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Source Visualization by Using Statistically Optimized Near-Field Acoustical Holography in Conical Coordinates

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SOURCE VISUALIZATION BY USING STATISTICALLY OPTIMIZED NEAR-FIELD ACOUSTICAL HOLOGRAPHY IN CONICAL COORDINATES

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INTRODUCTION

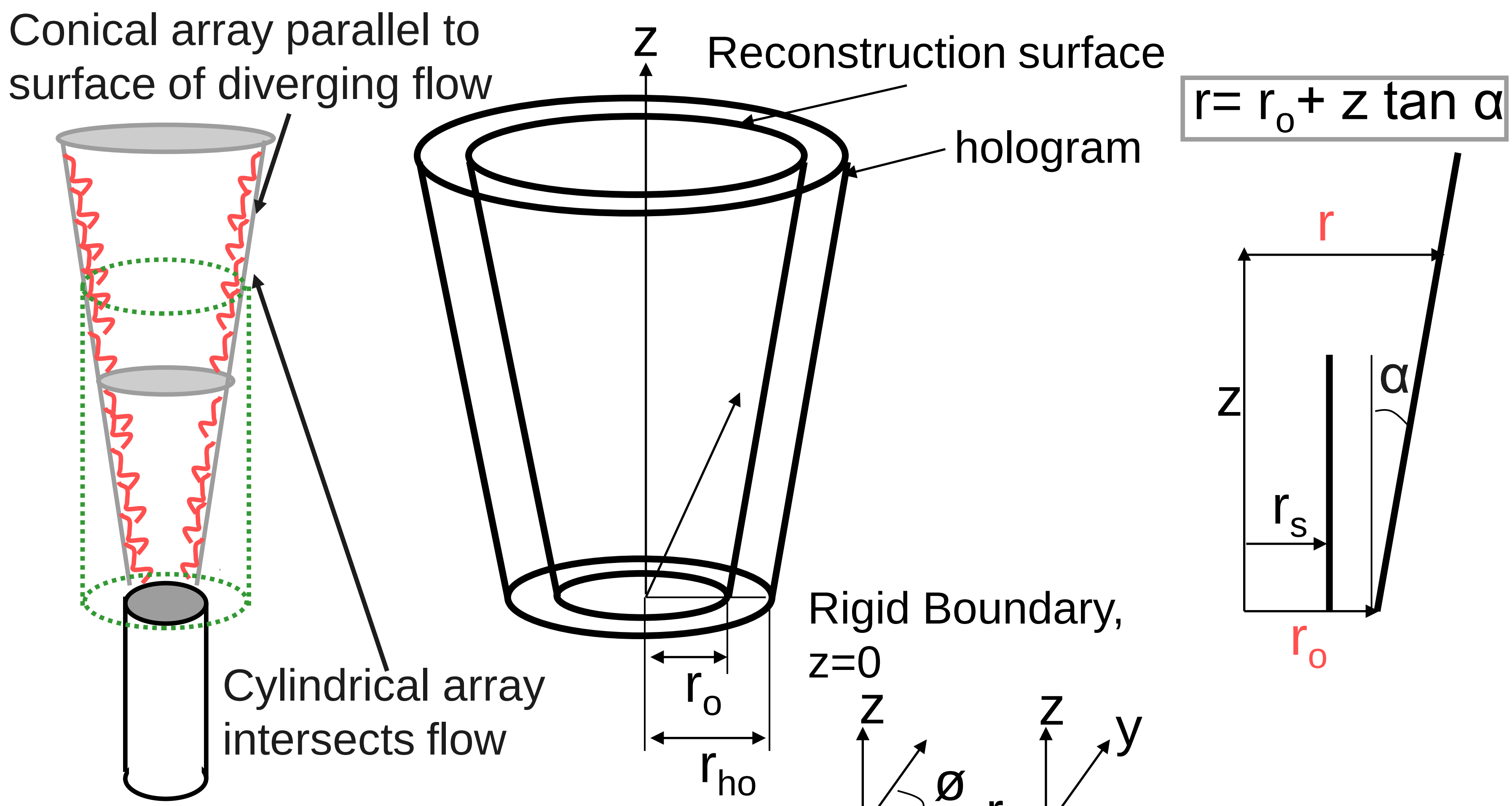
- NAH is a useful tool for visualizing noise sources throughout a 3D space.
  - Fast since implemented using spatial Fourier transform.
  - Needs zero padding of measurement results to avoid wrap-around error.
  - Meaningless velocity results close to measurement edge due to discontinuity.
- Statistically Optimized Nearfield Acoustical Hologrpahy
  - First introduced by Jørgen Hald in planar coordinates
  - No spatial Fourier transform involved.
  - More accurate result over entire measurement area.

