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Prediction-based Adaptive Robust Control for a Class of Uncertain Time-delay Systems

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Abstract: This paper presents an integrated control design approach for a class of dynamical systems that satisfy a certain matching condition subject to known input time-delay, unknown parameters, and time-varying disturbances, simultaneously. A novel nonlinear predictor adaptive robust control (PARC) is proposed to track a desired state trajectory. The controller uses predictor-based model compensation to attenuate the effect of input time-delay, gradient type projection with prediction-based learning mechanisms to reduce the parameter uncertainties, and prediction-based nonlinear robust feedback to attenuate the effect of model approximation errors and disturbances, simultaneously. The controller guarantees a prescribed transient performance (with global exponential convergence) and final steady-state tracking error with an ultimate bound proportional to the time-delay, the disturbances, and the switching gain. The effectiveness of the proposed control design is illustrated with a simple tumor growth example.

Keywords: Predictor feedback, Adaptive control, Sliding mode control, Tracking, Stability

1. INTRODUCTION

The problem of developing tracking controllers with guaranteed transient and asymptotic performance for rigorously handling a wide range of time-varying disturbances that enter through an uncertain nonlinear vector field in the presence of input time-delay and parameter uncertainties has remained largely intractable, see Krstic [2009]. Some practical examples include tumor growth suppression, see Eladdadi et al. [2014] and high-speed/high-accuracy position motion control with linear motor drives for precision machining, telemanipulation, see Yao and Jiang [2010], Zhang et al. [2015].

Predictive techniques have been popular to handle long input delay for linear time invariant systems with no uncertainties and have been extended to linear time varying systems in Mazenc et al. [2014], and to nonlinear systems in Bekiaris-Liberis and Krstic [2013]. An output feedback adaptive posicast control was proposed by combining finite spectrum assignment and adaptive control to deal with input delay and unknown parameters, see Niculescu and Annaswamy [2003], Yildiz et al. [2010]. In Niculescu and Annaswamy [2003], time-varying disturbances are handled using add-on σ modification to damp the drift of parameter estimates. In Yildiz et al. [2010], an adaptive feedforward term was added to handle the constant disturbances. In Bresch-Pietri and Krstic [2009], Bekiaris-Liberis and Krstic [2010], adaptation of both input delay and plant parameters has been studied. However, the above works do not comment on the transient performance of the controllers. This means that the actual system may have large initial tracking errors or have slower response. In Zhang et al. [2015], disturbance observer method was proposed to ensure that the teleoperation of motion control systems was robust to model uncertainties and time-delay. This involved the design of ad hoc filters, namely, a high-pass filter to handle high-frequency noises and a low-pass filter to compensate for the effect of delay and disturbances assumed to have low-frequency variations. In Bresch-Pietri et al. [2012], an adaptive control scheme was proposed to completely reject a constant disturbance in the presence of unknown but bounded constant input delay and uncertain parameters. In Han et al. [2012], a sliding mode control (SMC) was proposed in the presence of time-varying input delay and bounded matched disturbances to achieve ultimate boundedness of the closed loop system using a singular perturbation approach. The idea of adaptive robust control (ARC) with rigorous stability analysis and its application to motion control problems was presented in Yao and Jiang [2010] and references therein. However, their control law does not address delays in the input. Recently, in Léchappé et al. [2015b], a new prediction scheme with full state feedback has been proposed that is robust to external disturbances in the presence of input delay, then extended to output feedback in Léchappé et al. [2015a], and then to an unknown delay case in Léchappé et al. [2016]. But they do not address the effect of parametric uncertainties explicitly.

In this paper, a direct nonlinear predictor-based adaptive robust control (PARC) framework is developed for high precision control of a class of n^{th} order dynamical systems to simultaneously handle input time-delay, unknown parameters and time-varying disturbance uncertainties. The salient features of the PARC control design is that

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it seamlessly extends and then integrates: i) a predictive input delay compensation using the modified finite integral state predictor feedback (Yildiz et al. [2010]); ii) a predictor-based projection type adaptation laws for controller parameters to reduce the conservativeness; and iii) a predictor-based smooth robust filter feedback with local high-gain using modified SMC framework to better attenuate a wide range of time-varying nonlinearities (Yao and Jiang [2010]). The prediction-based parameter adaptation with model compensation is used to reduce the effect of various parametric uncertainties, while the prediction-based robust feedback is used to handle the cumulative effect of disturbances/uncertain nonlinearities and parameter estimation errors from uncertain prediction. These features yield good performances without requiring ad hoc delay compensation strategies or time-consuming/expensive off-line identification of system parameters. Thus, the structural information of the system dynamics and a priori information are effectively utilized to obtain high-performance tracking control. Quadratic Lyapunov functions are used to prove global exponential convergence of the tracking error with an ultimate bound that depends on time-delay, disturbance bounds, and controller gain, and to show boundedness of the controller parameter adaptation.

