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The Use of Non-Collocated Higher Order Sources in the Equivalent Source Method

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Quieting the World's Cities

The Use of Non-Collocated Higher Order Sources in the Equivalent Source Method

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Introduction

❑ General Process of Equivalent Source Method:

Measure sound field at some locations → Assume the structure of the sound field components → Estimate the parameters associated with each component → Predict the sound field everywhere

Lower Order Sources with Different Locations (Fixed)

- Monopole Distribution
- Monopole and Dipole Distribution

Higher Order Sources with a Collocated Location (Fixed)

- Spherical Harmonics
- Plane Wave Decomposition

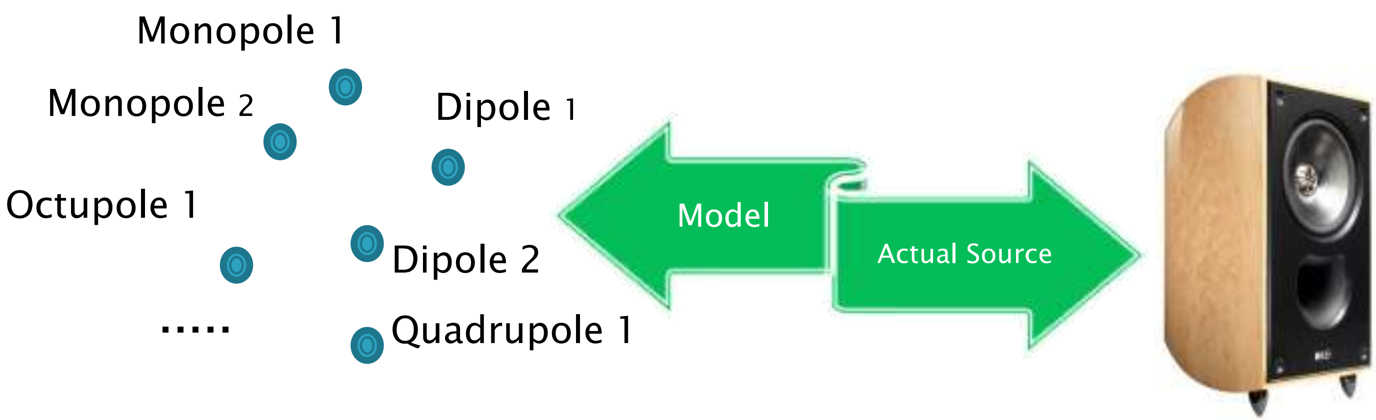
Fixed Location Methods

Parameter estimation process is linear.

(Only estimate source strength)

Introduction to Non-Collocated ESM

❑ Idea of the Non-Collocated ESM:



Generate the same sound field

1) Another set of multipole sources
2) Use non-collocated, unfixed source locations

Sources with more physical meaning: Monopole, dipoles, quadrupoles, etc.

More flexibility may reduce the number of parameters in the model.

Parameter estimation process is Nonlinear.

(Estimate both source strength and locations)

Construction of Non-Collocated ESM

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❑Model Formulation

M_0 monopoles M_1 dipoles   ...

Strength of each source

$$\begin{bmatrix} P_1(\vec{\zeta}_1,\omega) \\ P_2(\vec{\zeta}_2,\omega) \\ \dots \\ P_M(\vec{\zeta}_M,\omega) \end{bmatrix} = \begin{bmatrix} g_1(\vec{\zeta}_1|\vec{X}_1,\omega) & g_2(\vec{\zeta}_1|\vec{X}_2,\omega) & \dots & g_W(\vec{\zeta}_1|\vec{X}_W,\omega) \\ g_1(\vec{\zeta}_2|\vec{X}_1,\omega) & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ g_1(\vec{\zeta}_M|\vec{X}_1,\omega) & \dots & \dots & g_W(\vec{\zeta}_M|\vec{X}_W,\omega) \end{bmatrix} \begin{bmatrix} Q_1(\omega) \\ Q_2(\omega) \\ \dots \\ Q_W(\omega) \end{bmatrix}$$

Sound field measurements at different locations.
($\vec{\zeta}$ – measurement location)

g_i is the sound field of each source with unit strength .
($\vec{X}$ – source location)

of the form: $\vec{P} = A(\vec{X})\vec{Q}$ → Least-square → $\min\left\|\vec{P} - A(\vec{X})\vec{Q}\right\|^2$ with respect to $\vec{X}, \vec{Q}$

⇒ Formula for g_i?

