7-2006

Multi-Patch Near-Field Acoustical Holography and Spatial Resolution Enhancement

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July 2006

Background

- Constraints of DFT-based NAH
  - A sound field should be sampled with a uniform spacing on a surface of constant coordinate in a separable geometry
  - The hologram surface should extend into a sufficiently large region to avoid spurious effects resulting from the undue truncation of the sound field → Difficult to implement NAH for large scale-structures
- Objective
  - To establish a procedure that is not subject to the latter constraints, thus relieving measurement effort related to the use of DFT
  - An iterative algorithm for recovering missing data and the detailed theoretical background are provided

Recent Approaches to Patch NAH (1)

- Statistically optimized NAH (SONAH) - *J. Hald*
  - A plane-to-plane propagation is performed in the spatial domain by two dimensional convolution with a propagation kernel
- Helmholtz equation least-squares (HELS) - *S. F. Wu*
  - An assumed solution is expressed by an orthonormal expansion of spheroidal functions that satisfy the Helmholtz equation
  - Solve the Helmholtz equation directly and minimize errors by the least-squares method
- Wavelet-based method - *J.-H. Thomas and J.-C. Pascal*
  - the sound field is decomposed by using multiresolution analysis, and spatial filtering is then performed in a selective way before the wave number spectrum is calculated

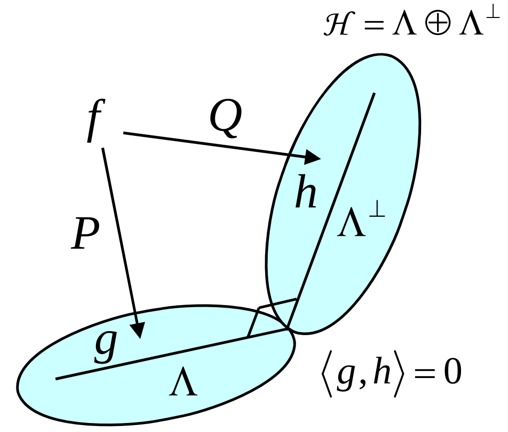
Recent Approaches to Patch NAH (2)

- Method of superposition - *A. Sarkissian*
  - The sound field is approximated by the superposition of fields produced by a number of sources
  - Can be used to either enlarge the finite measurement aperture (extrapolation) or fill the gap in the measurement aperture (interpolation)
- Iterative patch NAH - *K. Saijyou and E. G. Williams*
  - The sound field is extended into the region outside the measurement aperture by successive smoothing procedures
  - Available for both DFT and SVD-based NAH
  - Will be described in detail in the present work

Orthogonal Projection

- Consider square integrable functions in a Hilbert space, $\mathcal{H}$
- Every function in $\mathcal{H}$ can be decomposed uniquely as

$$f = g + h = Pf + Qf$$



where

$g \in \Lambda$

$h \in \Lambda^{\perp}$: orthogonal complement

- Properties of the orthogonal projection operator
  - $\|P\|, \|Q\| \le 1$
  - $P = P^*$ and $Q = Q^*$ (self-adjoint)
  - $P^2 = P$ and $Q^2 = Q = 1 - P$

Data Restoration from Partially Known Information

- When only its projection $g = P_a f$ onto the known subspace Λ_a is given
- Can f be reconstructed from g ?
- To enable the latter task, it is required that an additional constraint be imposed on the nature of the signals
- suppose that f belongs to the known subspace Λ_b : i.e., $f = P_b f$ then,

$$g = P_a f = P_a P_b f = \left(1 - Q_a\right) P_b f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

- f can be uniquely determined from the latter relation if the inverse of $A = P_a P_b = \left(1 - Q_a\right) P_b$ exists
- Λ_b and $\Lambda_a^{\perp}$ should have only the zero vector in common for solutions to be unique (Youla, D. C., 1978)

Iterative Method for Missing Data Restoration

- Rearrangement of Eq. (1) gives $f = Q_a P_b f + g$
- f can be obtained in an iterative way by using a method of successive approximations for finding the fixed point that satisfies the latter relation (method of alternating orthogonal projections): i.e.,

$$f^{(k+1)} = Q_a P_b f^{(k)} + g, \quad k = 1, 2, \cdots, \quad f^{(1)} = g \quad \ldots\ldots\ldots\ldots \quad (2)$$

