1998

Development of Axisymmetric Finite Elements for Poroelastic Materials

J Stuart Bolton
Purdue University, bolton@purdue.edu

Yeon June Kang
Seoul National University, Korea

In Hwa Jung
Seoul National University, Korea

Bryce K. Gardner
Automated Analysis Corp.

DEVELOPMENT OF AXISYMMETRIC FINITE ELEMENTS FOR POROELASTIC MATERIALS

Yeon June Kang and In Hwa Jung

School of Mechanical & Aerospace Engineering

Seoul National University, Korea

J. Stuart Bolton

Ray W. Herrick Laboratories, Purdue University, U.S.A.

Bryce K. Gardner

Automated Analysis Corporation, Ann Arbor, MI, U.S.A.

OUTLINE

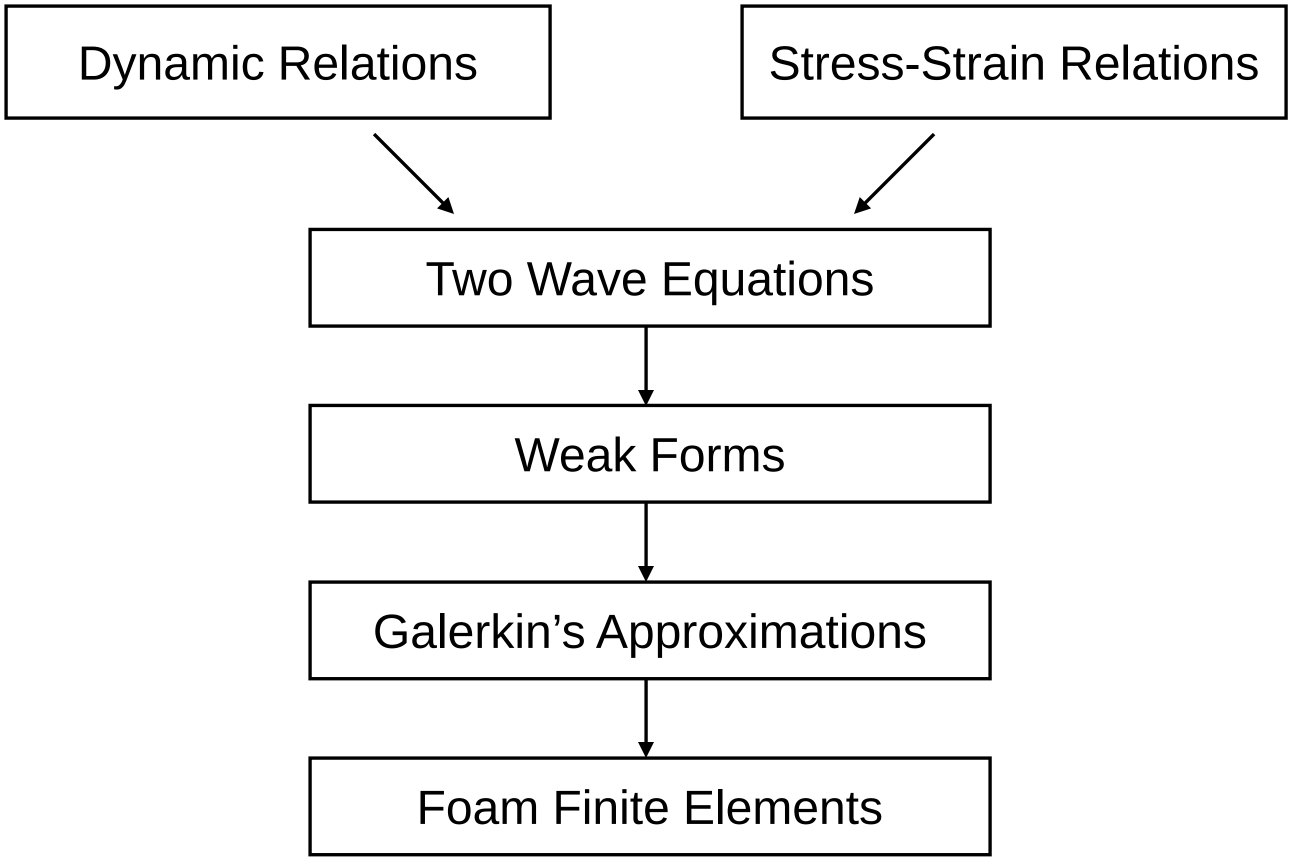
- Introduction
- Axisymmetrical Foam Finite Elements
- Sound Transmission through Cylindrical & Conical Foam Plug
  - validation with 3-D Cartesian solution
  - comparison with experimental results
  - effect of finite termination impedance
- Sound Attenuation in Foam-Lined Duct
  - comparison with Morse's solution
  - comparison with experiment results
  - effect of circumferential boundary condition

INTRODUCTION

- Cartesian Finite Elements of Poroelastic Materials
  - Normal incidence absorbtion coefficient
    (Y. J. Kang and J. S. Bolton, *J. Acoust. Soc. Am.* **98**, 1995)
  - Normal incidence sound transmission loss
    (Y. J. Kang and J. S. Bolton, *J. Acoust. Soc. Am.* **99**, 1996)
    (J. P. Coyette and H. Wynendaele, Inter-Noise 95)
    (N. Attala and R. Panneton, Inter-Noise 95)
  - Sound transmission through poroelastic wedges
    (Y. J. Kang and J. S. Bolton, *J. Acoust. Soc. Am.* **102**, 1997)
- Sound Propagation along Lined Ducts
  - Axisymmetric circular ducts, Locally reacting
    (Y. Kagawa *et al., J. Sound & Vib.* **53**, 1977)
  - Rectangular ducts, Extended & Locally reacting
    (R. J. Astley and A. Cummings, *J. Sound & Vib.* **116**, 1987)

Elastic Porous Material Theory based on Biot

3D (r, θ, z) $\longrightarrow$ 2D (r, z)

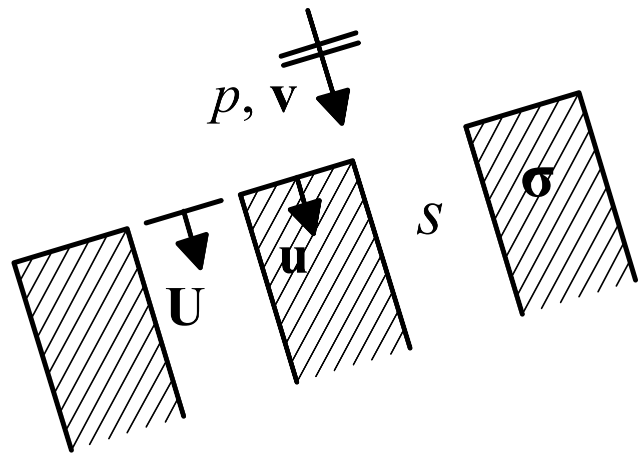


- Uncoupled System Equations

$$\begin{bmatrix} [\boldsymbol{K_a}] & \\ & \boldsymbol{K_f} \end{bmatrix} \begin{Bmatrix} p \\ u_r \\ u_z \\ U_r \\ U_z \end{Bmatrix} = \begin{Bmatrix} -j\omega\rho_0 2\pi\int_{\Gamma} r\phi_i (n_{ar}v_r + n_{az}v_z)d\Gamma \\ \int_{\Gamma^e} r\phi_i (n_r\sigma_r + n_z\tau_{zr})d\Gamma \\ \int_{\Gamma^e} r\phi_i (n_r\tau_{zr} + n_z\sigma_z)d\Gamma \\ \int_{\Gamma^e} r\phi_i n_r s\, d\Gamma \\ \int_{\Gamma^e} r\phi_i n_z s\, d\Gamma \end{Bmatrix}$$

→ need to be coupled using appropriate boundary conditions at the interface of two systems

