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Calculation of the Hydrodynamic Lubrication of Piston and Piston Rings in Refrigeration Compressors

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CALCULATION OF THE HYDRODYNAMIC LUBRICATION OF PISTON AND PISTON RINGS IN REFRIGERATION COMPRESSORS

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1. INTRODUCTION

The calculation of the lubricating conditions of a piston is, compared with a sliding bearing, much more difficult, because the configuration of the oil film and the operating conditions are much more complicated. Whereas the profile of the oil film in journal bearings can be described by eccentric circles, that of a piston is essentially of a more complicated form (Fig.1).

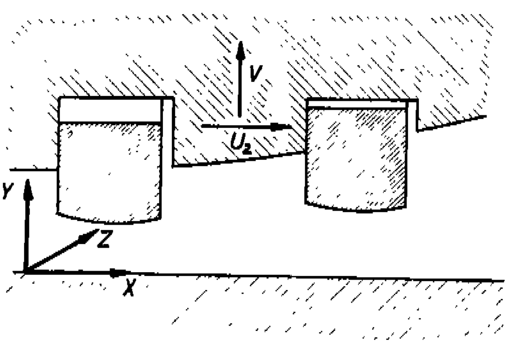

Fig.1 Configuration of the oil film between piston and liner

The shape of the oil film there is determined on the one hand by the surface profile of piston and piston rings which are mathematically difficult to describe, on the other hand by their position in the cylinder, whereby the piston rings are radially and axially moveable in the ring groove relative to the piston.
As regards the operating conditions, the variable gas pressure acting on the lubricant film and the quantitatively unknown filling of the clearance between piston and liner with oil, presents the greater complexity of the problem, which leads to the question, to what extent the theory of hydrodynamic lubrication is at all applicable.
From measurements of internal combustion engines (1), (2), (3), boundary friction occurs in the region of dead centers at piston rings, whereas over a wide range of the piston stroke liquid friction takes place.
Concerning the piston in scientific publications in general, the conclusion of a greater degree of fluid friction is drawn from the slight wear.
These assumptions can especially be made for refrigeration compressors, the pistons of which can be more lubricated in comparison to internal combustion engines, because at least with oil soluble refrigerants the lubricating oil is not lost, but is circulated back into the compressor. In spite of the assumption of fluid friction, the complexity of the problem has led to the situation where hydrodynamic calculations for pistons (4), (5), and piston rings (6), (7), (8), (9), (10), have been made almost exclusively separately.
Since however the two components form an interacting unit, it seems reasonable not to consider them separately in the calculation of the lubrication conditions.
Eilon and Saunders (11) although assuming the unit of piston and piston rings, apply the hydrodynamic lubrication film theory only to the piston ring. Although they regard the piston clearance as filled with oil and friction-producing, they do not take into account a hydrodynamic production of pressure in the lubrication film.

2. CALCULATION OF PISTON LUBRICATION

Here, as already indicated by the author (12) the hydrodynamic calculations are used for the common lubrication film on the piston and piston rings, which are derived from the general Reynolds Differential Equation for lubrication films.

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right) = 6\eta(U_1-U_2)\frac{\partial h}{\partial x} + 12\eta V \quad (1)$$

Since there is no analytical solution available for this equation, there are two possible methods for the calculation:

1. Simplification of the problem, so that a analytical solution is possible, and
2. nonanalytical methods by changing this equation into a difference equation and solving it with the help of a digital computer.

To begin with the first method was used, as basically the same physical fundamentals remain and influence of the various geometrical and operational parameters on the lubrication conditions of the piston can be studied more clearly and with less calculating effort, as by an approximate solution through a calculation programme which demands a large memory and calculation time.
Later, in addition, such an nonanalytical solution was used to study parameters which

could not be dealt with by simplified calculation methods, and to confirm to what extent they could be considered by means of approximation through an analytical solution. In order to simplify the Reynolds Differential Equation (1) it was at first assumed, as with the other authors mentioned (4),(5),(6),(7),(8) that

a) the piston moves coaxially in the cylinder with uniform velocity $U_2 = U$ and also the assumptions generally made are valid, that
b) the viscosity η in the lubricating film is constant,
c) the cylinder is circular cylindrical,
d) the cross section of the piston is circular, and
e) the piston rings posess an equal radial pressure distribution over the circumference.

For an analytical solution of the Reynolds Differential Equation, the lubricant film height configuration between the liner and the piston ring is described by straight lines parallel and inclined to the cylinder wall with an inclination m.
Piston ring profiles are also often approximated mathematically by parabolic expressions, yet wedge surface statements as in (7) give results which agree more with measured values.
Although an real sliding surface profile on pistons and piston rings can be better dealt with by approximation calculations by describing the lubricant film configuration as points in a height array, however here they ought to be approximated by mathematical equations, in order to make clearer the influence of the geometric parameters on the lubrication conditions.
Under the above mentioned conditions, the two right hand parts of both sides of equation (1) disappear, so that for the one dimensional problem without side leakage and radial movement it is simplified into equation (2)

