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An Improved Technique for the Valuation of Structural Dynamic Response

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Photo de couverture : Machine à équilibrer d'atelier à microprocesseur avec mémorisation des paramètres de réglage de plusieurs rotors (SCHENCK S.A.).

An improved technique for the valuation of structural dynamic response

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In the experimental modal survey of structures, employment of *rapid swept sine excitation* through a forcing vibrator is well established and, providing care is taken to ensure that the vibrator is of small relative mass and can impose an adequate force, then the signal to noise ratio at remote stations presents no problems. However, on very light structures, the vibrator must have a correspondingly small mass and therefore the magnitude of the imposed force is limited, leading to less well defined response signals. Under these conditions the frequency response of the vibrator is likely to be significantly affected by feedback from the structure under test, thus distorting the signal as applied to the structure. This further exaggerates signal to noise problems.

In this paper we report on the use of *shaped repetitive digital excitation signals* which both facilitate signal averaging in the time domain to eliminate extraneous noise components, and compensate for vibrator-structure interactions.

Prior to evaluating the structural response, the effect of coupling between the vibrator and the light structure is assessed by measuring the *in situ* transfer function of the vibrator. The transfer function is then modified and inverted to yield the shaping filter which gives either a flat or band limited spectral response at the structure forcing point and an appropriately modified swept sine signal is generated digitally. The computed signal is loaded into a digital transient signal generator and by synchronously counting out successive samples and using the same counting signal to drive the analogue to digital converter, the time-phase relationship between force and response is maintained. In addition, repeated output cycles can be sampled at precisely the same points with respect to the time origin of the output sequence. Thus if each acquisition cycle is properly triggered, the signal to noise levels of the measured force and response may be significantly improved by time domain averaging, effected simply by 'stacking' the results of successive acquisitions.

In the work reported here it has also proven advantageous to use a *modified* swept sine as the basic test signal. Conventional swept sine signals are nominally bandlimited and flat in their passband, but, in fact, their spectral level falls relatively slowly out of the band (especially noticable at the lower cutoff), and in the passband their spectrum shows a significant ripple. Techniques have been developed at the Data Analysis Centre to modify the swept sine to produce a signal whose spectrum falls off arbitrarily quickly out of band and which is perfectly smooth in the passband, while preserving the desirable characteristics of the linear sweep, ie, small crest factor and finite duration. A particularly useful feature of the modified sweep is that it is possible to avoid exciting lightly damped low frequency modes immediately below the frequency range of interest, and thus produce well-defined transient responses.

1. INTRODUCTION

Experimental determination of the modal parameters of a lightly damped structure showing resonant behaviour is sometimes hampered by the finite dynamic range of the signal generation and measurement transducers. Specifically, when the structure exerts a strong coupling influence on the excitation vibrator, the nominal forcing function is significantly distorted; this may cause noise related errors to appear in the calculated transfer function. In this paper we describe how closed loop modification of digital input signals can be used to limit the amplitude range of the force input to a structure during transient testing. This allows the experimenter to apply specified force levels uniformly across a frequency band. A prerequisite for work of this kind is a digital signal generator of the type described in section 4 below.

2. MODAL TESTING

Practical modal testing involves the generation of a test signal to drive the electromechanical, electrohydraulic or electropneumatic shaker or vibrator. The resultant mechanical motion in one axis is coupled to the structure via a force gauge. The corresponding structural motion is normally measured using accelerometers due to their low mass and therefore minimal loading of the structures.

The linear frequency sweep is often used to evaluate transfer functions. This method involves measurement of the system transient response to a short frequency sweep over a given frequency band. By capturing the entire force and response time histories, calculating the Fourier transforms of each and then dividing the response spectrum by the force spectrum, the transfer function can be evaluated. As normally employed this method suffers some limitations. Theoretically the spectrum of the swept sine is approximately flat between the lower and upper frequency limits and then falls off rapidly outside this band. In practice the spectrum of a swept sine does not fall off very quickly at the lower edge of the band. In addition, the frequency response of a vibrator (which may not be flat to begin with) is substantially altered when it is coupled to a structure having finite input impedance, and so a shaker does not normally reproduce a given input spectrum with great fidelity. As a result the level of the applied force may show large variations within a frequency band. Moreover, since the mass loading of the vibrator on the structure should be small, and the vibrational energy which a vibrator can generate is closely allied to its size, the energy input to the system under test may also be small. As the dynamic range of the measurement transducers is necessarily finite, the presence of one or more dominant resonances means that the responses at other frequencies may be poorly resolved or distorted by noise if the total signal is to not be clipped. Figure 2 shows a typical force input spectrum measured on a lightly damped plate when the shaker was driven with a swept sine function (200 Hz to 400 Hz in 0.2 s). The structure is highly resonant causing the input force levels to vary by ±20 dB within the band of interest and the force level does not fall off very quickly out of band. The latter feature is due to the nature of the swept sine spectrum. If there is significant out of band excitation the measured transient force and acceleration responses may be poorly defined.

