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A Review of Compressor Lines Pulsation Analysis and Muffler Design Research -Part II: Analysis of Pulsating Flows

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A REVIEW OF COMPRESSOR LINES PULSATION ANALYSIS AND MUFFLER DESIGN RESEARCH

PART II - ANALYSIS OF PULSATING FLOWS

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The suction and discharge system pressure fluctuations are inherent in the air, gas and refrigeration compressor installations. Their role in compressor performance and operation has already been discussed in Part I of this paper. It has become essential that the exact level of pressure pulsations and their effect be predicted by analytical or experimental studies so that the suitable remedial measures can be suggested. Also pulsating flow equations should be included in the overall compressor computer simulation programs for more accurate and realistic models.

VALVE INTERACTION

Valves do not only magnify the fluctuating flows as produced by the piston movement but also interact with the suction and discharge system uneven flows. First the interaction mechanism between the valve operation and the pressure pulses shall be discussed to illustrate the fact that valve flow and valve dynamics equations have to be solved simultaneously with the suction and discharge lines gas flow equations.

Motion is imparted to the valve as a result of pressure inequalities on either side of the valve leaf. Suction and discharge valve openings are delayed because of the inertia of the valves. When the valves start opening, an elastic restoring force, proportional to its displacement, also acts on the valve, thus giving rise to the vibrations of the valve. As the valves are opening, both flow equation and dynamics equations are applicable but when the valves reach the stop, only the flow equation is used to compute the pressure difference across the valve.

Flow equation

$$\dot{m}_{vi}(t) = A_{vi}\, p_{ui}\sqrt{\frac{2\gamma g_c}{(\gamma-1)RT_{ui}}}\sqrt{(r_i)^{2/\gamma}-(r_i)^{\frac{\gamma-1}{\gamma}}} \quad (1)$$

where $\dot{m}_v$ is mass flow rate through the valve, A_v is valve flow area, γ is abiabatic constant or ratio of specific heats, p_u is upstream pressure, T_u is downstream pressure, r is ratio of downstream to upstream pressure, and the subscript i indicates the suction or discharge valve. Refer Fig. 1 for physical model of the compressor. All the fluid variables have been listed there. For the suction lines (p_s) is upstream and (p_{cyl}) is downstream pressure. For mass flow rate through the discharge valve, cylinder conditions (p_{cyl}) become upstream conditions and the pressure in the discharge lines (p_d) is the downstream pressure. Conditions for maximum mass flow rate (choked flow) and back flow (a possibility because of the pressure fluctuations in the lines) can also be added to the flow equation.[1,2].

Dynamics Equation

Forced vibration of the valve, due to the pressure differential across the valve, in its simplest form can be expressed as,

$$M_i\ddot{q}_i(t) + C_i\dot{q}_i(t) + K_i q(t) = C_{Di}A_{fi}\Delta p_i(t) \quad (2\text{-a})$$

where q(t) is the valve displacement, M is effective valve mass, C is effective valve damping, K is effective valve stiffness, C_D is valve drag coefficient, A_f is valve force area, Δp is pressure difference across the valve and subscript i indicates suction (s) or discharge (d).

$$\Delta p_s(t) = p_s - p_{cyl}(t) \quad (2\text{-b})$$

$$\Delta p_d(t) = p_{cyl}(t) - p_d \quad (2\text{-c})$$

The above equation assumes the valve to be a single degree of freedom case, it is taken here only for the illustration of the valve interaction. More sophisticated valve dynamics models exist [2,4] where the valve is treated to be a multi-degree of freedom case or by a continuous system approach. From the above two equations it is clear the mass flow rate through the valves and the valve displacements are functions of the pressure differentials as shown below,

$$\dot{m}_{vi}(t)\alpha\sqrt{\Delta p_i} \qquad \text{and } q_i(t)\ \alpha\ (\Delta p_i)$$

If the pressures in the suction and discharge lines are changing continuously, so will be the mass flow rates and the valve displacements.

EQUATION OF MOTION IN COMPRESSOR LINES

Fluid flow in the suction and discharge lines has to be modeled to provide the following:[3]

i) Exact time varying suction and discharge pressures at the valves for computer simulation model so that the valve behavior and operation, mass flow rates, pressure-volume relationship in cylinder and capacities can be calculated and predicted precisely. Simulation models as given in references (4,6,11,17,18,22,24,25 & 26) include the pulsating flow analysis.

ii) Pressure distribution and pressure pulsation level in both suction and discharge system. Also it is a good tool for compressor muffler design & performance evaluation. Several investigators [5,7,8,9,10,12,13,14,15,19,20] have analysed pulsation systems for this objective.

Note that the flow in the lines is composed of two parts: mean flow and the pulsating flow. Pressure pulses are propagated in the form of waves travelling at the speed of sound, in advance of the moving mean flow itself. Thus, when the mean flow enters the new zone, it finds that the pressure there has already been changed by the proceeding waves. These waves transmit deformations and pressures at a finite speed (sonic speed). Each medium has a definite speed of sound depending upon its compressibility and density. The motion of the gas is governed by the laws of fluid mechanics: the equation of fluid motion (Navier-Stokes equation) and law of conservation of mass (continuity equation). These equations are non linear in their general form and their solution presents difficult problems. In order to model the compressor lines, it is therefore necessary to make simplifying assumptions which permit the solutions to only a certain degree of accuracy.

ACOUSTIC WAVE EQUATION

The wave equation can be derived by starting with the following basic equations of fluid mechanics and thermodynamic assumptions. Since, virtually all the research work is confined to one dimensional models, we shall also restrict discussion to one dimension problem. Also one dimensional models are easy to linearize.