- Boundary Conditions



- Velocity continuity : $\mathbf{v}_a = j\omega(1-h)\mathbf{u} + j\omega h\mathbf{U}$

- Force equilibrium (fluid part) : $hp\mathbf{n}_a = s\mathbf{n}_f$

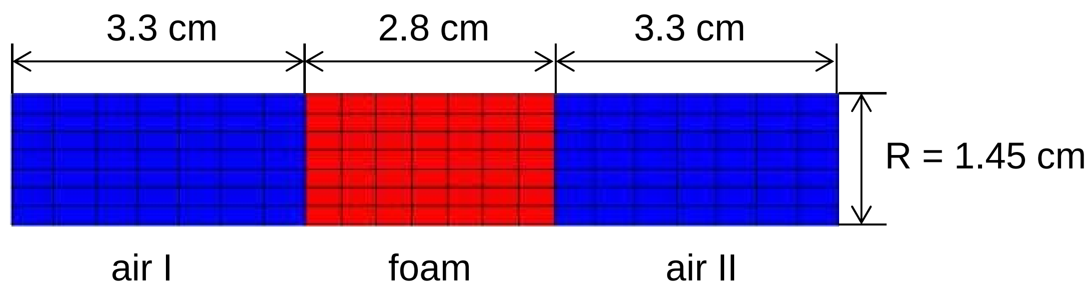
- Force equilibrium (frame part) : $(1-h)p\mathbf{n}_a = r(\sigma_r n_{fr} + \tau_{zr} n_{fz})\mathbf{i} + r(\tau_{zr} n_{fr} + \sigma_z n_{fz})\mathbf{k}$

- Coupled Acoustic-Foam System Equations

$$\begin{bmatrix} [\boldsymbol{K}_a] & [\boldsymbol{K}_{af}] \\ [\boldsymbol{K}'_{af}] & [\boldsymbol{K}_f] \end{bmatrix} \begin{Bmatrix} p \\ u_r \\ u_z \\ U_r \\ U_z \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{Q} \\ \boldsymbol{F}^1 \\ \boldsymbol{F}^2 \\ \boldsymbol{F}^3 \\ \boldsymbol{F}^4 \end{Bmatrix}$$

SOUND TRANSMISSION THROUGH CYLINDRICAL FOAM PLUG

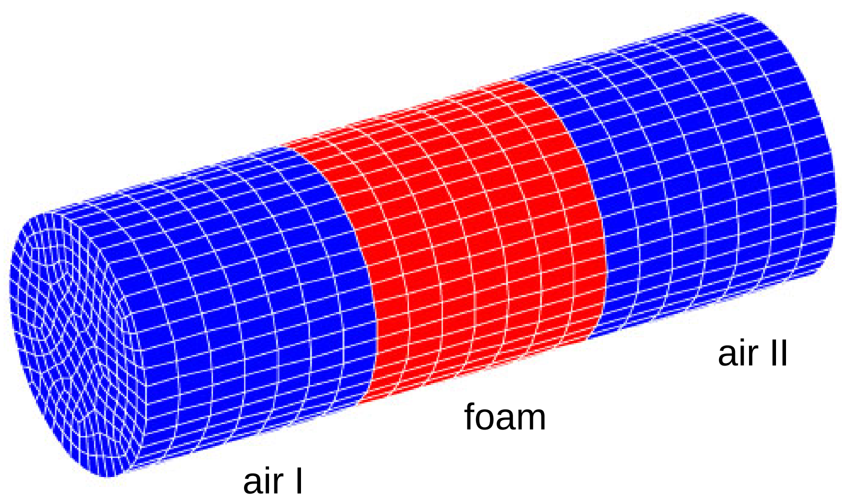
- Axisymmetric vs. 3-D Cartesian



49 foam elements

98 air elements

192 nodes



1456 foam elements

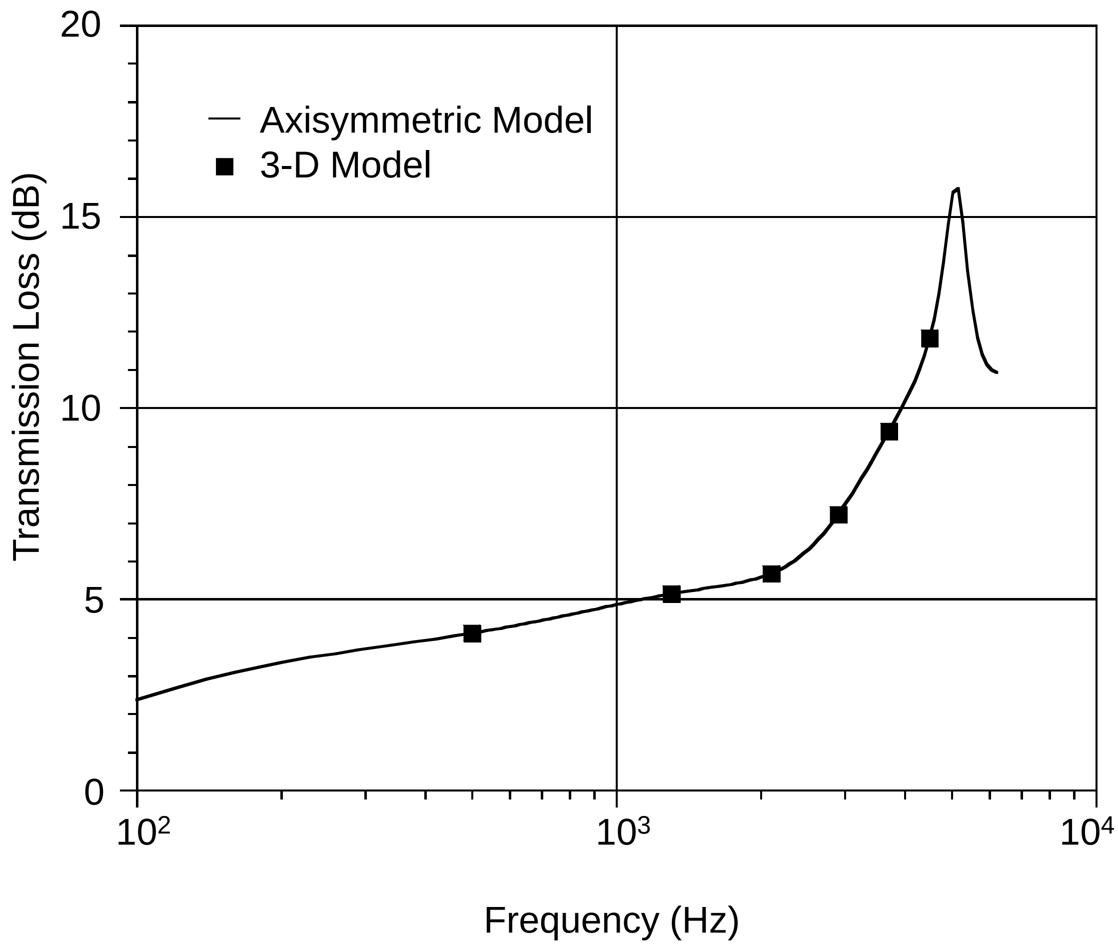
2912 air elements

5082 nodes

* It takes 5500 times longer solution time at each frequency !

