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Absorption of Finite-Sized Microperforated Panels with Finite Flexural Stiffness at Normal Incidence

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Absorption of finite-sized microperforated panels with finite flexural stiffness at normal incidence

Taewook Yoo

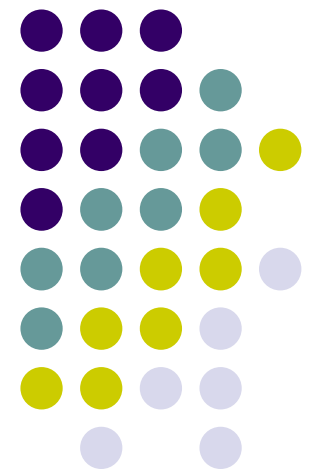
J. Stuart Bolton

Ray W. Herrick Laboratories

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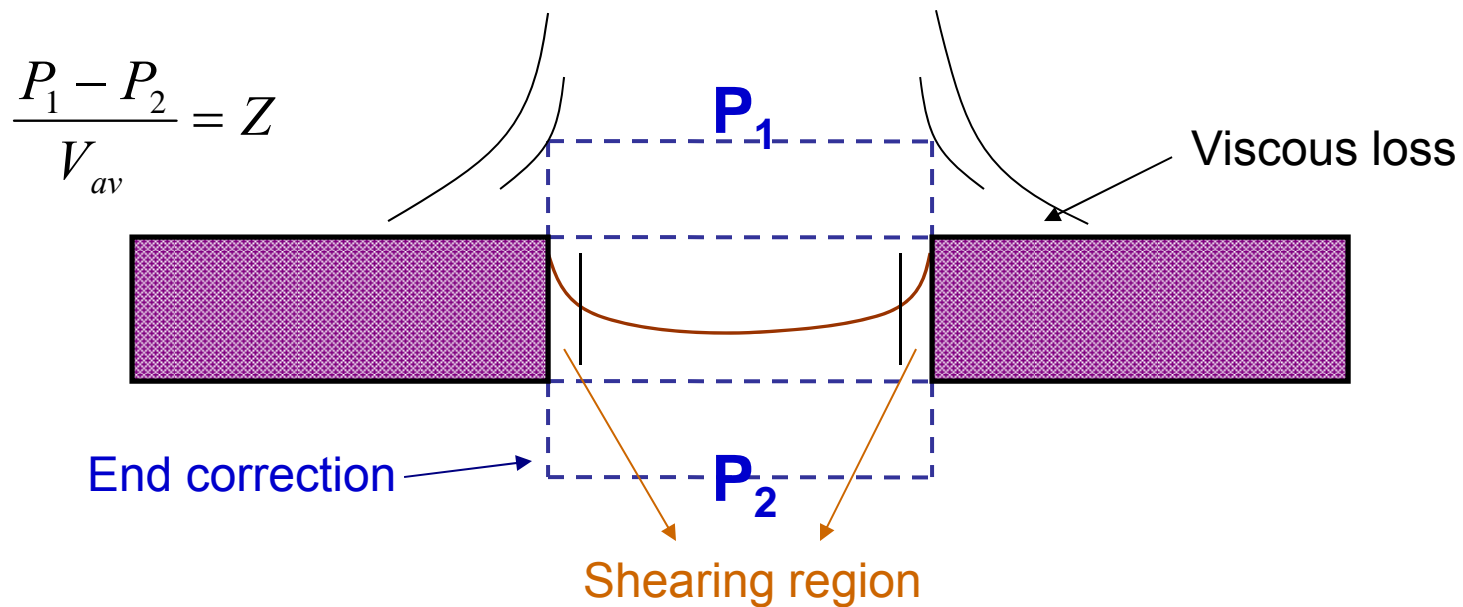
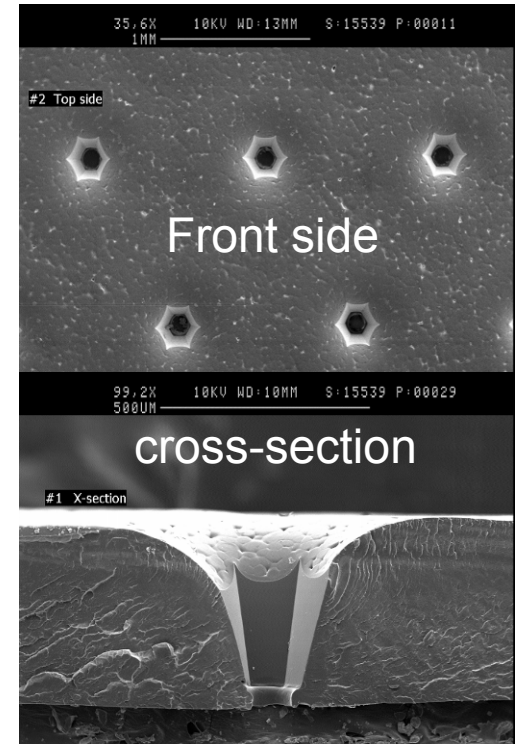
3M Center



28 July 2008

Micro-perforated panel

- Advantages over fibrous material
 - stiffness (self supporting), robust, weatherproof
- Energy dissipates when sound moves through small holes
 - $d=0.1\sim 0.9$ mm, $t=0.1\sim 2$ mm, $N=10^3\sim 10^6$ per m^2





Objectives

- Build models to predict the performance of micro-perforated samples in various conditions and validate the models with measurements
 - Finite-sized sample in duct with normal incidence plane wave
 - Two boundary conditions
 - Clamped-Clamped-Clamped-Clamped
 - S-S-S-S
 - Mode shapes
 - Energy dissipation



Maa model

The panel is assumed to be **rigid**

- Maa model:

Perforation constant : $k = r_0 \sqrt{\frac{\rho_0 \omega}{\eta}} = d \sqrt{\frac{\rho_0 \omega}{4\eta}}$