- Convergence conditions
  - $\Lambda_b \cap \Lambda_a^{\perp} = \{\phi\}$
  - $\left\| Q_a P_b \right\| < 1$

  then $\lim_{k \to \infty} f^{(k)} = f$
- f in an explicit form

$$f = \sum_{n=0}^{\infty} \left(Q_a P_b \right)^n g \quad \Rightarrow \quad f^{(k)} = \sum_{n=0}^{k-1} \left(Q_a P_b \right)^n g$$

Orthogonal Projection Operator (1)
Sampling Operator

- In NAH applications, the known information is the sound pressure measured over a partial region of the hologram surface
- The measured pressure is expressed in terms of the sound pressure over the complete hologram surface

$$p_m(\vec{r}) = \begin{cases} p(\vec{r})\ , & \text{when } \vec{r} \in \Lambda_a \\ 0\ , & \text{when } \vec{r} \in \Lambda_a^{\perp} \end{cases}$$

- The spatial sampling operator

$$p_m = Dp \quad \text{where} \quad D(\vec{r}) = \begin{cases} 1\ , & \text{when } \vec{r} \in \Lambda_a \\ 0\ , & \text{when } \vec{r} \in \Lambda_a^{\perp} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (3)$$

- No restriction to the spatial distribution of measured data

Orthogonal Projection Operator (2)
Band-limiting Operator

- Related to the additional constraint regarding the nature of signals
- Signals in many practical cases satisfy a certain constraint rather than being arbitrary
- Suppose p is a function band-limited in *k*-space to $(k_{1,c}, k_{2,c})$
- The band-limiting operator

$$B = F^{-1}LF \quad \text{where} \quad L(\vec{k}) = \begin{cases} 1\,, & \text{when } \vec{k} \in (k_{1,c}, k_{2,c}) \\ 0\,, & \text{when } \vec{k} \notin (k_{1,c}, k_{2,c}) \end{cases} \quad \text{..............} \ (4)$$

- The complete hologram pressure then satisfies $p = Bp$
- Since high wave number, evanescent sound field components decay quickly, the sound pressure measured on the hologram surface usually (at least weakly) satisfies the latter assumption

Iterative Formulation

- By combing the two orthogonal projection operators with Eq. (2), the iterative relation for recovering a missing part of a band-limited signal is obtained: i.e.,

$$p^{(k+1)} = (1-D)Bp^{(k)} + p_m, \quad k = 1, 2, \cdots, \quad p^{(1)} = p_m \; \ldots\ldots\ldots\ldots \; (5)$$

: Papoulis-Gerchberg algorithm

- Since continuous functions cannot be both band-limited and space-limited at the same time, the closed subspace onto which p is projected by $(1\text{-}D)B$ contains only the zero signal
- Therefore, in continuous-continuous problems, the result obtained by using Eq. (5) converges to the desired solution over a complete region in the absence of noise

Discrete-Discrete Problems

- Equation (5) is expressed in matrix-vector form
- A unique solution does not exist due to the effects of the artificial truncation of an infinite domain and the discretization of continuous functions
- Since the inclusion of measurement noise is inevitable in practical cases, the band-limitedness assumption is not likely to be strictly valid
- The iteration should be terminated after the finite number of iterations: e.g., when