SONAH in Conical Coordinates

- **Aeroacoustic sources are more closely defined in conical geometry**
- **NAH: non-regular geometries X**
  SONAH: conical geometry O
- **Wave functions in conical geometry are formulated by modifying cylindrical wave functions**

SONAH in Conical Coordinates

- Conical geometry definition



SONAH in Conical Coordinates

- Conical SONAH formulation (1)

- The sound pressure, $p(\mathbf{r})$, can be expressed as linear combination of the measured sound pressure $p(\mathbf{r}_\mathrm{n})$,

$$p(\mathrm{r}) \approx \sum_{n=1}^{N} c_n(\mathrm{r}) p(\mathrm{r}_\mathrm{n})$$

- If a good representation of the sound field can be obtained by using a finite subset of wave functions, the coefficients c_n can be determined.

$$\Phi_{\mathrm{Km}}(\mathrm{r}) \approx \sum_{n=1}^{N} c_n(\mathrm{r}) \Phi_{\mathrm{Km}}(\mathrm{r}_\mathrm{n}), \qquad m = 1 \ \ldots \ M$$

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SONAH in Conical Coordinates

- Conical SONAH formulation (2)

$$p(r,\phi,z) = \sum_{m=-\infty}^{m=\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{H_m^{(1)}(k_r r)}{H_m^{(1)}(k_r r_s)} P_m(r_s, k_z) e^{im\phi} e^{ik_z z} dk_z$$

- Defining wave function $\Phi_{kz,m}(\mathbf{r})$ in conical coordinates,

$$\Phi_{kz,m}(\mathbf{r}) \equiv \frac{H_m^{(1)}(k_r r)}{H_m^{(1)}(k_r r_s)} e^{im\phi} e^{ik_z z}, \qquad k_r = \begin{cases} \sqrt{k^2 - k_z^{\,2}} & \text{for } |k| \geq |k_z| \\ i\sqrt{k_z^{\,2} - k^2} & \text{for } |k| < |k_z| \end{cases}$$

$$\Phi_{k_z,m}(\mathbf{r}) = \Phi_{k_z,m}(r,\phi,z) \equiv \frac{H_m^{(1)}(k_r(r_o + z\tan\alpha))}{H_m^{(1)}(k_r r_s)} e^{im\phi} e^{ik_z z}, \quad r \geq r_s.$$

SONAH in Conical Coordinates

- Conical SONAH formulation (3)

$$\left[\mathbf{A}^{+}\mathbf{A}\right]_{ji} = \sum_{m=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \Phi^{*}_{k_{zq},m}(\mathbf{r}_{h,j})\Phi_{k_{zq},m}(\mathbf{r}_{h,i})$$

$$= \sum_{m=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \left\{ \frac{H_m^{*(1)}(k_r(r_{ho}+z_{h,j}\tan\alpha))H_m^{(1)}(k_r(r_{ho}+z_{h,i}\tan\alpha))}{\left|H_m^{(1)}(k_r r_s)\right|^2} e^{i\left(m\left(\phi_{h,i}-\phi_{h,j}\right)+k_{zq}\left(z_{h,i}-z_{h,j}\right)\right)} \right\}$$

$$\left[\mathbf{A}^{+}\boldsymbol{\alpha}\right]_{j} = \sum_{m=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \Phi^{*}_{k_{zq},m}(\mathbf{r}_{h,j})\Phi_{k_{zq},m}(\mathbf{r})$$

$$= \sum_{m=-\infty}^{\infty}\sum_{q=-\infty}^{\infty} \left\{ \frac{H_m^{*(1)}(k_r(r_{ho}+z_{h,j}\tan\alpha))H_m^{(1)}(k_r(r_o+z_i\tan\alpha))}{\left|H_m^{(1)}(k_r r_s)\right|^2} e^{i\left(m\left(\phi_i-\phi_{h,j}\right)+k_{zq}\left(z_i-z_{h,j}\right)\right)} \right\}$$