Transfer Impedance

$$z_t = \frac{Z_1}{\Omega_s \rho_0 c} = r + jx_m = r + j\omega m$$

$$r = \frac{32\eta t}{\sigma \rho_0 c d^2} k_r = \frac{32\eta t}{\sigma \rho_0 c d^2} \left(\sqrt{1 + \frac{x^2}{32}} + \sqrt{\frac{2kd}{8t}} \right)$$

$$\omega m = \frac{\omega t}{\Omega_s c} k_m = \frac{\omega t}{\Omega_s c} \left\{ 1 + \frac{1}{\sqrt{9 + \frac{k^2}{2}}} + 0.85 \frac{d}{t} \right\}$$

d : diameter of hole

η : viscosity of air (17.9e-6)

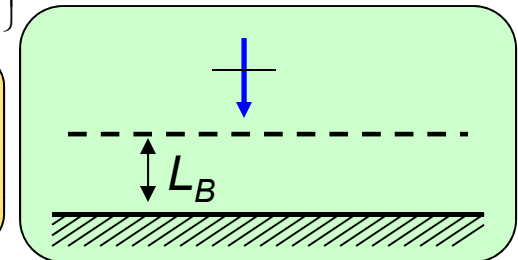
σ : porosity

t : hole depth

L_B : depth of backing space

Normal incidence
Absorption coefficient

$$\alpha_n = \frac{4r}{(1+r)^2 + (\omega m - \cot(\frac{\omega L_B}{c}))^2}$$



Flow resistance (R_t): $\frac{32\eta t}{\sigma d^2} \left(\sqrt{1 + \frac{x^2}{32}} + \sqrt{\frac{2kd}{8t}} \right)$,

End correction (δ): $\delta = \frac{1}{2} \left\{ \frac{t}{\sqrt{9 + \frac{k^2}{2}}} + 0.85d \right\}$

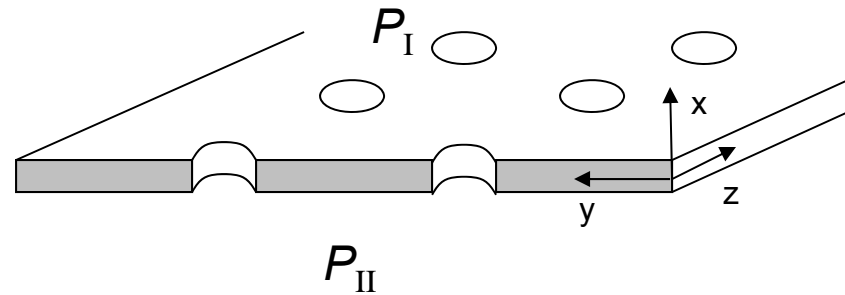
R_t and δ are used in finite-sized models

Equations of motion and velocity conditions



Volume velocity continuity at $x=0$

$$\begin{cases} -\frac{1}{j\omega\rho_o} \frac{\partial p_I}{\partial x} \Big|_{x=0} = (1-\Omega) \frac{\partial d_s}{\partial t} + \Omega \frac{\partial d_f}{\partial t} \\ -\frac{1}{j\omega\rho_o} \frac{\partial p_{II}}{\partial x} \Big|_{x=0} = (1-\Omega) \frac{\partial d_s}{\partial t} + \Omega \frac{\partial d_f}{\partial t} \end{cases}$$



Force equilibrium at $x=0$

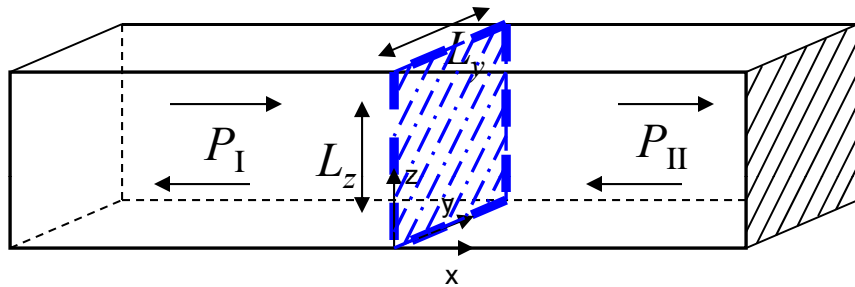
$$\begin{cases} \text{Solid} & p_I - p_{II} + R_f \frac{\Omega^2}{1-\Omega} \frac{\partial(d_f - d_s)}{\partial t} = D\nabla^4 d_s - T\nabla^2 d_s + \rho_s \frac{\partial^2 d_s}{\partial t^2} \\ \text{Fluid} & p_I - p_{II} - R_f \Omega \frac{\partial(d_f - d_s)}{\partial t} = j\omega\rho_o h' \frac{\partial d_f}{\partial t} \end{cases}$$

- p_I : Pressure at source side
- p_{II} : Pressure behind the panel
- d_s : Displacement of solid part
- d_f : Displacement of fluid part
- ρ_s : Membrane mass per unit area
- R_f : Flow resistance
- D : Flexural stiffness
- T : Tension
- h' : Effective thickness
- Ω : Porosity



Multi-mode solution in duct and membrane

- 3-dimensional model



Only symmetric modes exist

Sound pressure in each region

$$P_I = e^{-jkx} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B_{mn} \cos(k_{2m}z) \cos(k_{2n}y) e^{jk_{x2m2n}x}$$

$$P_{II} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_{mn} \cos(k_{2m}z) \cos(k_{2n}y) \left(e^{-jk_{x2mn}x} + e^{jk_{x2mn}(x-2L)} \right)$$

$$k_{2m} = \frac{2m\pi}{L_z} \quad k_{2n} = \frac{2n\pi}{L_y} \quad k_{x2m2n} = \begin{cases} \sqrt{k^2 - k_{2m}^2 - k_{2n}^2} & (k > k_{2n} + k_{2m}) \\ -j\sqrt{k^2 - k_{2m}^2 - k_{2n}^2} & (k < k_{2n} + k_{2m}) \end{cases}$$

($m, n = 0, 1, 2, \dots$)

Displacement of membrane

for simply supported BC

Solid part $d_s = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin(k_{2m-1}z) \sin(k_{2n-1}y)$