$$\left\| p^{(k+1)} - p^{(k)} \right\| \leq \varepsilon$$

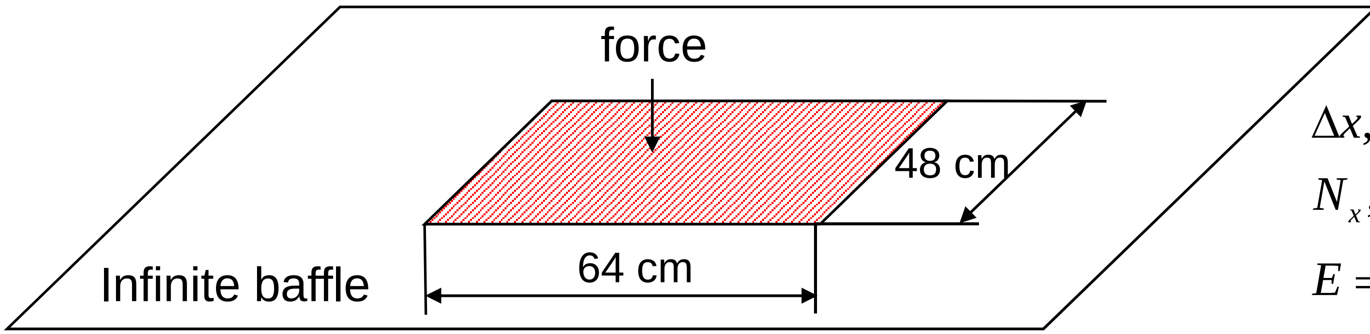
- As a result, the region in which accurate recovery is ensured is limited (usually observed when data is missing in a large contiguous region, i.e., extrapolation problems)

Comments

- The procedure can be used regardless of the spatial distribution of measured data
  - The convergence rate depends on the latter distribution
    - When data is missing in a large contiguous region (e.g., extrapolation) → slow convergence
    - When data is well distributed (e.g., interpolation) → fast convergence
- The actual bandwidth of a signal is usually unknown
  - In practice, however, the selection of a cutoff of the band-limiting operator within a sensible range usually yields results with reasonable accuracy, and the latter range can usually be identified by examining the wave number spectrum of the patch pressure
  - Is a time-consuming regularization procedure necessary?

Numerical Simulation Model

Point-driven, simply supported plate within an infinite baffle



$\Delta x, \Delta y = 0.5\,\text{cm}$

$N_x, N_y = 512$

$E = 2\times 10^{11}\,\text{Pa}$

$\nu = 0.28$

$\rho = 7860\,\text{kg/m}^3$

$h = 5\,\text{mm}$

$$\dot{w}(x,y,\omega) = \frac{j\omega}{\rho h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\Phi_{mn}(x_0,y_0)\Phi_{mn}(x,y)}{\omega^2-\omega_{mn}^2}$$

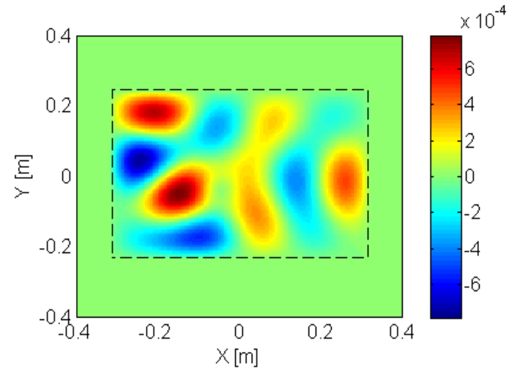
where

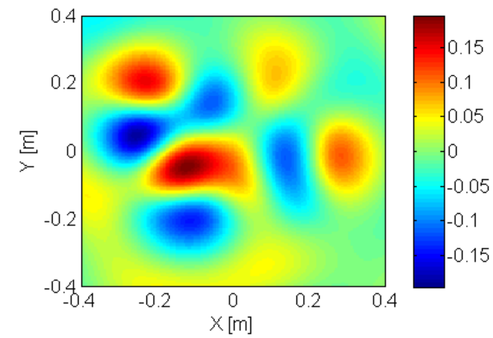
$$\Phi_{mn} = \frac{2}{\sqrt{L_x L_y}}\sin\left(\frac{m\pi x}{L_x}\right)\sin\left(\frac{n\pi y}{L_y}\right) \qquad \omega_{mn} = \sqrt{\frac{Eh^2}{12\rho\left(1-\nu^2\right)}}\left[\left(\frac{m\pi}{L_x}\right)^2+\left(\frac{n\pi}{L_y}\right)^2\right]$$

