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Distance Estimation Algorithm for Stereo Pair Images

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Abstract

Various algorithms have been proposed for estimating camera distance from objects in digital stereo pair videos and images. Existing algorithms assume that digital images were taken with pan and tilt cameras or assume that the angles between the cameras and the object are known. The algorithm that we are proposing has the ability to calculate distance from the camera plane to the object plane, where the cameras do not have the ability to pan and tilt nor the angles between the cameras and the object are known beforehand.

1 Introduction

Camera distance estimation from objects in images has been a versatile area for computer vision and image processing researchers. Different algorithms exist to calculate distance between the camera and the object from digital images and videos. These methods depend on the knowledge of the angle between the cameras and the object, therefore limiting the application of such algorithms. Existing algorithms utilize methods such as pan and tilt or laser rangers to measure the angles. This limits the ability of the algorithm to the object that the cameras or laser are currently focused on.

The purpose of this paper is to present a different approach to camera distance estimation from objects in digital images taken with stereo pair cameras. The method that we are proposing does not depend on the knowledge of the angle between the cameras and the object. Therefore this method can be applied to stereo pair images taken with generic digital cameras and eliminate the need for expensive laser rangers.

2 Camera Model and Assumptions

The pinhole camera model is assumed for our algorithm. This model allows for more simplistic calculation of camera properties such as angle of view. We are also assuming a perfect lens model to simplify the calculation related to the digital image produced by the camera. Our method of distance calculation relies on the notion that if the size of an object, h_O, is changed by δ in one direction (h_O' = $h_O + \delta$, where h_O' is the new size of the object), then the size of the image, h_I, is also changed by a function of δ (h_I' = h_I + f(δ), where h_I' is the new size of the image). Equation (1) shows that the image size is determined by the focal length f of the lens and the distance of the lens from the object R.

$$h_I = h_O \times \frac{f}{R-f}, R > f \quad (1)$$

If we increase the size of the object by δ and keep the object at the same distance from the lens, then the size of the image becomes:

$$h'_I = h_I + \delta \times \frac{f}{R-f}, R > f \quad (2)$$

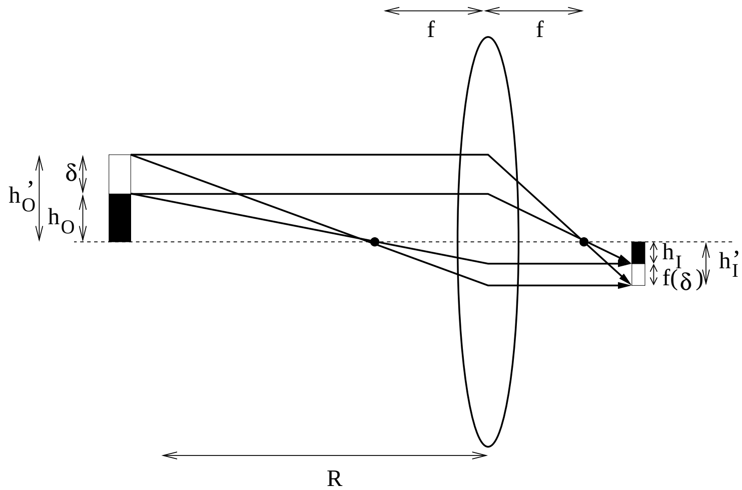

Figure 1: Lens and Object Magnification

When we keep the object at the same distance, $\frac{f}{R-f}$ is a constant and therefore the change in size of an object will cause a linear change in the size of the image. Therefore, the conversion from pixel difference to actual distance in meter or feet is governed by a linear relationship.

Before using our method, digital image processing algorithms need to be applied to the stereo pair images to find a spatial point in the two images. This spatial point is the pixel that is present in both images of the object and describes the same point on the real object. The pixel coordinates of the spatial point will then be used for distance calculation in our method and from this point forward will be referred to as "common pixel". Our method also requires that the distance between the cameras is known.