3. "IDEAL" OBJECTIVES

In principle we wish to measure signal distortion due to both the frequency response of the vibrator and vibrator/structure interaction effects. Then by use of inverse filtering techniques, for which a procedure is outlined below, we should be able to modify a given excitation signal to compensate for these effects and any deficiencies of the signal itself.

4. THE DIGITAL SIGNAL GENERATOR

To achieve these objectives, the Data Analysis Centre (DAC) at the ISVR has designed and built a digital signal generator based upon a standard microcomputer system. At present this is marketed for use in conjunction with DATS 11 the software package which originated at the DAC and which is currently distributed by Prosig Computer Consultants of Fareham. Plans are advanced for the necessary analogue to digital circuits to be added to give self contained on-line closed loop signal adaptation. The present device has the capacity to generate within itself up to a dozen types of mathematically defined signals as well as the ability to accept signals in digital form from an external computer. We have used the latter feature extensively in the work reported here.

The D/A subsystem generates an analogue signal to twelve bit resolution at user selected sampling rates. A smoothing filter is optional. Present configurations provide for up to one hundred thousand 12 bit samples to be stored in memory and for the D/A clock to be available to synchronously drive associated analogue to digital circuits. It is thus possible to generate repetitively and synchronously an excitation signal of desired characteristics and to sample the resultant response at one or more locations, with the sampling locked precisely to the time base of the excitation signal. Digital signals, whether generated on-board or externally, can be stored permanently using a built-in digital cassette drive, and therefore it is possible to load into memory any of a number of pre-recorded forcing functions.

5. TRANSFER FUNCTION EVALUATION

The experimental apparatus shown in figure 1 is set up as for conventional measurements. After appropriate calibration and gain trimming, the force gauge to theoretical signal transfer function (effectively the identification of the combined vibrator/structure frequency response) is measured and an inverse filter is designed to compensate for it. This is then convolved with the original signal to produce a flat bandlimited force spectrum at the input to the structure. The transfer function between the modified forcing function and each of the response sites can then be measured, and at this stage the input signal may be further modified to decrease the energy input at frequencies which show an excessively large response. Note that since the transfer functions used in this procedure are between the theoretical digital signal and a measured output, it is essential that the signal generation and data acquisition rates are precisely aligned. Once the design of an appropriate input signal has been finalized, the transfer function between input force and the acceleration at any point on the structure is measured in the normal way.

6. INVERSE FILTERING

The signal that we start with is the linear frequency sweep [1] defined by

$$s(t) = \sin\Phi(t)$$

where $\Phi(t) = at^2 + bt$, $a = (\omega_2 - \omega_1)/2T_s$, $b = \omega_1$, ω_1 and ω_2 are the start and finish radian frequencies of the sweep, T_s is the sweep duration and t is time. First, this signal is modified to increase its out of band rolloff rate, and to compensate for the passband ripple inherent to the swept sine. The steps in the production of this modified bandpass sweep are diagrammed in figure 3. $S(\omega)$, the Fourier transform of $s(t)$ is computed, and the modulus of its inverse is formed, i.e., $|S^{-1}(\omega)|$. The result is then multiplied by a target function $|T(\omega)|$ having the desired spectral characteristics; the parameters defining $|T(\omega)|$ are explained in figure 4. The minimum phase corresponding to the modulus $|T(\omega)||S^{-1}(\omega)|$ is then formed (see chapter 7 of reference [2] for a minimum phase algorithm) and is combined with the modulus to give the real and imaginary parts of the filter $[T(\omega)\ S^{-1}(\omega)]_{mp}$. This function is multiplied by $S(\omega)$ and the result is inverse transformed to produce the modified sweep $s_m(t)$. The use of a minimum phase filter guarantees that $s_m(t)$ is causal.