Sound field of Multipole Sources

❑Proposed Definition of Multipole Sources:

Monopole (zero order) → Source with sound field expressed as:

$$P_{S0}(\vec{X}\,|\,\vec{X}_0,\omega) = S\frac{e^{-jk\|\vec{X}-\vec{X}_0\|}}{4\pi\|\vec{X}-\vec{X}_0\|},$$

- S is monopole strength
- $\vec{X}_0$ is source location

Higher order (Order: n) → Two sources of order $n-1$, equal amplitude, out-of-phase, closely located at:

$$\vec{X}_0 \pm \vec{u}_n\, d/2,$$

- $\vec{u}_n$ is directional vector (unit vector)
- d is separation distance (small)

Sound field of Multipole Sources

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❑Sound field expression for simple sources:

Sound field of order $n+1$ source and order n source can be related by directional derivative:

$$P_{Sn+1}(\overrightarrow{X}|\overrightarrow{X_0},\omega) = d < \nabla P_{Sn}(\overrightarrow{X}|\overrightarrow{X_0},\omega), \vec{u}_{n+1} >$$

$$= d < \left[\frac{\partial P_{Sn}(\overrightarrow{X}|\overrightarrow{X_0},\omega)}{\partial x_0}, \frac{\partial P_{Sn}(\overrightarrow{X}|\overrightarrow{X_0},\omega)}{\partial x_0}, \frac{\partial P_{Sn}(\overrightarrow{X}|\overrightarrow{X_0},\omega)}{\partial x_0} \right]^T, \vec{u}_{n+1} >$$

Dipole: $P_{S1} = Sd_1 < \nabla P_0, \vec{u}_1 >,$ $\quad \nabla P_0 = [\frac{\partial P_0}{\partial x}, \frac{\partial P_0}{\partial y}, \frac{\partial P_0}{\partial z}]^T$

Dipole strength: $S\,d_1$

Quadrupole: $P_{S2} = Sd_1 d_2 \vec{u}_2^T R_2 \vec{u}_1,$

$$R_2 = \begin{bmatrix} \frac{\partial^2 P_0}{\partial x^2} & \frac{\partial^2 P_0}{\partial x \partial y} & \frac{\partial^2 P_0}{\partial x \partial z} \\ \frac{\partial^2 P_0}{\partial y \partial x} & \frac{\partial^2 P_0}{\partial y^2} & \frac{\partial^2 P_0}{\partial y \partial z} \\ \frac{\partial^2 P_0}{\partial z \partial x} & \frac{\partial^2 P_0}{\partial z \partial y} & \frac{\partial^2 P_0}{\partial z^2} \end{bmatrix}$$

Quadrupole strength: $Sd_1 d_2$

Order n:

$$P_{Sn} = Q_n g_n$$

Source strength:

$$Q_n = Sd_1 d_2 \ldots d_n$$

$$g_n = R_n(P_0) \cdot \overrightarrow{u_1} \cdot \overrightarrow{u_2} \ldots \cdot \overrightarrow{u_n},$$

$$R_n = \nabla^{\otimes n}$$

$\otimes$ – tensor outer product

$\cdot$ – tensor inner product

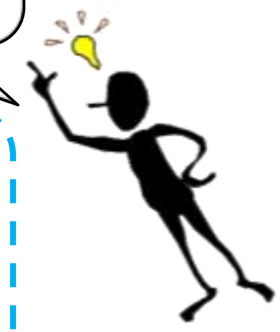
Decomposition of the Multipoles

For a source of order n ($n > 1$), there could be infinitely many types because of the orientation vectors.

Which source orientations should be included in the model?

Even more nonlinear if the orientations are unknown.

Orientations' effect is just a linear combination of components in R_n.



Order n: $P_{Sn} = Q_n g_n$

Source strength: $Q_n = S d_1 d_2 \ldots d_n$

$g_n = R_n(P_0) \Box \vec{u}_1 \Box \vec{u}_2 \ldots \Box \vec{u}_n$, $R_n = \nabla^{\otimes n}$

$\otimes$ – tensor outer product $\Box$ – tensor inner product

A source of arbitrary orientation can be decomposed into several standard source configurations.

Components in R_n are differentiations with respect to x, y, z different number of times.

Derivative does not depend on the sequence of differentiation

The number of independent components is less than 3^n.

Decomposition of the Multipoles

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- Standard Source Configurations

3 Standard Dipoles

$\frac{\partial P}{\partial x}$		$\frac{\partial P}{\partial z}$
[1 0 0]	[0 1 0]	[0 0 1]