SONAH in Conical Coordinates

- Conical SONAH formulation (4)

- Estimated pressure $p(\mathbf{r})$ is,

$$p(\mathbf{r}) \approx \sum_{n=1}^{N} c_n(\mathbf{r})\, p\alpha(\mathbf{r}_n) = \mathbf{p}^T \mathbf{c}(\mathbf{r}) = \mathbf{p}^T \left(\mathbf{A}^+\mathbf{A} + \theta^2 \mathbf{I}\right)^{-1} \mathbf{A}^+ \; \mathbf{(\;)}$$

where, $\mathbf{p}^T$ is measured pressure vector at $\mathbf{r}_n$

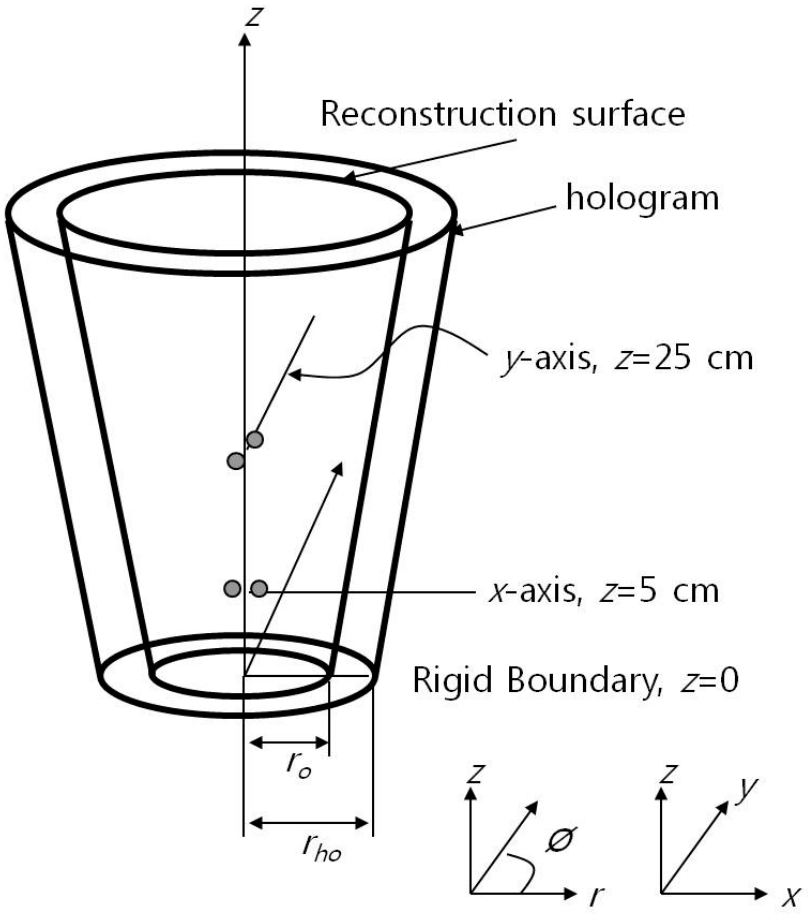
- Estimated radial particle velocity $u_r(\mathbf{r})$ is,

$$u_r(\mathbf{r}) \approx \mathbf{p}^T \beta\left(\mathbf{A}^+\mathbf{A} + \theta^2 \mathbf{I}\right)^{-1} \mathbf{A}^+ \; \mathbf{(\;)}$$

where, $\mathbf{A}^+\boldsymbol{\beta}(\mathbf{r})$ is a correlation vector that relates measured pressure and particle velocity.

