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Experimental Measurements and Modeling of Convective Heat Transfer in the Transitional Rarefied Regime

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Abstract. We present experimental measurements and numerical simulations of convective heat transfer performance in the transitional rarefied regime for an isolated rectangular beam geometry. Experiments were performed using single crystalline silicon beam elements having width-to-thickness aspect ratios of 8.5 and 17.4. Devices were enclosed in a vacuum chamber and heated resistively using a DC power supply. A range of pressures corresponding to Knudsen numbers between 0.096 and 43.2 in terms of device thickness were swept, adjusting applied power to maintain a constant temperature of 50 K above the ambient temperature. Both parasitic electrical resistance associated with the hardware and radiative exchange with the environment were removed from measured data, allowing purely convective heat flux to be extracted. Numerical simulations were carried out deterministically through solution of the Ellipsoidal Statistical Bhatnagar-Gross-Krook collision model of the Boltzmann equation. Results agree with experimental data, revealing a strong coupling between dissipated heat flux and thermal stresses within the flowfield as well as a nonlinear transition between the free-molecule and continuum regimes.

Keywords: convective heat transfer, ellipsoidal statistical BGK model, transitional flow, vacuum

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INTRODUCTION

Presently, convective heat transfer in the transitional rarefied regime is not well characterized. With a growing number of thermally sensitive micro and nanodevices operating beyond the continuum limit, the ability to accurately evaluate thermal dissipation is critical to achieving desired performance. In recent years, substantial interest has been given to applications involving heated microcantilevers [1], namely dip-pen nanolithography [2], nanotopography [3], data storage [4], and thermogravimetry [5], however the precise design of these systems from a heat transfer standpoint remains a formidable challenge. Both experimental [6, 7] and numerical [8] studies have been presented which aim to quantify heat transfer performance for thin plate or cantilever geometries at transitional Knudsen numbers, however a complete reconciliation among results has yet to be established. The purpose of the present study is therefore to both numerically and experimentally characterize convective heat transfer performance for an isolated rectangular geometry in the transitional rarefied regime to provide insight into the mechanisms governing heat transfer at intermediate Knudsen numbers.

BACKGROUND

For bodies immersed in a fluid medium exhibiting continuum behavior, convective heat flux, q''_{net} , is sufficiently described by Newton's law of cooling.

$$q''_{net} = h(T_b - T_\infty) \quad (1)$$

where h is the convective heat transfer coefficient, T_b is the temperature of the body and T_∞ is the temperature of the surrounding fluid. In stagnant mediums driven only by the influence of hydrostatic pressure, convection is said to be natural or free, showing strong dependence on device geometry, material, and orientation. The convective heat transfer coefficient for continuum fluids is generally difficult to evaluate theoretically and must be obtained through empirical means. Typically, this parameter is determined from the nondimensional Nusselt number, Nu , having functional dependence on both the Rayleigh, Ra , and Prandtl, Pr , numbers.

$$Nu = \frac{hL_c}{k_f} = f(Ra, Pr) \quad (2)$$

FIGURE 1. Silicon beam fabrication process.

where k_f is the thermal conductivity of the fluid and L_c is the characteristic length of the system under consideration. Correlations describing heat transfer performance for a variety of geometries and orientations can be found in the reference [9, 10, 11]. When evaluating heat transfer problems involving convection in the transitional regime, these classical correlations are generally invalid, owing to the reduced influence of buoyancy in such flows.

For systems operating in significantly rarefied conditions, heat transfer can be described in terms of equilibrium heat fluxes. For the simple rectangular beam elements considered in the present study, heat flux can be evaluated analytically through balance of the energy exchange among incident and emitted species. In terms of the convective heat transfer coefficient:

$$h_{FM} = \frac{1}{T_b - T_\infty} \frac{\gamma + 1}{\gamma - 1} \frac{\rho_\infty}{8\sqrt{\pi}} \left(\frac{1}{\beta_\infty \beta_b^2} - \frac{1}{\beta_\infty^3} \right) \quad (3)$$

where γ is the specific heat ratio of the gas, ρ_∞ is the density of the far-field, and β_b and β_∞ are the inverse most probable speed for temperatures associated with the beam and far-field respectively. For proximal bodies in flows with noncontinuum Knudsen numbers, Torczynski et al. have proposed the following correlation for the convective heat transfer coefficient [12],

$$h_{model} = \left(1 + \frac{\zeta}{4} \right) \left(\frac{\sigma}{S_1 S_2} \right) \left(\frac{p \bar{c}}{T} \right) \quad (4)$$