The rest of the paper is organized as follows: Section II formulates the problem and states the required assumptions. In Section III, a trajectory tracking PARC design is developed consisting of adaptive and robust feedback controllers with predictive state formulation. Section IV discusses theoretical stability and performance analysis of the closed loop system. Section V validates the proposed controller with an illustrative tumor growth example. Section VI provides some conclusions and future work.

2. PROBLEM FORMULATION

Consider an uncertain dynamical system with input time-delay of the form:

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t-\tau) + \Delta(x,t) \\ &x(s) = p(s), s \in [-\tau, 0],\ u(v) = q(v), v \in [-2\tau, 0] \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ represents the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $\tau \geq 0$ is the known time-delay in the control channel, A is a system matrix of appropriate dimension with unknown parameters $\bar{\theta}$, B is a known, full column rank input matrix of appropriate dimension, (A, B) is controllable, $\Delta(x,t)$ is the lumped effect of unknown time-varying disturbances, and $p(s)$ and $q(s)$ denote the initial conditions on the state and control input, respectively.

Our objective is to design a bounded control law u so that the system state $x(t)$ tracks the desired trajectory $x_d(t)$ as close as possible. The desired trajectory $x_d(t)$ is generated by the following stable reference model:

$$\dot{x}_d = A_d x_d + Bu_c(t-\tau) \tag{2}$$

where A_d is a suitable Hurwitz matrix, and $u_c(t-\tau)$ is the delayed reference command input.

Without loss of generality, the following practical assumptions about the system and environment are made:

Assumption 1. The unknown true plant parameter vector $\bar{\theta} = [\theta_1 \ \cdots \ \theta_N]^T \in \mathbb{R}^N$, $N = n^2$ at most, of system matrix A lies in a known bounded region Ω_θ and the disturbance $\Delta \in \mathbb{L}^1_{loc}(\mathbb{R}^n \times \mathbb{R}_{\geq 0}, \mathbb{R}^n)$ is an unknown locally integrable function that is bounded as:

$$\begin{aligned} \theta_i \in \Omega_\theta &:= \{\theta_i \mid \theta_{i,min} \leq \theta \leq \theta_{i,max}, i = 1, \cdots, N\} \\ \Delta \in \Omega_\Delta &:= \{\Delta \mid \|\Delta(x,t)\| \leq \ \delta_M\} \end{aligned} \tag{3}$$

where $\theta_{i,min}$ and $\theta_{i,max}$ are known lower and upper bounds of parameter θ_i and δ_M is assumed to be known.

Assumption 2. The input time-delay $\tau > 0$ is assumed to be constant and known, similar to the works of Yildiz et al. [2010], Pyrkin et al. [2014], Léchappé et al. [2015b].

Since the predictor feedback design tries to imitate the nominal (i.e., delay-free) system, the following reasonable assumption is required as demonstrated in the ARC design for systems subject to both unknown parameters and uncertain nonlinearities, see Yao and Jiang [2010]:

Assumption 3. For the nominal system without time-delay, i.e.,

$$\dot{x}(t) = Ax(t) + Bu(t) + \Delta(x,t) \tag{4}$$

the system trajectory x tracks the desired state trajectory $x_d(t) \in C^1(\mathbb{R}_{\geq 0}, \mathbb{R}^n)$ with a locally Lipschitz nonlinear control law $u(t) \in \mathbb{L}^\infty_{loc}(\mathbb{R}_{\geq 0}, \mathbb{R}^m)$ in the presence of parametric uncertainties and uncertain nonlinearities.

3. PREDICTOR ADAPTIVE ROBUST CONTROL DESIGN

The following controller structure for u is proposed consisting of three terms: i) a predictor-based adaptive model compensation u_m to achieve perfect tracking; ii) a command following feedforward u_f ; and iii) a predictor-based nonlinear robust feedback u_r to overcome parametric and disturbances uncertainties under time-delay to achieve robust stability and performance of the closed loop system:

$$u(t) = u_m(t) + u_f(t) + u_r(t) \tag{5}$$

First, the feedforward command input u_f and delay compensation u_m are designed as:

$$u_f(t) = u_c(t) \tag{6}$$

$$u_m(t) = -\hat{\theta}(t)^T x_p(t+\tau) \tag{7}$$

where $\hat{\theta}$ is the control parameter vector that needs to be updated on-line by an appropriate adaptive control law, specified later, and $x_p(t+\tau)$ is the predicted state vector at time t of the state at time $t+\tau$ given as:

$$x_p(t+\tau) = e^{A\tau}x(t) + \int_{-}^{0} e^{-A\eta} Bu(t+ \quad)\mathrm{d} \tag{8}$$

$$= x(t+\tau) - \int_{-}^{0} e^{-A\eta} \Delta(x, t+\tau+ \quad)\mathrm{d} \tag{9}$$