SONAH in Conical Coordinates

- Dipole numerical simulation



r_{ho}= 14.15 cm
r_o = 9 cm
N_Φ = 32
N_z = 17
α = 15°

SONAH in Conical Coordinates

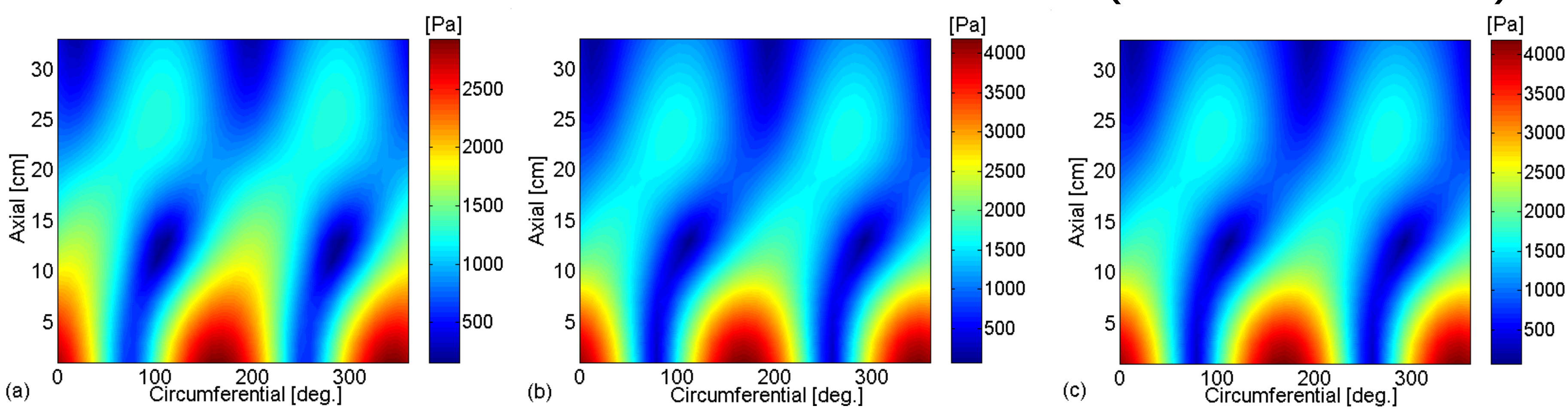
- Dipole numerical simulation (1000 Hz)

  Directly measured and backward projected pressure

Directly measured p, r_{ho}= 14.15 cm

Directly measured p, r_{ho}= 9 cm

Back projected p, r_o= 9 cm (MSE : 0.074 %)



SONAH in Conical Coordinates

- Dipole numerical simulation (1000 Hz)

Directly measured and backward projected particle velocity

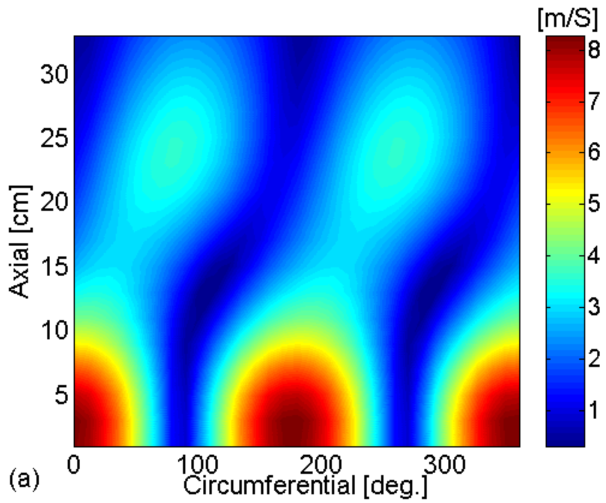
Directly measured u_r

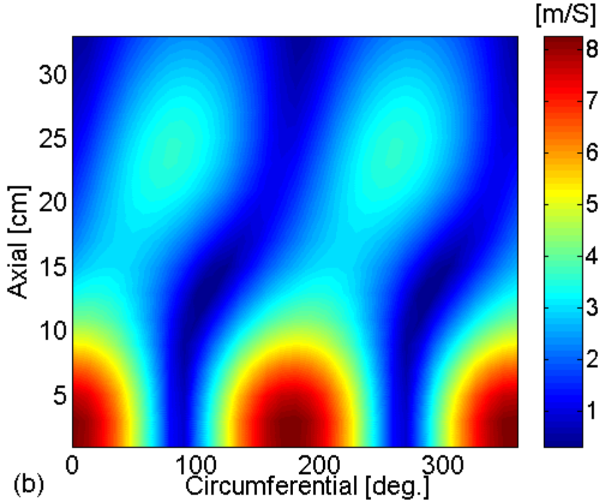
r_{ho}= 9 cm

Back projected u_r

r_o= 9 cm

(MSE : 0.17 %)