Fluid part $d_f = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} F_{mn} \cos(k_{2m}z) \cos(k_{2n}y)$

for clamped BC

Solid part $d_s = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \{\cos(k_{2m}z) - 1\} \{\cos(k_{2n}y) - 1\}$

Fluid part $d_f = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} F_{mn} \cos(k_{2m}z) \cos(k_{2n}y)$

$$k_{2m-1} = \frac{(2m-1)\pi}{L_z}, \quad k_{2n-1} = \frac{(2n-1)\pi}{L_y} \quad (m, n = 1, 2, \dots)$$

(Modeling) Application of boundary conditions



$$\textcircled{1} \quad -\frac{1}{j\omega\rho_0} \left. \frac{\partial p_I}{\partial x} \right|_{x=0} = (1-\Omega) \frac{\partial d_s}{\partial t} + \Omega \frac{\partial d_f}{\partial t}$$

$$\rightarrow j(1-\Omega)\omega^2 \rho_0 \sum \sum A_{mn} \sin(k_{2m-1}z_1) \sin(k_{2n-1}y_1) + j\Omega\omega^2 \rho_0 \sum \sum F_{mn} \cos(k_{2m}z_1) \cos(k_{2n}y_1) + \sum \sum B_{mn} k_{x_{2m2n}} \cos(k_{2m}z_1) \cos(k_{2n}y_1) = k$$

$$\textcircled{2} \quad -\frac{1}{j\omega\rho_0} \left. \frac{\partial p_{II}}{\partial x} \right|_{x=0} = (1-\Omega) \frac{\partial d_s}{\partial t} + \Omega \frac{\partial d_f}{\partial t}$$

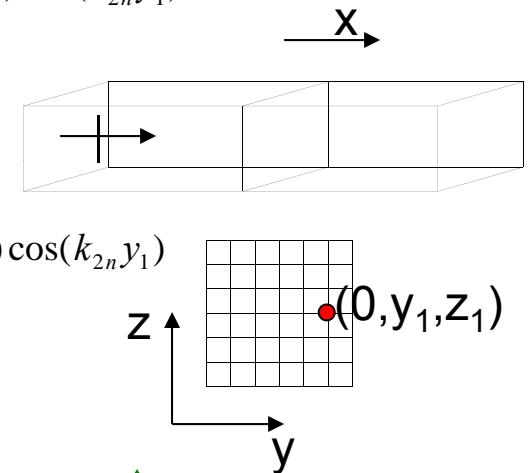
$$\rightarrow j(1-\Omega)\omega^2 \rho_0 \sum \sum A_{mn} \sin(k_{2m-1}z_1) \sin(k_{2n-1}y_1) + j\Omega\omega^2 \rho_0 \sum \sum F_{mn} \cos(k_{2m}z_1) \cos(k_{2n}y_1) - \sum \sum C_{mn} k_{x_{2m2n}} \cos(k_{2m}z_1) \cos(k_{2n}y_1) = 0$$

$$\textcircled{3} \quad p_I - p_{II} + R_t \frac{\Omega^2}{1-\Omega} \frac{\partial (d_f - d_s)}{\partial t} = D\nabla^4 d_s - T\nabla^2 d_s + \rho_s \frac{\partial^2 d_s}{\partial t^2}$$

$$\rightarrow \sum \sum A_{mn} \sin(k_{2m}z_1) \sin(k_{2n}y_1) \left\{ D(k_{2m-1}^2 + k_{2n-1}^2)^2 + T(k_{2m-1}^2 + k_{2n-1}^2) + j\omega R_f \frac{\Omega^2}{1-\Omega} - \omega^2 \rho_s \right\} - \sum \sum B_{mn} \cos(k_{2m}z_1) \cos(k_{2n}y_1) + \sum \sum C_{mn} \cos(k_{2m}z_1) \cos(k_{2n}y_1) - R_f \frac{\Omega^2}{1-\Omega} (j\omega) \sum \sum F_{mn} \cos(k_{2m}z_1) \cos(k_{2n}y_1) = 1$$

$$\textcircled{4} \quad p_I - p_{II} - R_t \Omega \frac{\partial (d_f - d_s)}{\partial t} = j\omega\rho_0 h' \frac{\partial d_f}{\partial t}$$

$$\rightarrow j\omega\Omega R_f \sum \sum A_{mn} \sin(k_{2m-1}z_1) \sin(k_{2n-1}y_1) + \sum \sum F_{mn} \cos(k_{2m}z_1) \cos(k_{2n}y_1) (\omega^2 \rho_0 h' - R_f \Omega j\omega) + \sum \sum B_{mn} \cos(k_{2m}z_1) \cos(k_{2n}y_1) - \sum \sum C_{mn} \cos(k_{2m}z_1) \cos(k_{2n}y_1) = -1$$





Building a matrix from BC's

$$X \times \begin{bmatrix} A_{mn} \\ F_{mn} \\ B_{mn} \\ C_{mn} \end{bmatrix} = Y \quad \longrightarrow \quad \begin{bmatrix} A_{mn} \\ F_{mn} \\ B_{mn} \\ C_{mn} \end{bmatrix} = X^{-1} \times Y$$