3 Distance Estimation Algorithm

Our method requires the knowledge of the half angle of view of the cameras. This is because our algorithm needs to know how big of an angle does the digital images taken with the camera represent. The half angle of view (θ) of a camera is given by:

$$\theta = \tan^{-1}\left(\frac{D}{2f}\right) \tag{3}$$

where D is the diameter of the camera's field stop and f is the focal length of the camera.

Our algorithm recognizes three different cases for distance estimation. These cases are: when the object is between two cameras, the object is either in the left or right of both cameras, and when the object is exactly in front of one of the cameras. The distinction is needed because each case requires a different geometrical approach hence resulting in different sets of formulas for each case.

3.1 Distance estimation for an object located between two cameras

One case for the distance estimation is when the object is located between the two cameras. This condition happens when the common pixel's coordinates in the stereo pair images is located in the right half of the left camera's image and in the left half of the right camera's image.

As explained in the previous section, conversion from pixel difference to actual distance in meter or feet is governed by a linear relationship. If we take ρ as the conversion factor from pixels to actual distance, thus, as seen in figure 2, for the left camera we can obtain the following relationships:

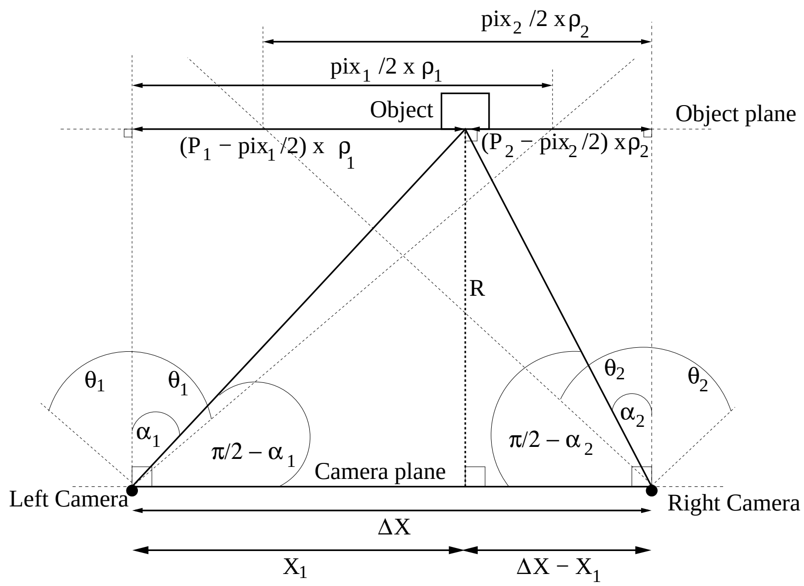

Figure 2: Geometry of an object between two cameras

$$\tan\theta_1 = \frac{pix_1/2 \times \rho_1}{R} \tag{4}$$

and

$$\tan\alpha_1 = \frac{(P_1 - pix_1/2) \times \rho_1}{R} \tag{5}$$

where P_1 is the coordinate of the common pixel in the left camera's image and $pix_1/2$ is the total number of pixels in the left camera's image divided by two. We do not need to know what is the conversion factor ρ_1 because if we equate both equations, ρ_1 will be canceled. All that is needed to be known is just the coordinates of the common pixel and the total number of pixels in the image. Therefore deriving from equations (4) and (5), the angle α_1 in figure 2 can be calculated using the following equation:

$$\alpha_1 = \tan^{-1}\left(\frac{P_1 - pix_1/2}{pix_1/2} \times \tan\theta_1\right) \tag{6}$$

Similarly, the angle α_2 can be calculated using the following equation:

$$\alpha_2 = \tan^{-1}\left(\frac{pix_2/2 - P_2}{pix_2/2} \times \tan\theta_2\right) \tag{7}$$

where P_2 is the coordinate of the common pixel in the right camera's image and $pix_2/2$ is the total number of pixels in the right camera's image divided by two.