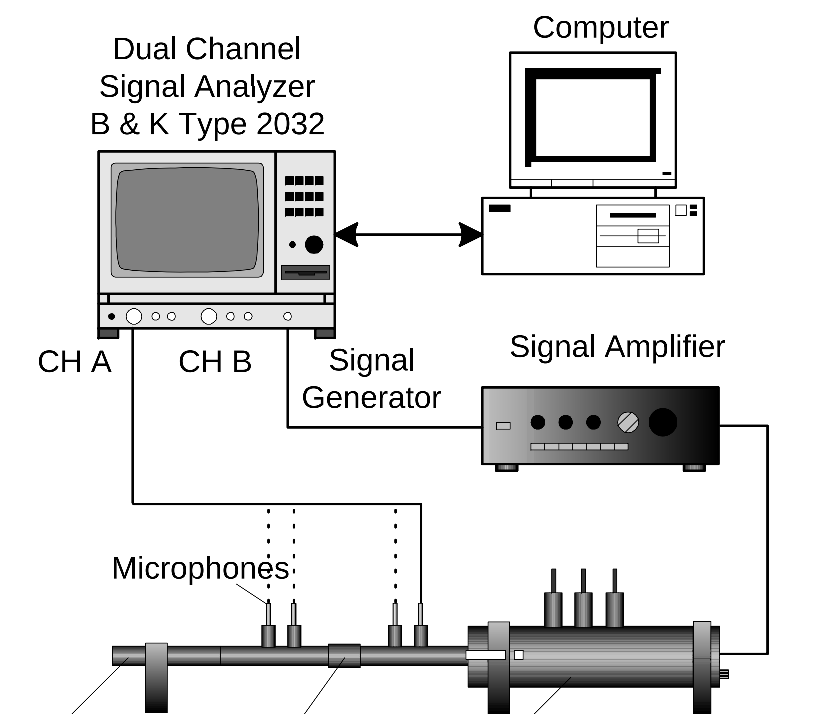
SOUND TRANSMISSION THROUGH CYLINDRICAL FOAM PLUG

- Validation with 3-D Cartesian Solution

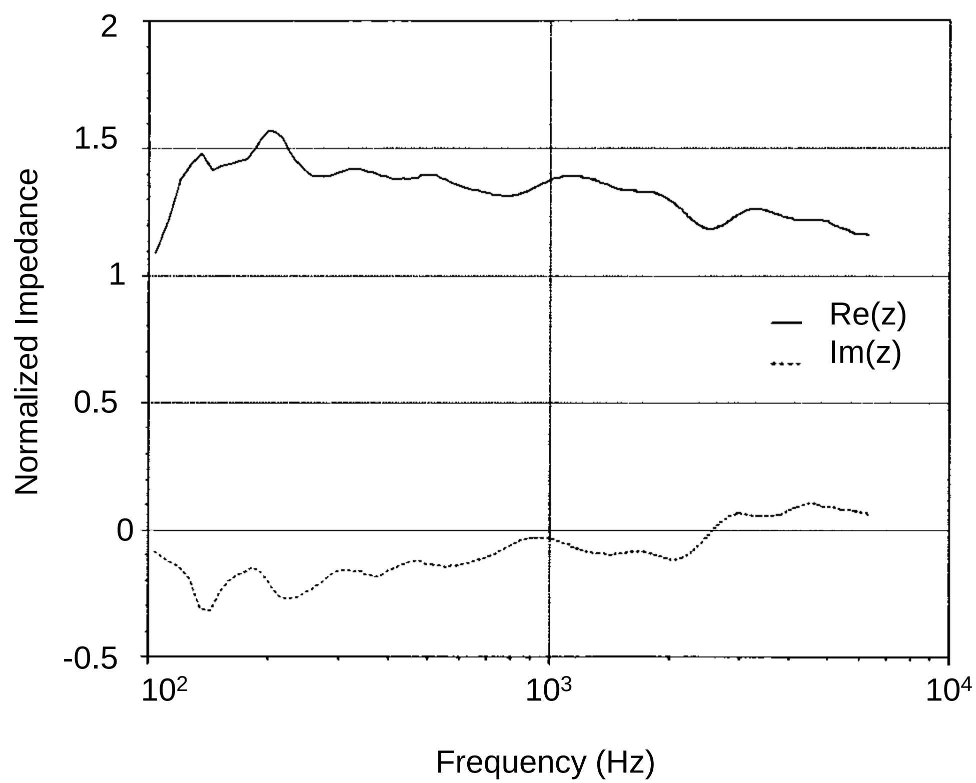


SOUND TRANSMISSION THROUGH FOAM PLUG

- Experimental Setup



- Anechoic Termination

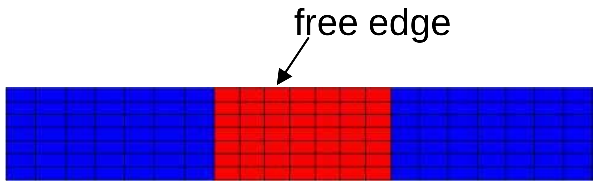


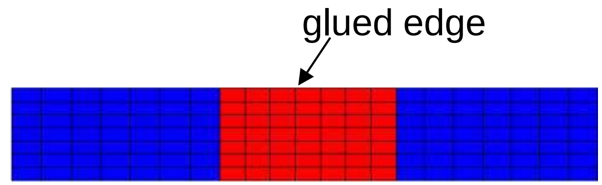
* Note:

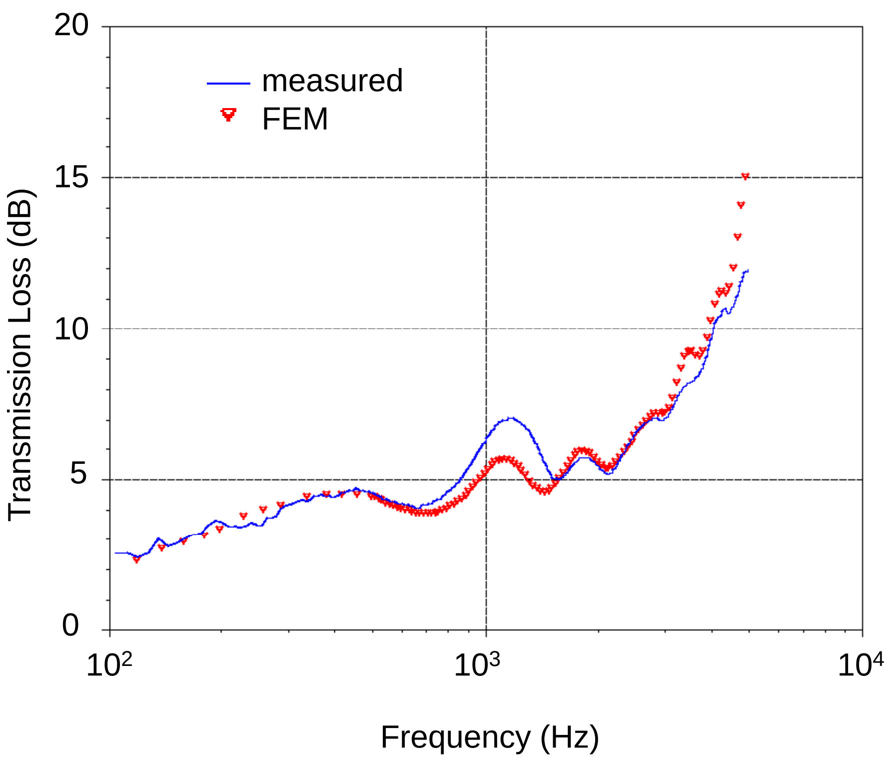
Measured impedance data was phase - corrected when it was applied to the model.

