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A MODEL FOR SOUND ABSORPTION BY AND SOUND TRANSMISSION THROUGH LIMP FIBROUS LAYERS

by

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INTRODUCTION

- A limp porous material model has been derived from Biot's theory by assuming zero stiffness of the frame.
- The simplified model yields a single second order wave equation governing the propagation of a single wave type.
- Experiments indicate that the predictions made using the present model more closely match experimental results related to limp fibrous materials than those made by using a rigid porous material model and give results equivalent to an elastic porous material model.

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TRANSMISSION LOSS RESULTS

Measured parameters :

$$\rho_b = 21.1 \text{ Kg} / \text{m}^3$$

$$\sigma = 16660 \text{ Rayls} / \text{m}$$

$$l = 5.7 \text{ cm}$$

Estimated parameters :

$$N = 20 \text{ N} / \text{m}^2$$

$$\eta = 0.005$$

$$\vartheta = 0.3$$

$$q = 1.2$$

$$\phi = 0.98$$

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REVIEW OF EXISTING THEORIES

- RIGID
 - Rayleigh (1877, 'Theory of sound')
 - Monna (1938, *Physica* 5, 129-142)
 - Morse (1952, 'Theoretical Acoustics')
 - Attenborough (1985, *JSV* 99 (4), 521-544)
- LIMP
 - Wood (1941, 'A textbook of sound')
 - Beranek (1947, *JASA* 19, 556)
 - Kosten and Jansen (1957, *Acustica* 7, 372)
 - Geertsma and Smit (1957, *Geophysics* 26, 169-181)
 - Ingard (1981, *ASME, J. of Engg. for Industry* 103, 302-313)

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REVIEW OF EXISTING THEORIES

- ELASTIC
 - Zwikker and Kosten (1949, ‘Sound absorbing materials’)
 - Biot (1956, *JASA* 28, 168-191)
 - Bolton, Shiau and Kang (1987, *AIAA*-87-2660 and *JSV* in press)
 - Allard (1993, ‘Propagation of sound in porous media’)

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OBJECTIVE OF THE PRESENT WORK

- To derive a limp porous material model (i.e., one that allows for motion of the solid phase) based on the Biot Theory.
- To show that the model gives the same result as the elastic porous theory for 'limp' fibrous materials.

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ELASTIC POROUS MATERIAL MODEL

GOVERNING WAVE EQUATIONS (BIOT)

Solid: $Q \nabla^2 \varepsilon + \omega^2 \rho_{12}^* \varepsilon = -P \nabla^2 e_s - \omega^2 \rho_{11}^* e_s$

Fluid: $R \nabla^2 \varepsilon + \omega^2 \rho_{22}^* \varepsilon = -Q \nabla^2 e_s - \omega^2 \rho_{12}^* e_s$

where (from Gassman)

$$P = \frac{(1-\phi) \left[1 - \phi - \frac{K_b}{K_s} \right] K_s + \phi \frac{K_s}{K_f}}{1 - \phi - \frac{K_b}{K_s} + \phi \frac{K_s}{K_f}} + \frac{4}{3} N,$$

$$Q = \frac{\left[1 - \phi - \frac{K_b}{K_s} \right] \phi K_s}{1 - \phi - \frac{K_b}{K_s} + \phi \frac{K_s}{K_f}}$$

and
$$R = \frac{\phi^2 K_s}{1 - \phi - \frac{K_b}{K_s} + \phi \frac{K_s}{K_f}}$$

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WAVENUMBERS IN ELASTIC MODEL

COMPRESSIONAL WAVES

$$k_{1,2}^2 = \frac{\omega^2}{2(PR - Q^2)} [P\rho_{22}^* + R\rho_{11}^* - 2Q\rho_{12}^* \pm \sqrt{\Delta}]$$

where,

$$\Delta = (P\rho_{22}^* + R\rho_{11}^* - 2Q\rho_{12}^*)^2 - 4(PR - Q^2)(\rho_{11}^*\rho_{22}^* - \rho_{12}^{*2})$$

SHEAR WAVE

$$k_t^2 = \frac{\omega^2}{N} \left(\frac{\rho_{11}^*\rho_{22}^* - \rho_{12}^{*2}}{\rho_{22}^*} \right)$$

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LIMP POROUS MATERIAL MODEL

- The frame bulk modulus is assumed to be zero. Therefore,

$$P = \frac{(1-\phi)^2 K_s}{(1-\phi) + \phi \frac{K_s}{K_f}}, \quad Q = \frac{(1-\phi)\phi K_s}{(1-\phi) + \phi \frac{K_s}{K_f}}, \quad R = \frac{\phi^2 K_s}{(1-\phi) + \phi \frac{K_s}{K_f}}.$$

- Applying these expressions within the governing equations of Biot's theory, we obtain,

$$(2\rho_{12}^* Q - \rho_{22}^* P - \rho_{11}^* R) \nabla^2 \varepsilon + \omega^2 (\rho_{12}^* - \rho_{11}^* \rho_{22}^*) \varepsilon = 0$$

$$\text{and} \quad e_s = \frac{(\rho_{12}^* Q - \rho_{22}^* P)}{(\rho_{12}^* P - \rho_{11}^* Q)} \varepsilon = \frac{1}{b} \varepsilon.$$

COMPRESSIONAL WAVENUMBER

$$k_p^2 = \frac{\omega^2 (\rho_{11}^* \rho_{22}^* - \rho_{12}^{*2})}{(\rho_{22}^* P + \rho_{11}^* R - 2\rho_{12}^* Q)}$$

- Single dilatational wave propagates through both phases of limp porous material.

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SOLUTION APPROACH FOR NORMAL INCIDENCE

$$\varepsilon = \nabla \cdot \mathbf{U}_y = e^{-ik_x x} \left(C_1 e^{-ik_y y} + C_2 e^{ik_y y} \right)$$

$$e_s = \nabla \cdot \mathbf{u}_y = \frac{l}{b} e^{-ik_x x} \left(C_1 e^{-ik_y y} + C_2 e^{ik_y y} \right)$$

Fluid stress : $s = R\varepsilon + Qe_s$

Solid stress : $\sigma_y = Pe_s + Q\varepsilon$

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BOUNDARY CONDITIONS

$$\underline{y=0}$$

$$-\phi p = s$$

$$-(1-\phi)p = \sigma_y$$

$$\underline{y=d}$$

$$u_y = 0$$

$$U_y = 0$$

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NORMAL SPECIFIC IMPEDANCE

$$Z_n = \frac{I}{\rho_0 c} \left(\frac{p}{v_y} \right)_{y=0} = - \frac{i(Rb + Q)k \cot kd}{\rho_0 c \phi \omega [1 - \phi(1 - b)]}$$

where,

$$v_y = i\omega(1 - \phi)u_y + i\omega\phi U_y$$

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COMPARISON WITH EXPERIMENT

TRANSMISSION LOSS COMPARISON

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COMPARISON WITH ELASTIC MODEL

TRANSMISSION LOSS

ABSORPTION COEFFICIENT

- How limp is 'Limp'?

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CONCLUSIONS

A theoretical model for limp fibrous materials has been constructed using Biot's theory. The new model has the following advantages over the two prevalent models :

- More accurate predictions than the rigid porous material model.
- Numerically robust compared to the elastic porous material model.
- Computationally faster compared to the elastic porous material model.

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