Surface Velocity and Hologram Pressure

< surface velocities > < hologram pressure >

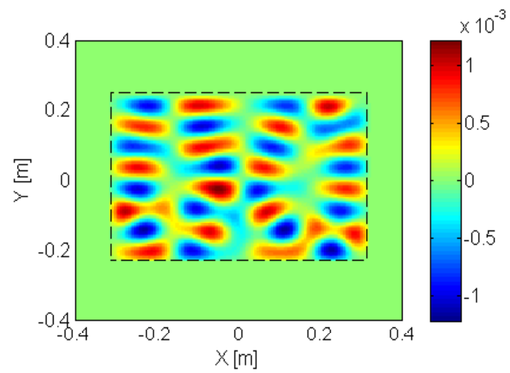
1 kHz

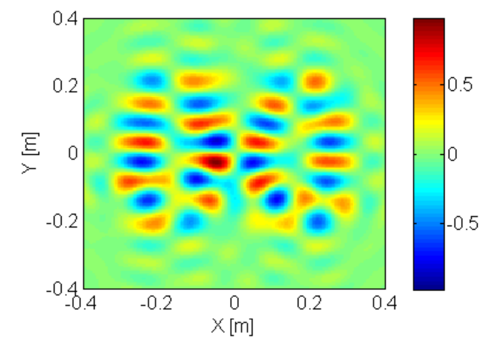




forward projection

3.8 kHz



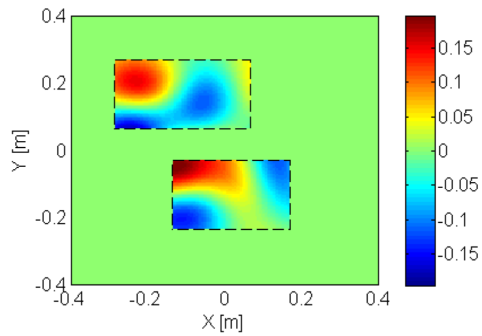


- The height of the hologram surface: 3 cm
- The SNR of the hologram pressures: 40 dB

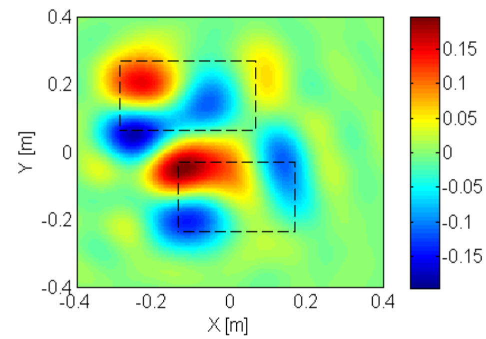
Multi-patch Holography (1)

< patch pressures > < extended pressures >

1 kHz

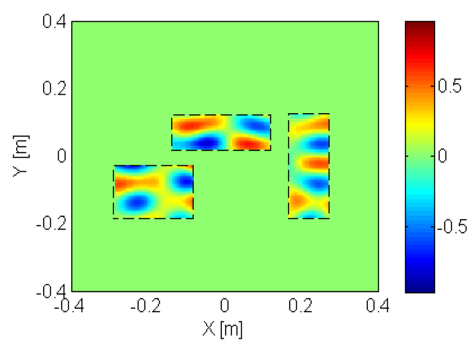


400 iter.

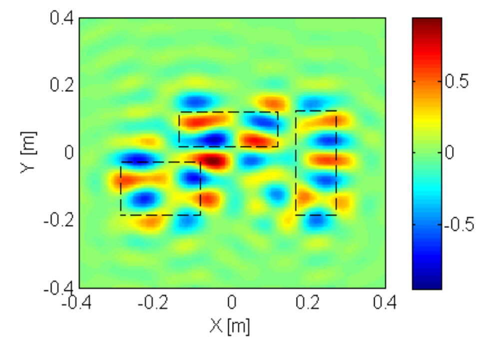


Patch extension

3.8 kHz



400 iter.



- The pressures at the locations near the measurement patches were recovered successfully, but the latter could not be achieved over the complete region

Multi-patch Holography (2)

< A comparison of the surface velocity reconstruction error >

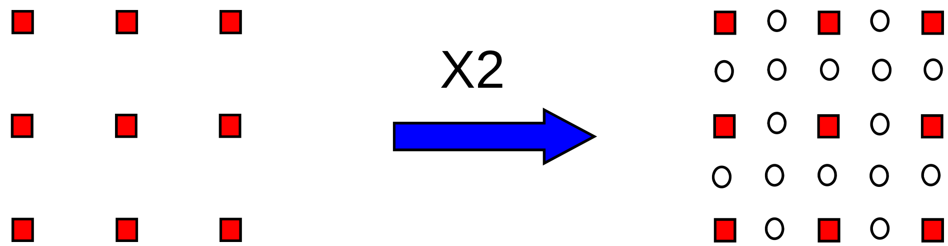
$$error = \left\| \dot{w}_{rec} - \dot{w} \right\| / \left\| \dot{w} \right\|$$