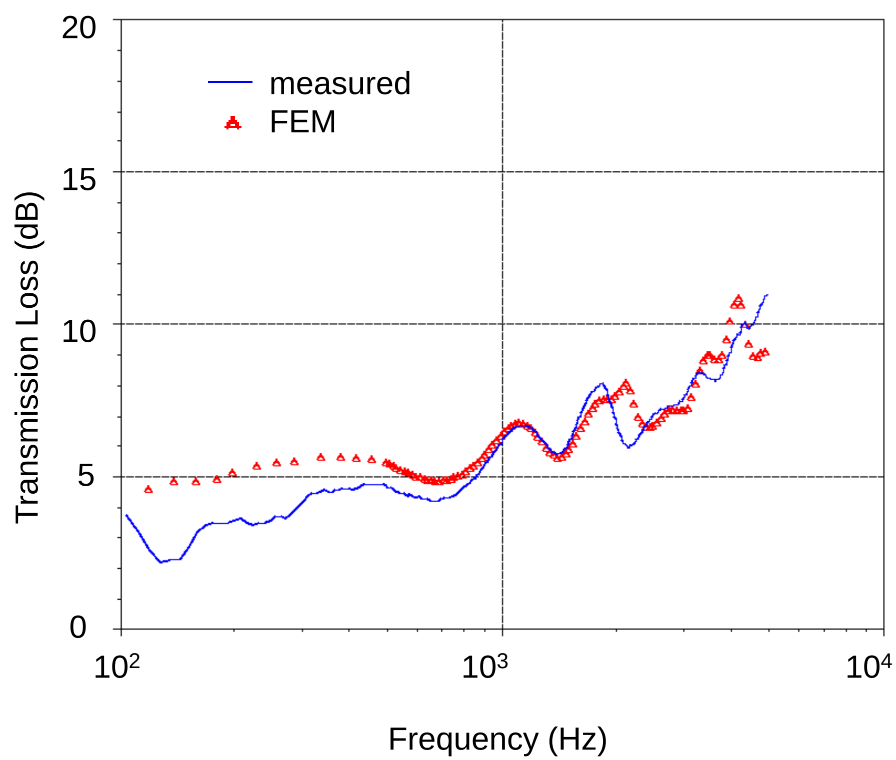
SOUND TRANSMISSION THROUGH CYLINDRICAL FOAM PLUG

- Effect of Circumferential Boundary Conditions

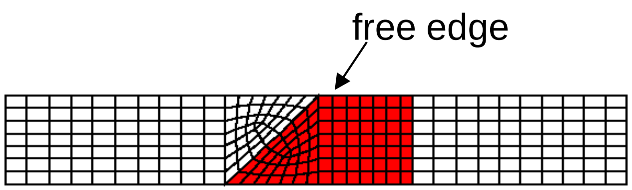


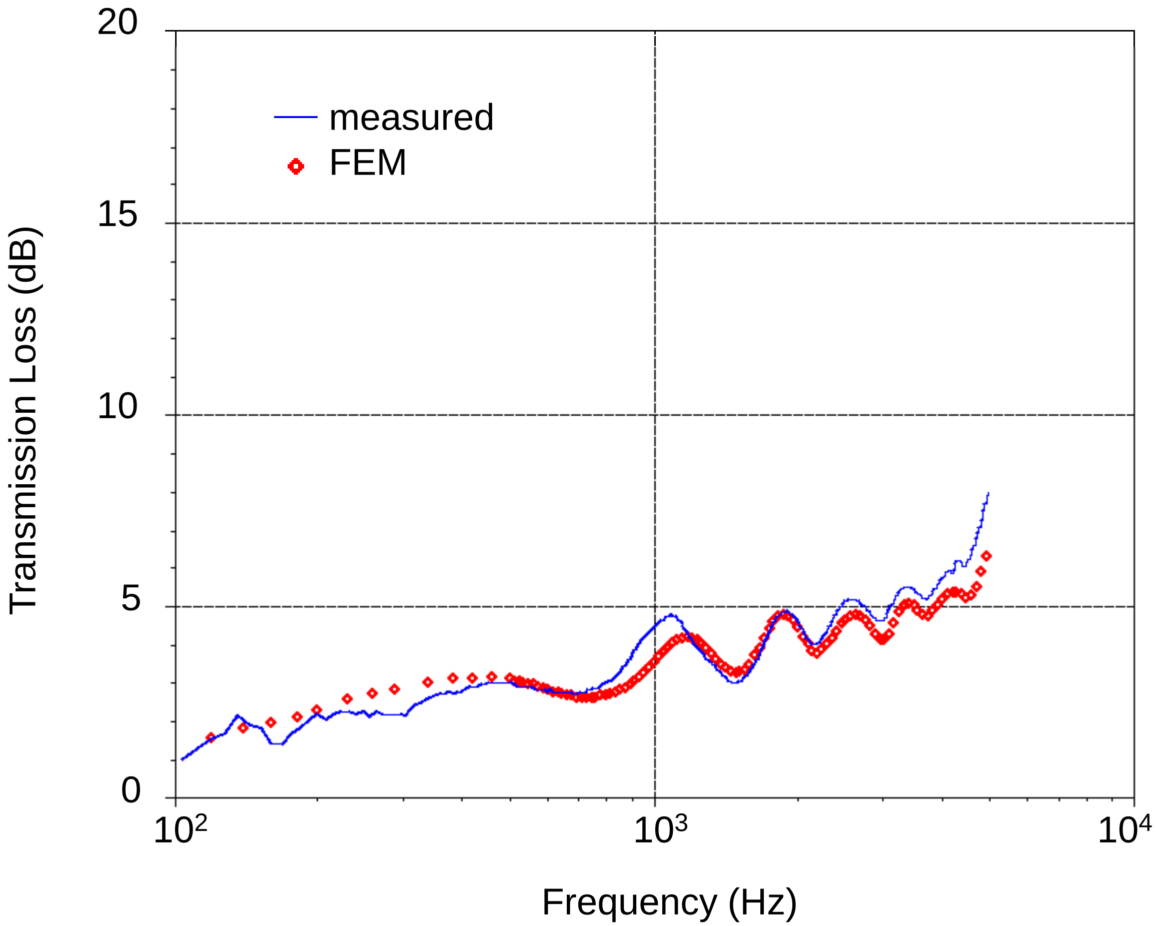






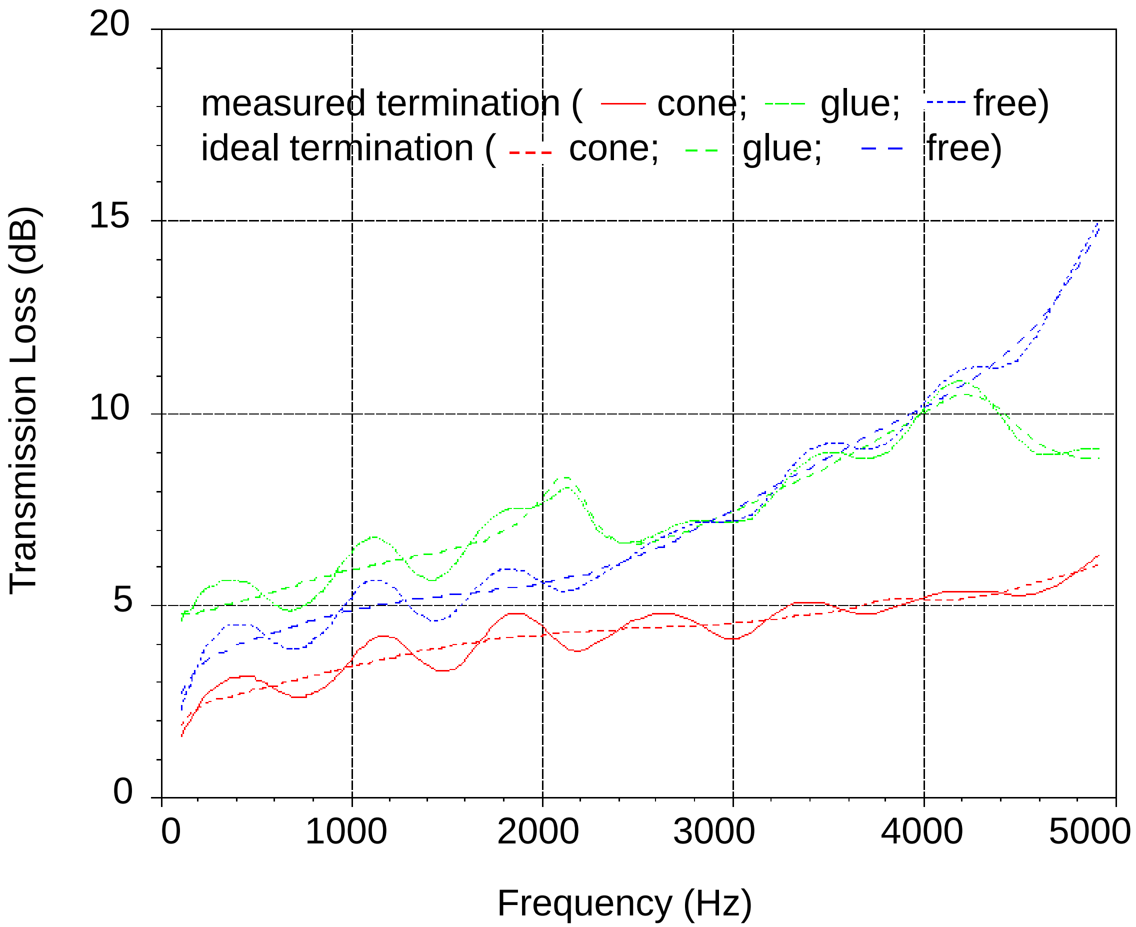
SOUND TRANSMISSION THROUGH CONICAL FOAM PLUG