$$\frac{d}{dx}(h^3 \cdot \frac{dp}{dx}) = 6\eta U \frac{dh}{dx} \qquad (2)$$

and reads after integration

$$\frac{dp}{dx} = 6\eta U \frac{\bar{h} - h}{h^3} \qquad (3)$$

Here $\bar{h}$ is an integration constant which stands for the thickness of the lubrication film at which the pressure gradient in the lubrication film is

$$\frac{dp}{dx} = 0$$

so that with equation (4) for the oil flow

$$q_x = h\frac{U}{2} - \frac{h^3}{12\eta} \cdot \frac{dp}{dx} \qquad (4)$$

when $h = \bar{h}$

$$q_x = \bar{h} \cdot \frac{U}{2} \qquad (5)$$

the "medium film thickness" $\bar{h}$ represents a rate of oil flow relative to half velocity of the piston.
The approximation of the real sliding surface profile through the surface lines parallel and inclined to the cylinder wall results in Fig.2 of a piston section with one piston ring.

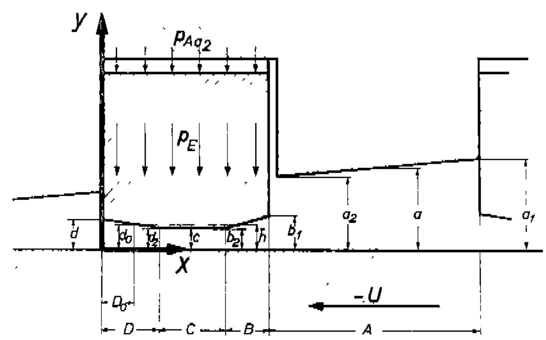

Fig.2 Sliding element of the piston

Here the real piston profile could be approximated not just by one but by a number of straight lines of various positive and negative inclination m_A , in the same way as the surface of the piston ring also has various inclinations m_B and m_D .
Between the piston and piston ring region there occurs a step in the lubricant film. Lord Rayleigh (13) first proposed step shaped sliding elements for bearings.
These ideas were later taken up by Archibald for thrust (14) and later for journal bearings and are intended here to be applied to the step in the lubrication gap of a piston.
By application of the equation (3) for each section A,B,C and D and the integration, taking into account the boundary conditions that the oil film pressures at both ends correspond to the ambient pressure and should be equal at the boundaries of each region, one gets as a result the equations giving the load capacity and frictional forces for the oil film pressure p and the shear stresses τ.
The film pressures in the different sections A,B,C,D are as follows

$$p_A = \frac{6\eta U}{m_A}\left(\frac{\bar{h}}{2}\left(\frac{1}{a_1^2} - \frac{1}{a^2}\right) - \left(\frac{1}{a_1} - \frac{1}{a}\right)\right) \qquad (6)$$

$$p_{Aa2} = \frac{6\eta U}{m_A}\left(\frac{\bar{h}}{2}\left(\frac{1}{a_1^2} - \frac{1}{a_2^2}\right) - \left(\frac{1}{a_1} - \frac{1}{a_2}\right)\right) \qquad (7)$$

$$p_B = \frac{6\eta U}{m_B}\left(\frac{\bar{h}}{2}\left(\frac{1}{b_1^2} - \frac{1}{b^2}\right) - \left(\frac{1}{b_1} - \frac{1}{b}\right)\right) + p_{Aa2} \qquad (8)$$

$$p_{Bb2} = \frac{6\eta U}{m_B}\left(\frac{\bar{h}}{2}\left(\frac{1}{b_1^2} - \frac{1}{b_2^2}\right) - \left(\frac{1}{b_1} - \frac{1}{b_2}\right)\right) \qquad (9)$$

$$p_C = p_{Aa2} + p_{Bb2} - 6\eta U\frac{\bar{h} - h}{c^3}(D + C - x) \qquad (10)$$

$$p_D = \frac{6\eta U}{m_D}\left(\frac{\bar{h}}{2d^2} - \frac{1}{d} + \frac{1}{2\bar{h}}\right) \qquad (11)$$

whereby additionally Gümbels boundary condition $dp/dx = 0$ is applied at the cavitation of the oil film in region D as a result of the film diverging. This equation states that the oil film pressure constantly turns into the ambient pressure here. The shear stresses are as follows

$$\tau = \eta U\left(\frac{4}{a} - 3\frac{\bar{h}}{a^2}\right) \quad (12) \qquad \tau = \eta U\left(\frac{4}{b} - 3\frac{\bar{h}}{b^2}\right) \quad (13)$$

$$\tau = \eta U\left(\frac{4}{c} - 3\frac{\bar{h}}{c^2}\right) \quad (14) \qquad \tau = \eta U\left(\frac{4}{d} - 3\frac{\bar{h}}{d^2}\right) \quad (15)$$

with the integration constant $\bar{h}$ as the rate of relative oil flow, by equalizing of the pressures p_C and p_D at x=D resulting to equation (16):

$$\bar{h} = \frac{\sqrt{\left[\frac{A}{a_1 \cdot a_2} + \frac{B}{b_1 \cdot c} + \frac{C}{c^2} + \frac{1}{m_D \cdot c}\right]^2 - \frac{1}{m_D}\left[A\frac{a_1+a_2}{a_1^2 \cdot a_2^2} + B\frac{b_1+c}{b_1^2 \cdot c^2} + C\frac{2}{c^3} + \frac{1}{m_D c^2}\right]} + \left[\frac{A}{a_1 a_2} + \frac{B}{b_1 c} + \frac{C}{c^2} + \frac{1}{m_D \cdot c}\right]}{\left[A\frac{a_1+a_2}{a_1^2 \cdot a_2^2} + B\frac{b_1+c}{b_1^2 \cdot c^2} + C\frac{2}{c^3} + \frac{1}{m_D \cdot c^2}\right]} \quad (16)$$