6 Standard Quadrupoles

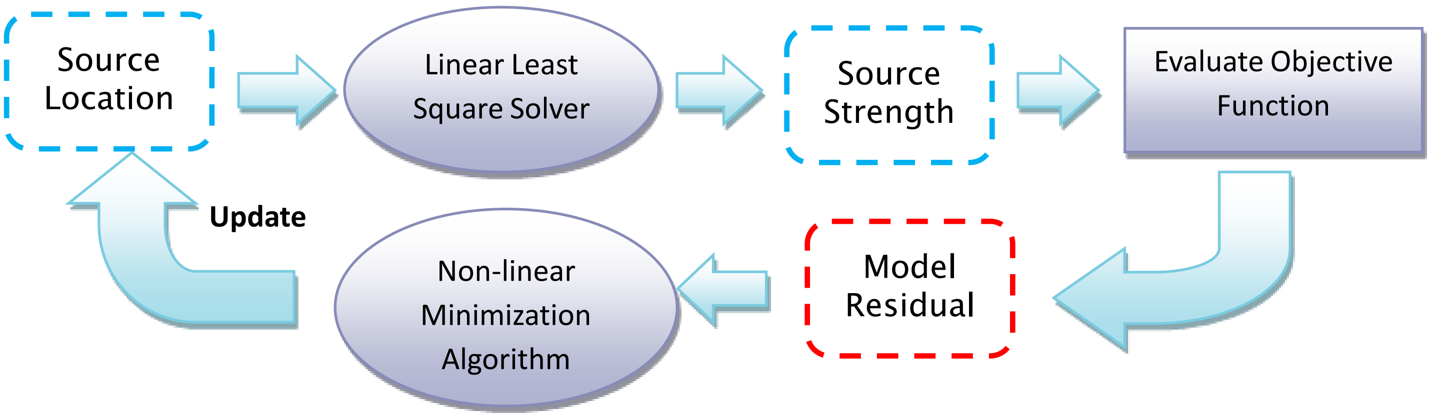
$\frac{\partial^2 P}{\partial x^2}$			$\frac{\partial^2 P}{\partial x \partial z}$	$\frac{\partial^2 P}{\partial y^2}$	$\frac{\partial^2 P}{\partial y \partial z}$	$\frac{\partial^2 P}{\partial y^2}$
[1 0 0] [1 0 0]	[1 0 0] [0 1 0]	[1 0 0] [0 0 1]	[0 1 0] [0 1 0]	[0 1 0] [0 0 1]	[0 0 1] [0 0 1]	

10 Standard Octupoles

$\frac{\partial^3 P}{\partial x^3}$	$\frac{\partial^3 P}{\partial x^2 \partial y}$	$\frac{\partial^3 P}{\partial x^2 \partial z}$	$\frac{\partial^3 P}{\partial x \partial y^2}$	$\frac{\partial^3 P}{\partial x \partial y \partial z}$	$\frac{\partial^3 P}{\partial x \partial z^2}$	$\frac{\partial^3 P}{\partial y^3}$	$\frac{\partial^3 P}{\partial y^2 \partial z}$	$\frac{\partial^3 P}{\partial y \partial z^2}$	$\frac{\partial^3 P}{\partial z^3}$
[1 0 0] [1 0 0] [1 0 0]	[1 0 0] [1 0 0] [0 1 0]	[1 0 0] [1 0 0] [0 0 1]	[1 0 0] [0 1 0] [0 1 0]	[1 0 0] [0 1 0] [0 0 1]	[1 0 0] [0 0 1] [0 0 1]	[0 1 0] [0 1 0] [0 1 0]	[0 1 0] [0 1 0] [0 0 1]	[0 1 0] [0 0 1] [0 0 1]	[0 0 1] [0 0 1] [0 0 1]

Parameter Estimation Process

❑ Process of estimating source strength and locations



$$\min \left\| \vec{P} - A(\vec{X})\vec{Q} \right\|^2$$

If $\vec{X}$ is fixed, the problem reduces to a linear least square problem.

❑ Algorithms in two parts of the above problem

Linear Part:

1. Standard Least-square Solution
2. Regularization (ill-posed)

(Different methods for regularization and choosing the regularization parameter were compared)

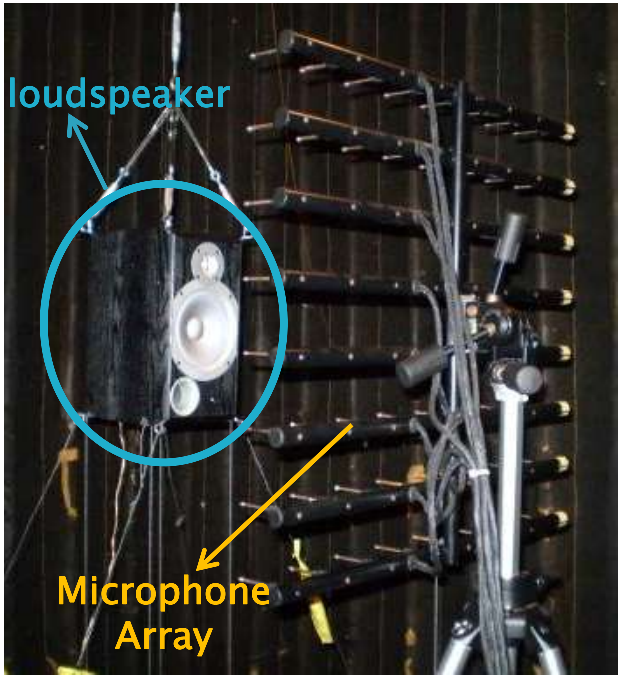
Non-linear Part: Trust Region Reflective algorithm

T.F. Coleman and Y. Li, "An Interior Trust Region Approach for Nonlinear Minimization Subject to Bounds", *SIAM J. Optimization*, **6**(2), (1996), pp. 418-445.