SOUND TRANSMISSION THROUGH CYLINDRICAL & CONICAL FOAM PLUG

- Effect Finite Termination Impedance



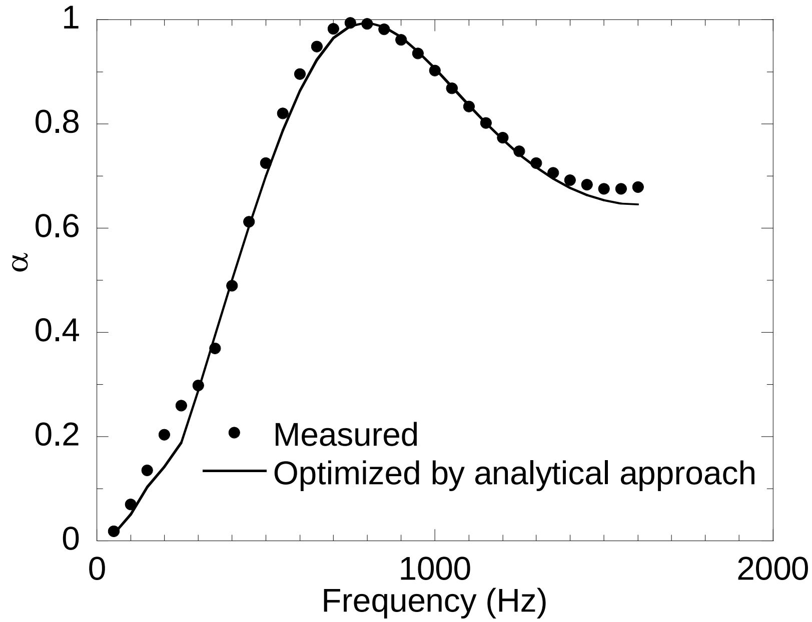
- Macroscopic Physical Properties of Foams Obtained by Measurement and Optimization

Foam Type / Parameter	Foam A (polyester)	Foam B (polyether)	Foam C (polyester)	
Flow resistivity (mks Rayls/m)	13666	30814	46417	measured
Tortuosity (Structure factor)	3.58	4.28	6.13	optimized
Porosity	0.96	0.96	0.96	assumed
Bulk density (kg/m^3)	32	29	32	measured
Bulk Young's Modulus (Pa)	30400	25200	85800	optimized
Loss factor	0.3	0.3	0.3	assumed
Poisson's ratio	0.4	0.4	0.4	assumed

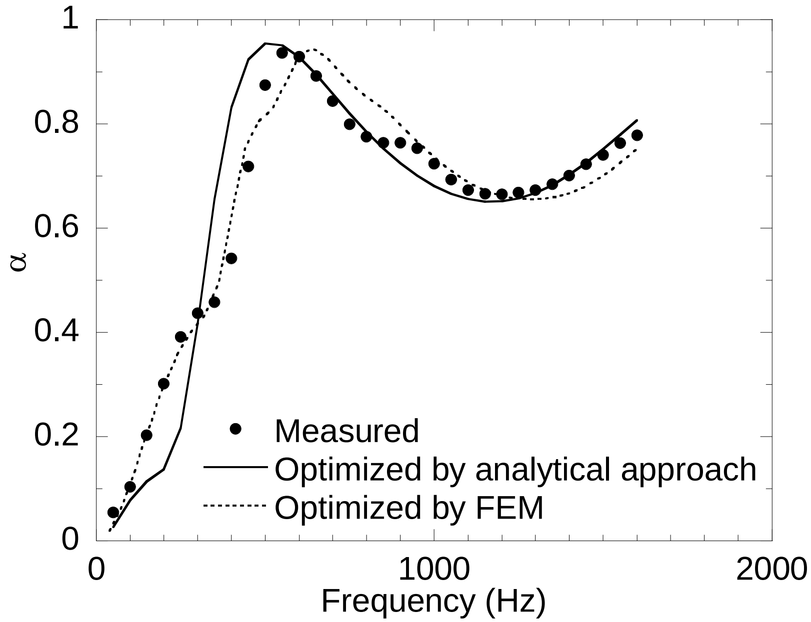
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