It contains, like the above mentioned equations , the unknown position represented by c , of the piston ring in the oil film, which is also included in the values b and d . This position is determined by the equilibrium of the forces acting radially on the ring, and is ascertained by application capacities P_B, P_C, and P_D in the region L=B+C+D

$$P_A = \int_{B+C+D}^{A+B+C+D} p_A \, dx = \frac{1}{m_A}\int_{a_2}^{a_1} p_A \, da = \frac{6\eta U}{m_A^2}\left[\ln\frac{a_1}{a_2} - \frac{\bar{h}}{2}\frac{(a_1-a_2)^2}{a_1^2 \cdot a_2} - \frac{a_1-a_2}{a_1}\right] \quad (17)$$

$$P_B = \int_{C+D}^{B+C+D} p_B \, dx = p_{Aa_2} \cdot B + \frac{6\eta U}{m_B^2}\left[\ln\frac{b_1}{c} - \frac{\bar{h}}{2} \cdot \frac{(b_1-b_2)^2}{b_1^2 \cdot b_2} - \frac{b_1-b_2}{b_1}\right] \quad (18)$$

$$P_C = \int_{C}^{D} p_C \, dx = (p_{Aa_2} + p_{Bb_2}) \cdot C - 6\eta U \frac{\bar{h}-c}{c^3} \cdot \frac{C^2}{2} \quad (19)$$

$$P_D = \int_{D_0}^{D} p_D \, dx = \frac{6\eta U}{m_D^2}\left[\frac{\bar{h}}{2c} - \frac{c}{2\bar{h}} - \ln\frac{\bar{h}}{c}\right] \quad (20)$$

in implicit form from the equation (21)

$$\frac{p_E \cdot L}{\eta \cdot U} = 6\left[\bar{h}\left(\frac{A \cdot D}{2} \cdot \frac{a_1+a_2}{a_1^2 \cdot a_2^2} + \frac{D^2}{2c \cdot \Delta d^2} - \frac{B \cdot C(b_1+c)}{2b_1^2 \cdot c^2} + \frac{C^2}{2c^3} - \frac{B^2}{2b_1^2 c}\right) - \left(\frac{D^2 \cdot c}{2\bar{h} \cdot \Delta d^2} + \frac{D^2}{\Delta d^2} \cdot \ln\frac{\bar{h}}{c} + \frac{B^2}{b_1 \cdot \Delta b} + \frac{A \cdot D}{a_1 \cdot a_2}\right) + \left(\frac{B^2}{\Delta b^2}\ln\frac{b_1}{c} + \frac{B \cdot C}{b_1 c} + \frac{C^2}{2c^2}\right)\right] \quad (21)$$

With this, therefore the oil film pressures, and, by integration of the shear stresses, the frictional forces in equations (22), (23), (24), and (25) are able to be calculated in the separate piston and piston ring regions.

$$R_{A0} = \frac{\eta U}{m_A}\left[4\ln\frac{a_1}{a_2} - 3\bar{h}\,\frac{a_1-a_2}{a_1 \cdot a_2}\right] \quad (22)$$

$$R_{B0} = \frac{\eta U}{m_B}\left[4\ln\frac{b_1}{c} - 3\bar{h}\,\frac{b_1-c}{b_1 \cdot c}\right] \quad (23)$$

$$R_{C0} = \eta U\left[\frac{4}{c} - 3\frac{\bar{h}}{c^2}\right] \cdot C \quad (24)$$

$$R_{D0} = \frac{\eta U}{m_D}\left[4\ln\frac{\bar{h}}{c} - 3\,\frac{\bar{h}-c}{c}\right] \quad (25)$$

3. STUDY OF THE INFLUENCING VARIABLES ON THE LUBRICATION CONDITIONS ON PISTONS

As can be ascertained from the equations, the relative oil flow $\bar{h}$, pressures p and frictional forces R are essentially determined by the geometric oil film dimensions and the values η, U, and p_E as operating conditions, along with changing gas pressures on both sides of the lubricating film and the partial filling of the lubrication gap with oil, which further influence the lubrication conditions. For the investigation of these various influencing factors the derived equations were progammed and the individual parameters were varied[1], in order to ascertain their effect on oil flow, pressure and friction in the lubrication gap of a piston of D=85 mm Ø, chosen as an example. The initial values of calculation were:

Lengths:	Thickness:		
A = 5.0 mm	a_1	= 0.15	mm
B = 1.0 mm	a_2	= 0.15	mm
C = 0.5 mm	b_1-b_2	= $2 \cdot 10^{-3}$	mm
D = 1.0 mm	d_1-d_2	= $2 \cdot 10^{-3}$	mm
Piston velocity	U	= 3	m/s
Lubricant viscosity	η	= 0.1162	kps/m^2
Piston ring tension	p_E	= 1.3	kp/cm^2

Of special interest are the results of a variation of the lubrication gap geometry in the region of piston and piston rings on the lubrication conditions.