Due to the parametric and disturbance uncertainties, x_p cannot be directly computed (8) and thus u_m is rewritten as follows:

$$\begin{aligned} u_m(t) = &\ -\hat{\theta}(t)^T \ \left(e^{A\tau}x(t) + \int_{-}^{0} e^{-A\eta} Bu(t+ \quad)\mathrm{d} \ \right) \\ = &\ -\hat{\theta}_z(t)^T x(t) - \int_{-}^{0} \hat{\ }_z(t, \quad)u(t+ \quad)\mathrm{d} \end{aligned} \tag{10}$$

where $\hat{\theta}_z(t) = e^{A^T} \ \hat{\theta}(t)$, $\hat{\ }_z(t, \ ) = \hat{\theta}(t)^T e^{-A\eta} B$. The controller parameters $\hat{\theta}_z(t)$ and $\hat{\ }_z(t, \ )$ are desired to attain their true values as time goes by, given as:

$$\theta_z^\star = e^{A^T\tau}\theta^\star \tag{11}$$
$$\lambda_z(\eta)^\star = \theta^{\star T}e^{-A\eta}B \tag{12}$$

where $\theta^\star$ is given by:

$$A - B\theta^{\star T} = A_d \tag{13}$$

Using the defined control parameters for u_f and u_m in (5)-(7) and (9) the closed loop system becomes:

$$\dot{x}(t) = Ax(t) + B\big(-\hat{\theta}(t-\tau)^T\{x(t) - \int_{-\tau}^{0} e^{-A\eta}\Delta(x,t+\eta) \mathrm{d}\eta\} + u_c(t-\tau) + u_r(t-\tau)\big) + \Delta(x,t) \tag{14}$$

Let $z(t) = x(t) - x_d(t)$ represent tracking error of the closed loop system. The error dynamics for $\dot{z}$ using (13)-(14) is:

$$\begin{aligned}\dot{z}(t) &= A_d z(t) + B\big(-\tilde{\theta}(t-\tau)^T x(t) + \hat{\theta}(t-\tau)^T\int_{-\tau}^{0} e^{-A\eta} \Delta(x,t+\eta)\mathrm{d}\eta + u_r(t-\tau)\big) + \Delta(x,t)\\ &= A_d z(t) + B\big(-\tilde{\theta}_z(t-\tau)^T x(t-\tau) - \int_{-\tau}^{0}\tilde{\lambda}_z(t-\tau,\eta) u(t-\tau+\eta)\mathrm{d}\eta + \theta^\star(t-\tau)^T\int_{-\tau}^{0} e^{-A\eta}\Delta(x,t+\eta)\mathrm{d}\eta + u_r(t-\tau)\big) + \Delta(x,t)\end{aligned} \tag{15}$$

where $\tilde{\theta}(t) = \hat{\theta}(t) - \theta^\star$, $\tilde{\theta}_z(t) = \hat{\theta}_z(t) - \theta_z^\star$, $\tilde{\lambda}_z(t,\eta) = \hat{\lambda}_z(t,\eta) - \lambda_z^\star(\eta)$. Simplifying (15), we obtain:

$$\dot{z}(t) = A_d z + B\big(-\tilde{\Theta}(t-\tau)^T\Psi(x,u,\tau) + u_r(t-\tau)\big) + \bar{\Delta}(x,t) \tag{16}$$

where $\Psi(x,u,\tau) := [x(t-\tau)\ \int_{-\tau}^{0} u(t-\tau+\eta)\mathrm{d}\eta]^T$, $\tilde{\Theta}(t-\tau) := [\tilde{\theta}_z(t-\tau)\ \tilde{\lambda}_z(t-\tau,\eta)^T]^T$, $\bar{\Delta}(x,t) = \Delta(x,t) + B\{\theta^{\star T}\int_{-\tau}^{0} e^{-A\eta}\Delta(x,t+\eta)\mathrm{d}\eta\}$. Thus, $\tilde{\Theta}^T\Psi$ is linearly parametrized by the parameter error vector $\tilde{\Theta}$ with the known basis function vector (regressors) Ψ.

The controller parameters $\hat{\theta}_z(t), \hat{\lambda}_z(t,\eta)$ are learned using a gradient type projection with prediction-based adaptation law to ensure bounded parameter estimates as follows:

$$\dot{\hat{\theta}}_z(t) = \mathrm{Proj}_{\hat{\theta}_z}(-\gamma_{\hat{\theta}_z} x(t-\tau)\rho(t+\tau)) \tag{17}$$
$$\frac{\partial\hat{\lambda}_z^T}{\partial t}(t,\eta) = \mathrm{Proj}_{\hat{\lambda}_z}(-\gamma_{\hat{\lambda}_z} u(t-\tau+\eta)\rho(t+\tau)) \tag{18}$$

where $-\tau \le \eta \le 0$, $\gamma_{\hat{\theta}_z}, \gamma_{\hat{\lambda}_z} > 0$ are the adaptation gains, $\rho(t+\tau) = z_p(t+\tau)^T PB$, $A_d^T P + PA_d = -Q, Q > 0$, and $z_p(t+\tau)$ is the predicted tracking error vector defined as:

$$z_p(t+\tau) = e^{A_d\tau}z(t) + \int_{-\tau}^{0} e^{-A_d\eta}Bu_r(t+\eta)\mathrm{d}\eta \tag{19}$$