From figure 2, we can obtain:

$$\tan\left(\frac{\pi}{2} - \alpha_1\right) = \frac{R}{X_1} \tag{8}$$

or

$$R = \tan\left(\frac{\pi}{2} - \alpha_1\right) \times X_1 \tag{9}$$

From the same figure, we can also obtain:

$$\tan\left(\frac{\pi}{2} - \alpha_2\right) = \frac{R}{\Delta X - X_1} \tag{10}$$

where ΔX is the distance between the two cameras. Substituting R in equation (10) with equation (9) yields:

$$\tan\left(\frac{\pi}{2} - \alpha_2\right) = \frac{\tan\left(\frac{\pi}{2} - \alpha_1\right) \times X_1}{\Delta X - X_1} \tag{11}$$

By solving for X_1, the equation becomes:

$$X_1 = \frac{\tan\left(\frac{\pi}{2} - \alpha_2\right) \times \Delta X}{\tan\left(\frac{\pi}{2} - \alpha_1\right) + \tan\left(\frac{\pi}{2} - \alpha_2\right)} \tag{12}$$

Therefore, substituting equation (12) back to equation (9) the distance from the camera plane to the object plane R can be obtained using the equation:

$$R = \frac{\tan\left(\frac{\pi}{2} - \alpha_1\right) \times \tan\left(\frac{\pi}{2} - \alpha_2\right) \times \Delta X}{\tan\left(\frac{\pi}{2} - \alpha_1\right) + \tan\left(\frac{\pi}{2} - \alpha_2\right)} \tag{13}$$

3.2 Distance estimation for an object on the left or right of both cameras

Another case for the distance estimation is when the object is located on the left or right of both cameras. This condition happens when the common pixel's coordinates in the stereo pair images is located in the left or right half of both the left camera's and the right camera's images.

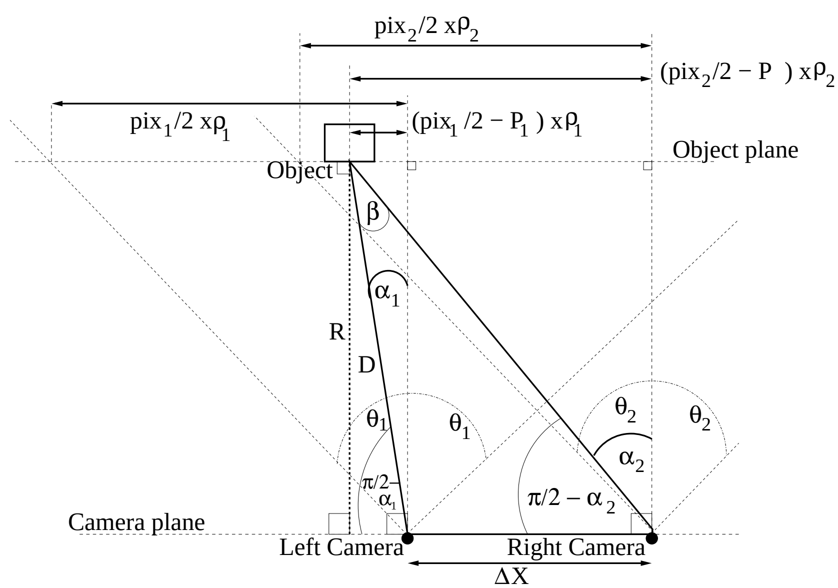

Figure 3: Geometry of an object on the left of both cameras

For an object in the left of both cameras, as described in figure 3, the angle α_1 can be calculated using the following equations:

$$\alpha_1 = \tan^{-1}\left(\frac{pix_1/2 - P_1}{pix_1/2} \times \tan\theta_1\right) \tag{14}$$

where P_1 is the coordinate of the common pixel in the left camera's image and $pix_1/2$ is the image's number of pixel divided by two. The angle α_2 can be calculated using equation (7). From figure 3 we can obtain the following equation:

$$\beta = (\alpha_2 - \alpha_1) \tag{15}$$

and using the sine equality property, we can obtain the equation:

$$\frac{D}{\sin\left(\frac{\pi}{2}-\alpha_2\right)} = \frac{\Delta X}{\sin\left(\alpha_2-\alpha_1\right)} \tag{16}$$

therefore, the distance from the left camera to the object, D, is:

$$D = \frac{\Delta X \times \sin\left(\frac{\pi}{2}-\alpha_2\right)}{\sin\left(\alpha_2-\alpha_1\right)} \tag{17}$$