Experiment and Numerical Results

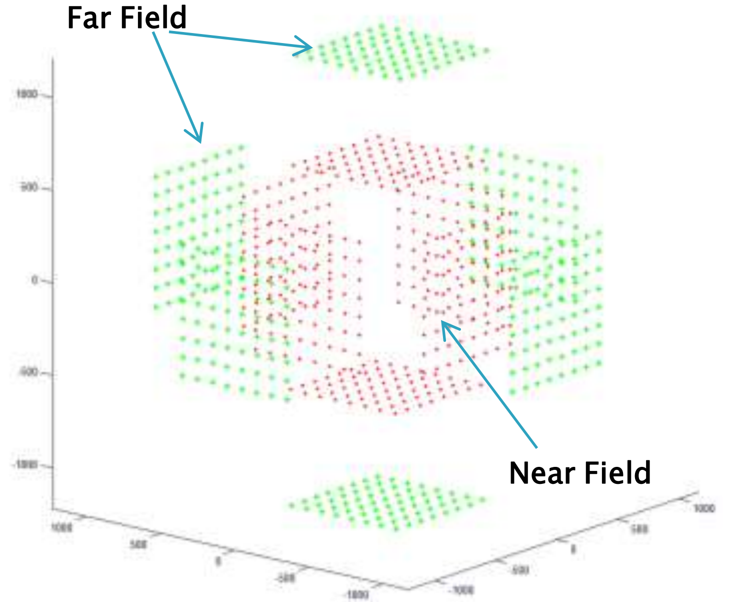
❑ Experiment Description

Measurement is performed **around** the loudspeaker (all six faces were covered).



Transfer Matrix Method:

Construct separately measured data into simultaneous data.



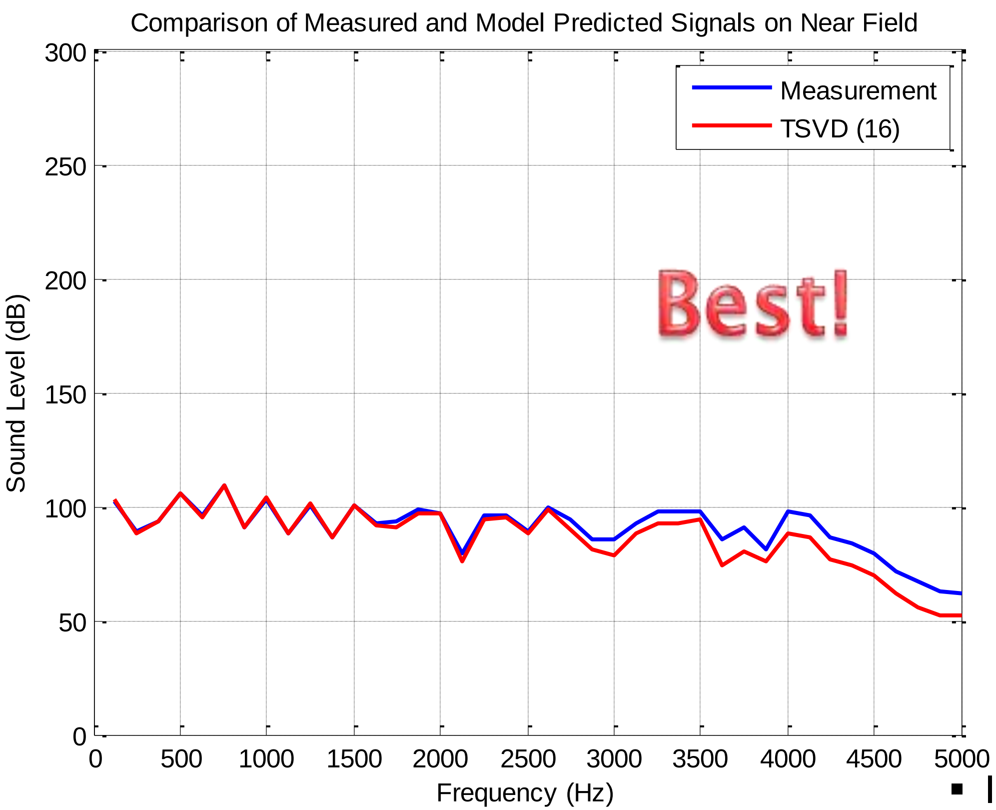
Coordinates of Microphones in the Measurement

