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Among the benefits of granular media as construction materials is the ability to fail – a breaking of particle-particle contacts and a sudden reduction in the stresses the aggregate can support – and then restructure and reform [1]. The cycle can repeat again and again without damage to the granular medium, in stark contrast to the permanent failure of more common building materials such as steel, glass, or concrete. This raises the question: in terms of this self-healing, what is in the realm of possibilities for granular materials if the properties of the individual particles can be designed?

With recent advances in 3D-printing and DEM simulations, the range of bulk properties open to granular materials has begun to be studied by varying the individual particles. Novel properties for granular media are emerging, such as angles of repose larger than 90° [2], even while maintaining flowability in the unjammed state to aid in recyclability [3]. Work has been done to find particle shapes that optimize packing density [4] or elastic moduli [5], and an exploration into the role of particle shape in low strain behaviour [6] broke ground for what is studied in this work.

Specifically, we compress granular materials in a standard soil mechanics triaxial test (Fig. 1) beyond yielding and well into plastic deformation. In these strain-controlled experiments, sudden drops in stress result from failure events where the particles rearrange locally and settle into a new configuration. The stress supported by the granular material then ramps back up, only to drop again when the next weak point is found. Hundreds of such failure events, spanning several orders of magnitude in unloaded stress, can occur in a single experiment compressed to 20% axial strain. We find that, over a wide range of particle shapes, 3D-printed and laser-cut in house, two measures of granular plasticity serve well to quantify the spectrum of plasticity behaviour observed.

The first is a standard soil mechanics measure of shear strength called the angle of internal friction ψ and defined by [7]

$$\sin \psi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}, \tag{1}$$

with σ_1 the largest principal component of stress and σ_3 the smallest, here the axial stress and the confining pressure, respectively. ψ is a measure of the baseline stress in plastic deformation, the stress at which interior rearrangements take place, and it varies by more than a factor of four across our range of particles, from spheres to platy disks.

The second measure of plasticity, borrowed from financial mathematics, is called volatility and used to quantify jumps away from a (possibly nonstationary) baseline in a time series. Since
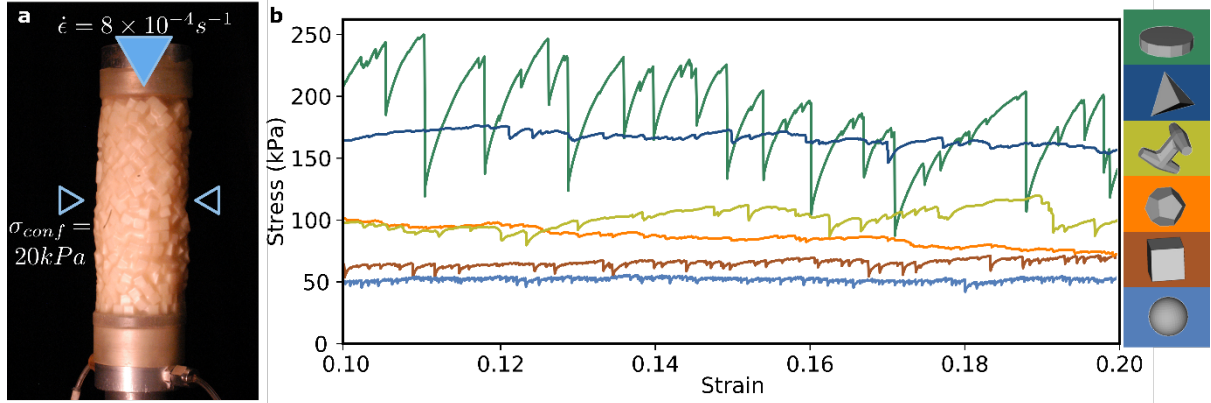
the fluctuations away from the mean plastic stress span decades in magnitude, volatility

Fig. 1. a) Photo of a granular column with experimental parameters. **b)** Raw plasticity data for shapes shown at right.

applies naturally to plastic stress data. It is defined by [8]

$$V = \frac{1}{N} \sqrt{\left(\sum_i^N (R_i - \bar{R})^2\right)}, \qquad R_i = \log\left(\frac{\sigma_i}{\sigma_{i-1}}\right), \qquad (2)$$

with N the total number of intervals measured and σ_i the stress at strain step i. V inherently depends on the strain interval $\Delta\epsilon$ between measurements, so we'll follow convention and "annualize" the volatility to 1% strain by multiplying by $\sqrt{0.01/\Delta\epsilon}$.

Using these two quantities as axes in an Ashby plot characterizing the plastic deformation of different granular materials, qualitative regions emerge (Fig. 2). Starting with rounded particles – spheres and the like – a low angle of internal friction means there is poor structural integrity in the packing to support shear stress. To improve upon shear strength, there are two options available, shown by the arms in the phase space. One is to use angular particles like tetrahedra and angular cubes which grip and grab each other to support shear stress. Plastic deformation proceeds via quiescent

Fig. 2. The Ashby plot for plasticity behavior for the particle shapes we tested. The particle sets with a black triangle are laser cut acrylic; the rest are 3D-printed hard plastic. The colored ellipses are qualitative.

grinding, as indicated by the low volatility. Another possible route to high shear strength is to use platy particles which can stack, such as cubes and disks. These have larger volatility, meaning violent stress drops where elastic strain energy is dissipated in discrete spurts. Presumably the strength arises from micro-columns formed in the interior of the granular packing, and the volatility results from their buckling.
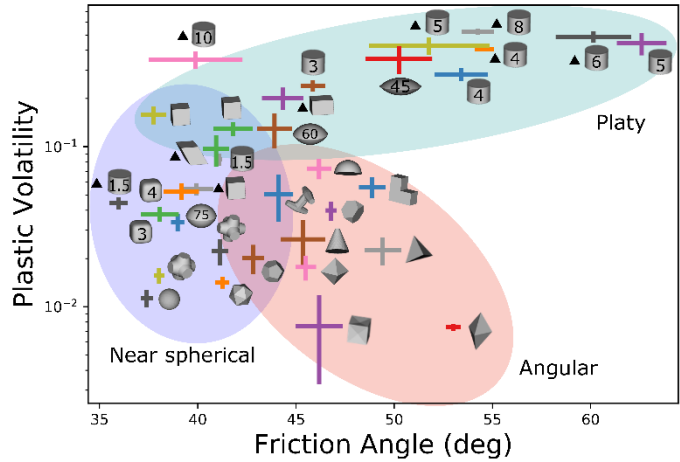
This novel phase space, which sets strength to shear stresses against violence of failure, needs much exploration. Where do interlocking particles, such as the entangling Zs, appear? How do surface properties (i.e., friction) enter into this picture? By mapping out the extents of

granular materials in this space of energy dissipation through repeated failure, we open up new avenues for material design.

Acknowledgments

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