where $A_d < 0$ and u_r is the nonlinear robust feedback designed later. The individual projection operator performs the adaptation with respect to its own parameter bounds. For example, the projection operator for $\hat{\theta}_z$, $\mathrm{Proj}_{\hat{\theta}_z}(\cdot)$, which guarantees that $\hat{\theta}_z \in \Omega_{\hat{\theta}_z}$ is given as:

$$\mathrm{Proj}_{\hat{\theta}_z}(\cdot) := \begin{cases} 0 & \text{if } \begin{cases}\hat{\theta}_z = \hat{\theta}_{z,max} \text{ and } \cdot > 0\\ \hat{\theta}_z = \hat{\theta}_{z,min} \text{ and } \cdot < 0\end{cases}\\ \cdot & \text{otherwise}\end{cases} \tag{20}$$

where $\hat{\theta}_{z,min} = \min\{e^{A^T\tau}\theta^\star\}$, $\hat{\theta}_{z,max} = \max\{e^{A^T\tau}\theta^\star\}$. Similarly, $\hat{\lambda}_{z,min}(\eta) = \min\{\theta^{\star T}e^{-A\eta}B\}$, $\hat{\lambda}_{z,max}(\eta) = \max\{\theta^{\star T}e^{-A\eta}B\}$, and min and max operators perform elementwise operations.

Re-writing (19), we obtain:

$$z_p(t+\tau) = e^{A_d\tau}z(t) + \int_{t-\tau}^{t} e^{A_d(t-\eta)}Bu_r(\eta)\mathrm{d}\eta \tag{21}$$

Differentiating (21) with Leibniz's rule and substituting (16) for $\dot{z}(t)$, we obtain the following delay-free dynamics regarding the predicted tracking error:

$$\begin{aligned}\dot{z}_p(t+\tau) &= e^{A_d\tau}\dot{z}(t) + A_d\int_{-\tau}^{0} e^{-A_d\eta}Bu_r(t+\eta)\mathrm{d}\eta + Bu_r(t) - e^{A_d\tau}Bu_r(t-\tau)\\ &= e^{A_d\tau}\big(A_d z(t) + B\{u_r(t-\tau) - \tilde{\Theta}(t-\tau)^T\Psi + \theta^{\star T}\int_{-\tau}^{0} e^{-A\eta}\Delta(x,t+\eta)\mathrm{d}\eta + \Delta(x,t)\}\big)\\ &\quad + A_d\int_{-\tau}^{0} e^{-A_d\eta}Bu_r(t+\eta)\mathrm{d}\eta + Bu_r(t) - e^{A_d\tau}Bu_r(t-\tau)\\ &= A_d z_p(t+\tau) + Bu_r(t) + e^{A_d\tau}f(x,t)\end{aligned} \tag{22}$$

where $f(x,t) = \Delta(x,t) + B\{-\tilde{\Theta}(t-\tau)^T\Psi + \theta^{\star T}\int_{-\tau}^{0} e^{-A\eta} \Delta(x,t+\eta)\mathrm{d}\eta\}$. Thus, the effect of input time-delay through predictor-based feedback can result in the cumulative uncertainty $f(x,t)$ (includes disturbances, uncertain prediction, and parametric errors) becoming unmatched due to the structure of $e^{A_d\tau}$. To ensure that $f(x,t)$ remains matched, we assume that $f(x,t)$ satisfy the following matching condition:

$$f(x,t) = e^{-A_d\tau}B\ \ (x,t) \tag{23}$$

where $\|\ \ (x,t)\| \le h(x,t)$ and $h(x,t)$ can be determined based on the bound information on the uncertain parametric and disturbance terms in Assumption 1. The above matching condition is satisfied for some practical systems such as in Yildiz et al. [2010], Léchappé et al. [2015a], Eladdadi et al. [2014]. This leads to the delay-free predictor error dynamics as:

$$\dot{z}_p(t+\tau) = A_d z_p(t+\tau) + Bu_r(t) + B\ \ (x,t) \tag{24}$$

Under the above matching condition, the bounds on $\ \ (x,t)$ satisfy the following inequality:

$$\begin{aligned}\|\ \ (x,t)\| &\le \|\bar{\Delta}(x,t)\| + \|B\tilde{\Theta}^T\Psi\|\\ &\le h(x,t) + h_0\end{aligned} \tag{25}$$

where $h(x,t) = (h_1(x,t) + h_2(x,t))/\|e^{-A_d\tau}B\|$ and $h_0 \ge 0$ is a design parameter. Here, $h_1 := max\{\|\bar{\Delta}_{min}\|, \|\bar{\Delta}_{max}\|\}$ where:

$$\bar{\Delta}_{min} = \min\{\Delta(x,t)\} + \min\{B\theta^{\star T}\}\min\{\Delta(x,t)\}\mu_{min}$$
$$\bar{\Delta}_{max} = \max\{\Delta(x,t)\} + \max\{B\theta^{\star T}\}\max\{\Delta(x,t)\}\mu_{max}$$