3.1.1. Variation of the Oil Film at the Piston

A variation of the piston clearance as in Fig. 3

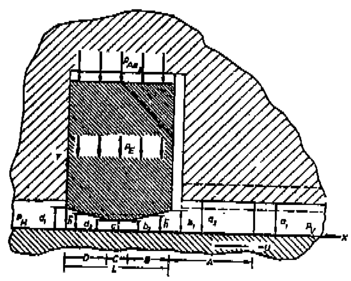

Fig. 3 Variation of piston clearance

influences as according to equation (6), the pressure of the oil film in the piston region A, and thereby also the pressure p_{Aa2} at the leading ring edge, which also, as seen in the equation for the other regions, influences the oil film pressure acting there. The results of the variation of piston clearance in Fig. 4 show with increasing piston clearance a strong decrease in the pressure p_{Aa2} on the leading edge of the ring. Since this pressure, supporting the force of radial pressure, acts on the ring, a decrease means an uplift of the piston ring, represented by the increase in the smallest lubrication film thickness c. Connected with it is an increase in the relative oil flow $\bar{h}$ and a decrease in the friction force R_{ges} which can also be seen from equation (22) to (25).

1) These calcutations have been carried out in the diploma works of F. Wrede and B. Wolter

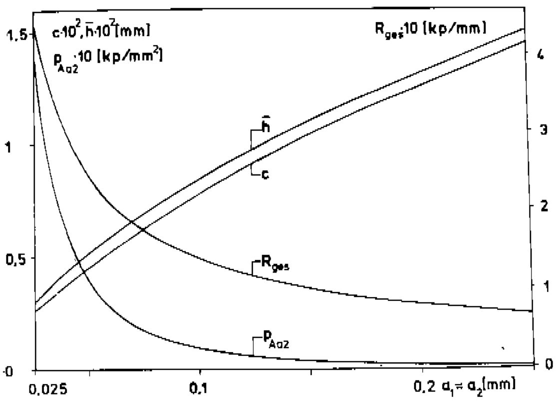

Fig.4 Results of variations versus piston clearance

A variation of the piston profile as in Fig. 5

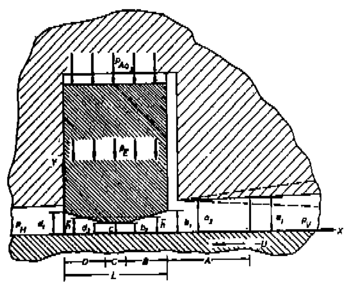

Fig.5 Variation of piston profile

means a simultaneous medium clearance variation and a variation in the inclination m_A. It can be seen (12) that when the medium clearance $(a_1 + a_2) / 2$ remains the same, a variation in inclination m_A in respect to the pressure p_{Aa2} at the leading edge of the ring and minimum and medium oil film thickness $\bar{h}$ effects no difference, but only in respect to pressure and load capacity in region A.
That means that the result in Fig.6 as regards the above mentioned values, only reflects the effects of an increase in the clearance, i.e., that with positive inclination and an increase in the clearance, a decrease in the pressure at the leading edge of the ring p_{Aa2} and an increase in the minimum and medium thickness of the oil film as a rate for the oil flow results.
The friction force R_{ges} changes only unessentially, for the change only affects piston region A, whereas the friction in the piston ring region stays unaffected because of the equal pressure on the leading edge.

3.1.2. Variation of the Lubrication Film on the Piston Ring

A variation of the piston ring profile according Fig.7 shows by calculation results in Fig.8 that with increasing tapered land at the piston ring end, b_1-b_2 and d_1-d_2 for

