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**A DISCRETE IMPLEMENTATION OF A
HANKEL TRANSFORM TECHNIQUE FOR PREDICTING
MULTIPOLE SOUND PROPAGATION OVER
PLANE ABSORBING SURFACES**

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INTRODUCTION

- **OBJECTIVE:**

- accurate prediction of sound propagation from a higher order source over outdoor surfaces.

- **REQUIREMENTS:**

- measurement of the reflection coefficients
- appropriate sound propagation theory

INTRODUCTION

PREDICTION OF SOUND PROPAGATION

- **PREVIOUS TECHNIQUES:**

- asymptotic approximation, limitation: accurate in a certain domain ($k_r r_2 \gg 1$ or $k_r r_2 \ll 1$)
- one-dimensional Hankel transform technique for a monopole source

- **PRESENT APPROACH:**

- apply the two-dimensional inverse Hankel transform

- **RESULT:**

- theory capable of predicting multipole sound propagation over a finite impedance plane

- **APPLICATION:**

- prediction of sound propagation from aerodynamic noise sources over outdoor surfaces

THEORY

- **WAVE EQUATION**

- scalar Helmholtz equation

$$\nabla^2 p(\mathbf{r}, \omega) + k^2 p(\mathbf{r}, \omega) = 0$$

note: *time dependence of $e^{-i\omega t}$ is assumed here.*

- the sound pressure in cylindrical coordinates

$$p(r, \phi, z) = \sum_{n=0}^{\infty} [Q_{1n}(r, z) \cos n\phi + Q_{2n}(r, z) \sin n\phi]$$

THEORY

- REPRESENTATION OF A SOUND FIELD BY USING TWO-DIMENSIONAL HANKEL TRANSFORM TECHNIQUE

- the wavenumber spectrum in Cartesian coordinates:

$$\tilde{P}(k_x, k_y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x, y, z) e^{-i(k_x x + k_y y)} dx dy$$

- the wavenumber spectrum in cylindrical coordinates:

$$\tilde{P}(k_r, \psi, z) = \frac{1}{(2\pi)^2} \int_0^{\infty} \int_0^{2\pi} r P(r, \phi, z) e^{-ik_r r \cos(\phi - \psi)} d\phi dr$$

where k_r is the radial wavenumber and ψ is the azimuth angle in the wavenumber domain

Cylindrical coordinates

Cartesian coordinates

THEORY

— forward 2-D Hankel transform:

$$\tilde{P}(k_r, \psi, z) = \sum_{n=0}^{\infty} \frac{(-i)^n}{2\pi} [\tilde{Q}_{1n}(k_r, z) \cos n\psi + \tilde{Q}_{2n}(k_r, z) \sin n\psi]$$

where

$$\tilde{Q}_{ln}(k_r, z) = \int_0^{\infty} r Q_{ln}(r, z) J_n(k_r r) dk_r$$
$$l = 1, 2; \quad n = 0, 1, 2, \dots$$

is the Hankel transform of n th-order and J_n is the Bessel function of order n .

— inverse 2-D Hankel transform:

$$P(r, \phi, z) = \sum_{n=0}^{\infty} [Q_{1n}(r, z) \cos n\phi + Q_{2n}(r, z) \sin n\phi]$$

where

$$Q_{ln}(r, z) = \int_0^{\infty} k_r \tilde{Q}_{ln}(k_r, z) J_n(k_r r) dk_r$$
$$l = 1, 2; \quad n = 0, 1, 2, \dots$$

THEORY

The Incident and Reflected Sound Fields

- the wavenumber spectrum of the reflected pressure fields at $z = 0$

$$\tilde{P}_r(k_r, \psi, 0) = R(k_r, \omega) \tilde{P}_i(k_r, \psi, 0)$$

- in which the plane wave reflection coefficient, $R(k_r, \omega)$, may be calculated using

$$R(k_r, \omega) = \frac{(z_n - \omega \rho / k_z)}{(z_n + \omega \rho / k_z)}$$

- the wavenumber spectrum of the reflected field at any z -plane

$$\tilde{P}_r(k_r, \psi, z) = \tilde{P}_r(k_r, \psi, 0) e^{ik_z z}$$

- the total pressure in the wavenumber domain

$$\tilde{P}(k_r, \psi, z) = \tilde{P}_i(k_r, \psi, z) + \tilde{P}_r(k_r, \psi, z)$$

PREDICTION OF SOUND FIELD

Know: Sources and Reflecting Surface

Objective: Prediction the Sound Field over a Finite Impedance Surface

Methodology:

- find $\tilde{P}_i(k_r, \psi, z)$ either analytically or numerically
- decompose $\tilde{P}_i(k_r, \psi, z)$ into $\tilde{Q}_{ln}^i(k_r, z)$'s
- multiply $\tilde{Q}_{ln}^i(k_r, z)$'s with $e^{i2k_z z} R_p(k_r)$ to obtain $\tilde{Q}_{ln}^r(k_r, z)$'s
- add $\tilde{Q}_{ln}^i(k_r, z)$'s and $\tilde{Q}_{ln}^r(k_r, z)$'s to give $\tilde{Q}_{ln}(k_r, z)$'s
- inverse transform $\tilde{Q}_{ln}(k_r, z)$'s to give $Q_{ln}(r, z)$'s
- synthesize $Q_{ln}(r, z)$'s to give $P(r, \psi, z)$

FAST HANKEL TRANSFORM

Summary:

- *FHT is not as accurate as DHT but is faster with the same numerical parameters*
- *performance ($|E|^2 \times T_{cpu}$) of the FHT is generally superior to that of the DHT*
- *most efficient to compute higher order azimuthal harmonic components (shifted sources)*

THEORY

THE SOUND FIELD OF A MONOPOLE

- the direct sound pressure from the monopole in the space domain

$$P_i(r, \phi, z) = \frac{A_m}{R} e^{ikR}$$

- the wavenumber domain spectrum of the monopole is

$$\tilde{P}_i(k_r, \psi, z) = \frac{A_m}{(2\pi k_z)} e^{ik_z(z_s - z)}$$

where $k_z = \sqrt{k^2 - k_r^2}$

THEORY

THE SOUND FIELD OF AN ARBITRARILY ORIENTED DIPOLE

- the direct sound pressure from the inclined dipole in the space domain

$$P_i(r, \phi, z) = A_d \frac{(1 - ikR)}{R^3} e^{ikR} [(z_s - z) \cos \gamma + r \sin \gamma \cos(\phi - \phi_o)]$$

- the wavenumber domain spectrum of the inclined dipole

$$\tilde{P}_i(k_r, \psi, z) = \frac{A_d}{(2\pi k_z)} [k_z \cos \gamma + k_r \sin \gamma \cos(\psi - \phi_o)] e^{ik_z(z_s - z)}$$

MODEL OF SURFACE

- **assumption:**

the test carpet is a finite-depth layer of an extended reaction Delany and Bazley-type material

- **the characteristic impedance z_{o1}**

$$z_{o1} = \rho c [1 + 0.0571(\rho f / \sigma)^{-0.754} + i0.087(\rho f / \sigma)^{-0.732}]$$

where σ is the flow resistivity of the material in MKS Rayls/m

- **the complex wavenumber k_1**

$$k_1 = k [1 + 0.0978(\rho f / \sigma)^{-0.70} + i0.189(\rho f / \sigma)^{-0.595}]$$

- **the surface normal impedance**

$$z_n = \frac{iz_{o1}}{\cos\theta_1} \cot(k_1 l \cos\theta_1)$$

where l is the layer depth and $\cos\theta_1 = \sqrt{1 - (k/k_1)^2 \sin^2\theta}$

EXAMPLE: VERTICAL DIPOLE

$$f = 4000 \text{ Hz} \quad z_s = 0.1 \text{ m} \quad z = 0.025 \text{ m}$$

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EXAMPLE: INCLINED DIPOLE

$$f = 4000 \text{ Hz} \quad \gamma_d = 30^\circ \quad \phi_d = 90^\circ \quad z_s = 0.1 \text{ m} \quad z = 0.025 \text{ m}$$

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PROPAGATION MEASUREMENTS

Geometry

Instrumentation

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PROPAGATION MEASUREMENTS

$f = 1953 \text{ Hz}$, $z = 0.025 \text{ m}$

$f = 1953 \text{ Hz}$, $z = 0.012 \text{ m}$

PROPAGATION MEASUREMENTS

Optimal Monopole Source Strength

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PROPAGATION MEASUREMENTS

$f = 3613 \text{ Hz}$, $z = 0.025 \text{ m}$

$f = 3613 \text{ Hz}$, $z = 0.012 \text{ m}$

PROPAGATION MEASUREMENTS

$z = 0.012 \text{ m}$

PROPAGATION MEASUREMENTS

$z = 0.025 \text{ m}$

CONCLUSIONS

- *The two-dimensional Discrete Hankel Transform approach presented here offers an effective means for predicting the sound radiation from multipole sources over plane absorbing surfaces.*
- *The approach may be extended to arbitrarily directional sources if those sources can be modeled as a superposition of multipoles.*
- *The performance of the two-dimensional Discrete Hankel Transform prediction procedure can be enhanced by the use of a Fast Hankel Transform algorithm.*
- *The procedure outlined here must be supplemented by a farfield prediction to allow predictions at arbitrarily large distances from the source.*