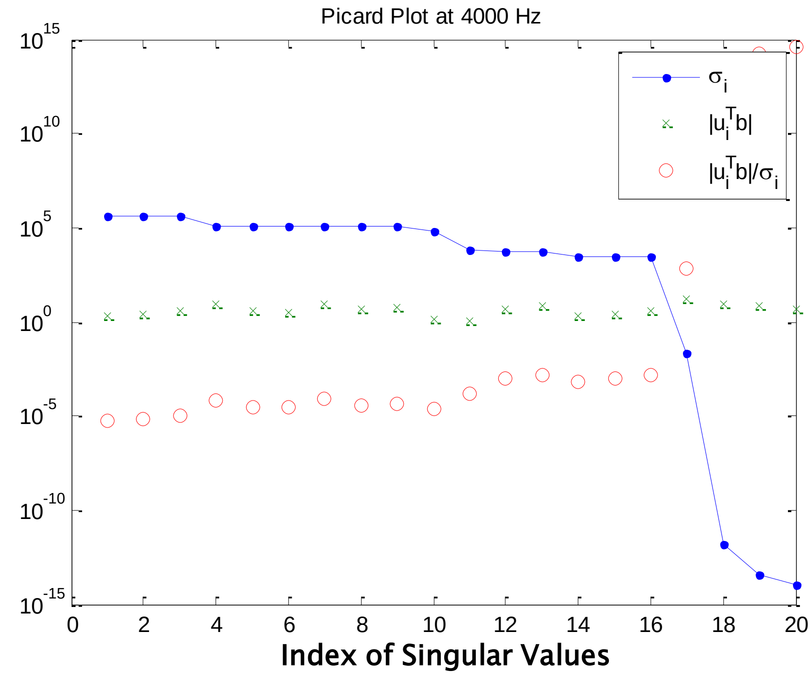
NOTES:

•Far Field sound field is also measured to compare with the model prediction.