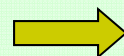
Solve for B_{00} and C_{00}

B_{00} : reflection coefficient

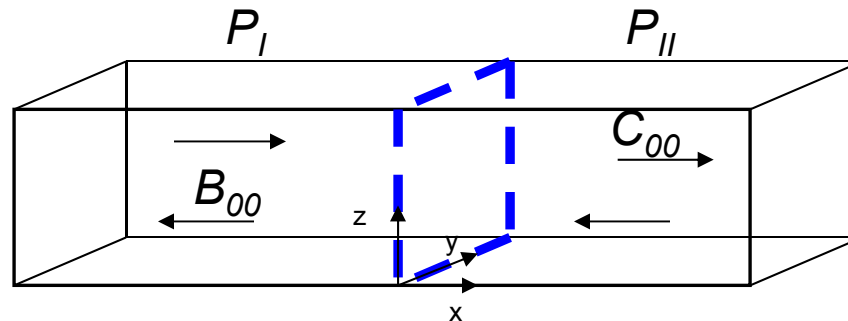


$$\alpha_n = 1 - |B_{00}|^2$$

C_{00} : transmission coefficient

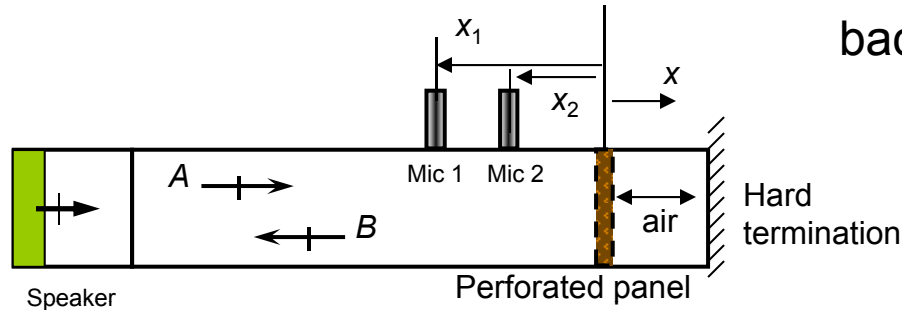


$$TL = 20 \log_{10} \frac{1}{|C_{00}|}$$



B_{00} and C_{00} are amplitudes of the plane waves

Absorption coefficient (ASTM E1050)



Using finite air
backing space

Normal Incidence
Absorption coefficient

1. Sound pressures

$$P_1 = (Ae^{-jkx_1} + Be^{jkx_1})e^{j\omega t}$$

$$P_2 = (Ae^{-jkx_2} + Be^{jkx_2})e^{j\omega t}$$

2. Measuring transfer function

$$H_{21} = \frac{Ae^{-jkx_2} + Be^{jkx_2}}{Ae^{-jkx_1} + Be^{jkx_1}}$$

$$H_{21} = \frac{e^{-jkx_2} + Re^{jkx_2}}{e^{-jkx_1} + Re^{jkx_1}}$$

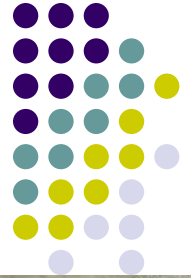
3. Solve for R

$$R = \frac{-H_{21}e^{-jkx_1} + e^{-jkx_2}}{H_{21}e^{jkx_1} - e^{jkx_2}}$$

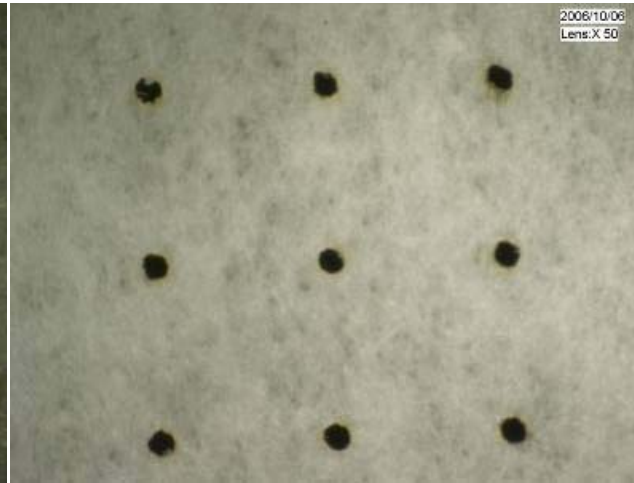
4. Absorption coefficient

$$\alpha = 1 - |R|^2$$

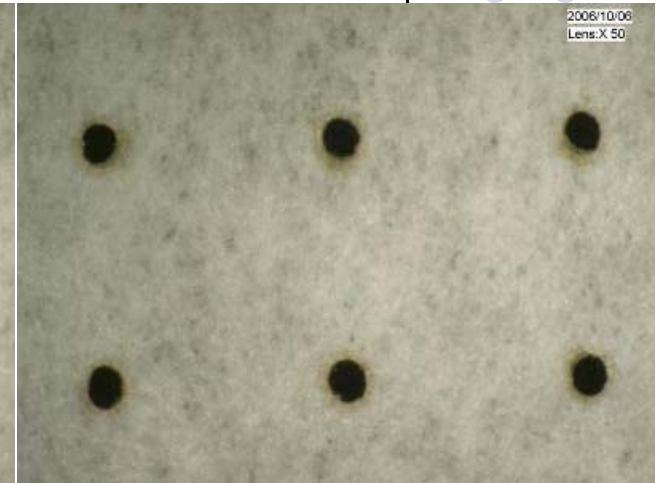
Measurement of Plexiglass samples



Sample 1 (x100)

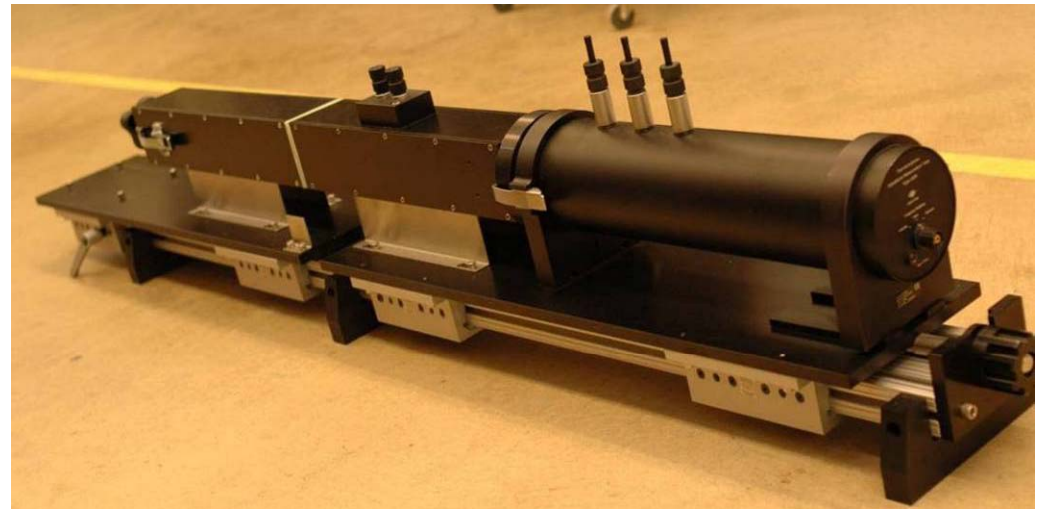


Sample 2 (x50)