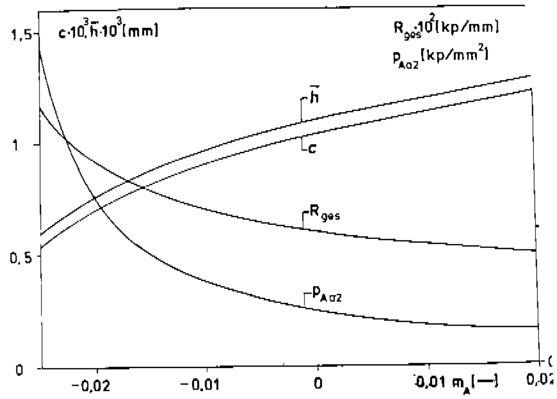

Fig.6 Results of variation versus pisto[n] profile

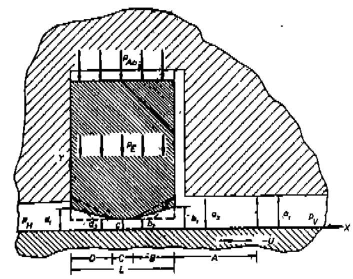

Fig.7 Variation of piston ring profile

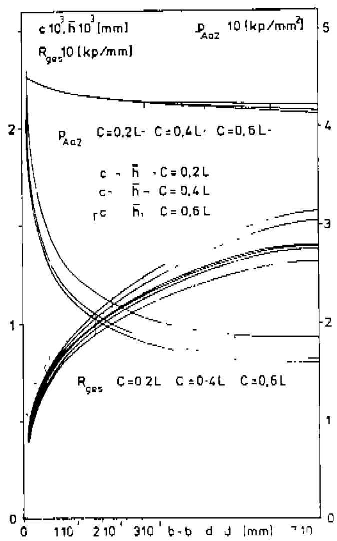

Fig.8 Results of variation versus piston ring profile

various middle flat lands C=0.2 L,0.4L,0.6L the pressure in front of the ring only decreases insignificantly, whereas the effects on friction and oil flow are essentially greater.
That means, that the piston ring profile gains through tapering a better dynamic load capacity, which results in a greater minimal oil thickness with greater oil flow and less friction.
A shortening of the middle flat land C at

the piston ring also improves its lubrication conditions.

3.2. Variation of Operating Conditions

The effects of a variation of η,U and p_E will not be dicussed in greater detail here, because it is easily seen and generally known,that increasing values of these three parameters lead to greater friction.They also result in greater oil film pressures, whereas the influence on the oil flow is different,for increasing internal tension of the piston ring throttles the oil flow,and increasing velocity and viscosity increase it,because with the increasing oil pressure the influence of the internal tension of the ring on the gap width,becomes relatively less.

Of essential influence on the lubrication conditions are the quantity of the oil in the clearance and varying ambient pressures as an significant expression of the operating conditions of the piston.Because these parameters are not clearly defined in size, the calculation of the lubrication of an running piston is made more difficult.

To investigate their influence,see Fig.9,

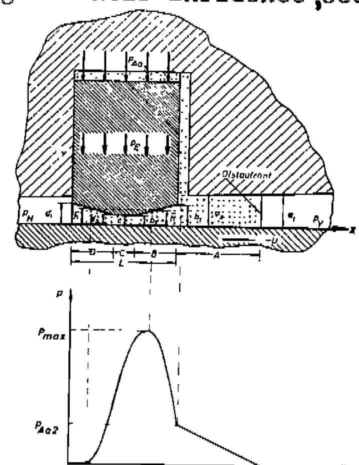

Fig.9 Variation of oil film lengths and gas pressure

the quantity of oil in the lubrication gap and the gas pressures,p_V in front,and p_H behind the oil film,were varied,in which case the given equations had to be slightly extended.

3.2.1. Variation of the Lubrication Gap Filling

A variation in the quantity of lubricant was simulated by systematic variation of the relative length S_A in piston region A,relative to the length L of the piston ring, and the length S_B,relative to the length B under the piston ring.

The qualitative pressure profile in the lubrication gap shown in Fig.9 varies for the different lengths,as in Fig.10,so,that the maximum pressures with decreasing length S_A at first decrease and increase with further decreasing length S_B.

The initially falling pressure in front of the ring and also above the ring means a fall off in oil film load under the ring.But at

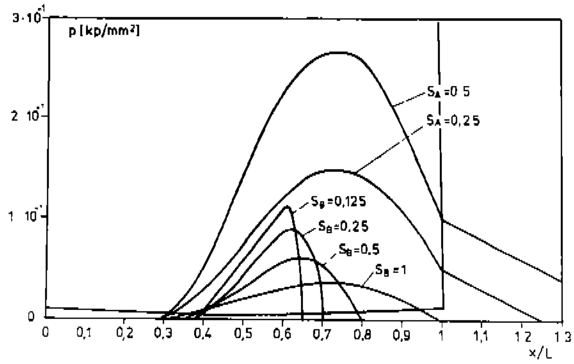

Fig.10 Pressure profile in the clearance between piston ring and liner

oil film pressure zero in front of the ring, the oil film load,which stays constant under further reduction in the oil supply,demands equal hydrodynamic load capacities, expressed through the integrals of area of the pressure profile under the ring,which leads to higher maximal pressures at decreasing lengths.

Plotting the results over of the relative lengths S_A and S_B, Fig.11,

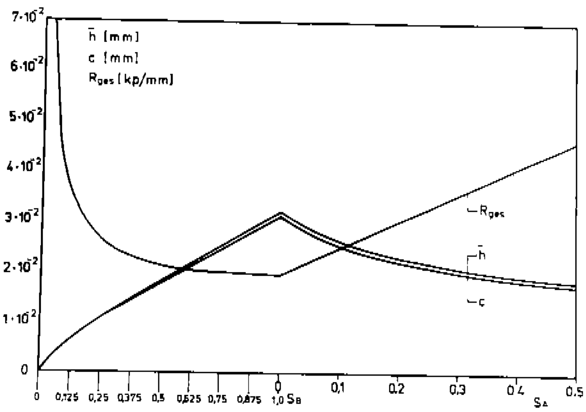

Fig.11 Variation in the quantity of oil in the lubrication gap

with an increase in the quantity of oil under the ring S_B a greater up-lift of the piston ring occurs,expressed through the minimum oil film thickness c,connected with a greater oil flow and less friction.If,however,the clearance in region A is also filled with oil over a length S_A,then the friction increases again.This,however,is not only because of the now additional piston friction,but mainly because the minimum lubrication gap width is again reduced.