Here, ζ is the number of internal molecular degrees of freedom, σ is the accommodation coefficient, p is the ambient pressure, $\bar{c}$ is the mean molecular thermal speed, and S_1 and S_2 are correction factors, giving the model flexibility for various system configurations and degrees of rarefaction. Expressions for these parameters can be found in the reference. In this work, the limiting values of 2 and 1 were taken for S_1 and S_2 respectively, corresponding to an isolated body in the free-molecule regime.

EXPERIMENTAL CONFIGURATION

Devices were fabricated from a 100 mm diameter single crystal silicon wafer having a thickness of 540 μm and resistivity of 0.01 Ω – cm. An illustration of the process is provided in figure 1. To provide electrical contacts, a 100 nm seed layer of copper was initially deposited over the wafer surface. Areas not used for the electrical interface were then masked using approximately 7 μm of AZ9260 photoresist. Following lithography, the wafer was placed in a copper sulfate bath whereby exposed surfaces were electroplated with 50 μm of additional copper. After deposition, the photoresist was removed and the wafer was submersed into a copper etching solution until the thin seed layer was stripped from all unplated surfaces. The wafer was then diced into rectangular beam segments of varying width. After dicing, two T-type thermocouples were affixed to the face of each beam using thermal adhesive, one near the center and the other near the electrical contacts. Lead wires were subsequently soldered to the copper contacts and the

FIGURE 2. Schematic of Silicon beam devices for both horizontal and vertical orientations (dimensions in *mm*).

FIGURE 3. Experimental apparatus used for heat flux measurements.

beams were mounted in the experimental apparatus. A schematic of the beams with associated dimensions is shown in figure 2. The hardware configuration is provided in figure 3.

Measurements were performed in a 4.3 m^3 vacuum chamber for ambient pressures between 0.37 and 124.8 *Pa*, corresponding to a Knudsen number range between 0.096 and 43.2 in terms of device thickness. Beams were suspended 50 *cm* above a leveled optical table using two stainless steel risers and were held in place using TDI reverse-action tweezers with CF30 insulating tips. Use of a ball-joint at the tweezers' fixtures permitted manual adjustment of beam angle relative to the table, allowing the influence of buoyancy to be evaluated for horizontal and vertical orientations.

The silicon beams were heated resistively by passing current through the electrical contacts. At each pressure, the bias was adjusted to maintain an average temperature difference of 50 *K* between thermocouples and the ambient. Heat flux was measured by monitoring total steady-state power through the beam. Due to the relatively large width-to-thickness aspect ratios of the devices, power dissipated through the lateral edges was neglected. Thus, total heat flux was computed as half of the total dissipated power divided by the area of the major face. Here the major face is taken as the surface having its perimeter defined by the beam width (9.4 - 18.4 *mm*) and length (71.64 *mm*) as defined in figure 2. Experimental data in terms of total dissipated power is shown in figure 4. As expected, increasing pressure generates higher capacity for convective heat transfer, requiring a larger input to maintain a constant temperature across the beams.

In all experiments, both current and voltage were monitored at the power supply terminals. Thus, a temperature-dependent parasitic component of electrical resistance, R_p , associated with the lead wires and vacuum chamber

FIGURE 4. Total dissipated power from silicon beams.

TABLE 1. Intercepts calculated from linear least squares fit

Aspect Ratio (Orientation)	q''_{rad} [W/m ²]	R ²
$w/t = 8.5$ (V)	373.5	0.990
$w/t = 8.5$ (H)	380.6	0.995
$w/t = 17.4$ (V)	331.5	0.973
$w/t = 17.4$ (H)	298.7	0.994

feedthrough must be addressed. This component was evaluated by placing a jumper across the beam and measuring the series resistance of the short circuit at the supply terminals. This parasitic component was measured as 1.10Ω and was assumed constant among all tests as room temperature was fixed at $28.0 \pm 0.5^\circ\text{C}$.