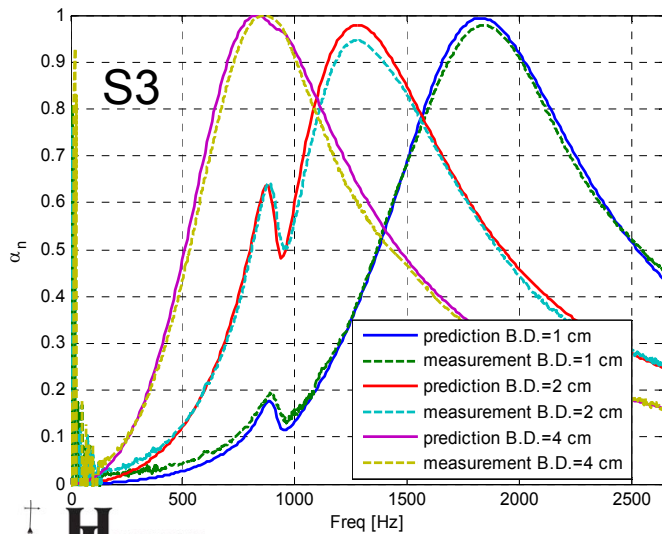
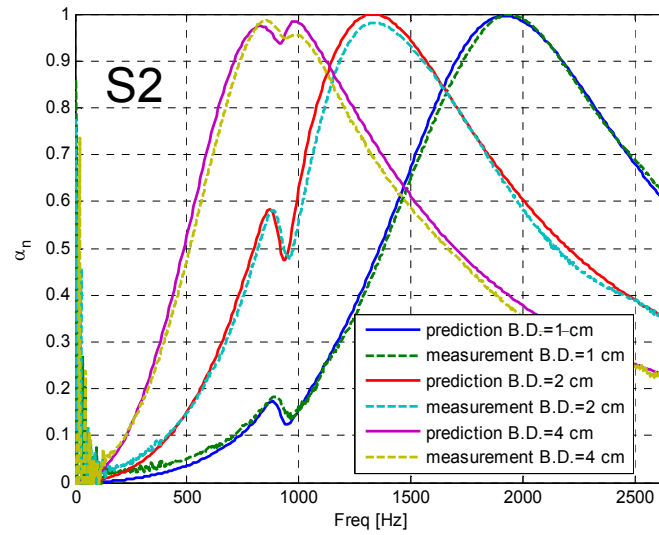
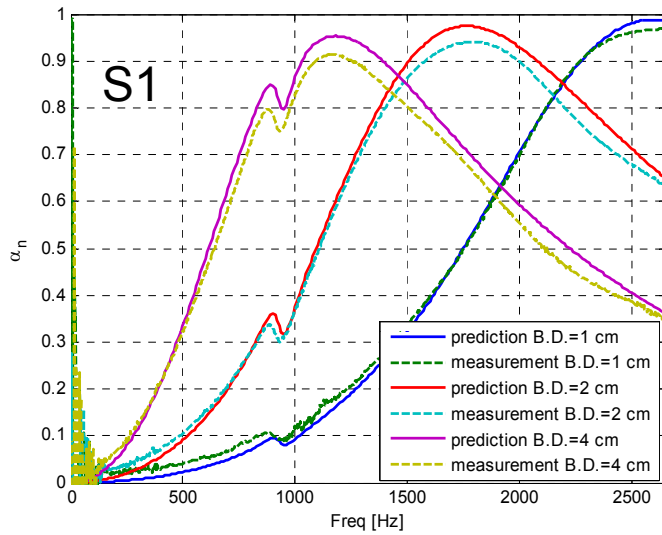


Sample 3 (x50)

	d (nominal) [mm]	t [mm]	ρ_s [kg/m ²]	N
Sample 1	0.254	1.588	1.584	722500
Sample 2	0.2667	1.588	1.627	291600
Sample 3	0.4064	1.588	1.631	160000



Comparison between predictions and measurements

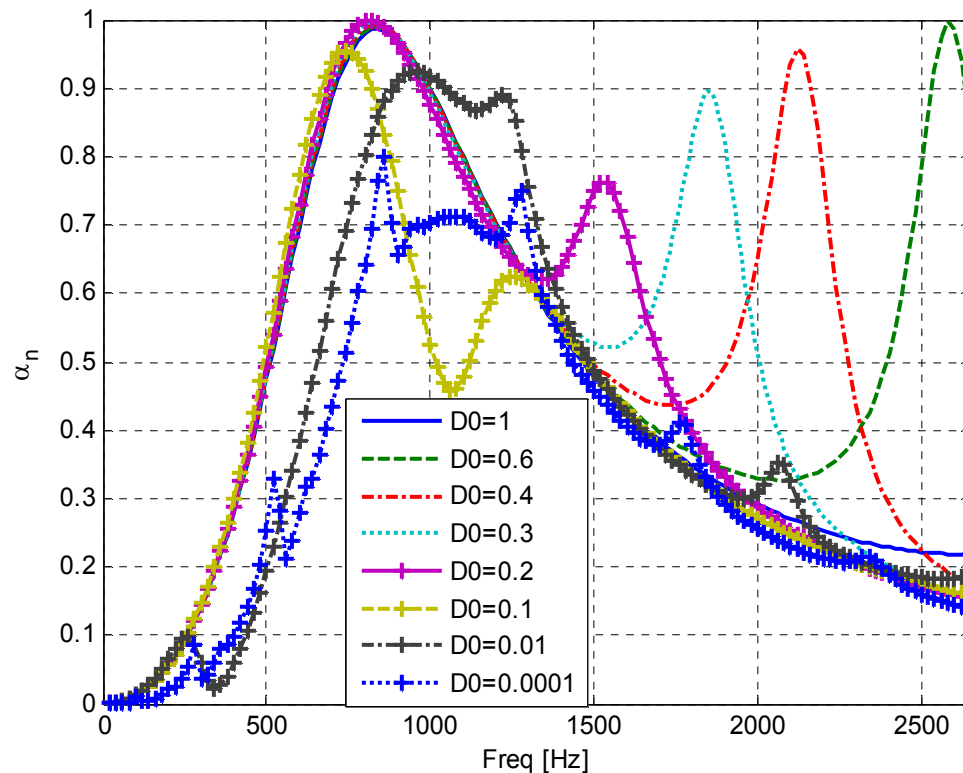


Good agreement is shown between measurements and predictions by using adjusted hole size