where $\mu_{min} = \frac{|e^{\|A\|_{min}\tau}-1|}{\|A\|_{max}}$, $\mu_{max} = \frac{|e^{\|A\|_{max}\tau}-1|}{\|A\|_{min}}$, and $h_2 := \|B(\tilde{\Theta}^T\Psi)_{max} - B(\tilde{\Theta}^T\Psi)_{min}\|$, and computed using the Holder's inequality as:

$$(\tilde{\Theta}^T\Psi)_{min} = \min\{\theta^{\star T}e^{A\tau}\}|x(t-\tau)| + \min\{B^T e^{-A^T\tau}\theta^\star\}\int_{-\tau}^{0}|u(t-\tau+\eta)|\mathrm{d}\eta$$
$$(\tilde{\Theta}^T\Psi)_{max} = \max\{\theta^{\star T}e^{A\tau}\}|x(t-\tau)| + \max\{B^T e^{-A^T\tau}\theta^\star\}\int_{-\tau}^{0}|u(t-\tau+\eta)|\mathrm{d}\eta$$

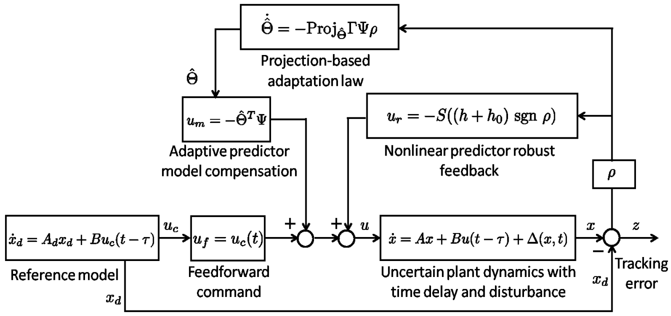

Fig. 1. PARC framework for an uncertain delayed system

Further, for some initial bounds on the state and input over $[t_0 - \tau, t_0)$ and $[t_0 - 2\tau, t_0)$, respectively, by inductive arguments similar to Yildiz et al. [2010], we have that $h(x,t)$ is bounded such that $\|h(x,t)\| \leq h_M(\tau), \forall x, t$ where h_M is a known bounded function. Detailed discussion is omitted for brevity. Using the above computed bound information $h(x,t)$, we design a predictor-based nonlinear robust feedback $u_r(t)$ as the ideal SMC law with delay compensation to dominate the effect of cumulative disturbances, as follows:

$$u_r(t) = -(h(x,t) + h_0)\mathrm{sgn}(\rho(t+\tau)) \tag{26}$$

where $h_0 > 0$ is a constant controller gain, and sgn is the signum function that performs elementwise operation as:

$$\mathrm{sgn}(\rho) = [\mathrm{sgn}(\rho_1), \cdots, \mathrm{sgn}(\rho_m)]^T$$

In reality, the ideal SMC law leads to severe actuator chattering due to the discontinuous nature of $\mathrm{sgn}(\rho)$. To address this problem, $(h + h_0)\mathrm{sgn}(\rho(t+\tau))$ is replaced with a continuous approximation $S((h+h_0)\mathrm{sgn}(\rho(t+\tau)))$ utilizing a boundary layer approach to obtain the nonlinear predictor robust feedback u_r with delay compensation as:

$$u_r(t) = -S((h(x,t) + h_0)\mathrm{sgn}(\rho(t+\tau))) \tag{27}$$

The approximation of the smoothed SMC law $S((h + h_0)\mathrm{sgn}(\rho))$ satisfies the following stabilization and approximation conditions:

$$\begin{aligned}&1.\ \rho u_r(t) \leq 0\\ &2.\ \rho\big[(h+h_0)\mathrm{sgn}(\rho) - S((h+h_0)\mathrm{sgn}(\rho))\big] \leq \epsilon(t)\end{aligned} \tag{28}$$

where $\epsilon(t)$ is any bounded time-varying positive scalar (i.e., $0 < \epsilon(t) \leq \epsilon_M$ for some ϵ_M) which is a measure of approximation accuracy. An example of S map is the saturation function $(h+h_0)\mathrm{sat}(\frac{z_p}{\phi_z})$, where the accuracy is determined by the width of the boundary layer ϕ_z.

The structure of the proposed PARC design is outlined in Fig. 1. The projection operator guarantees that $\hat{\Theta}(t) \in \Omega_\Theta, \forall t$. The predicted tracking error z_p, thus ρ, is determined through a predictive scheme given in (19).

4. PERFORMANCE AND STABILITY ANALYSIS

In this section, the theoretical results on the stability and performance of the closed loop PARC design are discussed.