This reduction in the width of the lubrication gap c also means a reduction in the relative lubricant flow h̄ through the lubrication gap.This means,that unstable conditions occur,when there is an accumulation of oil in front of the ring,to the extent that the piston gap tends either to fill up or to empty,because the ring gradually closes, as the oil supply increases,and opens up,as the oil supply decreases. This statement appears essential for the study of lubricating conditions on pistons and has to be checked by experimentation, which is why an experimental set up is at the moment being developed.

3.2.2. Variation in Gas Pressure

A variation in gas pressure on both sides of the oil film was chosen of a type where their differences alone were studied, so that the excess pressure in front of the lubrication gap p_V, or that behind the gap p_H was varied from the other opposite film pressure.
At positive values of p_V, pre-pressure predominated, at positive values of p_H after-pressure, whereas at the values of zero, equal pressure on both sides was present.
Attention must be paid to the effect of the vaiation of these differences in pressure on the axial position of the piston ring in the ring groove. Whereas under the predomination of pre-pressure as in Fig.12

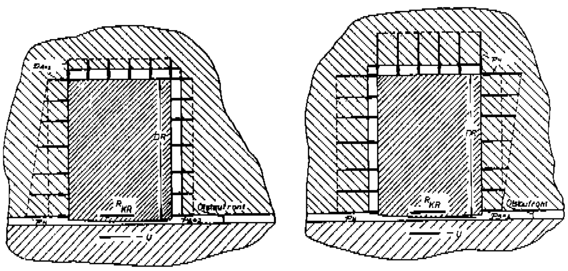

Fig.12 Possibilities of axial piston ring position

the piston ring in the left hand part of the figure lies , as in previous studies with its trailing edge in the ring groove when looked at in the direction of movement, by a predomination of after pressure over the oil film pressure and the frictional force, can so change its position, that it lies to the groove with its leading edge.
In the first case, as in Fig.13, at high pre-pressure, the lubrication gap produces a similar pressure profile to that seen previously without pre-pressure, (Fig.10), the only deviation being, that the pressure profile on the leading edge is raised all in all by a gas pressure of, in this case, 20 kp/cm^2, and the hydrodynamic oil film pressures are considerably higher.
The raised gas pressure in front of the ring edge, also has an effect above the ring, and there supplements the internal tension of the ring in pressing the ring against the cylinder wall.
This must be compensated for by means of an improved bearing capacity, produced by higher pressure in the oil film.
In order to generate this load capacity, the area integrals of the pressure profile under

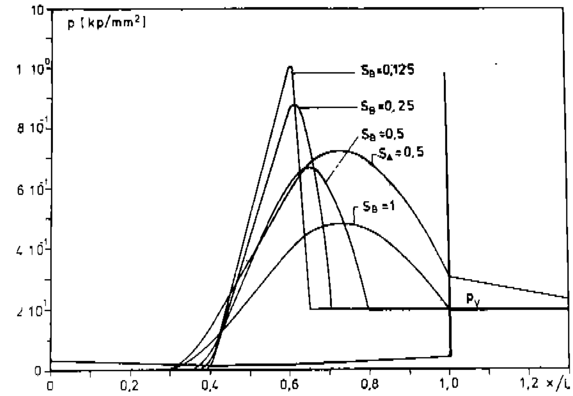

Fig.13 Pressure profile in the clearance between piston ring and liner at higher pre-pressures

the ring must be equal, as long as there is no accumulation of oil in front of the ring. Under conditions of a limited oil supply this leads to higher maximal pressures, and vice versa, under increasing lengths S_B, decreasing maximal pressures. When there is additional oil accumulation S_A in front of the ring together with the consequently raised pressure on the edge of the ring, higher pressure, and therefore a load on the oil film occurs above the ring in addition to the gas pressure and to the internal tension of the ring, so that the maximal pressure must rise again in order to maintain the equilibrium of load.
If the pressure behind the lubrication film is so much greater than the pre-pressure that the ring changes its axial position (as in Fig.12) counteracting to this, the oil pressure at the leading edge p_{Aa2} and the frictional force R_{KR} of the piston ring; then the oil film pressure profile between the piston ring and cylinder changes as in Fig.14

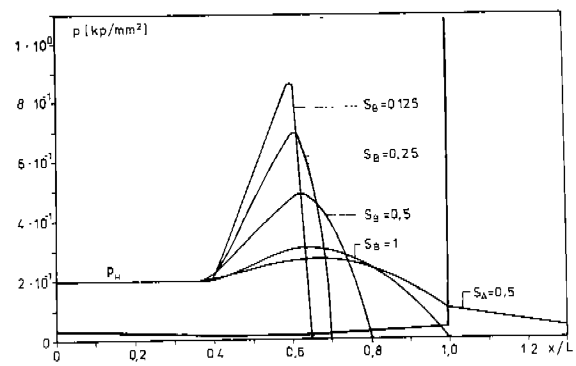

Fig.14 Pressure profile in the clearance between piston ring and liner at higher after-pressure