	d (nominal) [mm]	d (adjusted) [mm]	t [mm]	$D /$ loss factor [N·m ²]	T [N]	ρ_s [kg/m ²]	N
S 1	0.254	0.305	1.588	0.7/ 0.07	0	1.584	722500
S 2	0.2667	0.35	1.588	0.7/ 0.07	0	1.627	291600
S 3	0.4064	0.45	1.588	0.7/ 0.07	0	1.631	160000



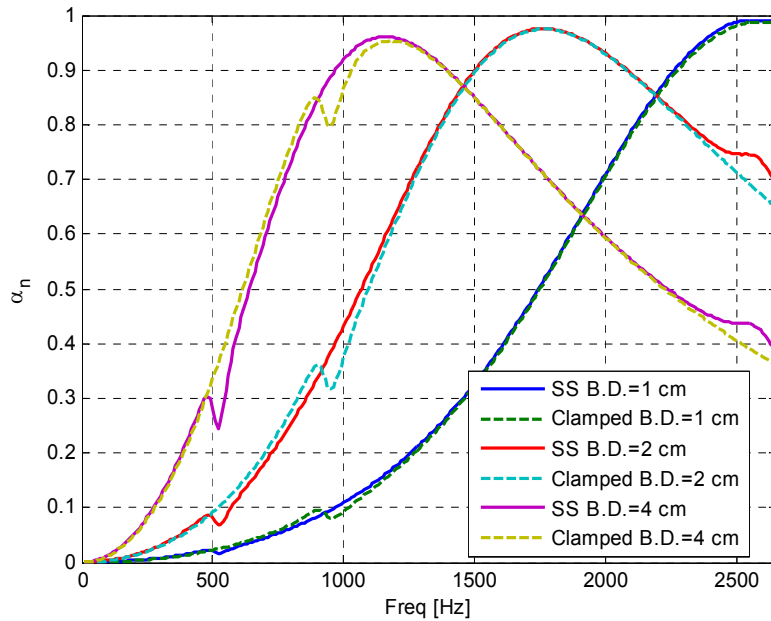
Flexural stiffness effect



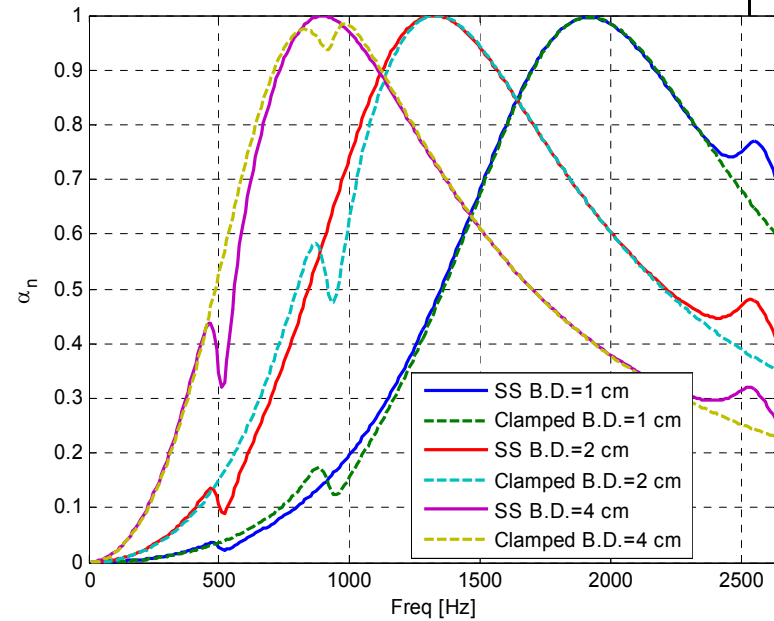
Depending on the flexural stiffness, the absorption performance can be enhanced with a proper loss factor

d [mm]	t [mm]	D [N·m ²]	loss factor in D	T [N]	Mass/area [kg/m ²]	N	Size [mm]
0.45	1.588	1, 0.6, 0.4, 0.3, 0.2, 0.1, 0.01, 0.0001	0.05	0	0.1631	160000	63.5 x 63.5

Boundary condition effect



Plexiglass Sample 1

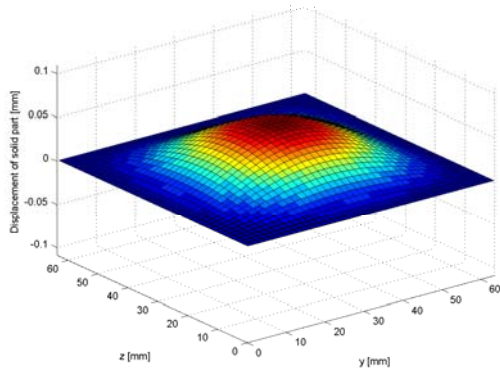
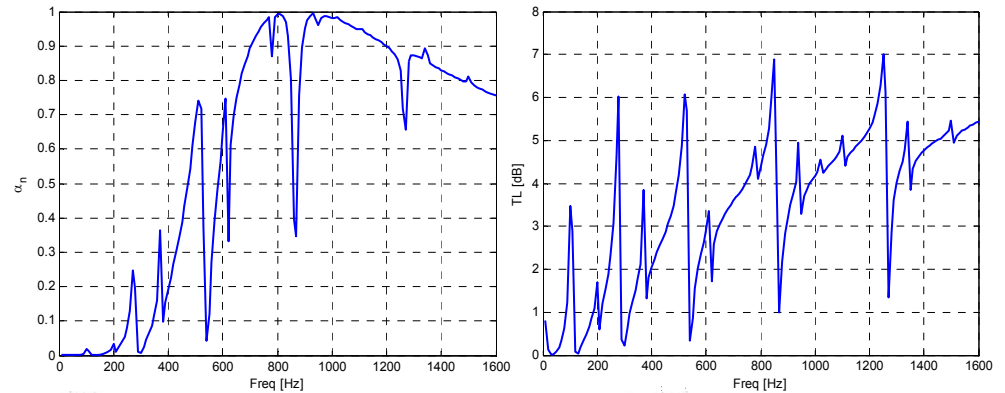