- Measured and Predicted Absorption Coefficient

Foam A

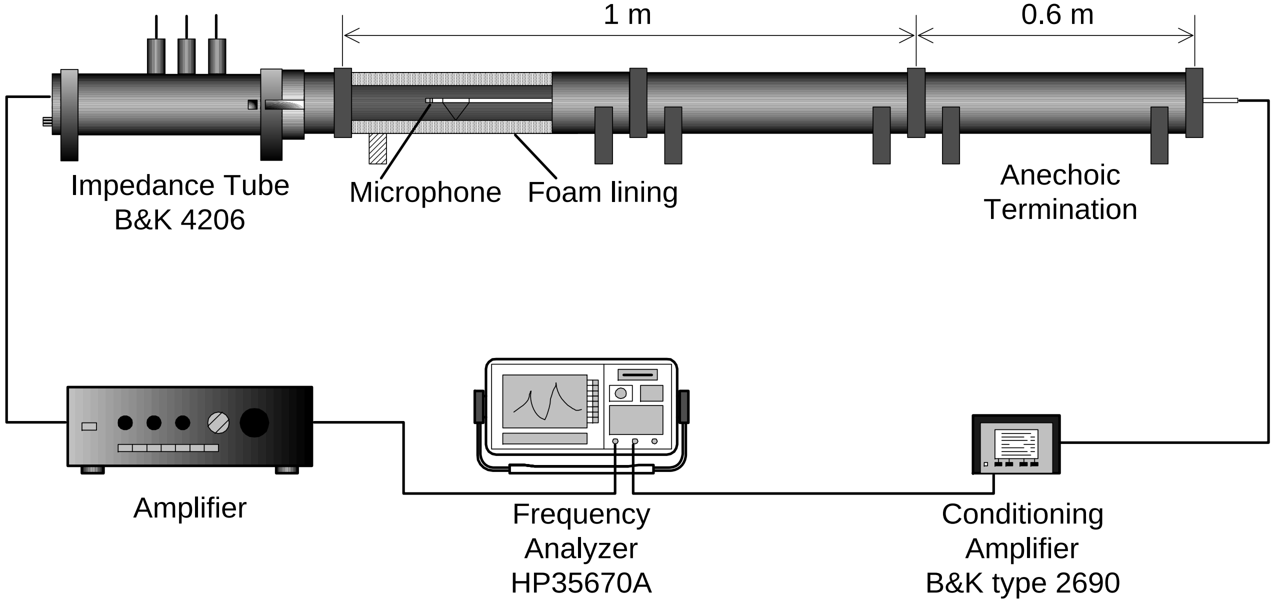


Foam C

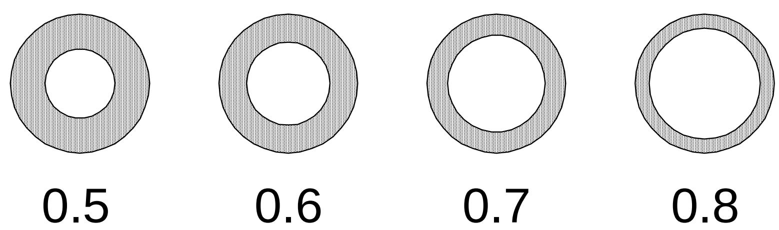


SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

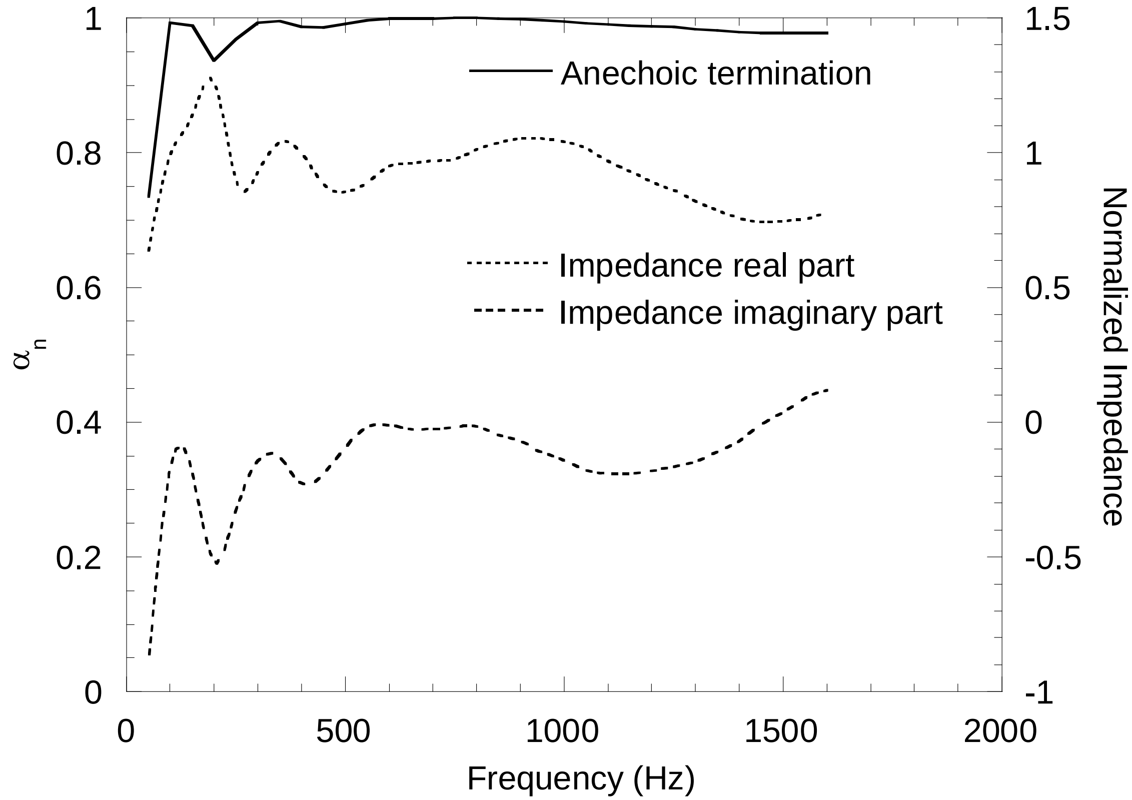
- Experimental Setup



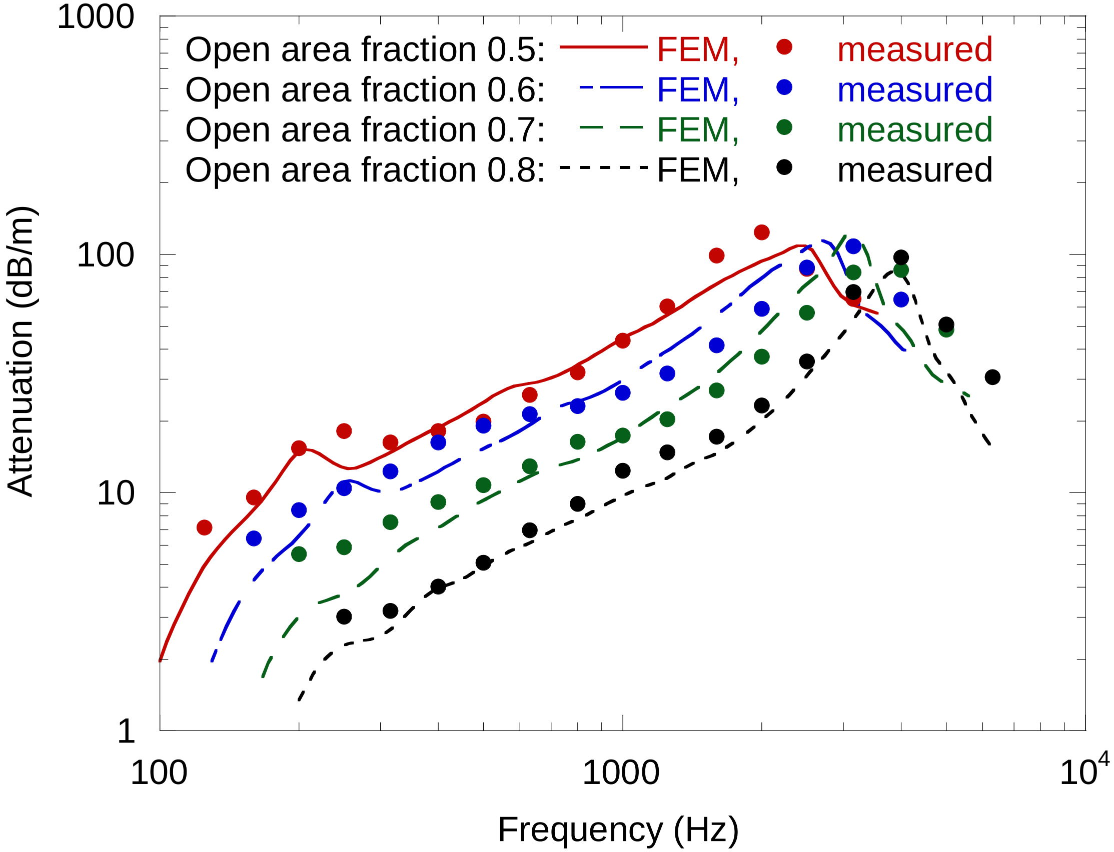
- Open Area Fraction (radius of airway / radius of tube)