%	Evaluated over the entire region			Evaluated over the region directly under the patches		
	Complete	Patch	Extended	Complete	Patch	Extended
1 kHz	13.0	119.5	49.2	13.8	104.9	24.8
3.8 kHz	8.9	92.4	51.9	4.7	83.4	10.7

- The use of the extended pressure resulted in significant improvement especially in the region directly under the measurement patches

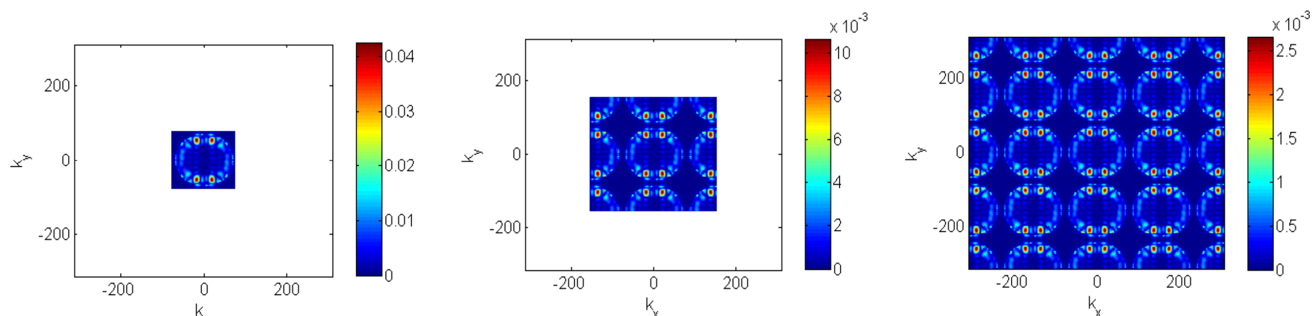
Spatial Resolution Enhancement (1)

- The initial pressure is prepared by inserting zeros on the uniform grids between the measured points



- The actual bandwidth of a signal is “known” in this case

Wave number spectra at 3.8 kHz



measured   zero-inserted (X2)   zero-inserted (X4)

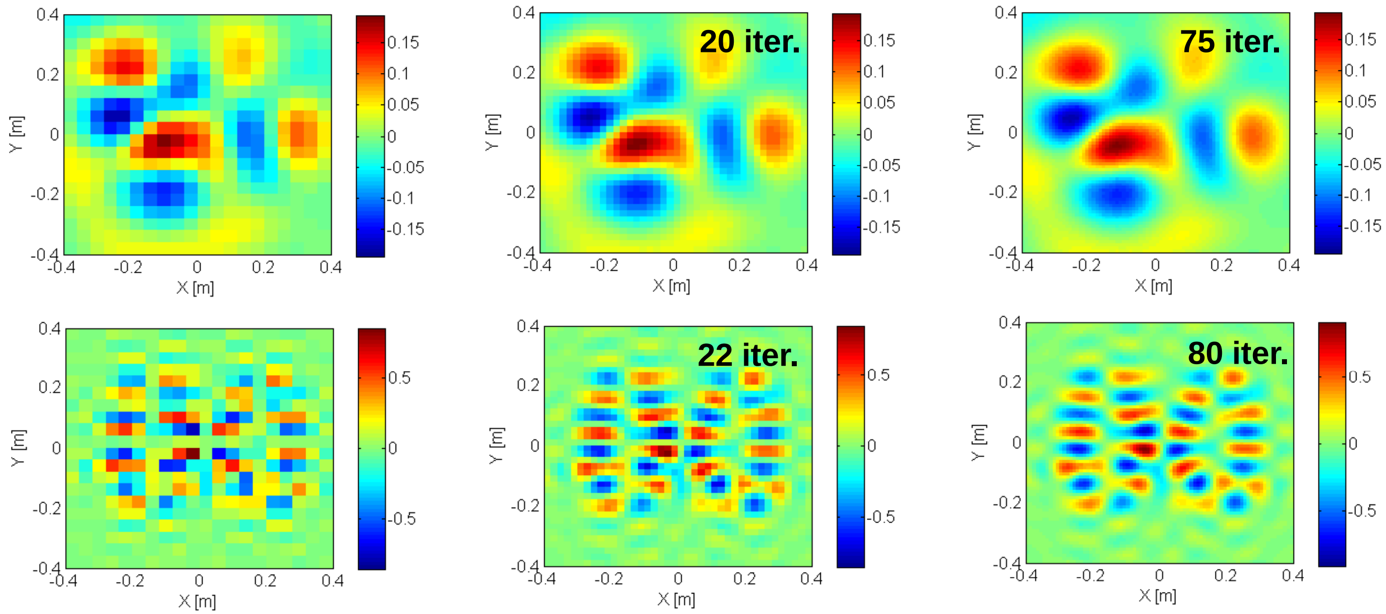
Spatial Resolution Enhancement (2)