Navier-stokes Equation

For non-viscous ideal flow, the equation of motion is

$$\frac{\partial u_t}{\partial t} + u_t \frac{\partial u_t}{\partial x} = -\frac{1}{\rho_t}\frac{\partial p_t}{\partial x} \qquad (3)$$

where u_t is the velocity, p_t is pressure, ρ_t is density, t is time and x is the longitudinal coordinate. The subscript t indicates the total instantaneous value of the fluid variables.

Continuity Equation

$$\frac{\partial \rho_t}{\partial t} + u_t \frac{\partial \rho_t}{\partial x} + \rho_t \frac{\partial u_t}{\partial x} = 0 \qquad (4)$$

Hooke's Law

We assume that Hooke's Law holds for a gas medium. According to this, stresses are proportional to the deformations (always true for small deformations). This assumption is based on two thermodynamic assumptions: (i) gas follows perfect gas law relationship, (ii) acoustic process is isentropic i.e. reversible adiabatic process. Fluid variables in (3) and (4) represent total instantaneous values i.e. sum of the mean and fluctuating parts, as shown below

$$u_t = u_o + u, \quad p_t = p_o + p \quad \text{and } \rho_t = \rho_o + \rho \qquad (5\text{-a})$$

where subscript o indicates mean value and the variables without any subscript are the fluctuating parts or the acoustic variables. According to Hooke's law

$$p = -K\ (\Delta V/V) = K\frac{\rho}{\rho_o} \qquad (5\text{-b})$$

Where V is the volume, ΔV is change in volume and K is the bulk modulas and is given by $K = \rho_o c^2$ when ρ_o is the mean density and c is the sonic velocity (also expressed as $c = \sqrt{\gamma g_c R T_o}$, where γ is adiabatic constant, R is gas constant, T_o is absolute mean temperature and g_c is gravitational constant .

If we assume that fluctuating variables are small compared to the mean variables and there is no mean flow ($u_o = o$), then from (3), (4), & (5) we get a linearized equation, known as the acoustic wave equation,

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \qquad (6)$$

Its harmonic solution is

$$p = Ae^{i(\omega t-kx)} + Be^{i(\omega t+kx)} \qquad (7)$$

where A and B are constants, ω is the circular frequency, k is the wave number and is given by $k = \frac{\omega}{c}$. The first part of the solution represents a positive x direction wave and the second part represents a negitive x direction wave. In forming the above equation, assumptions made may limit the applications but the experience has shown that it gives a quite precise description of the wave phenomena and the deviation from the laws governing the general propagation are small corrections to wave equations in the majority of cases. Many investigators [6,7,8,9,10,11,12,14,17,19,20,22,25,30] have used this equation successfully for modeling. According to Elson [16,17] and Soedel [14] wave equation is applicable even up to $\frac{p}{p_o} = 0.15$ to 0.18. They have reported good experimental and analytical correlations. For a more exact approach to the problem it must be borne in mind that acoustic processes take place in the viscous media and also the wave amplitudes frequently may build up to a finite value, comparable to the mean flow variables. Carpenter [19] has reported one extreme condition where $\frac{p}{p_o}$ was 50%, in one inter stage condition. But the fact that normally $\frac{p}{p_o}$ is always below 20%, and in this range, the wave equation can be used for modeling satisfactorly from an engineering point of view. Benson [23,24], Brablik [11], and MacLaren [18] used simulation model that accounted for finite amplitudes. Viscous effects have been taken into account by Chen [5], Abe [30], Grover [12] and Brablik [11] etc.

FACTORS TO BE INCLUDED IN WAVE EQUATION

Wave equation (6) does not consider the effects of the friction, mean flow, turbulence, thermal conductivity, heat transfer and the finite wave amplitudes. These effects can be either directly added to the wave equation or applied in the form of the correction factors to the solution of the wave equation. Also, some situation may demand the consideration of three dimensional wave equation, acoustically nonlinear elements and absorption material lining aspects.

1. <u>Friction, Thermal Conductivity and Turbulence</u> [5,20]: For the accuracy of calculation, it may be necessary to calculate the damping of the waves and this is done by taking the viscous and thermal effects into account. Friction is considered proportional to the velocity and when included in the wave equation, (6) changes to

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} - \frac{R}{\rho_o} \frac{\partial p}{\partial t} \qquad (8)$$

where R is the coefficient of friction. The solution of equation (8) is,

$$p = Ae^{-\alpha x} e^{i(\omega t-kx)} + B\, e^{\alpha x} e^{i(\omega t+kx)} \qquad (9)$$

where the first term represents a positive x direction and second term indicates a negative x direction travelling wave. α is the damping factor and is expressed as $\alpha = R/2c\rho_o$. The damping factor, α, can be calculated from the Binder's [21] empirical expression which includes the effect of mainly gas viscosity μ and also of gas thermal conductivity k_t and turbulence, in the form of eddy viscosity ϵ, as follows:

$$\alpha = \frac{1}{dc}\sqrt{\frac{\omega}{\pi}\left(\frac{\mu}{\rho_o} + \frac{\epsilon}{\rho_o} + \frac{k_t}{g\rho_o C_p}\right)} \qquad (10)$$

where d is the diameter of pipe, C_p is the specific heat of the gas at constant pressure and g is the acceleration due to the gravity. The damping effects, as witnessed from above, depend upon the frequency. Eddy viscosity, ϵ, obviously depends upon Reynolds number and thus on mean flow which shall be discussed next.

2. <u>Mean Flow</u> [5]: The greatest effect of the mean flow could be the convective effect. For the wave travelling in the direction of flow, the sound speed is increased by u_o, the mean flow velocity, and for the wave propagating opposite to the flow direction the sound speed is decreased by u_o. These corrected sonic velocities may be used in the solution of the wave equation, as shown below in equation (11).