Plexiglass Sample 2

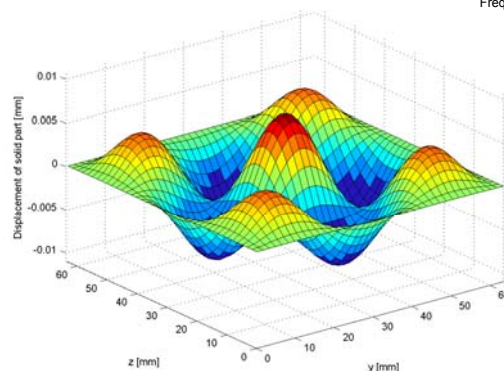
Although same material properties were used, very different flexural natural frequency is shown depending on the boundary condition.

Simply supported boundary condition has lower natural frequency

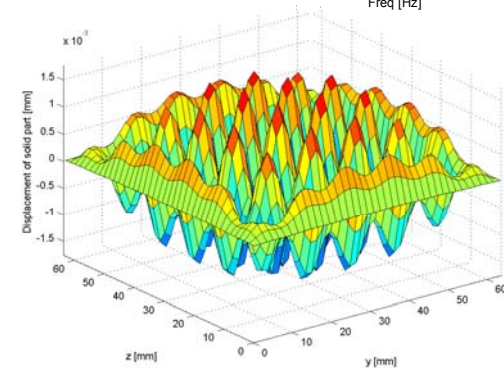
3D Clamped-Clamped Displacements of fluid and solid parts



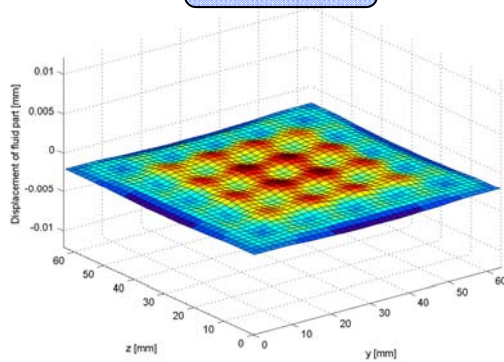
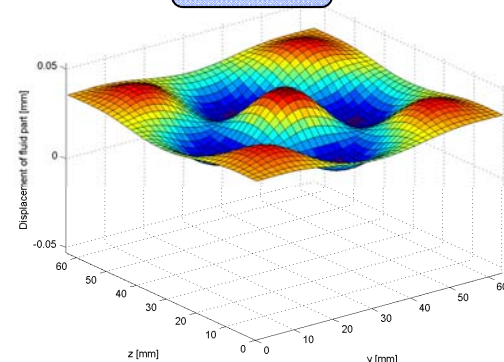
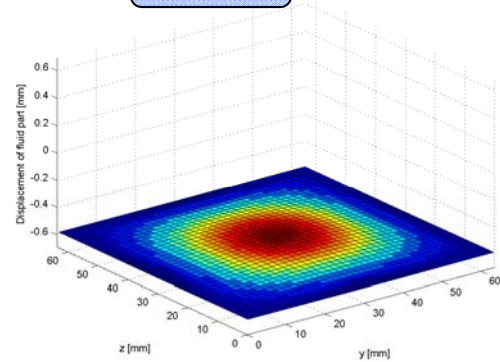
10 Hz



200 Hz



1250 Hz

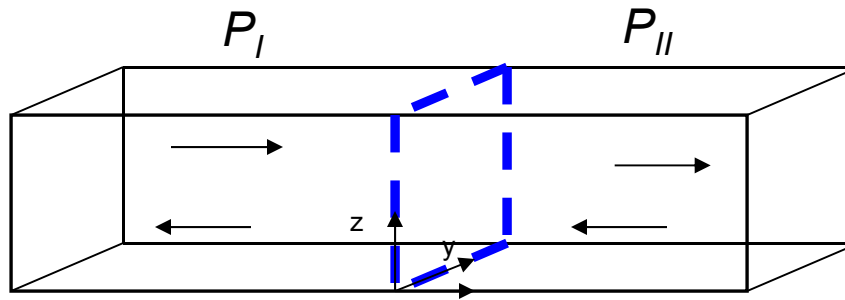


d [mm]	t [mm]	D [N·m ²]	loss factor	T [N]	ρ_s [kg/m ²]	N	Size [mm]
0.15	0.4	0.0001	0.001	0	0.1631	756589	63.5 x 63.5

Energy dissipation



$$W_I = \frac{1}{2} \operatorname{Re} \left\{ \iint P_I u_I^* dydz \right\}$$



$$W_{II} = \frac{1}{2} \operatorname{Re} \left\{ \iint P_{II} u_{II}^* dydz \right\}$$

From sound fields

$$W_{E.D._{field}} = W_I - W_{II}$$

$$W_{E.D._{solid}} = \frac{1-\Omega}{2} \operatorname{Re} \left\{ \iint \left(D\nabla^4 d_s - T\nabla^2 d_s + \rho_s \frac{\partial^2 d_s}{\partial t^2} - R_t \frac{\Omega^2}{1-\Omega} \frac{\partial(d_f - d_s)}{\partial t} \right) \left(\frac{\partial d_s}{\partial t} \right)^* dydz \right\}$$