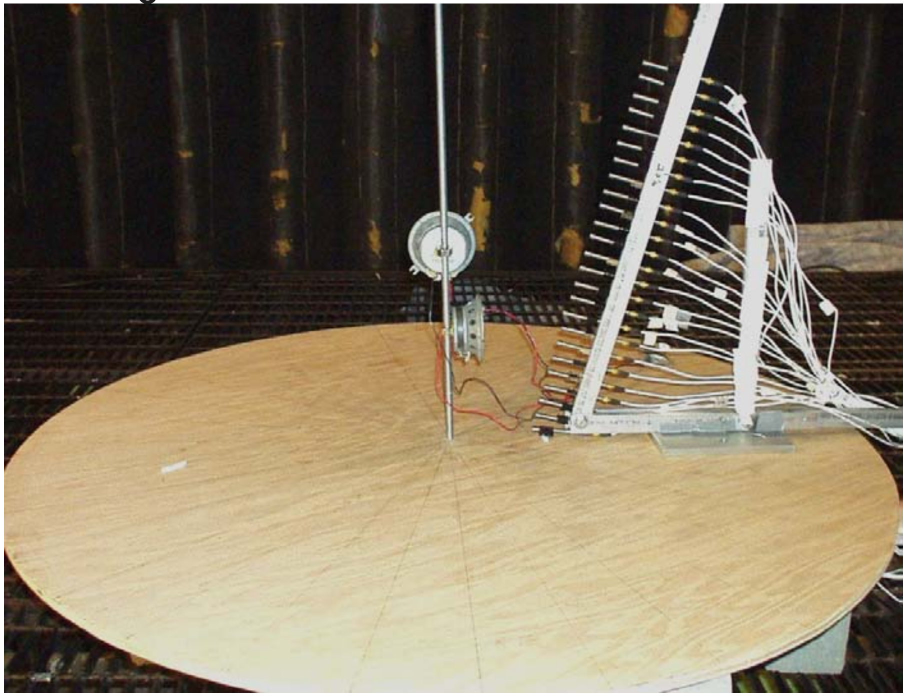


SONAH in Conical Coordinates

- Conical SONAH measurement

Microphone arrays and loudspeakers

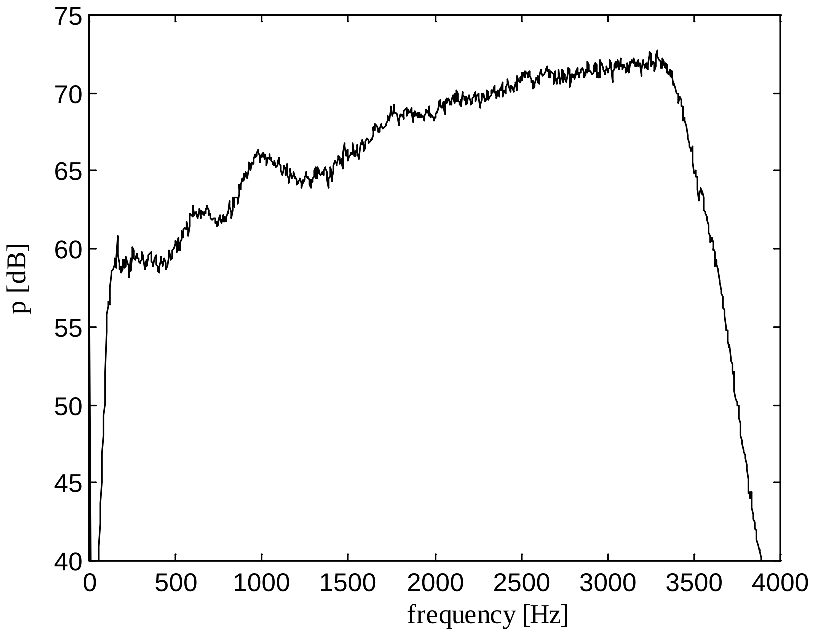
(r_o=10.6 cm, 15.6 cm, α =15°)