Another approach could be to use the equations (3), (4) and (5) as such without ignoring mean flow velocity terms. This approach also results in the following solution of wave equation

$$p = A\, e^{i\omega(t-x/c+u_o)} + B\, e^{i\omega(t+x/c-u_o)} \qquad (11)$$

However it should be mentioned here that generally in the compressor lines, u_o/c is rarely above 5%.

3. Finite Amplitudes [11,18,23,24]: In some cases, the amplitudes of the wave may become large and hence it cannot be treated by the linear wave equation (6). Again we shall consider the one dimensional case for gas obeying the adiabatic law

$$p_t / \rho_t^{\gamma} = \text{const}$$

$$c^2 = \frac{\partial p_t}{\partial \rho_t} = \text{const}\ \gamma \cdot \rho^{\gamma-1} \qquad (12)$$

Using Navier-Stokes equation (3) with (12), we get (3) as

$$\frac{\partial u_t}{\partial t} + u_t \frac{\partial u_t}{\partial x} + \frac{2c}{\gamma-1} \frac{\partial c}{\partial x} = 0 \qquad (13)$$

Similarly using (12) in continuity equation (4), we get

$$\frac{2}{\gamma-1}\left(\frac{\partial c}{\partial t} + u_t \frac{\partial c}{\partial x}\right) + c\frac{\partial u_t}{\partial x} = 0 \qquad (14)$$

Now if we let

$$a = \frac{c}{\gamma-1} + \frac{1}{2} u_t \simeq \frac{c}{\gamma-1} + \frac{1}{2} u$$

and

$$b = \frac{c}{\gamma-1} - \frac{1}{2} u_t \simeq \frac{c}{\gamma-1} - \frac{1}{2} u \qquad (15)$$

Thus equations (13) and (14) reduce to a pair of equations,

$$\frac{\partial a}{\partial t} + (fa + gb)\frac{\partial a}{\partial x} = 0 \qquad (16)$$

$$\frac{\partial b}{\partial t} - (fb + ga)\frac{\partial b}{\partial x} = 0 \qquad (17)$$

where $f = \frac{1}{2}(\gamma+1)$ and $g = \frac{1}{2}(\gamma-3)$

The quanities a and b are called the Riemann invariants. If one of these is a constant, then one equation of the pair (16) and (17) is an identity, and the other is a first order equation by means of which the other invariant may be determined. The gas flow corresponding to the solution so obtained is called a simple wave. From the initial and the boundary conditions for waves in a pipe, the pressure distribution at any time or position along the pipe can be calculated. This method is generally called the method of characteristics. It is, in spite of being potentially accurate, severely limited by the cost of the analysis. Also pressure fluctuations in the system, as stated previously, are fairly small. The simplicity of plane wave model (6) justifies the approach of several investigators without sacrificing much accuracy. Benson [23,24] has developed a computer program for the method of characteristics and has used it successfully in analysing unsteady flows in compressors and I.C. Engines.

4. Heat Transfer [5,23]: We have to consider two heat transfer cases. The first is heat transfer due to the mean flow. Suction lines and discharge lines may exchange heat with the environment and with each other (if they are sufficiently close, as may be found in compact compressors). This heat transfer affects the mean temperature of the flow and since sonic speed is based on mean temperature or mean conditions, an allowance has to be made for mean temperature variations. The second heat transfer case is in the basic assumption of wave motion that it is an isentropic process. However, in reality because of the friction, some entropy change is always there. The thermal conductivity of the gas has been considered in the equation (10). Benson [23] has made correction for this by adjusting the entropy change across the valves. Chen [5] has solved the problem by taking gas columns at different temperatures.

5. Three-dimensional effects: Wave equation for a three dimensional case is,

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad (18)$$

In compressor suction and discharge lines, generally we encounter tube elements and the oscillatory motion is axisymmetrical, hence, the above equation reduces to,

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad (19)$$

Where r is the radial distance and z is longitudinal coordinate. Generally the pipe diameter is small compared to the wave length of the sound and hence one-dimensional model holds good. However, in plenum chambers there is a possibility of chamber dimensions being larger than the wavelength at higher frequencies. In such cases, we have to consider two dimensional models and cross modes in the piping. For compressors, no one has taken cross modes effect into account but in I.C. engines, Alfredson [28] and others have included this possibility.

6. Absorption Lining: Generally lined ducts are not used in refrigerating compressors but however, the possibility exists for the gas compressors. Lining can be included in the analysis by using three dimensional wave equation (18) or (19) with suitable boundary conditions, either in the form of impedance information or by taking the energy loss into account.

7. Nonlinear elements: the linearized wave equation model may break down in the case of certain nonlinear elements like orifices where the linear range is valid only for the low amplitudes. At high amplitudes according to Ingard [27], the acoustic pressure is a quadratic function of the particle velocity.

APPLICATION OF WAVE EQUATION

The wave equation as such can be either directly coupled to the valve dynamics and valve flow equations or calculated separately and then resulting pressures can be used in equation (1) & (2) for mass flow rate and value response. For simulation purposes it is an iterative procedure because the wave equation solution requires one boundary condition at the valve and since all the equations are coupled, the solution has to be a simultaneous one. However, for the pulsation level prediction and muffler performance evaluation, the equation can be solved directly with given boundary conditions. Note that the solution of the valve dynamics and flow equations is in the time domain and the harmonic solution of the wave equation is in the frequency domain. Pressure and volume velocity (velocity times area) can be broken down into Fourier series components as discussed in Part I of the paper. The next step now, is the discussion of boundary conditions as the wave equation constitutes a boundary valve problem and then we shall discuss the various solution techniques, as available in the literature, for pressure pulsation analysis.