Theorem 1. Given the uncertain time-delay system (1) under Assumptions 1 - 3 and the initial conditions $\hat{\theta}_z(0), \hat{\lambda}_z(s,\eta), x(s)$ for $s \in [-\tau, 0]$ and $u(v)$ for $v \in [-2\tau, 0]$, $\exists \tau^\star$ such that $\forall \tau \in [0, \tau^\star)$, the proposed PARC law (5) and the projection based adaptation laws (17) - (18) allow tracking a desired reference trajectory (2) in the presence of known input time-delay, unknown parameters, and uncertain nonlinearities, with the following result:

– All signals are bounded and the tracking error has a prescribed transient and steady-state performance with global exponential convergence to a ball $\{z(\infty) \mid \|z(\infty)\| \leq \left\| {}^{\dagger}_{d} \right\| \sqrt{\frac{2\epsilon_M}{c}} + \ (h_M(\tau) + h_0)\}$ at a convergence rate $c = \frac{1}{2}\lambda_{min}(P^{-1}Q)$, where $P = \ {}^T$, $= \frac{\|\zeta B\|}{\|A_d\|}(e^{\|A_d\|\tau} - 1)$, ${}_d = \ e^{A_d \tau}$, ${}^{\dagger}_{d} = ({}^T_d\ {}_d)^{-1}\ {}^T_d$.

Proof. Consider the following Lyapunov function regarding the predicted tracking error behavior, where P is the positive definite matrix satisfying the Lyapunov equation given by $A_d^T P + P A_d = -Q, Q > 0$:

$$V_s = \frac{1}{2} z_p(t+\tau)^T P z_p(t+\tau) \tag{29}$$

From (16), (24), (27), and (28), we obtain $\forall t, \tau \geq 0$:

$$\begin{aligned}\dot{V}_s &= \frac{1}{2}\dot{z}_p(t+\tau)^T P z_p(t+\tau) + \frac{1}{2} z_p(t+\tau)^T P \dot{z}_p(t+\tau)\\ &= \frac{1}{2} z_p^T(A_d^T P + P A_d) z_p + z_p^T P B(u_r(t) + \sigma(x,t,\tau))\\ &= -\frac{1}{2} z_p^T Q z_p + \rho(t+\tau)\big(\sigma(x,t,\tau) - S((h(x,t,\tau) + h_0)\\ &\quad sgn(\rho(t+\tau)))\big)\\ &\leq -\frac{1}{2} z_p^T Q z_p + \rho(t+\tau)\big((h+h_0)sgn(\rho(t+\tau)) -\\ &\quad S((h(x,t,\tau) + h_0)sgn(\rho(t+\tau)))\big)\\ &\leq -\frac{1}{2} z_p^T Q z_p + \epsilon(t)\end{aligned} \tag{30}$$

where $\rho(t+\tau) = z_p(t+\tau)^T P B$. Further, $\forall P > 0, Q > 0$, the following holds true:

$$x^T Q x \geq \lambda_{min}(P^{-1}Q) x^T P x \tag{31}$$

where $\lambda_{min}(P^{-1}Q) > 0$ is the smallest eigenvalue of $P^{-1}Q$. This allows us to re-write (30) as:

$$\begin{aligned}\dot{V}_s &\leq -\frac{1}{2}\lambda_{min}(P^{-1}Q) z_p^T P z_p + \epsilon(t)\\ &\leq -\frac{1}{2}\lambda_{min}(P^{-1}Q) V_s + \epsilon(t)\end{aligned} \tag{32}$$

Using the Comparison Lemma, see Krstic [2009], we obtain the following bound on V_s:

$$\begin{aligned}V_s(t) &\leq e^{-ct} V_s(0) + \int_0^t e^{-c(t-v)}\epsilon(v)\mathrm{d}v\\ &\leq e^{-ct} V_s(0) + \frac{\epsilon_M}{c}(1 - e^{-ct})\end{aligned} \tag{33}$$

where $c = 0.5\lambda_{min}(P^{-1}Q)$. Let $\xi(t+\tau) = \ z_p(t+\tau)$ where $\in \mathbb{R}^{m \times n}$ is full column rank chosen such that $P = \ {}^T$. Then using (33), the bounds on the predicted tracking error is given by:

$$\|\xi(t+\tau)\|^2 \leq e^{-ct}\|\xi(0)\|^2 + \frac{2\epsilon_M}{c}(1 - e^{-ct}) \tag{34}$$

From this, we have the uniform ultimate bound of $\xi(t)$ that exponentially converges as:

$$\|\xi(\infty)\| \leq \sqrt{\frac{2\epsilon_M}{c}} \tag{35}$$

In practice, however, there always exists a lower bound when choosing ϵ_M, since it can lead to local high-gain feedback exciting the neglected high-frequency dynamics, thus limiting the achievable tracking accuracy.

Remark 1. If the ideal SMC law is used for $u_r(t)$, then we have the following inequality:

$$V_s(t) \leq e^{-ct}V_s(0) \Rightarrow \|\xi(t+\tau)\| \leq e^{-ct}\|\xi(0)\|, \forall t \quad (36)$$

From this, the uniform ultimate bound of ξ exponentially converges to 0 as $t \to \infty$.