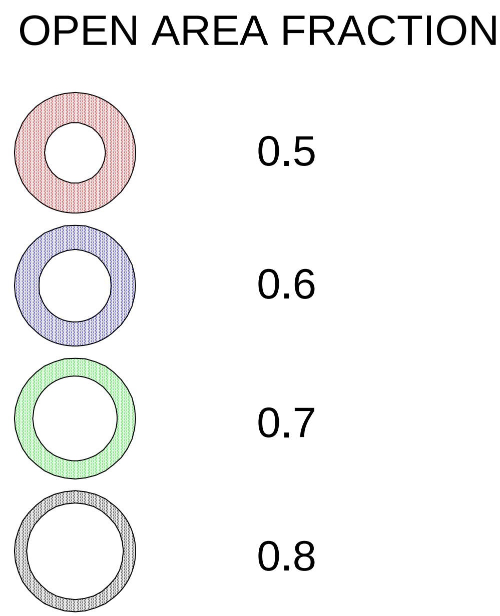


- Performance of Anechoic Termination

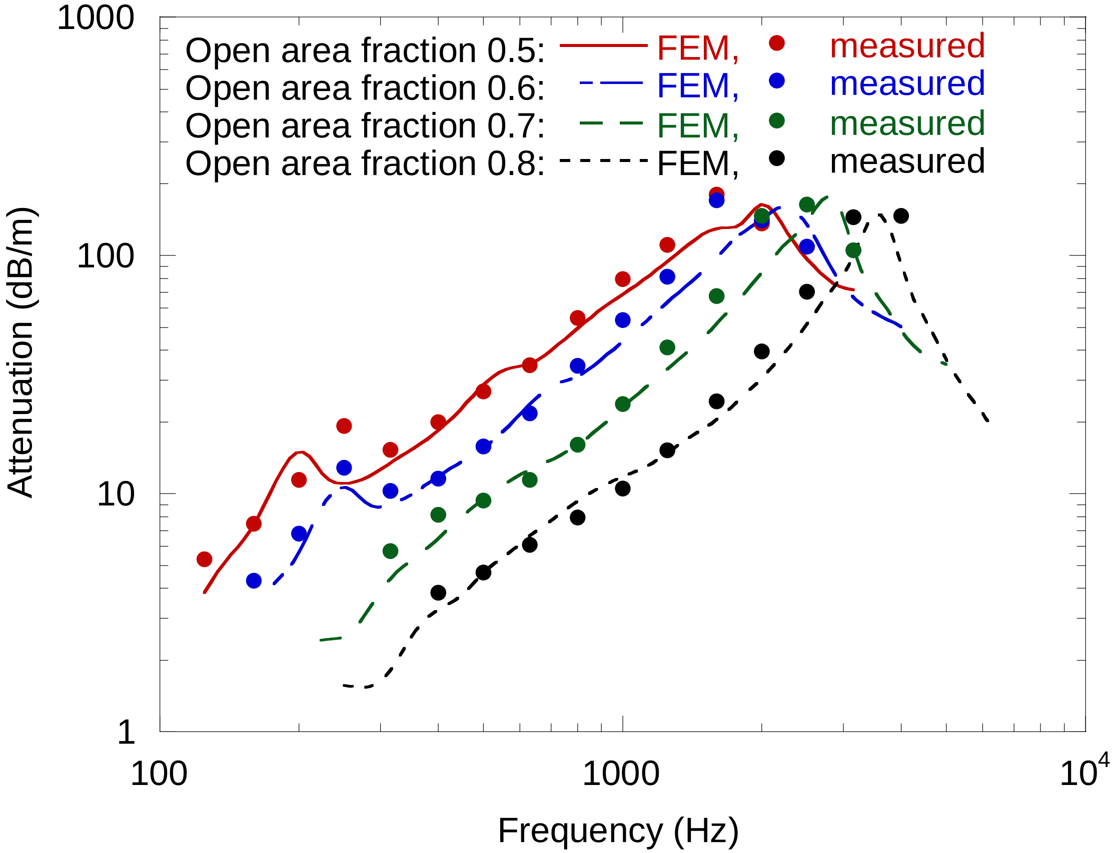


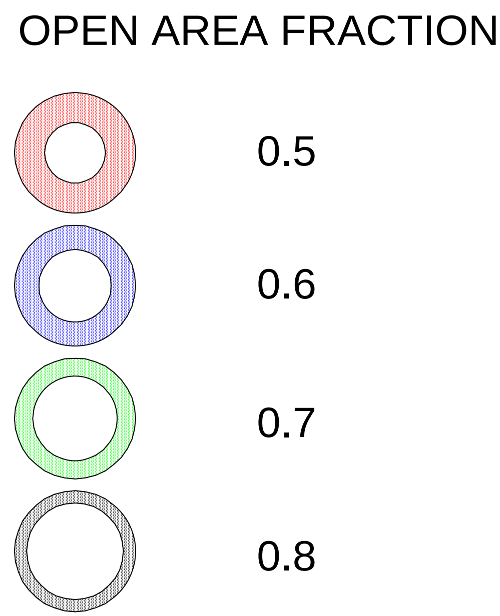
- Comparison with Experimental Results (Foam A)



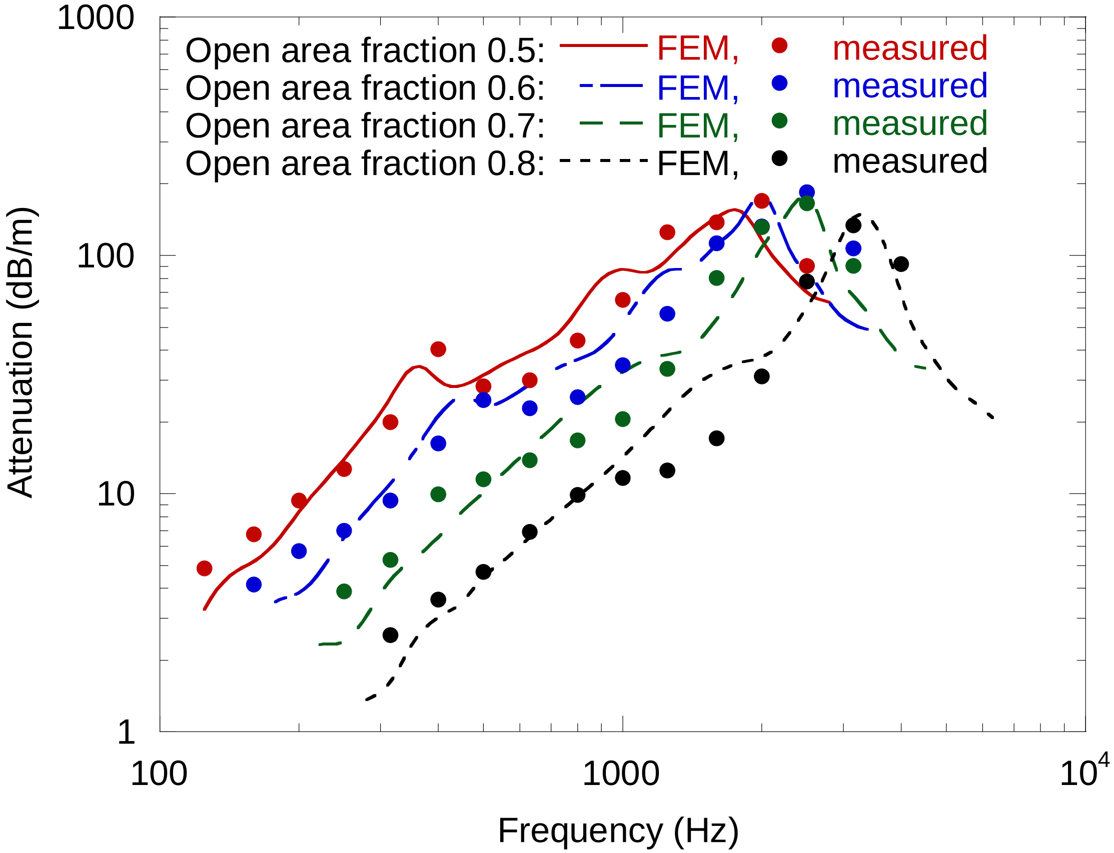


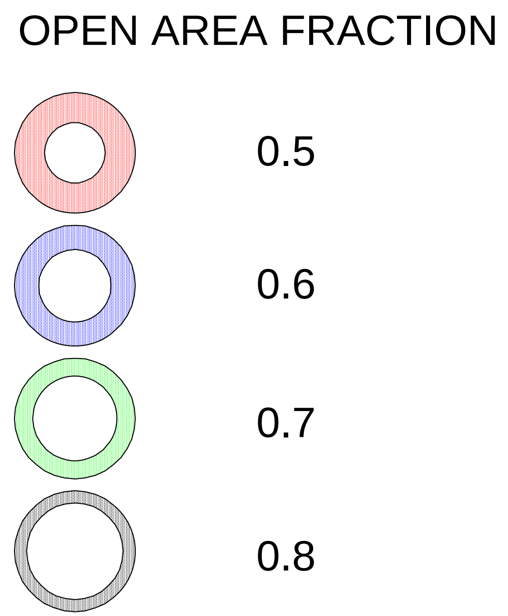
- Comparison with Experimental Results (Foam B)