BOUNDARY CONDITIONS

In the wave equation (6), the acoustic variable is p. The acoustic particle displacement and velocity can also be represented by the similar equations. Pressure p and particle velocity u are related by the characteristic impedance ($\rho_o c$) as, $p = \rho_o c\, u$ for positive x direction wave fronts and $p = -\rho_o c\, u$ for negative x direction wave fronts. Since here the acoustic propagation is limited to pipes and volumes, we have to add another variable, volume velocity Q, to take into account the area of the piping S where Q is given by $Q = uS$. Note that here p, u and Q may be complex quantities. The complex quotient of p and Q is called acoustic impedance Z and is expressed as the sum of real and imaginary numbers,

$$Z = \frac{p}{Q} = Z_R + i\, Z_I \tag{20}$$

The boundary condition may be expressed in the form of any one of these variables p, Q or Z. Various boundary conditions existing in the air and the refrigeration compressors are discussed below.

1. Starting Point of Piping: The starting point for both the suctions and the discharge system piping is at the valves. Acoustic behavior is the same whether the mean flow is sucked into and exhausted out of the cylinder by the reciprocating piston. At the valve, the mass flow rate, $\dot{m}_v$ is given, then volume velocity will be

$$Q_i(0,\theta) = \frac{\dot{m}_{vi}}{\rho_{o_i}}\ ; \quad i = s, d \tag{21}$$

where 0 means that at x = 0(Fig. 2) and θ is the crank angle. θ is given by $\theta = \omega t$. Since the mass flow through the valve can be analysed by Fourier series, Q_i can also be expressed similarly.

$$\dot{m}_{vi} = \sum_{n=0}^{\infty} \dot{m}_{ni} \cos(n\omega t - \psi_{ni})\ ; \quad i = s,\ d$$

$$Q_i = \sum_{n=0}^{\infty} Q_{ni} \cos(n\omega t - \phi_{ni})\ ; \ i = s, d \tag{23}$$

where $\dot{m}_{ni}$ and Q_{ni} are the mass flow rate and the volume velocity amplitudes respectively and ψ_n and ϕ_n are phases of $\dot{m}_v$ and Q respectively. n is the order of harmonic.

2. Suction Line End Point: (Refer Fig. 3) For an air compressor, suction piping is of finite length and the end is generally open to the atmosphere. Thus at the end, pressure is known. Ignoring radiation effects from the end, we may assume p to be zero at the end or very small value.

$$p(L,\ t) \cong 0 \tag{24}$$

In the case of a gas compressor, suction line is attached to a receiver wherein the pressure pulsation will be negligible. Thus p, at the end, is also zero in this case. However, in a refrigerating compressor, the suction line has the non-reflecting end i.e. it is an anechoic line and is specified by their characteristic acoustic impedance $\frac{\rho_o c}{S}$, where S is the area of tube joined to the evaporator, thus the end condition for an anechoic line is acoustic impedance Z,

$$\Big[Z(w)\Big]_{x=L} = \rho_o c/S \tag{25}$$

3. Discharge Line End Point: (Refer Fig.4) For a refrigerating compressor, discharge tubes connected to conclusions constitute anechoic terminations and hence the impedance Z at the end is equal $\frac{\rho_o c}{S}$ An air compressor piping is connected to a receiver where all pulsations are smoothed out, thus pressure at the end will be zero

$$\text{Air compressor:} \quad p(L,\ t) \cong 0 \tag{26}$$

$$\text{Refrigerating compressor:} \Big[Z(w)\Big]_{x=L} = \rho_o c/S \tag{27}$$

4. Boundary Conditions for Geometry Changes: For the area change, as shown in Fig. 5, at the boundary, following conditions prevail at the junction,

$$p_1 = p_2 \quad \text{and } Q_1 = Q_2 \tag{28}$$

Thus the acoustic impedance at the junction is continuous. In the case of the branching, the following conditions are used at the junction,

and
$$p_1 = p_2 = p_3 \quad (29)$$
$$Q_1 = Q_2 + Q_3 \quad (30)$$

5. Conditions for Compressor Staging: In the case of staging of compressors, the discharge of the low pressure compressor (1) is connected to the suction of the high pressure compressor (2), by means of the piping (an inter cooler may also be included in between) as shown in Fig. 6. Apply the harmonic solution to the piping by considering one source of the pulsation at a time and then apply the following boundary conditions,

$$Q_1(L, t) = Q_2(L, \theta_2) = \frac{\dot{m}_{v2}}{\rho_2} \quad (31)$$

$$Q_2(0, t) = Q_1(0, \theta_1) = \frac{\dot{m}_{v1}}{\rho_1} \quad (32)$$

Where subscript 1 & 2 refer to the low and high pressure compressors respectively. Note that the proper crank angle for each compressor is taken in its solution.

6. Condition for Multi-Cylinder Case: For a compressor with multiple cylinders delivering gas into a common pipe, the pulse of the equal order can be superimposed directly. Of course, the phase difference of several harmonics have to be taken into consideration. Pressure pulsations in such a system are more pronounced because of the cavity interactions. One calculates pressure and volume velocity due to each cyliner and then superimposes the solution. At the front of each valve, Q has to be equal to the valve mass rate divided by the gas density.

SOLUTION TECHNIQUES

Now the various methods, as applied by several investigators shall be discussed. A lumped parameter representation of the acoustic wave equation has been used by Miller & Hatten [25], Nimitz [9], Wallace [10], Brunner [6], Chilton & Handley [15] and Touber [26] and Soedel et al [4]. Whereas, a distributed parameters approach has been utilized by Elson & Soedel [17], Brablik [11], Benson [23,24], Abe [30], Miller & Hatten [25], Grover [12], and Chen [5], etc. All the analytic methods available under these two broad categories shall be outlined with the degree of applicability and limitations.