Finally, to determine the ultimate boundedness of $z(t)$, we note that ζ is of full column rank and exploit the relationship between $z_p(t+\tau)$ and $z(t)$ in (21) as:

$$\xi(t+\tau) = S_d z(t) + \int_{t-\tau}^{t} S e^{A_d(t-\eta)} B u_r(\eta) \mathrm{d}\eta \quad (37)$$

where $\zeta_d = \zeta e^{A_d \tau}$. Due to the full column rank of ζ_d, we have an empty nullspace for $z(t)$. By taking the Moore-Penrose pseudoinverse of ζ_d denoted as $\zeta_d^\dagger = (\zeta_d^T \zeta_d)^{-1}\zeta_d^T$, we obtain the following inequality using the bound on ξ:

$$\|z(t)\| \leq \left\|\zeta_d^\dagger \xi(t+\tau)\right\| + \beta \sup_{t-\tau \leq \eta \leq t} \|(h(x,\eta) + h_0)\| \quad (38)$$

where $\beta = \frac{\|\zeta B\|}{\|A_d\|}(e^{\|A_d\|\tau} - 1)$. Then, we have the following results:

- Uniform Boundedness: From the exponential stability of $\xi(t+\tau)$, $\exists \alpha_1 > 0$, a constant, such that:
  $$\|\xi(t+\tau)\| < \alpha_1, \ t \geq 0 \quad (39)$$
  Hence, from (38), we obtain:
  $$\|z(t)\| \leq \alpha_2, \forall t \geq 0 \quad (40)$$
  where $\alpha_2 = \left\|\zeta_d^\dagger\right\| \alpha_1 + \beta(h_M(\tau) + h_0)$
- Uniform Ultimate Boundedness: Again, from the exponential stability result of $\xi(t+\tau)$ given in (35), we have from (38) as $t \to \infty$:
  $$\|z(\infty)\| \leq \left\|\zeta_d^\dagger\right\| \sqrt{\frac{2 \ _M}{c}} + \beta(h_M(\tau) + h_0) \quad (41)$$

Thus, $\forall \tau \geq 0$ the tracking error $z(t)$ has global exponential convergence to a ball with a uniform ultimate bound given in (41) where $_M$ and c are design parameters which can be freely adjusted to predetermine the transient and steady state performance. Further, the projection-based adaptation laws in (17) - (18) render the parameter estimates $\hat{}_z$, $\hat{}_z$ bounded $\forall t \geq 0$. Also, $z(t)$ and $x_d(t)$ are bounded implies that $x(t)$ and the control input $u(t)$ are bounded. Finally, from (38), we note that for $\beta < 1$ i.e., for $\tau < \tau^\star = \frac{ln(1+\frac{\|A_d\|}{\|\zeta B\|})}{\|A_d\|}$, in addition to the ultimate boundedness result, the effect of the cumulative uncertainty on the tracking error is guaranteed to be a fraction less than 1 of the maximum uncertainty $(h_M(\tau) + h_0)$ affecting the closed loop system.

Remark 2. It can be observed from (41) that for $\tau = 0$, we retrieve the results of the ARC, see Yao and Jiang [2010].

5. SIMULATION

To demonstrate the proposed PARC controller, a second order tumor growth model is considered, see Eladdadi et al. [2014], where $x_1(t)$ and $x_2(t)$ represent the volume of tumor cells in the quiescent and the active phases of the cell cycle at time t, respectively, $u(t)$ is the amount of injected oncolytic virotherapeutic vesicular stomatitis virus (VSV), τ is the total biological time for VSV to travel through the cell cycle and act on the tumor cell [1], $\Delta(x,t)$ is the neglected chemical interactions and VSV dynamics:

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 0 \\ a_1 & a_2\end{bmatrix}\begin{bmatrix}x_1(t) \\ x_2(t)\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix} u(t-0.3) + \begin{bmatrix}0 \\ 1.2214\end{bmatrix} \Delta(\boldsymbol{x},t) \quad (42)$$

The simulation is run with $a_1 = 0.4, a_2 = 0.3$, $\Delta(x,t) = 0.2sin(3t)$, $x(0) = [1, 0.6]^T$. The parameters bounds $a_1 \in [0.3, 0.5]$ and $a_2 \in [0.25, 0.35]$ (i.e., 25% and 16.67% variability, respectively) are known to the controller. Here, a_1 is the rate at which cells move from quiescent to active phase and a_2 is the rate at which cells die in active phase.

A stable, desired reference model (2) with $a_{d1} = 0.5, a_{d2} = 0.6$, $u_c(t) = 3.5sin(t)$, $x_d(0) = [0.5, 0]^T$ is given as:

$$\begin{bmatrix}\dot{x}_{d1} \\ \dot{x}_{d2}\end{bmatrix} = \begin{bmatrix}-1 & 0 \\ -a_{d1} & -a_{d2}\end{bmatrix}\begin{bmatrix}x_{d1}(t) \\ x_{d2}(t)\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix} u_c(t-0.3) \quad (43)$$

The goal here is to prevent the unbounded growth rate of tumor cells so that the condition of the patient can be quickly stabilized as desired. The adaptation gains are set as $\gamma_{\hat{\theta}_z} = 100 * \mathbb{I}_2, \gamma_{\hat{}_z} = 100$, and $_M = 0.01$. From (13), we have the true parameter $^\star = [a_1 + a_{d1} \ a_2 + a_{d2}]^T$. It can be checked that the cumulative uncertainty

$$f(x,t) = \begin{bmatrix}0 \\ 1.2214\end{bmatrix} \Delta(x,t) + \begin{bmatrix}0 \\ 1\end{bmatrix}\big(-\tilde{\Theta}^T\Psi + {}^{\star T} \int_{-\tau}^{0} e^{-A\eta}\begin{bmatrix}0 \\ 1.2214\end{bmatrix}\Delta(x,t+\eta)\mathrm{d}\eta\big) \quad (44)$$

lies in the column space of $e^{-A_d\tau}B$, thus satisfies the matching condition in (23), with an easily computable bound $h(x,t)$ on $\sigma(x,t)$ based on the known parameter and disturbance bounds.

