A transfer matrix approach for estimating the sound speed and attenuation constant of air in a tube

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Outline

- Sound propagation in a pipe
  - Traces back to the early period of acoustic research

- Standing wave tube technique
  - 2-microphone standard versus 4-microphone standard

- Estimation of complex wave number
  - Iterative procedure based on four-microphone measurement
Sound propagation in the pipe

- Classical Problem
  Kirchoff, Zwiker, Kosten, Morse and Ingard
- “Wall Effect”
  Viscous, thermal effect
- CFD
  Computational fluid dynamics
Standing wave tube techniques

- Standard test method for estimating impedance and absorption coefficients of acoustical materials
  - ISO 10534-2
  - ASTM E 1050

- Sources of errors: tube attenuation

- Two-Microphone absorption measurement
  - Newton-Raphson iteration scheme
Transfer matrix approach

- Four-microphone measurement is in the process of standardization by ASTM

\[
P_{x=0} = (A + B) \\
V_{x=0} = (A - B)/\rho_0 c \\
\]

\[
P_{x=d} = (Ce^{-jkd} + De^{jkd}) \\
V_{x=d} = (Ce^{-jkd} - De^{jkd})/\rho_0 c \\
\]

Transfer Matrix:

\[
\begin{bmatrix}
P \\
V
\end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix}
P \\
V
\end{bmatrix}_{x=d}
\]
Estimation of complex wave number

For acoustic materials, typically foam and fiberglass, the complex wave number can be obtained from the transfer matrix

\[
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix} =
\begin{bmatrix}
\cos k_p d & j \rho_p c_p \sin k_p d \\
-j \sin k_p d / \rho_p c_p & \cos k_p d
\end{bmatrix}
\]

One Load Approach

\[T_{11} = T_{22}\]
\[T_{11}T_{22} - T_{12}T_{21} = 1\]

\[k_p = \frac{1}{d} \cos^{-1} T_{11}\]

Treat empty air column as tested samples

Anechoic termination
Post-process iterative algorithm

Iterative Algorithm Flow Chart

\[ A = \frac{j(P_1 e^{jk_2} - P_2 e^{jk_3})}{2\sin k(x_1 - x_2)} \]
\[ B = \frac{j(P_2 e^{-jk_2} - P_1 e^{-jk_3})}{2\sin k(x_1 - x_2)} \]
\[ C = \frac{j(P_4 e^{jk_4} - P_3 e^{jk_3})}{2\sin k(x_3 - x_4)} \]
\[ D = \frac{j(P_4 e^{-jk_4} - P_3 e^{-jk_3})}{2\sin k(x_3 - x_4)} \]

\[ P_{x=0} = A + B, \quad P_{x=d} = Ce^{-jkd} + De^{jkd} \]
\[ V_{x=0} = \frac{A - B}{\rho_0 c}, \quad V_{x=d} = \frac{Ce^{-jkd} - De^{jkd}}{\rho_0 c} \]

\[ k_p = \frac{1}{d} \cos^{-1} T_{11} \]
Experimental Setup

- Semi-empirical prediction: Temkin formula
  \[ k = \frac{\omega}{c} + (1 - i)\delta \]
- Experimental estimation: Four-microphone measurement based on the transfer function combined with our iterative algorithm
- B&K 4206 standard small and large transmission loss tube kit
Measurement results - Large Tube

- Advantage: Simple, Fast, Flexible
- Disadvantage: Half Wave Length Problem
Corrected Temkin Formula

- Temkin formula can match the experimental results well if we modify the measured temperature.
Measurement results - Small Tube

Sound Speed

Tube Attenuation
Conclusions

- Semi-empirical Temkin formula for complex wave number estimation can be adjusted to match the experimental results very well.

- The iterative algorithm based on four-microphone technique gives a quick and reliable estimation of sound speed and attenuation in the tube.

- The approach has the potential to be used for the accurate measurements of gases properties and duct lining performance.
Thanks You!