Lumped Parameters Approach

It is based on the analogy to electrical circuits and mechanical vibrations theory. From the acoustic wave equation, it can be shown that an acoustic element either has inertia property or elastic property. If the element has only inertia property, then it is represented by acoustic mass M and the elastic elements are represented by the acoustic compliance C where

$$M = \frac{\rho_o L}{S} \qquad \text{and} \qquad C = \frac{V}{\rho_o c^2} \quad (33)$$

where L = length of the element, V is volume and S is the cross sectional area. M is analogous to inductance in electrical circuits and mass in vibrations while C is analogous to the capacitance in the electrical circuits and is equivalent to the stiffness in vibration theory. Thus an acoustic system can be reduced to an electrical network or a simple mass-spring system and the standard results of these can be applied directly (refer Part I)[3]. Acoustic resistive element, in analogy with the electrical resistance and the damping in vibrations, can also be added. Lumped parameters analysis, being very simple and straight forward, is very popular for the design of mufflers or filters for the compressors. Also, it has also been used successfully for suction and discharge system modeling for simulation purposes. In this context one interesting modeling is based on Helmholtz resonators approach. In many industrial applications, irregular shapes of plenum chambers and cavities may be encountered and it will be difficult to write the boundary conditions and solve wave equation for such a case. In Helmholtz resonator approach [4], any irregular shape can be handled easily. Soedel et al [4], not only modeled the discharge system of a two cylinder compressor but accounted for the cylinder cavity interactions by Helmholtz resonator approach. This approach has been extended further to add the modeling of anechoic lines, as encountered in the refrigeration compressors.[30]

A tube or a passage is generally considered to be composed of the acoustic mass only and a plenum or a volume is handled as the acoustic compliance in the lumped parameter approach. However, at low frequencies, tube stiffness effects will be more pronounced than the tube mass and similarly at high frequencies, plenum mass will affect the system performance more than the plenum stiffness. This deficiency can be removed by considering both the inertia and the elastic properties of each acoustic element. A finite element approach may also be used i.e. each tube may be divided into various acoustic mass and compliance elements rather than treating the tube as a whole. The accuracy of the procedure will depend upon the selection of elements. If very large numbers of such elements are taken, then the accuracy will approach the distributed parameters approach.

Distributed Parameters Approach

While the lumped parameters method is analogous to the electrical circuits, the distributed parameters approach resembles electrical transmission lines theory. The basic philosophy of the approach is that the wave equation, with or without corrections, is solved by applying the boundary conditions. Depending upon these, the procedure may be very time consuming but the results are precise. Following are some of the methods used for the modeling of lines.

1. Finite Elements Approach [12,17]: Suction or discharge system is divided into numerous finite elements or pressure calculation stations. For each station, existing boundary conditions are specified. The wave equation is reduced to algebric equations by applying the calculus of differences or any other integeration procedure and are solved on digital computer. Elson [17] and Grover [12] have used this technique.

2. Transfer Matrix and Transfer Equations [20,25]: The pulsating system is divided in its main elements like pipes, chambers, branching points and each of these is analysed either by the wave equation or by its solution. Acoustic conditions at the beginning and at the end are calculated and then solutions of individual elements are connected by means of either transfer equations or a transfer matrix to form the general solution of the system. Miller & Hatten [20] analysed the refrigerating compressor by transfer matrix method whereas transfer equations were used by Abe et al [25] for a large air compressor installation. Although both have used digital computer for calculations, the transfer equations method involved lesser number of unknown coefficients [25].

3. Impedance Approach [17]: Pulsating flow systems can be described in terms of their system impedances. This approach is also based on the wave equation but according to Elson, [16,17] it has advantages over other methods because it is a steady state solution, is efficient and can readily be extended to a complicated system. A method of coupling the impedance description of line systems with the nonlinear response characteristics of valves is available. [17]

4. Graphical Method [5]: Chen [5] has used a graphical method for the calculation of the pulsations. The solution of wave equation can be presented in a graphical form by means of the vectors. Chen's method not only solves the basic wave equation but can also take into account the effect of the friction, temperature variations, flow velocity and the simultaneous exitations at different points in the system.

5. Green's Functions Method: Green's function is like a dynamic influence coefficient. This method is applicable for any type of the pressure and the velocity impulse input and can be used for the solution of acoustic wave equations with given boundary conditions. Soedel [30] has illustrated the use of Green's function for discharge systems. Hiramatsu [29] has applied the concept of dynamic stiffness for piping vibrations.

6. Method of Characteristics [11,12,23,24]: As outlined for the finite amplitude case.

There is no doubt that the distributed parameters approach is more accurate and describes the system performance precisely. The lumped parameter approach is not advocated in general but, as stated earlier, in practical applications it might be attractive because it's simple and less time consuming (both computer and human labor). Depending upon the system configuration, objective of analysis and the degree of accuracy desired, any suitable method could be chosen. Historically speaking, the earlier pulsation studies were performed on analog computers [6,14,19]. These were mainly based on lumped parameters approaches and used the concept of the analogies to solve the system. But, the approximations of the method, and inability of determining exact electrical analogous of the acoustic systems and finally the advent of high speed digital computers have attracted the researchers to digital computation. [4,7,11,12,17,18,20,22,23,24,25] Hybrid computers [26] may be even more desirable from the design point of view because wave forms of the pulsating flow can be directly seen on the output screen.

The role of experimental investigations should not be underestimated. For complex geometries and three dimensional wave effects, where it is not easy to model the system, acoustic characteristics and the nature of the pulsating flows can be determined experimentally. Gately & Cohen [8] have developed a general experimental method of the performance evaluation and the design of compressor muffler elements.

