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Introduction

Grazing Incidence Applications

- Porous materials are often used to line “channels”
- Previous work shows that the properties that control grazing direction attenuation are somewhat different from those that control normal absorption.

Here introduce anisotropic theory to account for different flow resistivity of the lining material in normal and grazing directions.
Duct lining geometry and coordinate system for poro-elastic lining

Direction of sound propagation

Stress in solid

Stress in fluid

Transversely isotropic poro-elastic displacement and stress field

\[ u_x = e^{-jkx}\left( \sum_{i=1}^{4} \alpha_i C_i e^{-j\beta_i z} + \sum_{i=5}^{6} C_i e^{-j\beta_i z} \right) \]

\[ u_z = e^{-jkx}\left( \sum_{i=1}^{4} C_i e^{-j\beta_i z} + \sum_{i=5}^{6} \alpha_i C_i e^{-j\beta_i z} \right) \]

\[ U_x = e^{-jkx}\left( \sum_{i=1}^{6} \beta C_i e^{-j\beta_i z} \right) \]

\[ U_z = e^{-jkx}\left( \sum_{i=1}^{6} \gamma_i C_i e^{-j\beta_i z} \right) \]

\[ \sigma_z = -je^{-jkx}\left[ \sum_{i=1}^{4} \{k_x(F\alpha_i + Q\beta_i) + k_z(C + Q\gamma_i)\}C_i e^{-j\beta_i z} \right. \]

\[ + \sum_{i=5}^{6} \{k_x(F + Q\beta_i) + k_z(C\alpha_i + Q\gamma_i)\}C_i e^{-j\beta_i z} \]

\[ \tau_{xz} = -je^{-jkx}\left[ \sum_{i=1}^{4} \{k_x(F\alpha_i + Q\beta_i) + k_z(C + Q\gamma_i)\}C_i e^{-j\beta_i z} \right. \]

\[ + \sum_{i=5}^{6} \{k_x(F + Q\beta_i) + k_z(C\alpha_i + Q\gamma_i)\}C_i e^{-j\beta_i z} \]

\[ \tau_{yz} = \sum_{i=1}^{6} \{k_x(F\alpha_i + Q\beta_i) + k_z(C\alpha_i + Q\gamma_i)\}C_i e^{-j\beta_i z} \]

\[ s = -je^{-jkx}\left[ \sum_{i=1}^{4} \{k_x(F\alpha_i + Q\beta_i) + k_z(C + Q\gamma_i)\}C_i e^{-j\beta_i z} \right. \]

\[ + \sum_{i=5}^{6} \{k_x(F + Q\beta_i) + k_z(C\alpha_i + Q\gamma_i)\}C_i e^{-j\beta_i z} \].
Boundary condition: fully-lined model

- 2-D porous formulation of the free-wave solution in a lined duct.
- Rigid wall boundary condition
- Simplified by symmetric condition
- Solve for $k_x$ to find attenuation rate in lining.

\[
\begin{align*}
\text{at } y = -d & \quad \text{at } y = d \\
(1) \ u_{x(y, \text{symm})} &= 0 & (4) \ u_{x(y, \text{symm})} &= 0 \\
(2) \ u_{y(y, \text{symm})} &= 0 & (5) \ u_{y(y, \text{symm})} &= 0 \\
(3) \ U_{y(y, \text{symm})} &= 0 & (6) \ U_{y(y, \text{symm})} &= 0 \\
\end{align*}
\]

- The characteristic dispersion equation

\[
A_f X_f = 0
\]

\[
X_f = \begin{bmatrix} C_1^* & C_3^* & C_5^* \end{bmatrix}^T
\]

\[
\det(A_f) = 0
\]
Boundary condition: partially-filled model (FA)

- 2-D porous formulation of the free-wave solution in a lined duct
- Rigid wall boundary condition
- Solve for $k_x$ to find attenuation rate in lining.

\[ \begin{align*}
  \text{at } y &= 0 \\
  (1) & u_{x,1} = 0 \\
  (2) & u_{y,1} = 0 \\
  (3) & U_{y,1} = 0 \\
  \text{at } y &= L_1 \\
  (4) & -\Omega_p P = s \\
  (5) & -(1-\Omega_p)P = \sigma_y \\
  (6) & v_y = j\omega(1-\Omega_p)u_y + j\omega\Omega_p U_y \\
  (7) & \tau_{yx} = 0 \\
  \text{at } y &= L_2 \\
  (8) & v_y = 0 \\
  A_p X_p &= 0 \\
  X_p &= \begin{bmatrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & A_1 & A_2 \end{bmatrix}^T \\
  \det(A_p) &= 0
\end{align*} \]
Square duct system and glass fiber layer

- 4-microphone transfer matrix method used to measure attenuation at grazing incidence
Effect of Material Anisotropy

Measurement of TL in two configurations for glass fiber (green)

**Case 1:**
9 pieces layered in vertical direction

- 9 pieces layered in horizontal direction

- Acoustic properties depend on material orientation – flow resistivity larger in normal direction
Anisotropic Duct Lining

• Prediction and measurement for **Yellow glass fiber** by using anisotropic poro-elastic model

• Different flow resistivities [MKS Rayls/m] in x- and y-directions

\[ \sigma_x = 7500 \quad \sigma_y = 15000 \]

![Diagram](attachment:image.png)

**Graph**

- 1st least attenuated mode (Prediction)
- 2nd least attenuated mode (Prediction)
- Measurement

**Axes**

- **X**: Frequency in Hz
- **Y**: Attenuation ratio in dB/0.5m
Sensitivity to transverse flow resistivity

Yellow glass fiber

- Iso. model: 15000
- Aniso. model: 15000(x), 7500(y)
- Aniso. Model: 7500(x), 15000(y)
- Iso. model: 7500

Green glass fiber

- Iso. model: 26000
- Aniso. model: 26000(x), 16000(y)
- Aniso. Model: 16000(x), 26000(y)
- Iso. model: 16000
Conclusions

• A free wave propagation model based on a transversely isotropic poro-elastic theory was implemented for both fully-lined and partially-lined duct systems.

• Performed experimental measurements of lined duct attenuation rates in x- and y-directions.

• Showed good agreement between predictions and measurements for partially-lined duct case.

• The study of flow resistivity shows that the x-direction is more effective than that of y-direction for controlling the sound attenuation performance in the duct system.

• Comparison of isotropic and anisotropic models shows that anisotropic model is more effective for predicting anisotropic real behavior of the lining materials.