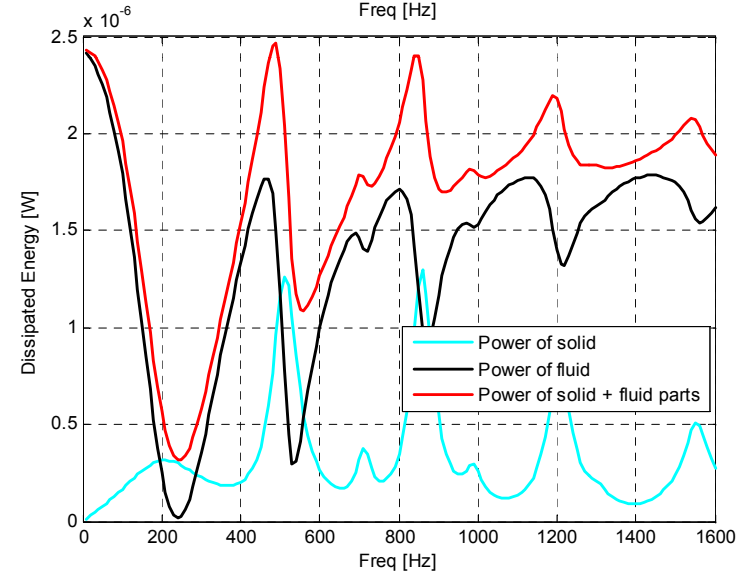
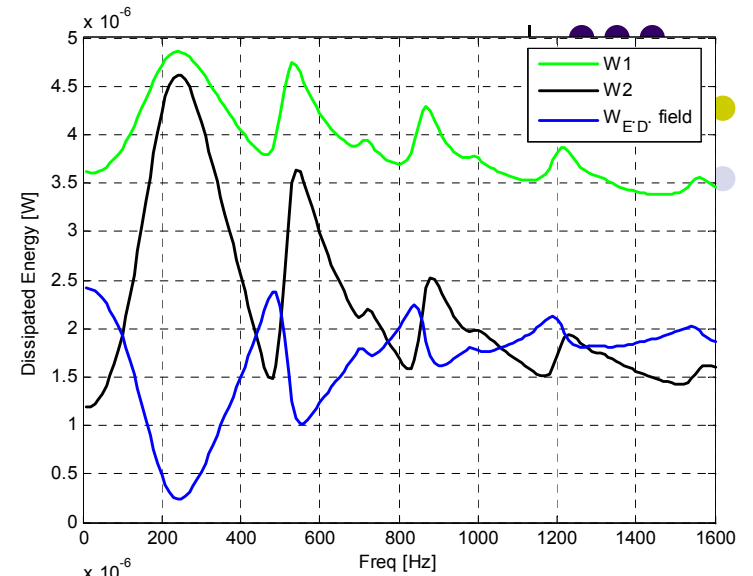
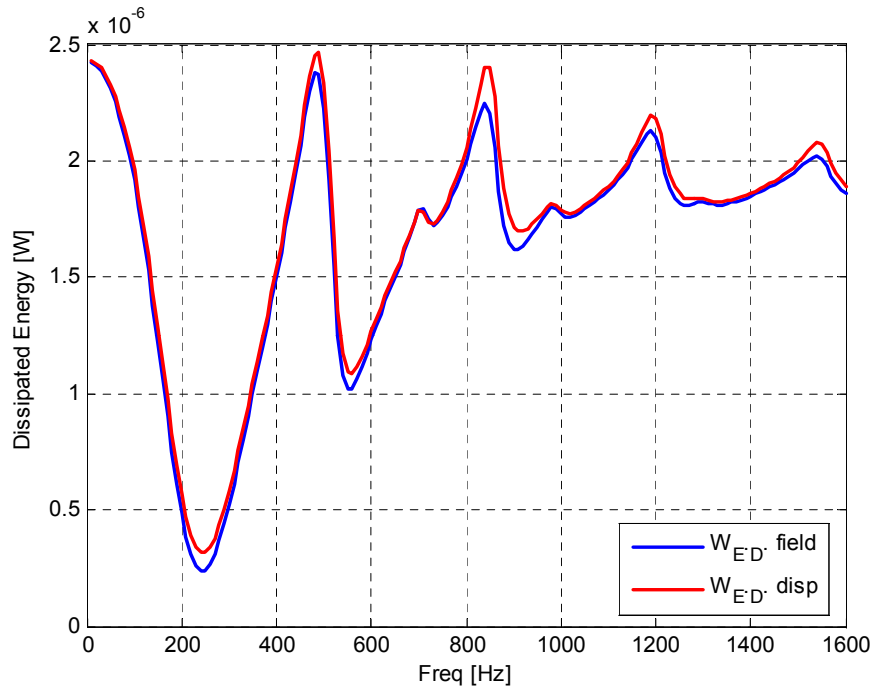
$$W_{E.D._{fluid}} = \frac{\Omega}{2} \operatorname{Re} \left\{ \iint \left(R_t \Omega \frac{\partial(d_f - d_s)}{\partial t} + j\omega \rho_o h' \frac{\partial d_f}{\partial t} \right) \left(\frac{\partial d_f}{\partial t} \right)^* dydz \right\}$$

From displacements of solid and fluid parts

$$W_{E.D._{disp.}} = W_{E.D._{solid}} + W_{E.D._{fluid.}}$$

$$W_{E.D._{field}} = W_{E.D._{disp.}}$$

Energy dissipation



d [mm]	t [mm]	Number of holes per unit area	Mass/area [kg/m ²]	Tension / loss factor	Flexural Stiffness/ loss factor	Panel size
0.15	0.4	756,589	0.174	85.41/0.05	0	63.5 x 63.5 mm



Conclusions

- Found that flexural vibration of finite micro-perforated panels can enhance absorption
- Developed initial model that accounts for main features of flexural resonances and their interaction with viscous dissipation in holes
- Verified this results with normal incidence absorption coefficient measurement
- Energy is dissipated both by viscous flow through holes and by flexural losses in the solid