Computations and Input Data

At the start of computations, generally the conditions are not known, so an iteration process is started by taking pressure equal to the normal suction and discharge pressures. As stated earlier, valve mass flow rate is one of the conditions, which is not known at the beginning of calculation. The iteration procedure is continued until convergence of the resulting variables takes place. The following information is required for the computation: cylinder kinematics data, piping geometric details, thermodynamic conditions and property data of medium.

CONCLUSION

The paper has attempted to present the basic philosophy of compressor lines pulsating flow analysis and review the research work with the latest information, as available in the literature. This area has attracted and fascinated both the academicians and the industrial engineers and a significant contribution in this area has been made in the last decade. Although some pneumatic and refrigeration industries have already accepted the role of such studies in the design procedure, a high degree of awareness is still desired as the gas pulsation studies are still far from being complete and require further intensive investigations.

NOMENCLATURE

c speed of sound

k wave number

L length

$\dot{m}$ mass flow rate

n integar

p pressure

Q volume velocity

S area

t time

T temperature

u velocity

V volume

x longitudinal coordinate

ω circular frequency

ρ density

γ adiabatic constant

θ crank angle

Subscript

o mean

t total

s suction

d discharge

v valve

Note that p and u without subscripts represent acoustic variables.

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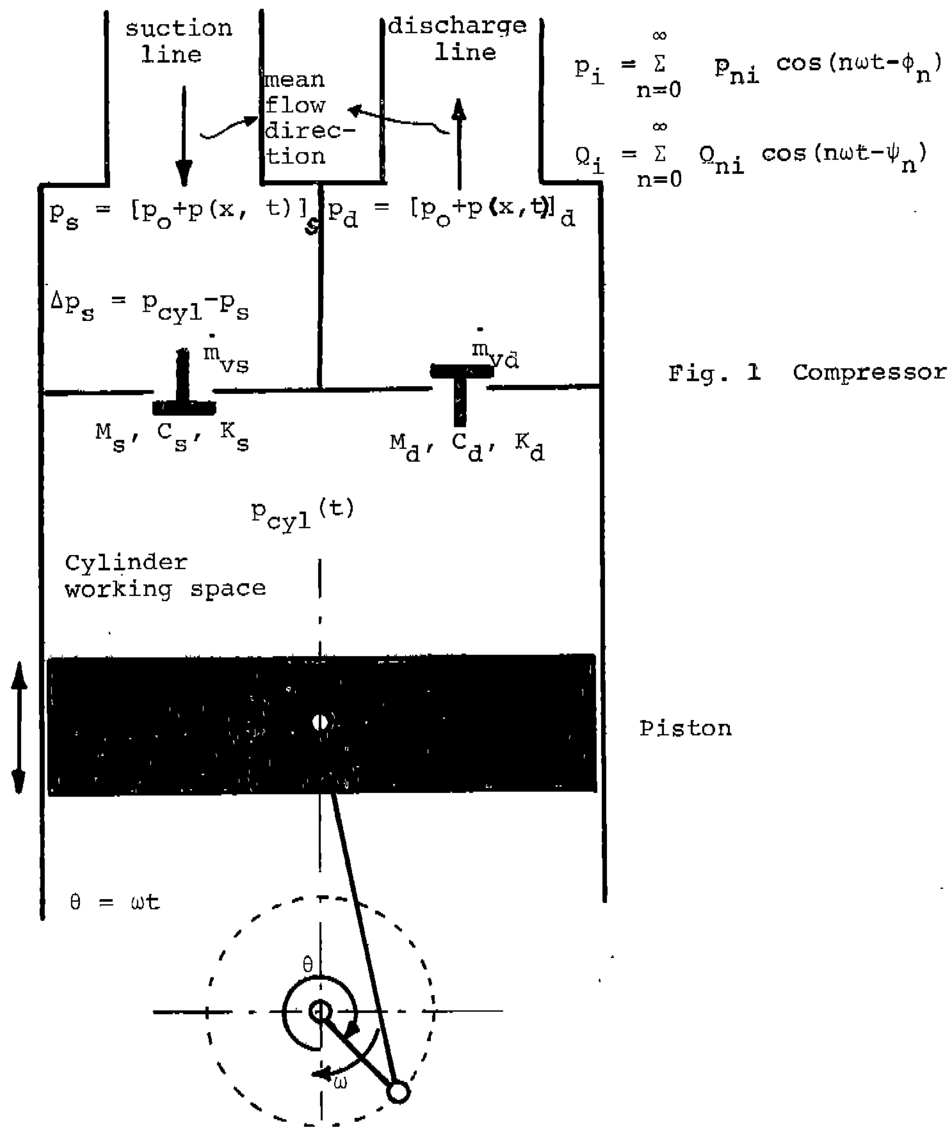