From figure 3 we can also obtain the equation

$$R = D \times \sin\left(\frac{\pi}{2}-\alpha_1\right) \tag{18}$$

Substituting D in equation(18) with equation (17), the distance from the camera plane to the object plane, R, can obtained using the equation:

$$R = \frac{\sin\left(\frac{\pi}{2}-\alpha_1\right) \times \sin\left(\frac{\pi}{2}-\alpha_2\right) \times \Delta X}{\sin\left(\alpha_2-\alpha_1\right)} \tag{19}$$

Conversely, the distance from the camera plane to the object plane, R, when the object is located on the right of both cameras can obtained using the equation:

$$R = \frac{\sin\left(\frac{\pi}{2}-\alpha_1\right) \times \sin\left(\frac{\pi}{2}-\alpha_2\right) \times \Delta X}{\sin\left(\alpha_1-\alpha_2\right)} \tag{20}$$

where α_2 can be obtained using the equation:

$$\alpha_2 = \tan^{-1}\left(\frac{P_2 - pix/2}{pix/2} \times \tan\theta_2\right) \tag{21}$$

and α_1 is obtained using equation (6). This condition happens when the common pixel's horizontal coordinates in the stereo pair images is located in the right half of both the left camera's and the right camera's images.

3.3 Distance estimation for an object that is exactly in front of one of the cameras

When the object is in front of one of the cameras (i.e. the common pixel's horizontal coordinate in one of the stereo pair images is equal to the midpoint of the horizontal pixels), as shown in figure 4, the calculation for object plane distance becomes simpler. Using basic trigonometry, the distance can be found using the equation:

$$R = \tan\left(\frac{\pi}{2}-\alpha_2\right) \times \Delta X \tag{22}$$

where the angle α_2 can be calculated using equation (7). Conversely, the distance of the object plane from the camera plane when the object is exactly in front of the right camera can be calculated with the equation:

$$R = \tan\left(\frac{\pi}{2}-\alpha_1\right) \times \Delta X \tag{23}$$

where the angle α_1 can be calculated using equation (6).

4 Conclusion

In this paper we have proposed a method to estimate the distance of an object from the camera plane that does not require equipments other than a pair of digital cameras, where their field stop diameters and focal lengths are known. Future works need to be done to find how accurate this method is and what its margin of error is.

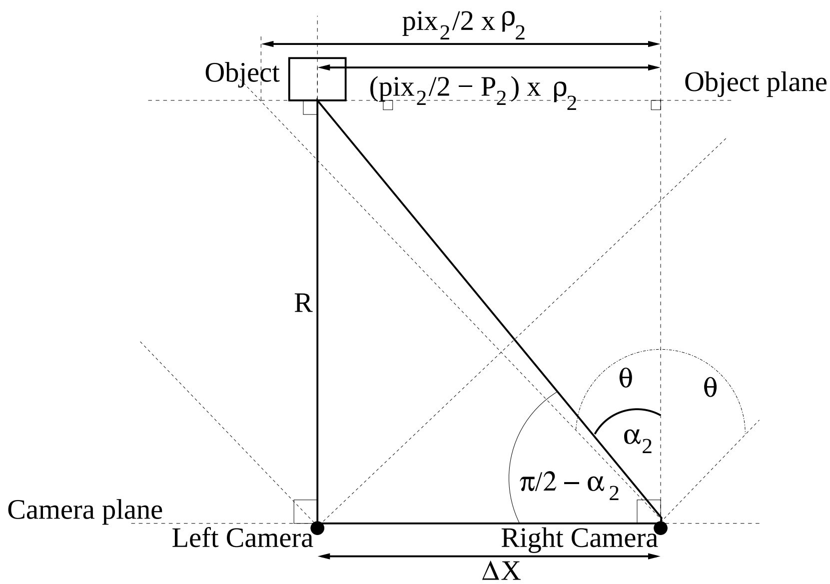

Figure 4: Geometry of an object that is exactly in front of the left camera