In this forward axial position the after pressure p_H also has an effect above the ring and intensifies the internal tension of the ring as a constant value. This constant film load demands, once more, constant load capacities at the various lengths under the piston ring S_B, so that here also the maximal pressures decrease with an increasing oil accumulation. Compared with the previous figure, however, they further decrease, if the oil supply is so great, that an accumulation of oil also occurs in front of the ring S_A because the consequently generated hydrodynamic pressure on the leading edge of the ring as a result of its forward position can not have an effect above the ring and therefore the film load stays constant.

Because of the higher pre-pressure on the leading edge, the pressure profile in this case under the ring is fuller, so that the maximal pressure for equal load capacity becomes less.

A similar case also occurs, when, although the after-pressure is greater than the pre-pressure, but still not sufficient to change the axial position of the ring. Then the raised after-pressure only acts upon the lubrication film, but is sealed off from the part above the ring by the rear position of the ring. As a result it raises the pressure level of the oil film under the ring, without, at the same time, loading the ring additionally from above the ring. In this position this result is an improvement of the friction conditions, because of greater load capacity and thereby greater thickness of the oil film. This case can be seen in the results of variation of the gas pressures, in Fig.15 in respect of the frictional force, and in Fig.16 in connection with the oil flow.

Fig.15 shows, how the frictional force changes with the variations in gas pressure acting on both sides of the oil film, whereby under the pressures p_V and p_H, excessive pressure compared with the opposite film boundary pressure are understood.

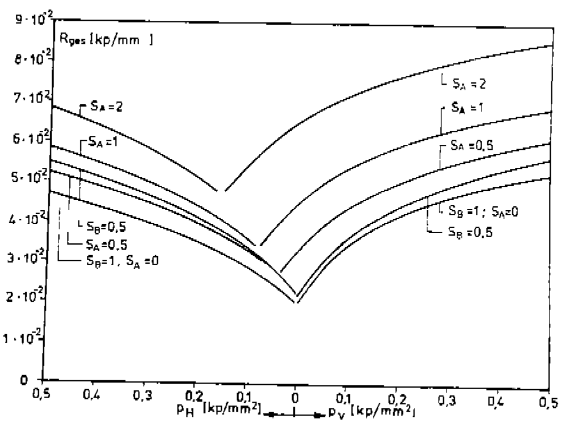

Fig.15 Frictional force versus gas pressures

It is seen, that with decreasing pre-pressure p_V, the frictional force decreases, because this gas pressure loads the ring in addition to its internal tension, and by removing the load, a greater thickness of lubricant under the ring, together with less friction, occurs. This reduction in friction goes up to pre-pressure $p_V=0$, when there is only accumulation of oil under the ring ($S_B \leq 1$). With additional oil accumulation in front of the ring $S_A>0$ the friction decreases further, if the after-pressure p_H already predominates, and that as expression of the previously described improvement in the lubrication conditions, as long as the ring stays in the rear position. If the ring changes its position, the after pressure additionally loads the ring from above and leads furthermore to an increase in friction.

The minima of the curves not shown, therefore, are evidence of a change in the position of the ring.

It is recognized that with an increasing length S_A, increasingly greater after-pressures are necessary, to effect this change in position of the ring.

Furthermore it is ascertainable from the diagram, that with increasing length under the ring S_B, the friction at first decreases, to later increase with a further increase in the length S_A in the piston region in front of the ring - a phenomenon, which has already been shown and explained in Fig.11. The reason lies in the various effects of the oil accumulation under the ring and in front of the ring on the uplift of the ring. These influencing factors can be seen in Fig.16

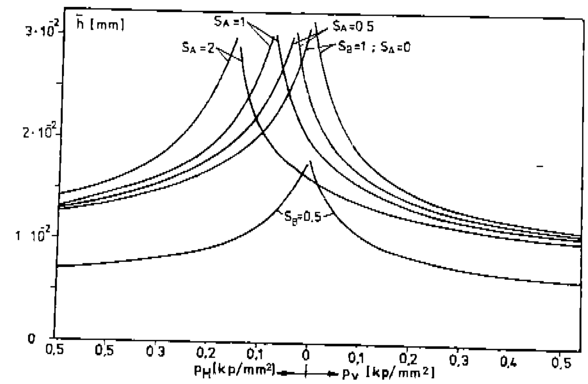

Fig.16 Oil flow versus gas pressures

in which the oil flow resp. the medium lubricant thickness $\bar{h}$ are plotted against the gas pressures in front of and behind the oil film. In general it is recognized, that with increasing gas pressures in front of or behind the lubricant film, the oil flow through the gas decreases.