z_{inc} = 2 cos α cm, N_Φ = 32, N_z = 25

Spatially averaged pressure

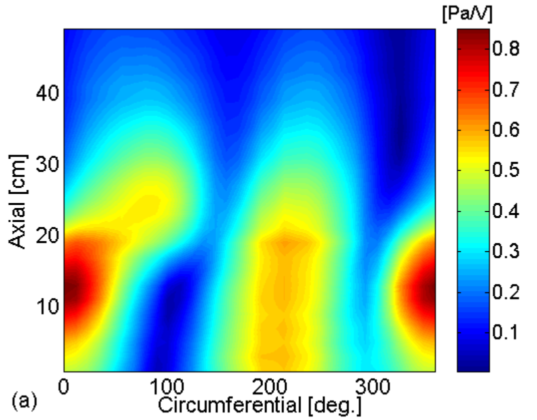
(r_o=10.6 cm)



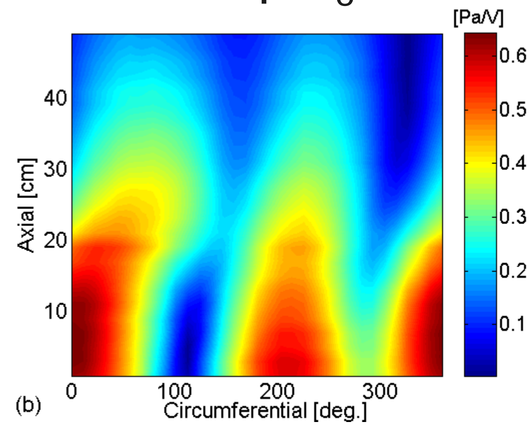
SONAH in Conical Coordinates

- Conical SONAH measurement result (1), 684 Hz

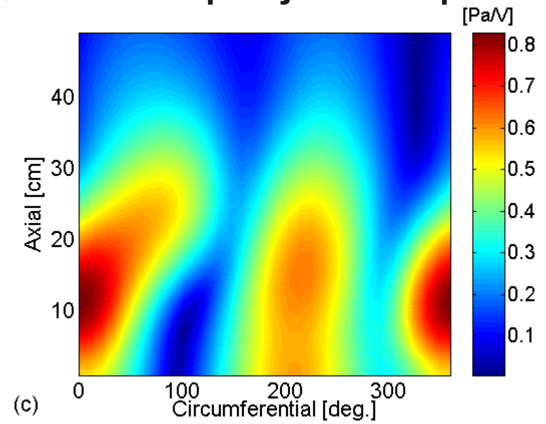
Measured p, r_o= 10.6 cm



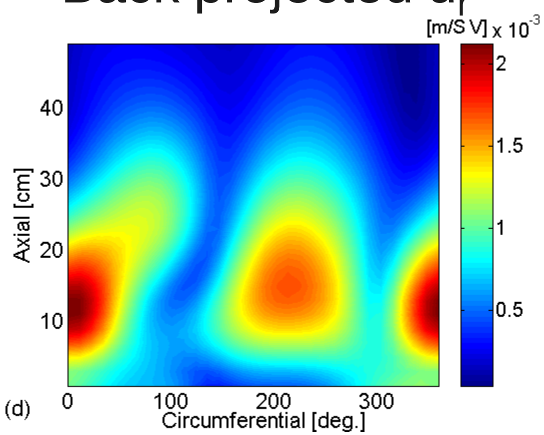
Measured p, r_o= 15.6 cm



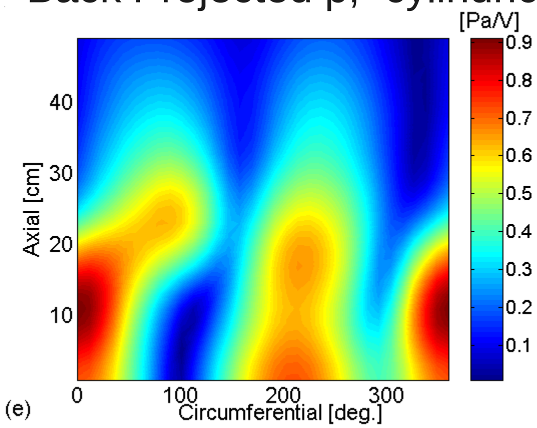
Back projected p



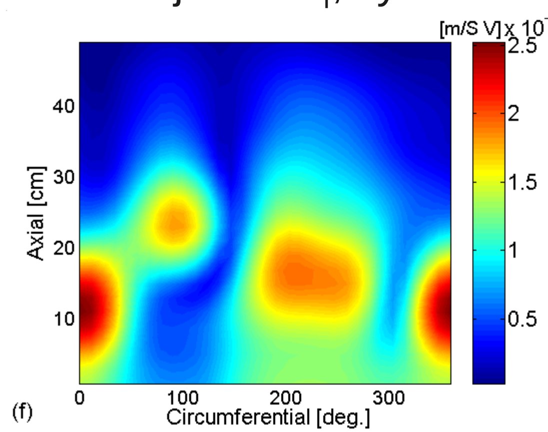
Back projected u_r



Back Projected p, cylindrical



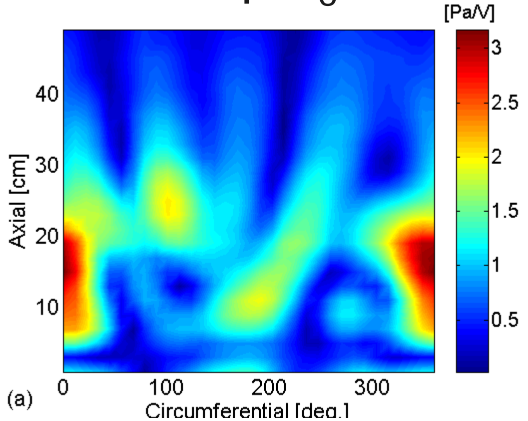
Back Projected u_r, cylindrical



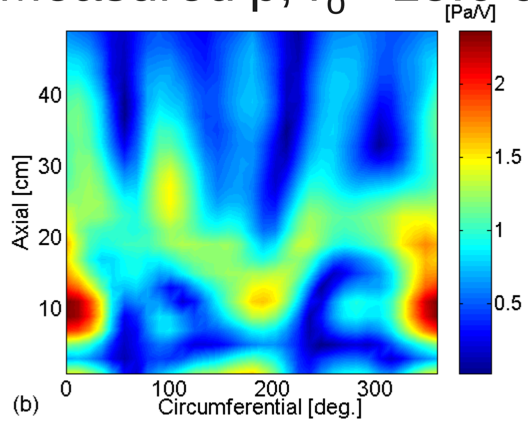
SONAH in Conical Coordinates

- Conical SONAH measurement result (2), 2648 Hz

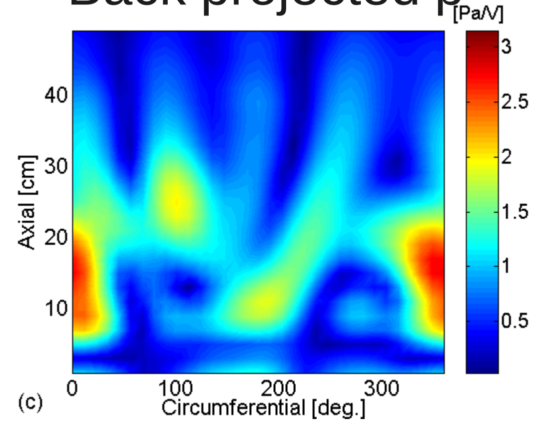
Measured p, r_o= 10.6 cm



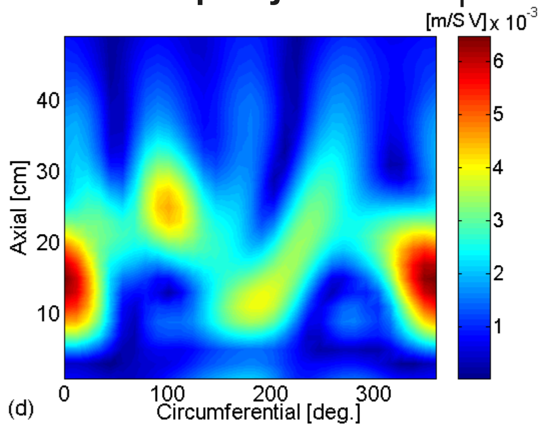
Measured p, r_o= 15.6 cm