This signal is then loaded into the signal generator and used to measure $F(\omega)$, the transfer function between the digital signal and the input force applied to the structure. From this measurement we proceed to design the inverse filter to compensate for both the shaker frequency response and shaker/structure interaction. First the modulus $|F^{-1}(\omega)|$ is formed. Because the out of band components of $F(\omega)$ are relatively small, they are correspondingly large in its inverse. Thus it is necessary to window $|F^{-1}(\omega)|$ using a function $|T'(\omega)|$ having the same frequency range as $|T(\omega)|$ but a much lower level in the stopband. This operation ensures that no significant stopband components are present in the final test signal. The modulus $|T'(\omega)|\ |F^{-1}(\omega)|$ is then combined with the phase of $F^{-1}(\omega)$ to give the inverse filter $F_m(\omega)$, i.e., $F_m(\omega) = |T'(\omega)|\ |F^{-1}(\omega)| \exp[j \angle F^{-1}(\omega)]$, where exp[j___] is the complex exponential and $\angle$ indicates the argument of the function following it. $F_m(\omega)$ is inverse Fourier transformed to give $f_m(t)$, the impulse response of the desired inverse filter. Because of linear phase components in $F_m(\omega)$, $f_m(t)$ is usually acausal in appearance and it is desirable to rotate the time history at this stage to centre it in the record. After this operation a sine squared taper, $w_t(t)$, is applied to the first and last ten percent of the the data record to de-emphasize starting and finishing transients. The resulting time history $f'_m(t) = w_t(t)\ f_m(t)$ is then Fourier transformed and multiplied by $S_m(\omega)$. A final inverse Fourier transform gives the test signal, $s'_m(t)$, which is now fully corrected for signal, shaker, and shaker/structure interaction characteristics.

7. RESULTS

This procedure was applied to measurements conducted on an undamped aluminium plate. The input force spectrum measured using a conventional linear sweep was shown in figure 2 above. The modulus of the spectrum of the modified linear sweep (200 Hz to 400 Hz) is shown in figure 5, and the measured input force which resulted when this signal was delivered to the shaker is shown in figure 6. A comparison with figure 2 shows that the out of band components have been significantly reduced. The shaker/structure inverse filter was then computed, applied to the original modified sweep and then used as the input signal. The resulting input force is shown in figure 7. The deviation in force level across the passband has been reduced from approximately ±20 dB to ±3 dB. The residual variation was due to a 0.07 percent frequency misalignment between the clocks used to drive the signal generator and the A/D's. This emphasizes the importance of driving both signal generation and acquisition devices from the same clock; this will be possible with the construction of a signal acquisition device to match the digital signal generator.

The acceleration measured at the driving point is shown in figure 8. If it were desired to reduce the amplitude variation in the acceleration spectrum at the input point, or at any other point on the structure, it would be possible at this stage to go back through the second stage of the filtering procedure to decrease the response levels at particular frequencies if the amplitude range in the acceleration results was observed to be unacceptably large. This operation was not performed in the current experiments.

In figure 9 we show the ratio of acceleration to force at the driving point (the input inertance) measured using the fully compensated signal (+++++) and using a conventional linear sweep (_____). In this instance there was no significant difference between the two since signal to noise levels were reasonably good and there were no apparent nonlinearities in the system. It is clear however that it would be advantageous to use the modified signal were it desired to reproduce specific force levels across a band, e.g., to simulate particular vibration environments or to investigate the effects of structural nonlinearities.

8. CONCLUSIONS

The digital signal generator has proven to be a useful tool for applying an improved technique for transfer function analysis, particularly on lightly damped structures. Its power will be enhanced by the addition of matching signal acquisition equipment. The procedures outlined here allow the application of specified force levels, uniform across a frequency band, to a structure. Thus particular vibration environments may be simulated and the effect of absolute force levels may be investigated.

ACKNOWLEDGEMENTS

The authors acknowledge the contributions of colleagues both in the suggestions for improvements of the technique and in the developement of the digital signal generator. Part of the work was supported by grant from the Science and Engineering Research Council.

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1 R.G.White and R.J.Pinnington 1982 *The Aeronautical Journal of the Royal Aeronautical Society* 89, 179-199. Practical application of the rapid frequency sweep technique for structural frequency response measurement.