In the free-molecule regime, heat flux varies linearly with pressure, allowing radiative heat flux, q''_{rad} , to be extracted from the measured data through removal of the intercept formed by a least-squares linear fit of total heat flux magnitude for Knudsen numbers greater than 10. A summary of extracted radiative heat transfer components is provided in table 1. Removing both the parasitic resistance and radiative heat transfer components from the measured data, the net convective heat flux, q''_{exp} , becomes

$$q''_{\text{exp}} = IV - I^2 R_p - q''_{\text{rad}} \quad (5)$$

Here I is the current through the beam and V is the potential across the supply terminals.

NUMERICAL MODELING

To evaluate the flow mechanisms governing convective heat transfer in the transitional rarefied regime, numerical simulations were carried out using a 2D2V Ellipsoidal Statistical Bhatnagar-Gross-Krook (ES-BGK) collision model of the Boltzmann equation. Solver details can be found in the reference [13]. All simulations assumed steady flow and neglected the influence of external forces. Physical space was discretized using a cartesian mesh containing 180×150 nonuniform quadrilateral elements. A spherical discretization was applied to velocity space consisting of a 16th-order Gauss-Hermite quadrature with 64 uniformly distributed velocity angles. A schematic of the computational domain is shown in figure 5. Corresponding modeling parameters are provided in table 2.

Flowfield results for Knudsen numbers of 1.28 and 0.18 are shown in the left and right half of figure 6 respectively. The uniform device temperature suggests streamline behavior is largely governed by the effects of thermal edge flow, the net inward motion of fluid from the outer edge towards the centerline [14]. As the inward flow encounters the fluid

FIGURE 5. Computational domain used in ES-BGK simulations.

TABLE 2. Input parameters used in ES-BGK simulations.

Parameter	Value
Working Fluid	Nitrogen
Domain Width (W)	40.0 [mm]
Domain Height (H)	40.0 [mm]
Beam Width (w)	9.4 [mm]
Beam Thickness (t)	540 [μ m]
Beam Temperature (T_w)	350 [K]
Ambient Temperature (T_∞)	300 [K]
Ambient Pressure (p_{ref})	0.37 - 124.7 [Pa]
Accommodation Coefficient	1.0
Prandtl Number	0.716

from the opposite edge, the streamlines are forced upwards and a high pressure region is formed above the beam. This effect, combined with the low pressure zone off the lateral edge formed by the inward flow results in the manifestation of the large vortical structure seen in the left half of figure 6. Flows of this type have been studied extensively for a comparable geometry in an enclosed container [15, 8], reporting a similar recirculating behavior. At higher pressures, the influence of the thermal edge flow diminishes, reducing the strength of the vortex until its eventual collapse near a Knudsen number of around 0.3. Here, flow behavior becomes largely governed by diffusion. This effect can be observed in the shear stress distributions normalized by the integrated average over the primary and lateral faces in the left and right halves of figure 7. For large Knudsen numbers, the thermally induced stresses are small, showing a localization at the edges of the beam element. Decreasing Knudsen number leads to a systematic increase in shear stress concentration near the edge with a corresponding decrease near the centerline. This behavior continues until $Kn = 2.44$ whereby the average shear stress over the primary face swings negative. Further decreasing Knudsen number maintains this reversal in flux profile orientation, contributing to the weakening of the edge effect and collapse of the vortex as seen in figure 6.

Net heat flux distributions normalized by the integrated average over both the primary face and lateral edge of the 8.5 aspect ratio beam element for Knudsen numbers between 0.096 and 32.49 are provided in figure 8. For large Knudsen numbers, the heat flux distribution is relatively uniform over all surfaces. This result stems from the largely collisionless nature of the flow, with the majority of incident species originating from the uniform temperature far-field. As pressure is increased, thermal dissipation becomes concentrated near the free-edge with a corresponding decrease

FIGURE 6. Temperature contours for $Kn = 1.28$ (left) and 0.18 (right).

