in Fig. 4 is described by the following relation:

$$v_1/t/=\begin{cases} 0 & 0\leqslant\omega t\leqslant\varphi_1 \\ A_p R\omega/\cos\,\omega t+\frac{\lambda}{2}\sin\,2\omega t/ & \varphi_1\leqslant\omega t\leqslant\pi \\ 0 & \pi\leqslant\omega t\leqslant 2\pi \end{cases} =$$

$$=\psi/t/ \quad ..../12/$$

where: $\lambda = \frac{R}{L}$;

symbol A_p denotes here cross-sectional area of a piston, R - crank radius, L - connecting rod length, ω - angular velocity of compressors crankshaft.

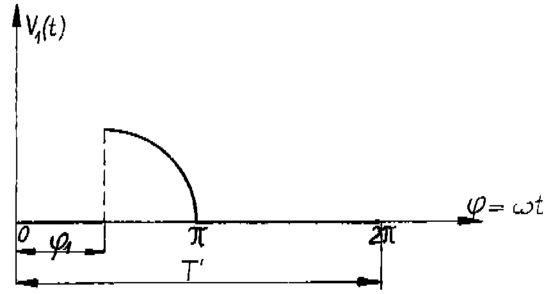


Fig. 4. Form of the excitation function

For the calculation it is necessary to decompose the excitation function /12/ into Fourier's series:

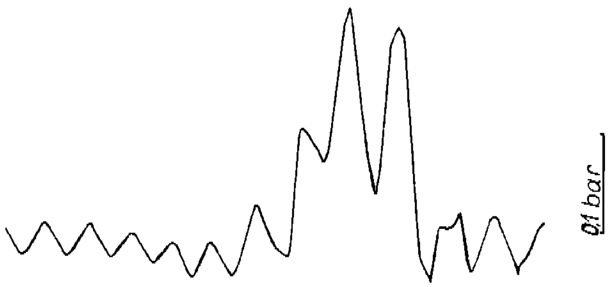
$$v_1/t/=\sum_{m=1}^{\infty}/B_m\cos\,m\omega t + A_m\sin\,m\omega t/ \quad ..../13/$$

where:

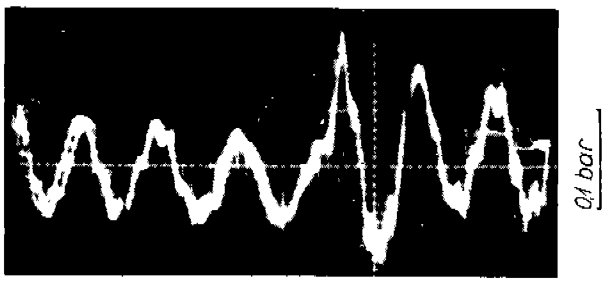
$$B_m=\frac{2}{T}\cdot\int_0^{T'} v_1/t/\cos\,m\omega t\quad dt$$

$$A_m=\frac{2}{T}\cdot\int_0^{T'} v_1/t/\sin\,m\omega t\quad dt$$

Fig. 5a illustrates pressure pulsation run in the distance x_1 of the considered compressed air installation which is shown in Fig. 3. Fig. 5b illustrates on the other hand pressure pulsation run in the same place registered by the use of piezoelectric pressure indicator. The presented runs refer to the case of compressor work at rotational speed 1100 rev/min with absolute pressure 3 bars in installation. For calculations was accepted speed of sound 408 m/s which corresponds with air temperature prevailing in the chamber of compressor outlet valve. The divergence between calculated and registered runs mainly result from disregarding valve action in calculations. Besides the opening and closing angles of the outlet valve which are a little different from the real ones were accepted to calculations. Speed of sound changes caused by temperature drop along the way of pipeline was disregarded as well.



a.



b.

Fig. 5. The runs of pressure pulsation changes in section x_1 of installation presented in Fig. 3, a - calculated, b - measured

CONCLUSION

The presented calculation way of the changes run of pressure pulsation in reciprocating compressors pipeline gives the results being in accordance with the measurement results. It gives the calculation possibility of changes run in pressure pulsation in time for any installation section. On the base of runs calculated in this way in some sections, the diagram of maximum amplitudes runs along the pipeline way may be drawn up, and the right choice of the damper type and its installation place may be estimated properly in the project phase.

Calculation time by means of presented method is relatively long. There is also necessity of making up separate calculation algorithms in principle for installation composed of different quantity of tubes. Therefore it is advisable to work out a number of some algorithms for typical schemes of inlet installations, interstage and outlet ones, which mostly meet the occuring needs.

REFERENCES

1. R.S. Benson, A.S. Üçer "Some Recent Research in Gas Dynamic Modelling of Multiple Single Stage Reciprocating Compressor Systems", Proceedings of 1972 Purdue Compressor Technology Conference, West Lafayette, Indiana, July 25 - 27, 1972, pp. 491-498.

2. R.S. Benson, R.D. Garg, D. Woollatt "A Numerical Solution of Unsteady Flow Problems", Int. J. Mech. Sci. 1964, Vol. 6, pp. 117-144.

3. J. Bráblik "Berechnung der Gasschwingungen in Rohrleitungen von Kolbenverdichtern", Die Schwerindustrie der Tschechoslowakei 1972, No. 8, pp. 39-44.

4. Y.N. Chen "Calculation of Gas Due to Simultaneous Excitations in Reciprocating Compressor Piping Systems with Allowance for Frictional Effekt and Temperature Change in the Flow", Journal of Sound and Vibrations 1967, 5/2/, pp. 215-256.

5. S.S. Grover "Analysis of Pressure Pulsations in Reciprocating Compressor Piping Systems", Journal of Engineering for Industry 1966, 88B. pp. 164-171.

6. W.A. Kozłow, A.A. Kozobkow, A.S. Messerman "Elektriczeskoje modelirowanie processa dwiżenija pulsirujuszczego potoka gaza", Wibracja tiechnołogiczeskich truboprowodow na nieftiechimiczeskich priedprijatiach 1967, pp. 98-104.

7. P. Kuhlmann "Berechnung von Schwingungen in den Rohrleitungen von Kolbenverdichtern", VDI - Forschungsheft 1966, No. 516, pp. 7-30.

8. M. Łuszczycki "Elektryczne modelowanie przepływu pulsującego strumienia gazu w rurociągu", Gaz, Woda i Technika Sanitarna 1974, tom XLVIII, No. 1, pp. 8-10.

9. W. Nimitz, G. Damewood "Compressor Installation Design Utilizing an Electro-Acoustic System Analog", ASME Paper No. 61-WA-290.

10. W. Nimitz "Pulsation and Vibration", Pipe Line Industry 1968, August pp. 36-39 and September pp. 39-42.

11. J.M. Sharp, E.N. Henderson "Pulsation Control and Maximum Compressor Efficiency", Oil and Gas Journal 1956, January 9, pp. 81-85 and January 16, pp. 114-118.