Experiment and Numerical Results

❑ Effect of Different Regularization Methods



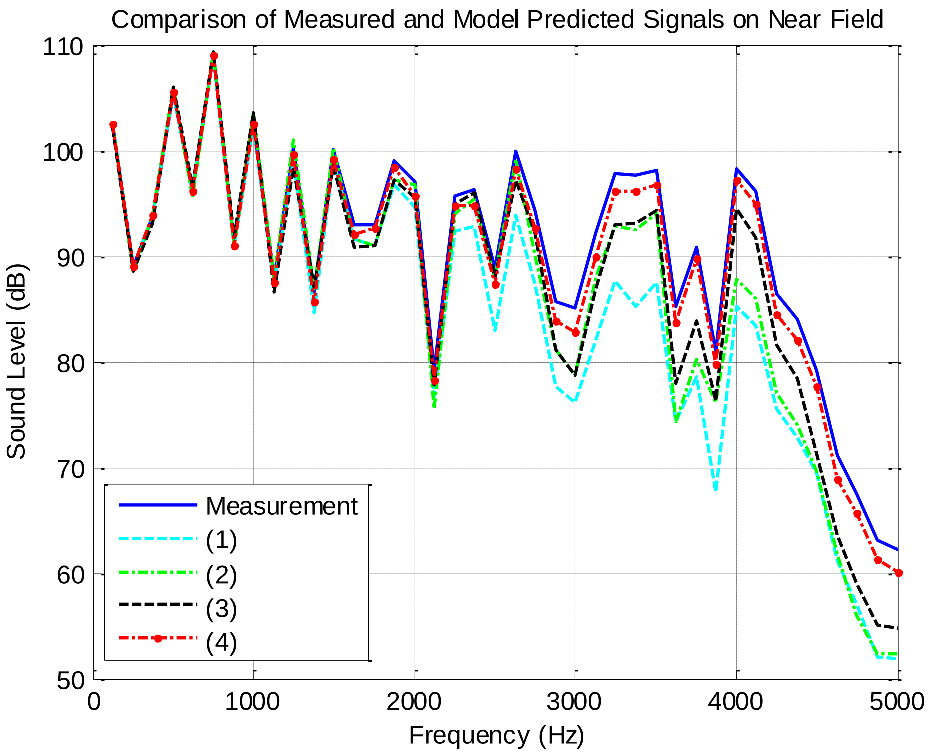


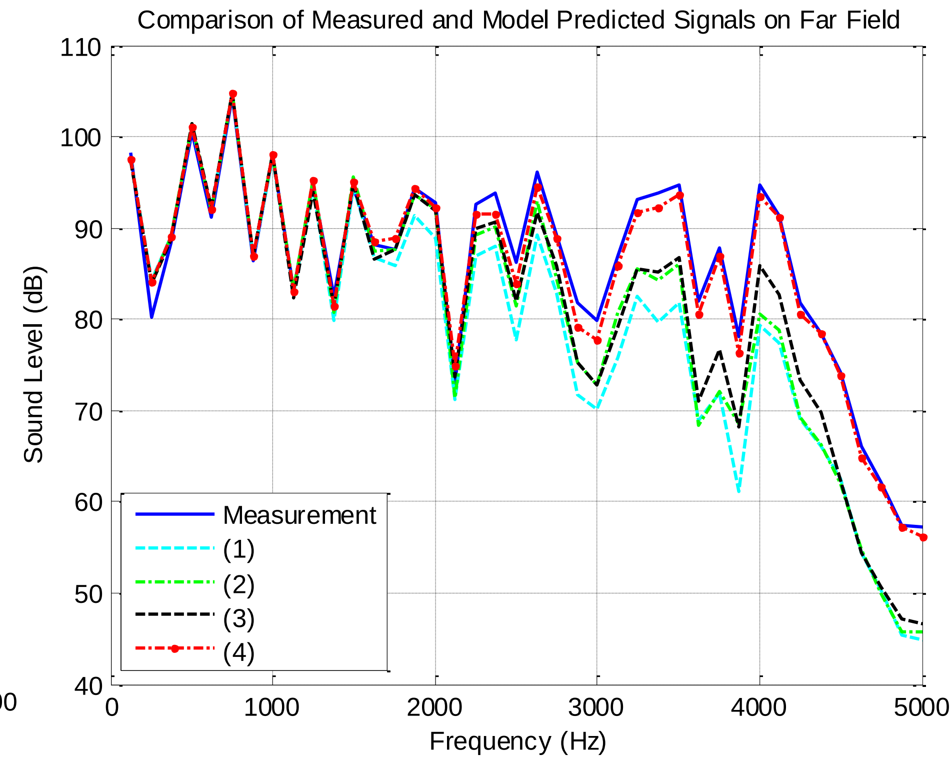
σ – Singular Value
$u^T b$ – Measurement Projected onto Singular vector

- Include all standard sources up to octupole.
- All sources have the same location but are allowed to move within the loudspeaker region.