FIGURE 7. Shear stress profiles over half width of primary (left) and lateral (right) faces.

in flux towards the center of the face. The enhancement in heat flux results from exposure of the beam free-edge to the lower temperature, higher number density fluid in the far-field, facilitating a larger capacity for thermal dissipation. Similar effects are observed along the lateral edge of the plate.

COMPARISON OF EXPERIMENTAL AND NUMERICAL RESULTS

Convective heat flux coefficients for aspect ratios (w/t) of 8.5 and 17.4 are shown in figure 9 with (V) representing the vertical orientation and (H) the horizontal orientation. The limiting continuum fluxes correspond to the heat transfer coefficient on the upper and lower faces of a horizontally oriented beam [10] as well as a vertically oriented device [9].

As expected, all data exhibit linear behavior near the free-molecule regime, showing no dependence on orientation or geometry. Comparing flux magnitude of experimental data to the analytical free-molecule solution for Knudsen numbers greater than 10 an average discrepancy of around 37% is present, accounting for the less-than-unity ac-

FIGURE 8. Dissipated heat flux profiles over half width of primary (left) and lateral (right) faces.

FIGURE 9. Convective heat transfer coefficient in terms of Knudsen number.

commodation coefficient associated with the gas-solid interface. Increasing Knudsen number, both experimental and numerical data show a kink in heat flux magnitude, however for the measured data this feature occurs at a Knudsen number roughly half that of the numerical results. A possible explanation for this apparent modeling error is the nonuniform temperature distribution along the axial direction associated with the real beam elements. During the experiments, temperature was maintained at 50 K above the ambient using the average between the central and peripheral thermocouples. For all tests, the temperature difference between these locations was measured as roughly 6 K, $\pm 6\%$ of the nominal value. Additional modeling error could also result from uncertainty in measured device geometry, with a maximum around 5% of the device thickness.

Over the range of measured Knudsen numbers, data for the aspect ratio of 8.5 are largely coincident whereas a discrepancy in heat transfer performance is observed for the larger device, most notably when placed in the horizontal configuration. Although the effects of buoyancy would be expected to show significance for the larger element, the disparity observed between extracted radiative heat transfer components suggest a conclusion based solely on these

effects may be incorrect. In terms of percent difference, a maximum error of 24.1% arises between the extracted intercepts of table 1. This result stems from the decreased convective dissipation at low pressures, contributing to a high sensitivity in measurement uncertainty. Ideally, this error could be rectified through an increase in sample density at free-molecule pressures, however the sensitivity in chamber pressure to valve position, the relatively high ultimate pressure (≈ 0.15 Pa) of the roughing pumps, as well as the modest resolution (10 mV) of the DC power supply prohibited the implementation of this solution. Errors in the least-squares fit evident from the correlation factors of table 1 also likely contribute to the mild discrepancy between experimental measurements and ES-BGK simulations.

CONCLUSIONS

Experimental measurements of convective heat transfer performance were carried out in a 4.3 m³ vacuum chamber using resistively heated 540 μ m thick single crystalline silicon beam devices of varying aspect ratios. In all tests, the beams were maintained at a constant 50 K above ambient temperature, varying Knudsen number directly with chamber pressure. Total dissipated power was monitored at the power supply terminals, allowing convective heat flux to be extracted through removal of both parasitic electrical resistance associated with the lead wires and radiative exchange with the environment. Results demonstrate a nonlinear transition in heat transfer performance between the free-molecule and continuum regimes.

As expected, all data exhibit linear behavior for large free-molecule Knudsen numbers, asymptotically approaching empirically defined continuum limits as pressure is increased. An abrupt change in heat transfer performance was observed for Knudsen numbers near 0.2 in experimental measurements, likely owing to the collapse of a large primary vortical structure formed by thermal edge effects. This collapse acts to enhance thermal dissipative performance for further increases in pressure. The effects of buoyancy do not appear to appreciably influence heat transfer performance, namely for the small aspect ratio device, however a larger sampling of data corresponding to Knudsen numbers in the free-molecule range are still needed to conclusively rule out these effects.

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