2 A.V.Oppenheim and R.W.Schafer 1975 *Digital Signal Processing*. Englewood Cliffs, New Jersey: Prentice-Hall.

FIGURES

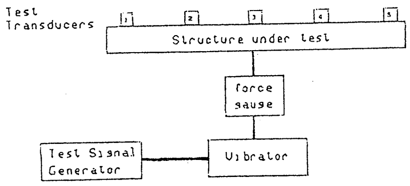


Figure 1: Experimental Configuration

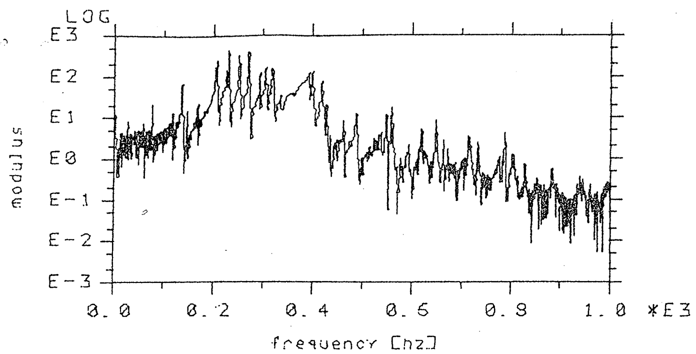


Figure 2: Input Force measured using Swept Sine Signal

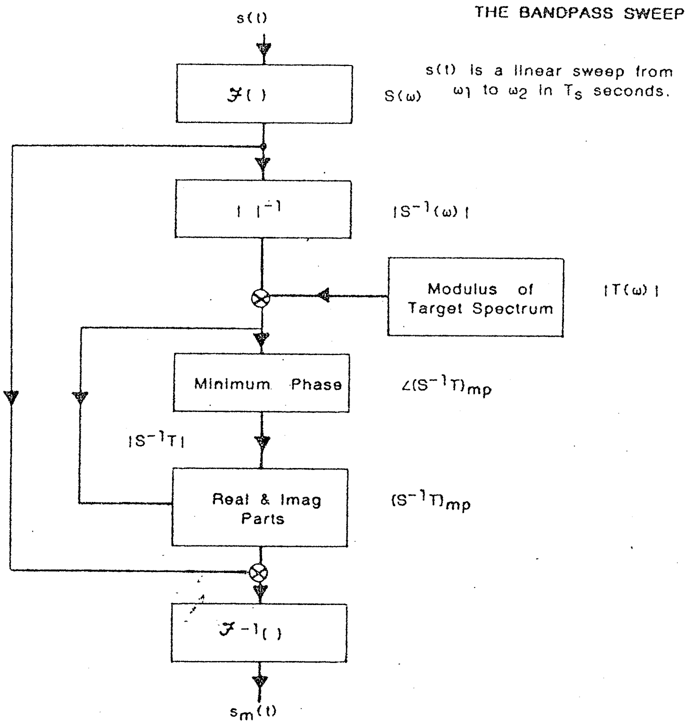
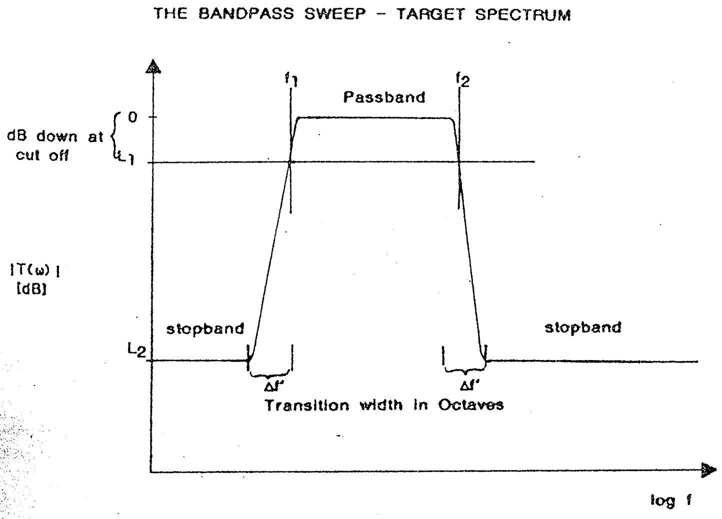


Figure 3: Computational Procedure for Signal Generation



In design procedure specify:

1. f_1 and f_2 cutoff frequencies
2. L_1 dB down at cutoff
3. L_2 dB down in stop bands
4. $\Delta f'$ transition width in Octaves