< Resolution enhanced hologram pressure >

measured ($\Delta x, \Delta y = 4\,\mathrm{cm}$) | enhanced (X2) | enhanced (X4)

1 kHz

3.8 kHz

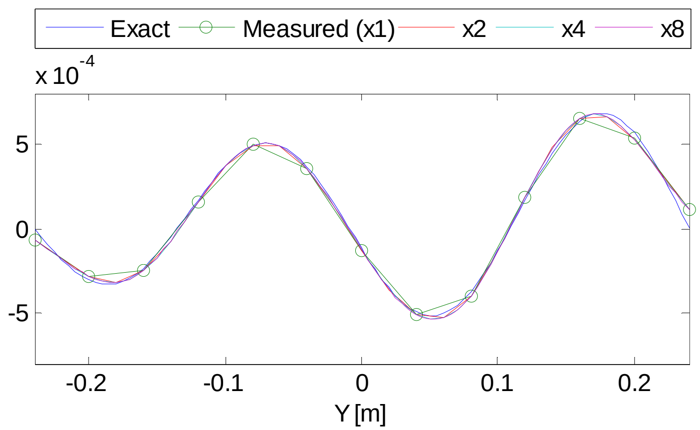


- The results converged far faster than in the extrapolation cases

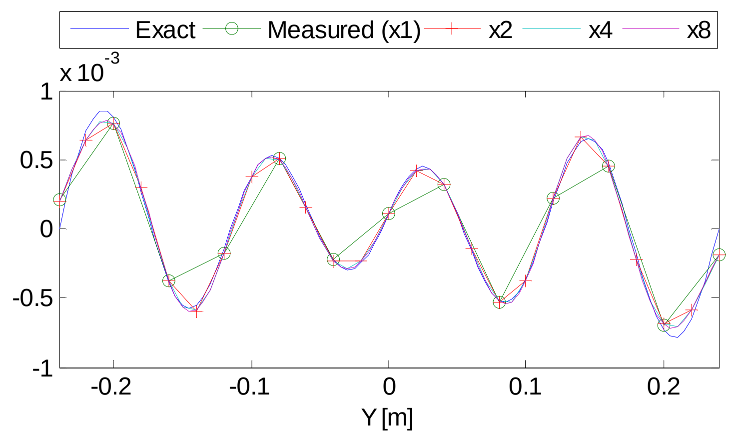
Spatial Resolution Enhancement (3)

< Comparisons of the surface velocities reconstructed by using the measured and resolution-enhanced hologram pressures >

1 kHz



3.8 kHz



- A larger number of samples per wavelength are necessary to describe the signal shape and locate its peaks correctly

Conclusions

- An iterative patch NAH algorithm was derived from a method of alternating orthogonal projections
- As assumed when the iterative algorithm was derived, it is important for signals to be band-limited in *k*-space for this procedure to be successful
- It was shown that the applications of the procedure described in the present work can be expanded to the hologram pressure measured over arbitrary locations
- The practical limitation of the procedure resulting from the effects of the artificial truncation of an infinite domain and the discretization of continuous functions was discussed
- Numerical simulation results obtained in two cases were presented: i.e., the extrapolation of the hologram pressure measured over multiple, distinct patches, and spatial resolution enhancement by interpolation between measured points