The reason lies in the additional gas pressure load on the ring, which supplements the internal tension in pressing the ring against the cylinder wall. The greater thickness in the oil film at less after-pressure, when the ring is still in its rear position (reason for the reduction of friction), is connected with an increase in the relative oil flow $\bar{h}$.

The maximal clearance $\bar{h}$ and the maximum oil flow are reached at the moment when the ring changes its position. That is the case at greater lengths S_A, and again at greater after-pressures, whereas when oil is only pre-

sent under the ring ($S_B \leq 1$) the ring actually changes its position with the change in pressure. Further it can be seen from the figure, that when the trailing edge of the ring lies against the groove, represented in the right hand arm of the curve, the oil flow, with increasing length S_B under the piston ring, becomes at first greater, and then with further increasing length in front of the rings S_A, less again.
This is an expression of the instability of the oil film, already explained in Fig.11, which leads to piston gap either filling, when the oil supply increases, or emptying, when it decreases. On studying the course of the left arm of the curve, it can be seen that with the forward position of the ring in the groove, stable conditions are present as, with increasing lengths S_A and S_B, the medium oil thickness $\bar{h}$ and thereby the oil flow, becomes greater, so that the lengths now at a standstill stabilize in the lubrication gap, because a greater oil supply causes both a larger ring up-lift and a greater oil flow through the lubrication gap.
From this, conclusions about the oil flow through the ring sealing system of a compressor can be drawn.
In the compression and discharge stroke there is definitely only an oil supply present on the liner, so that probably no oil accumulates on the rings. Then, with increased oil supply, the ring would open and hinder the passage of oil into the discharge chamber, whereas with a decreased oil supply, the ring closes, so that the oil necessary for hydrodynamic lubrication of the ring cannot flow away. In the reexpansion and suction stroke, the ring, at first in the forward position and with an increased oil supply, would allow more oil pass through and leave an oil film on the cylinder wall for the following stroke However, after changing position in the course of the stroke, it would have a scraping effect and prevent an overgreat oil flow into the system.
The calculations described and presented here and their results are valid for a coaxial piston operating in a cylinder, and are certainly suited to represent the influence of the parameters under discussion on the lubrication conditions.
Trunk pistons run eccentrically and inclined in the cylinder, which leads to the question, how the lubrication conditions change, and what value the calculation described has in this case.

4. APPLICATION OF THE CALCULATION TO TRUNK PISTONS

Burmeister [15], in expanding this calculation recently solved the Reynolds Differntial Equation by approximation on a digital computer, for various eccentricities and angles of inclination of a piston of the same size as the one taken as an example here.
In Reynolds Differential Equation, the second term of the left hand side can no longer be ignored, that is, that which describes the circumferential pressure generation. The second term on the right hand side of the equation was ignored, which takes into account a radial movement of the piston or the ring, just as an alternating piston velocity was also ignored. Changing piston velocities with the piston in a concentric position were studied by Furuhama with respect to radial movement of the ring.
Here are the essential results of the approximation solution for eccentric and inclined pistons:
If the piston runs eccentrically in the cylinder, then the pressure at the point of minimum oil thickness increases to a greater extent at the circumferencs than it decreases on the side of the maximum gap. In medium clearance in the vicinity of the piston pin holes, which has the same clearance as the piston running concentrically, the lubrication film pressure is obtained at the value of the one dimensional solution. Similar statements about the minimum lubrication gap width, the oil flow, and the friction, can be made. The question is, what value the one dimensional calculation has for trunk pistons, and in particular, if the lubrication conditions, at various single points on the circumference, paying attention to the oil film thickness present there, so calculated, give sufficiently exact results.
A comparison of such a one dimensional calculation with a two dimensional approximation solution results in deviation of less than 1 % in respect of the oil film pressure and the minimum oil thickness, if the changing oil film thickness on the piston circumference is substituted into the one dimensional calculation.
The result of this, is that the one dimensional calculation can be applied to trunk pistons with good accuracy.

5. REFERENCES

[1] POPPINGA, R. Deutsch. Kraftf. Forsch. H. 54
[2] COURTNEY-PRATT, Proc. Inst. Mech. Eng. 155
[3] FURUHAMA, S. Bullet. Jap. Soc. Mech. Eng. 3,4
[4] LISCHEWSY, Energomaschinostr. 5 (1958)
[5] WASSILJEW, B.N., Kraftf. Techn. 5 (1955)
[6] EWEIS, M. VDI-Forsch. Heft 371 (1935)
[7] HORGEN, H. Diss. ETH Zürich 1942
[8] CASTLEMAN, R.A. Physics 7 (1936)
[9] FURUHAMA, S. Bullet. Jap. Soc. Mech. Eng. 2
[10] WAKURI, Y. Bullet. Jap. Soc. Mech. Eng. 13
[11] EILON-SAUNDERS, Inst. Mech. Eng. Proc. 171
[12] KRUSE, H., Motortechn. Z. 10+12 (1965), 5+ 9 (1966).
[13] RAYLEIGH, O.M. Phil. Mag. 35 (1918)
[14] ARCHIBALD, F.R., ASME Trans. 72 (1950)
[15] BURMEISTER, J., Diss. TU Hannover 1973