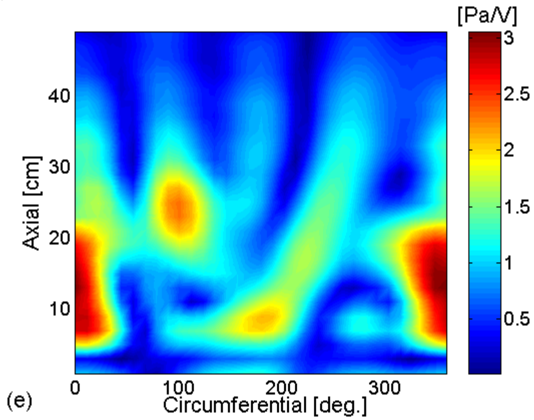
Back projected p



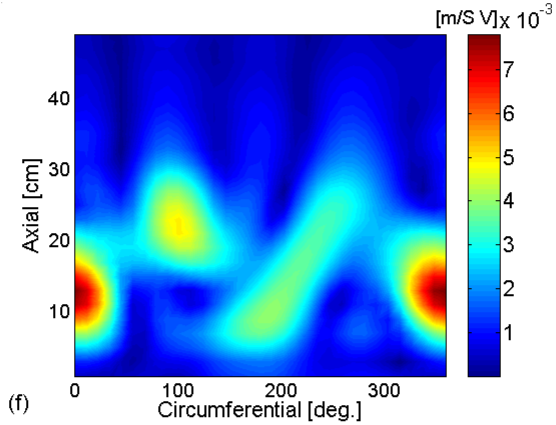
Back projected u_r



Back Projected p, cylindrical



Back Projected u_r, cylindrical



SONAH in Conical Coordinates

- Conical SONAH mean square error

- **Directly measured and back projected pressure**

  Two loudspeakers

  r_o=15.6 cm to 10.6 cm (α =15°)

Frequency (Hz)	684	976	1764	2128	2648	3288
MSE (%)	12.44	13.83	14.81	15.72	15.72	12.74

- **Directly measured and back projected pressure, velocity**

  Numerical dipole simulation

  r_o=14.15 cm to 9 cm (α =15°, 1000 Hz)

	Pressure	Velocity
MSE (%)	0.074	0.17

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SONAH in Conical Coordinates

- Computation effort required

Estimate $2N_{\varnothing}N_z$ by $2N_{\varnothing}N_z$ square matrices $\left[A^{+}A\right]_{ji}$, $\left[A^{+}\right]_{j}$

number of elements in SONAH matrices

Hologram	Square	Cylindrical	Conical	Arbitrary
Elements	$N_{\varnothing}N_z$	$2N_{\varnothing}N_z$	$4N_{\varnothing}N_z^2$	$4N_{\varnothing}^2N_z^2$
$N_{\varnothing}$=32, N_z=17	544*	1088	36992	1183744
$N_{\varnothing}$=32, N_z=25	800*	1600	80000	2560000
$N_{\varnothing}$=32, N_z=34	1088* (0.5)	2176 (1)	147968 (68)	4734976 (2176)

SONAH in Conical Coordinates

- Conclusions

- **Conical SONAH accuracy confirmed**
  Numerical simulation and loudspeaker measurement
- **Reasonable to use cylindrical wave functions for conical geometry**
- **More detailed visualization of sources**
  by back projection from conical to cylindrical surfaces
- **Cylindrical and conical SONAH matrix**
  quite different computation time
  conical SONAH >> cylindrical SONAH
- **Finite difference calculation would be required to calculate particle velocity normal to conical surface (and hence intensity and sound power)**