Fig. 1 Compressor Model

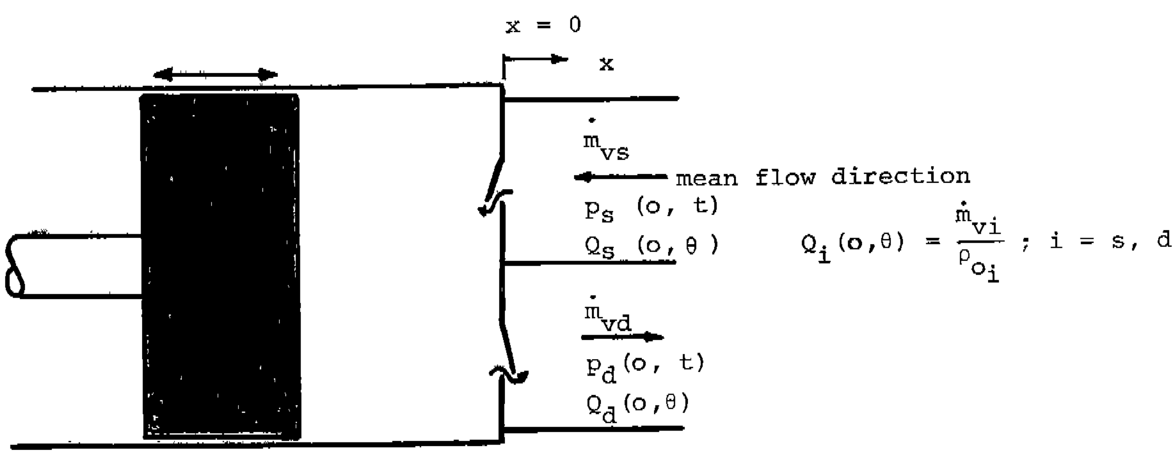

Fig. 2 Boundary Condition at the Compressor Valves - Starting Point of Piping.