Figure 4: Target Function Parameters

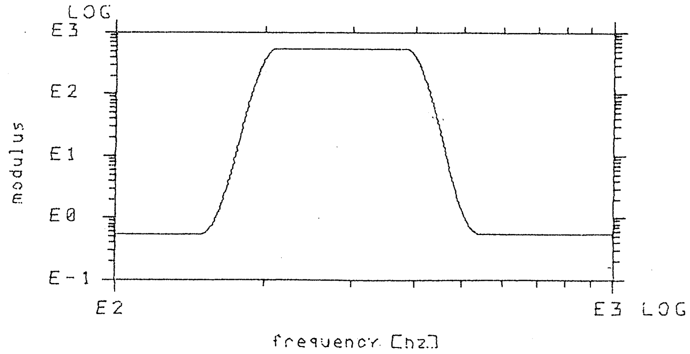


Figure 5: Spectrum of Modified Linear Sweep

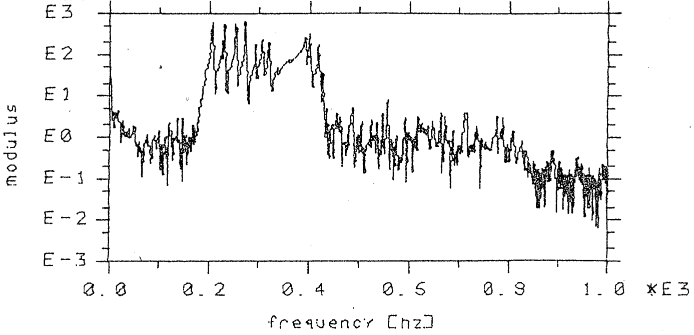


Figure 6: Input Force measured using Modified Linear Sweep

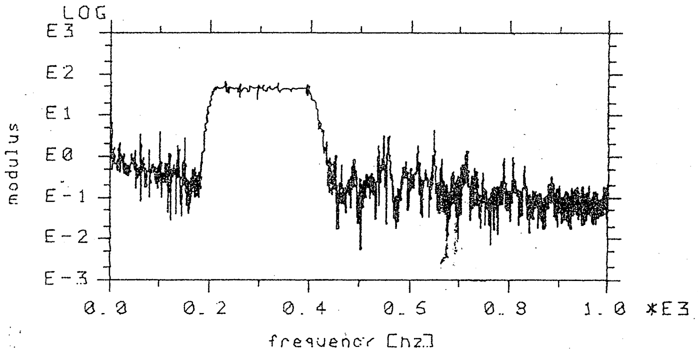


Figure 7: Input Force using Fully Compensated Signal

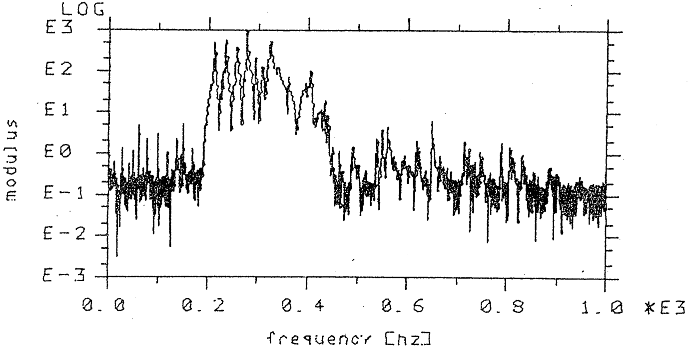


Figure 8: Acceleration at Input Point Measured using Fully Compensated Signal

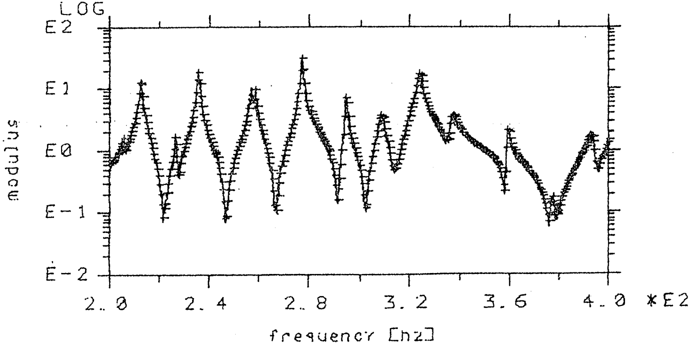


Figure 9: Input Inertance Measured using Fully Compensated Signal (++++), and Conventional Linear Sweep (———)