- All frequencies have similar Picard Plots (sharp transition at the 16th singular value)

Experiment and Numerical Results

- Comparison of Different Degrees of Non-collocation





(1) All sources were fixed at the center of the loudspeaker

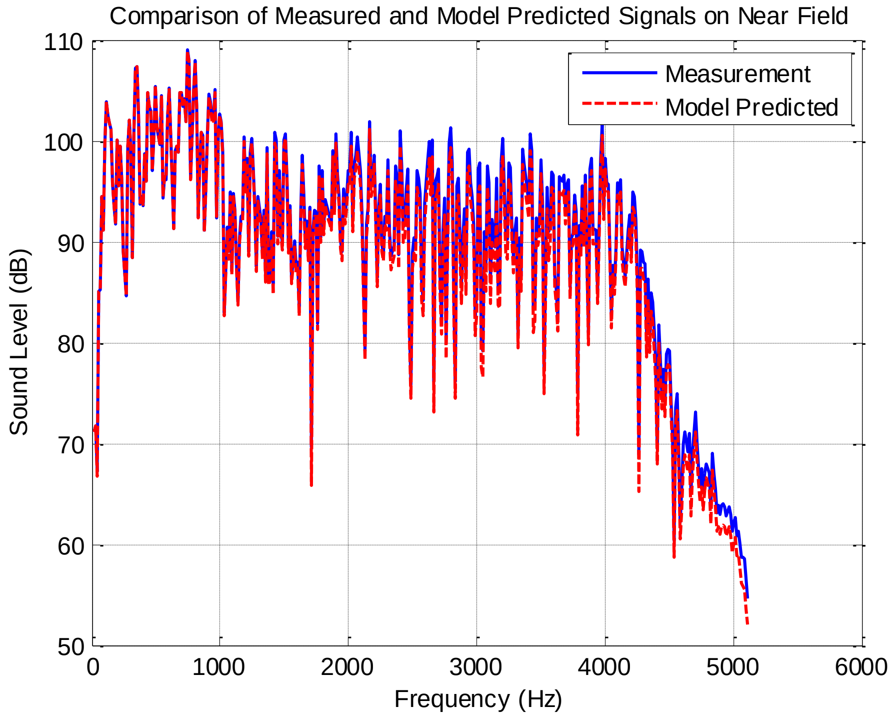
(2) Sources are collocated (Initial guess is the center)

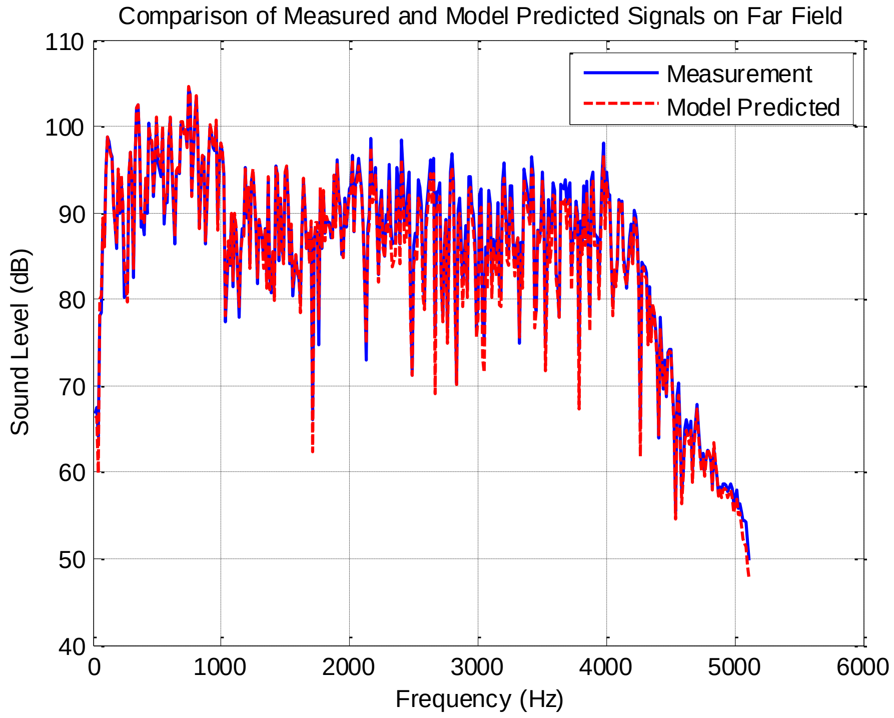
(3) Sources are collocated (Initial guess is updated from the previous frequency)