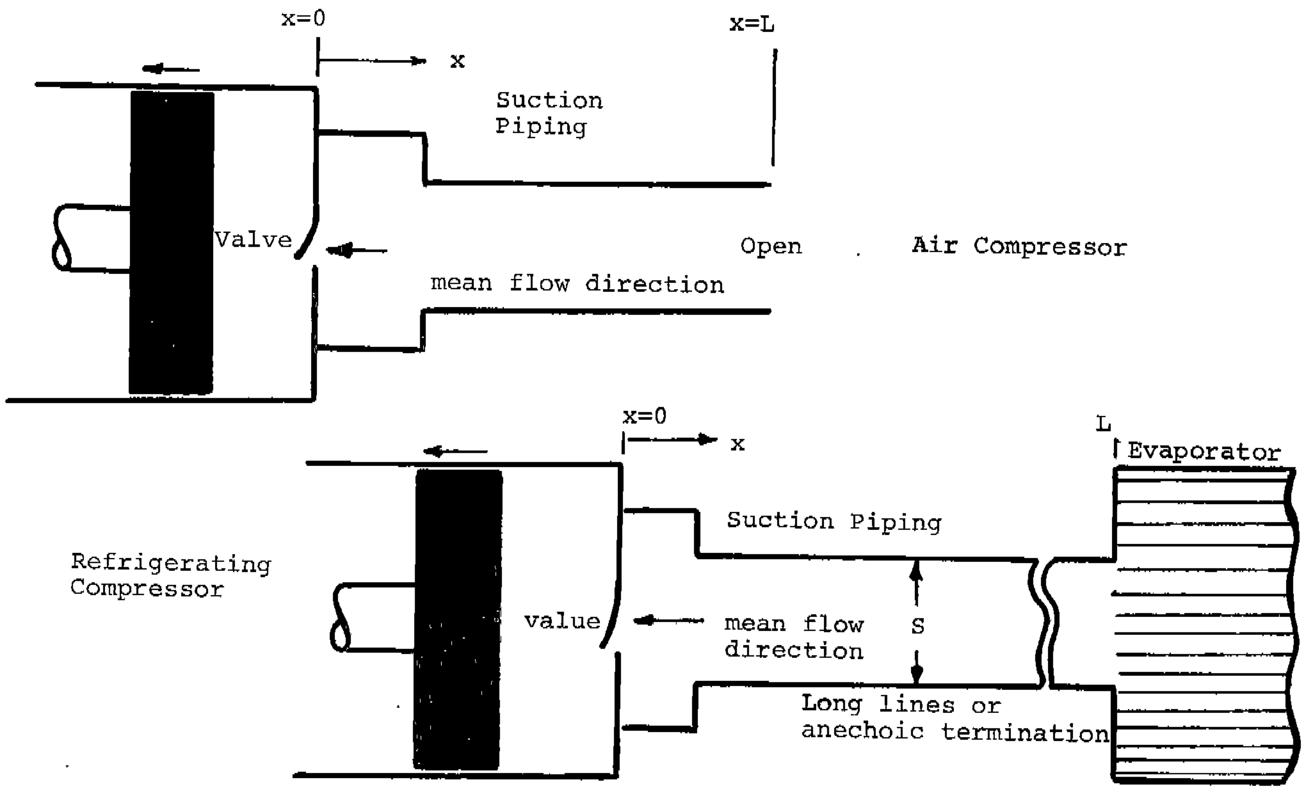

Fig. 3 Suction Line End Conditions

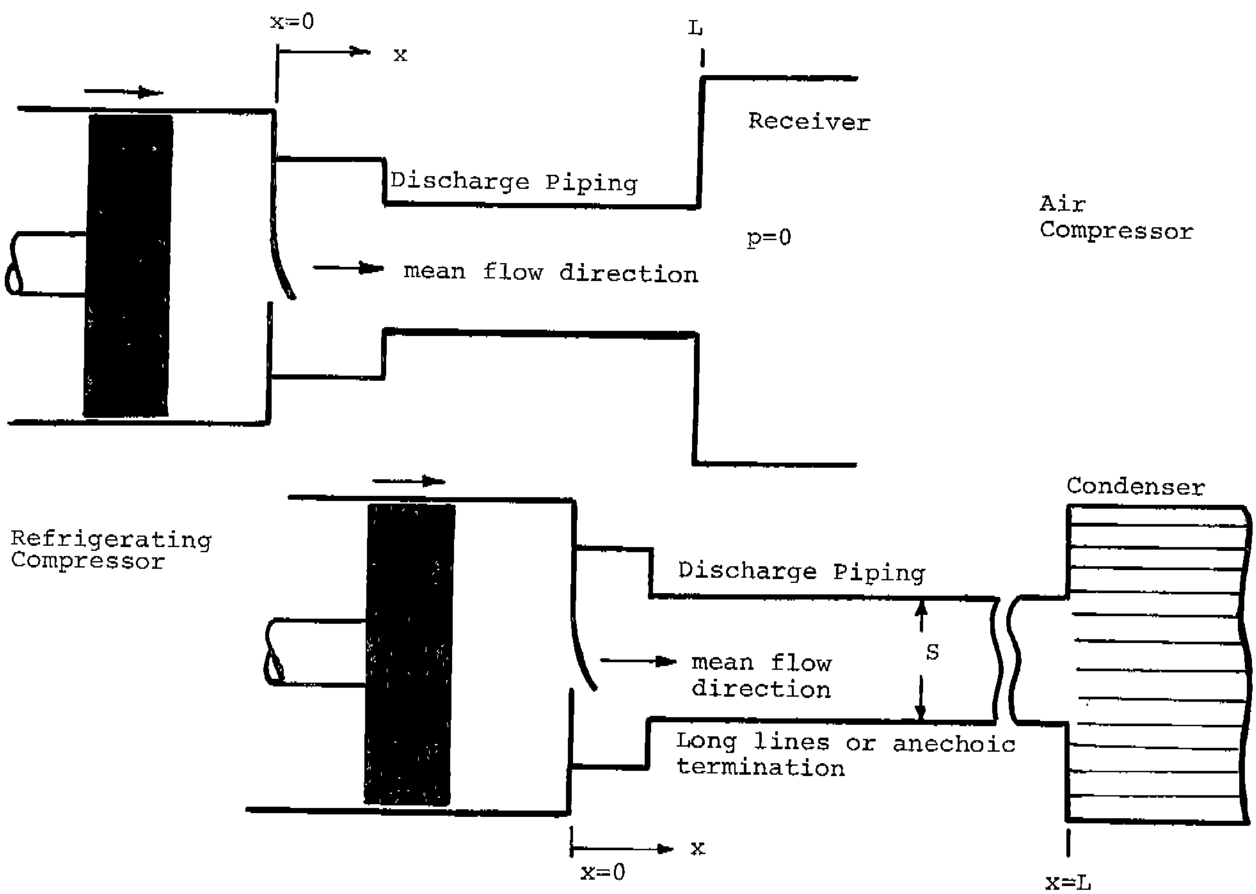

Fig. 4 Discharge Line End Condition

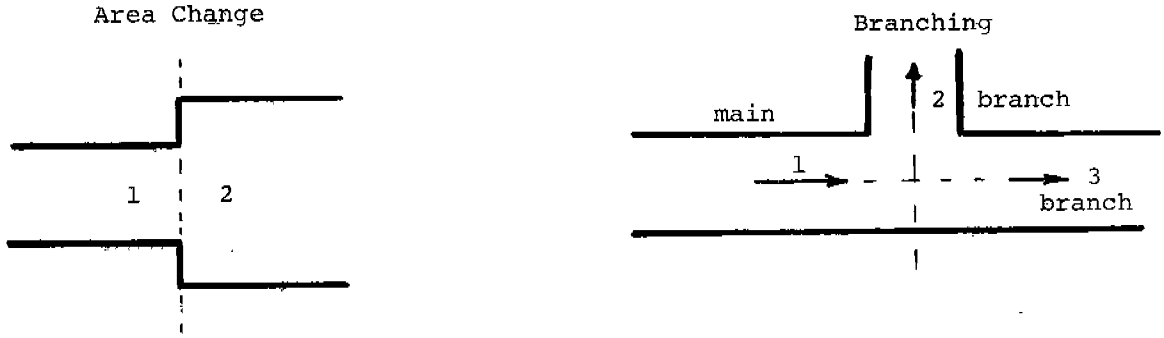

Fig. 5 Boundary Conditions for Geometry Changes

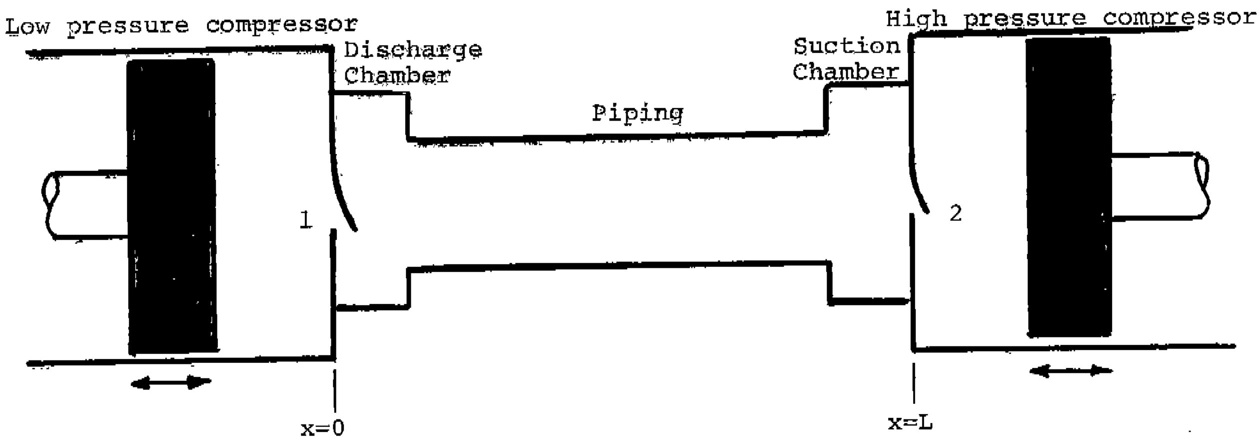

Fig. 6 Boundary Conditions for Compressor Staging