Using (5), we design the PARC controller of the form:

$$u(t) = u_c(t) - \hat{}_z(t)^T x(t) - \int_{-\tau}^{0} \hat{}_z(t,\eta) u(t+\eta)\mathrm{d}\eta - h(x,t)\mathrm{sat}\Big(\frac{z_p^T PB}{\phi_z}\Big) \quad (45)$$

where $\phi_z = \frac{4 \ _M}{h}$, and P is solved from the Lyapunov equation $A_d^T P + PA_d = -Q$, $Q = 5 * \mathbb{I}_2$. The parameters $\hat{}_z(t)$ and $\hat{}_z(t)$ are learnt using (17) - (18). To implement the PARC control law, a discretized approximation of integration as noted in (Yildiz et al. [2010]) is performed and Euler approximation is carried out for parameter adaptation with the sampling rate set as 1 kHz.

Figure 2 shows trajectory tracking for both x_1 and x_2 by the proposed PARC and the baseline ARC, where the PARC's tracking is excellent and smooth, while the ARC's tracking is poor with larger transient and phase lag.

Figure 3 shows superior tracking error performance of the PARC compared with the baseline ARC. The PARC's tracking error for x_2 exponentially decreases to a small value of 0.0860, unlike the ARC's tracking error of 0.6251. This indicates that tumor growth in active phase is better suppressed with the PARC than the ARC. The tracking error for x_1 goes to 0 for both the controllers indicating that

[1] VSV is an oncolytic virus which can only infect tumor cells when they are in the active phases of the cell cycle. The ensuing side-effects of VSV injection are currently disregarded in this paper

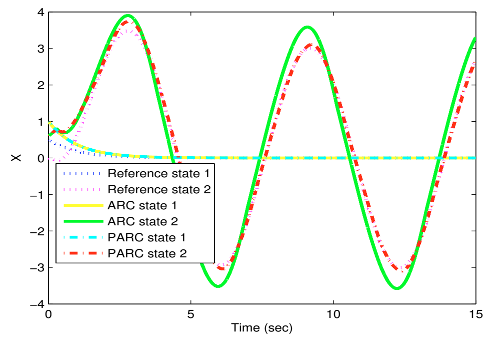

Fig. 2. Trajectory Tracking (baseline ARC vs. PARC)

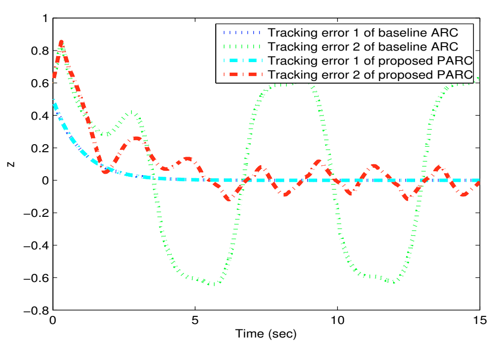

Fig. 3. Tracking Error (baseline ARC vs. PARC)

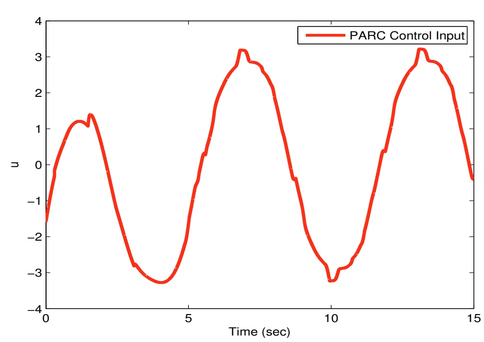

Fig. 4. Control Input History (baseline ARC vs. PARC)

tumor cells in quiescent phase is eliminated. It can also be verified that the theoretical ultimate error bound $\|z(\infty)\|$ of PARC is 0.6670 and $\tau^\star$ is 0.4101, confirming that the simulation results satisfy the analytical derivations.

Figure 4 shows the smooth control input of the amount of VSV injection using the PARC to treat the tumor growth.

6. CONCLUSIONS

This paper has developed a predictor adaptive robust controller (PARC) for n^{th} order dynamical system that satisfy matching condition to simultaneously handle unknown parameters, known input delay, and time-varying uncertain but bounded disturbances, while guaranteeing superior transient and steady state tracking performance. Our future work is to relax the matching condition restriction and extend to unknown delay case so that our PARC framework is applicable to even broader class of systems.

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