(4) Non-collocated sources can move in the front face of the loudspeaker (updated initial guess)

Experiment and Numerical Results

- Performance of the 2D Non-collocated Model

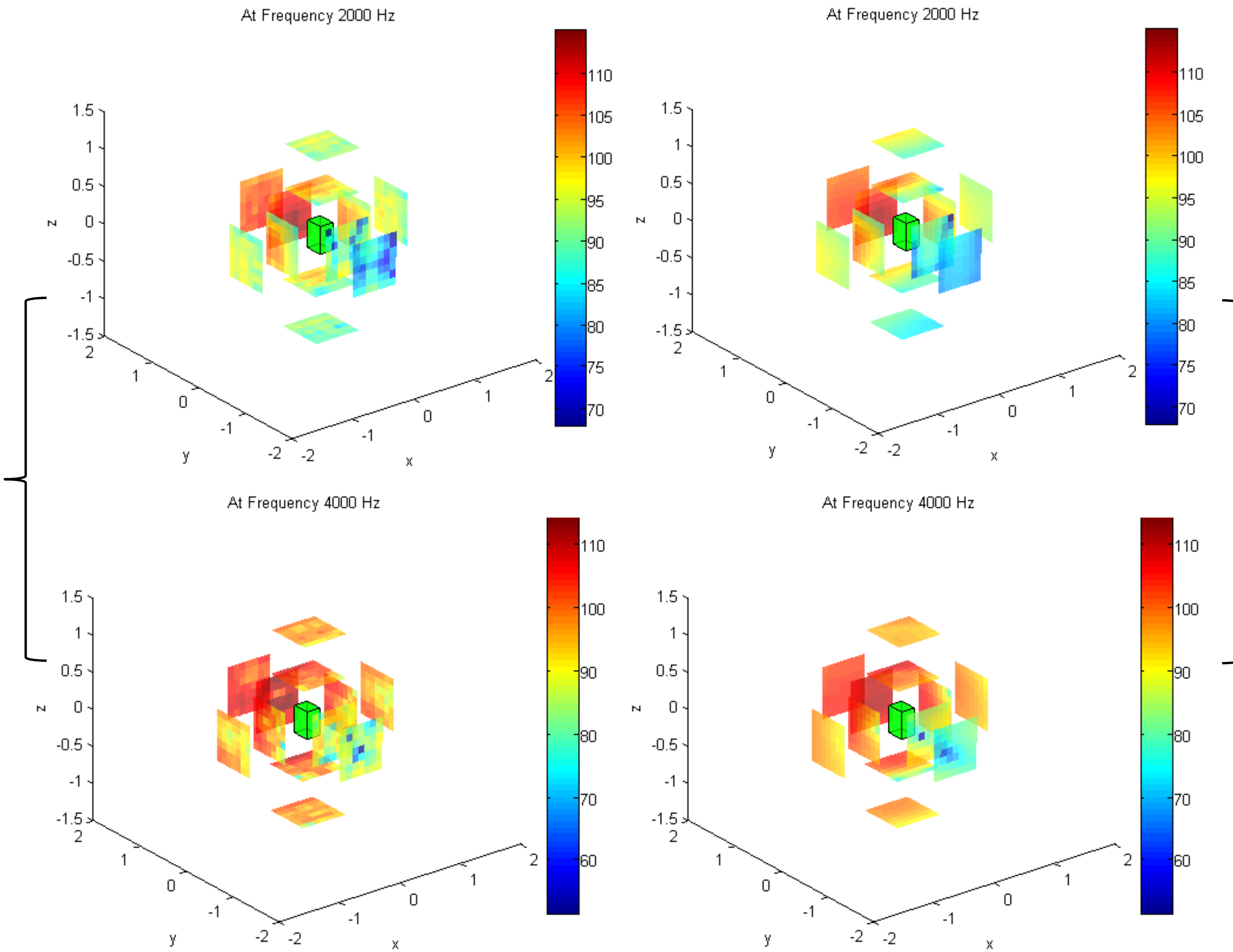




Experiment and Numerical Results

- Performance of the 2D Non-collocated Model

Measurement

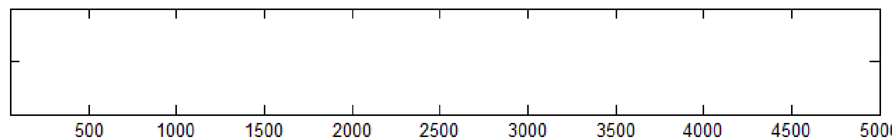


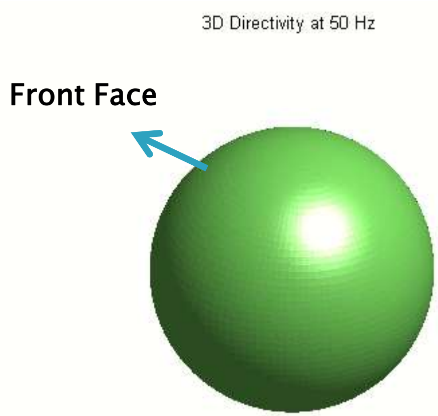
Prediction

Experiment and Numerical Results

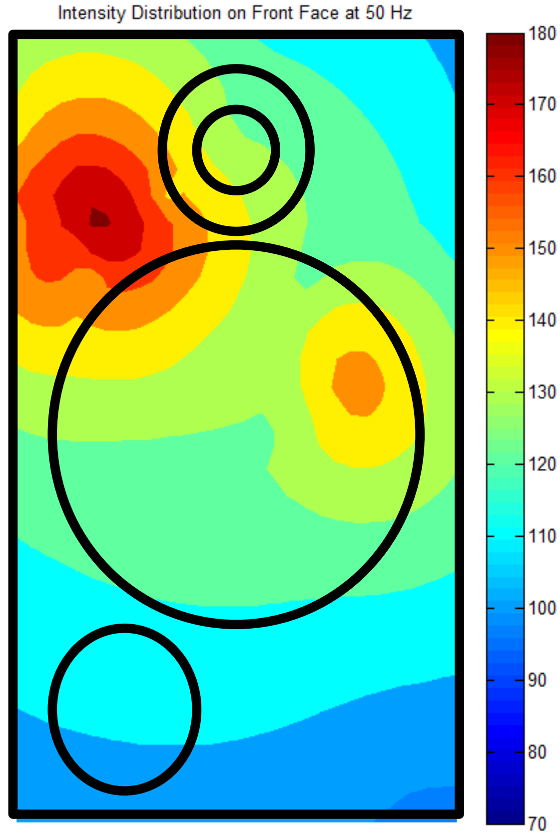
- Implementation of Non-collocated Model

Front Face

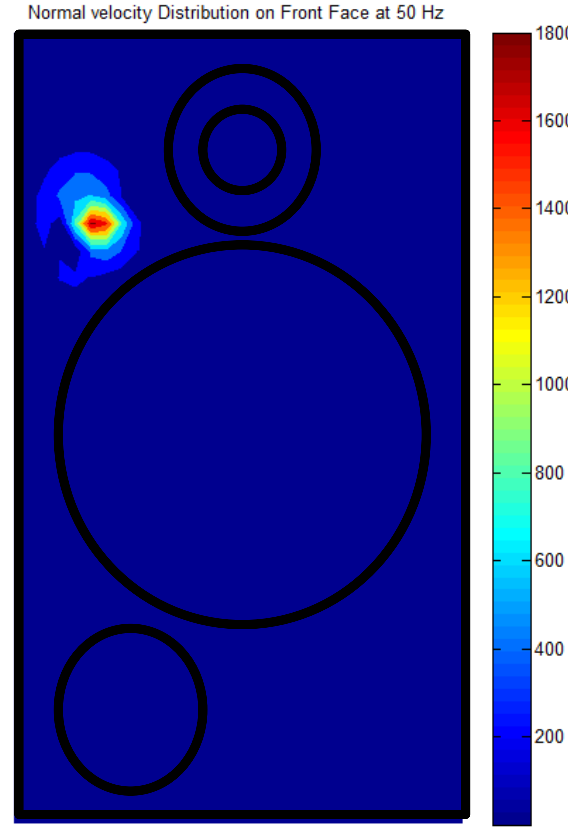




3D Directivity



Intensity Distribution



Normal Velocity Distribution

Conclusion

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1) A Equivalent Source Model using non-collocated higher order multipoles with unfixed source locations was proposed.
2) Source locations can be determined by a non-linear optimization approach.
3) The model was validated by a measurement of loudspeaker sound radiation in an anechoic environment (showing a good prediction up to at least 5000 Hz).
4) The model can be easily implemented to the prediction and visualization of sound intensity, normal velocity, directivity, etc.

Thank you