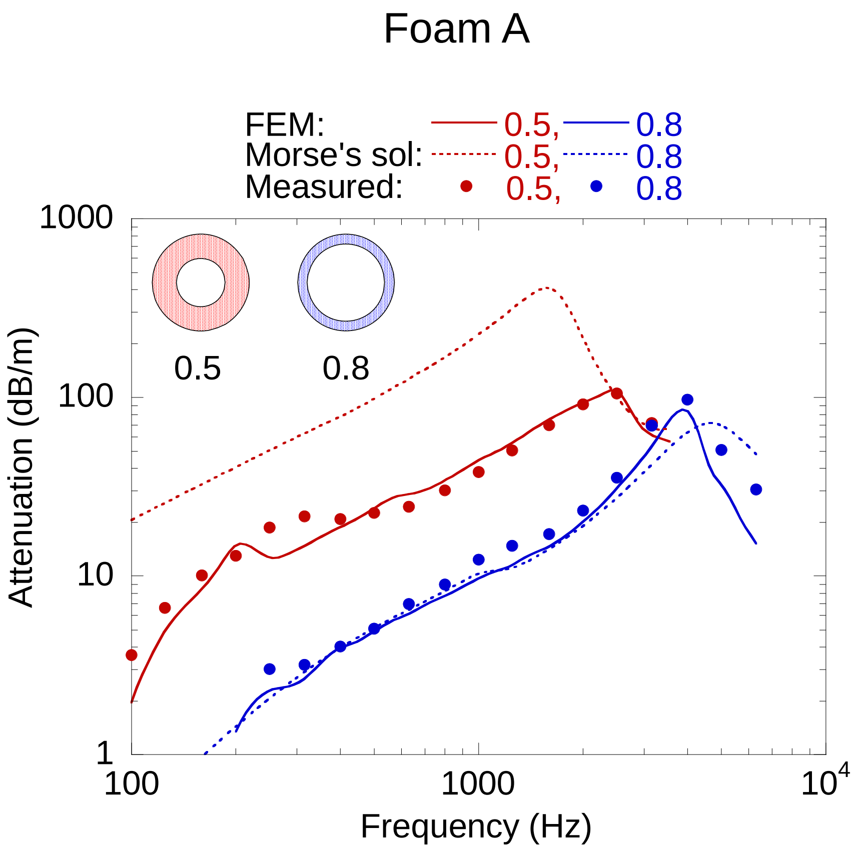
- Comparison with Experimental Results (Foam C)

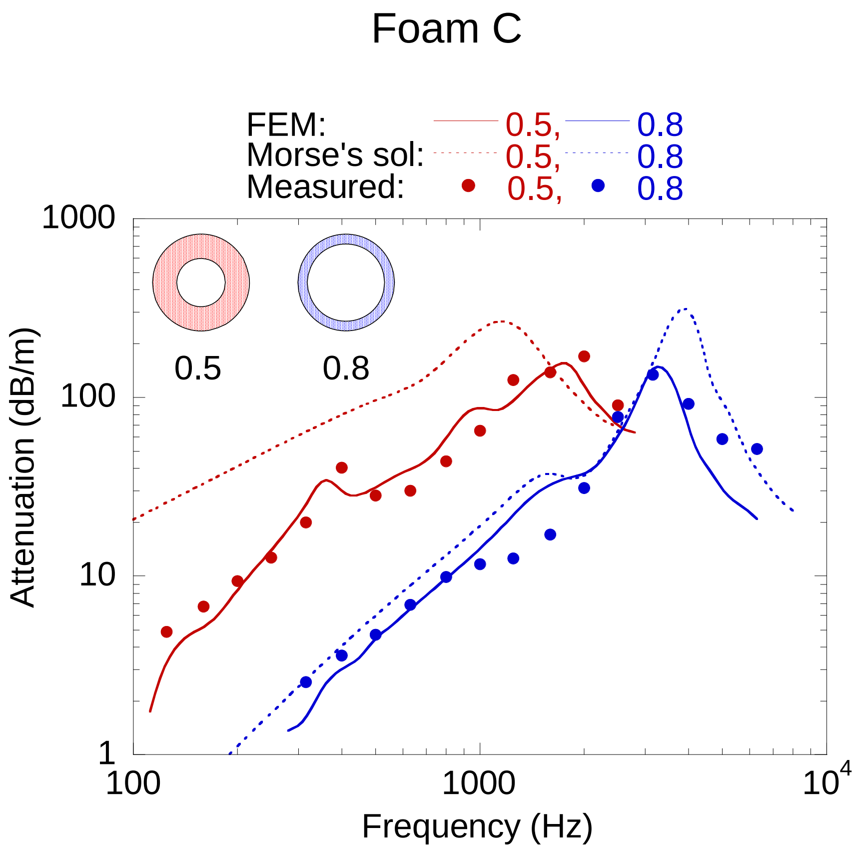




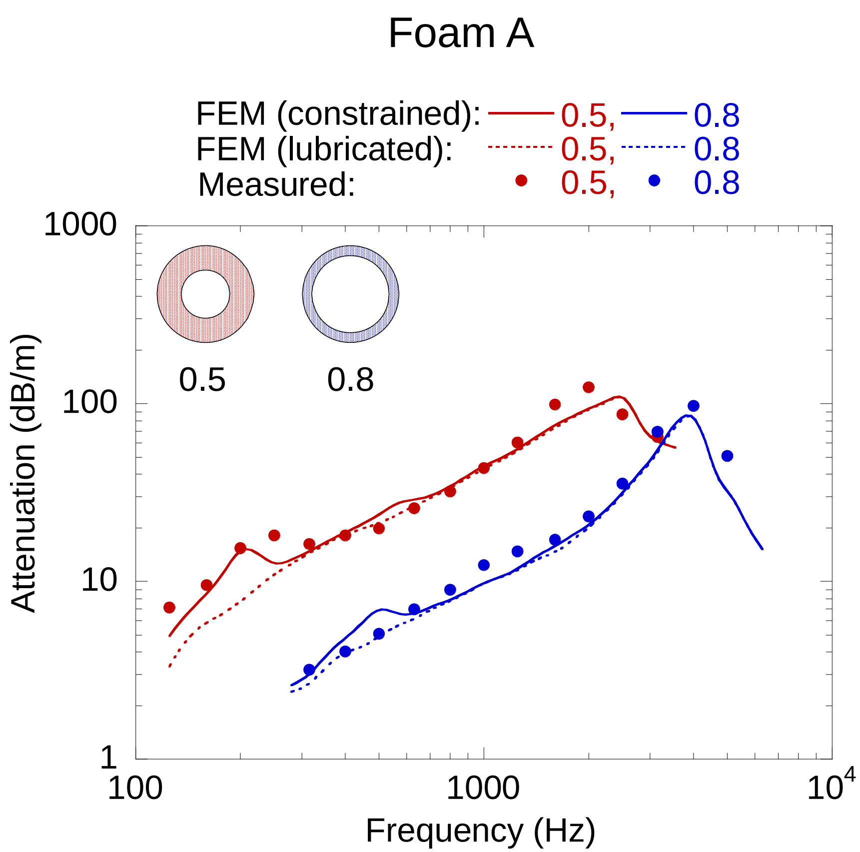
SOUND ATTENUATION IN FOAM-LINED CIRCULAR DUCT

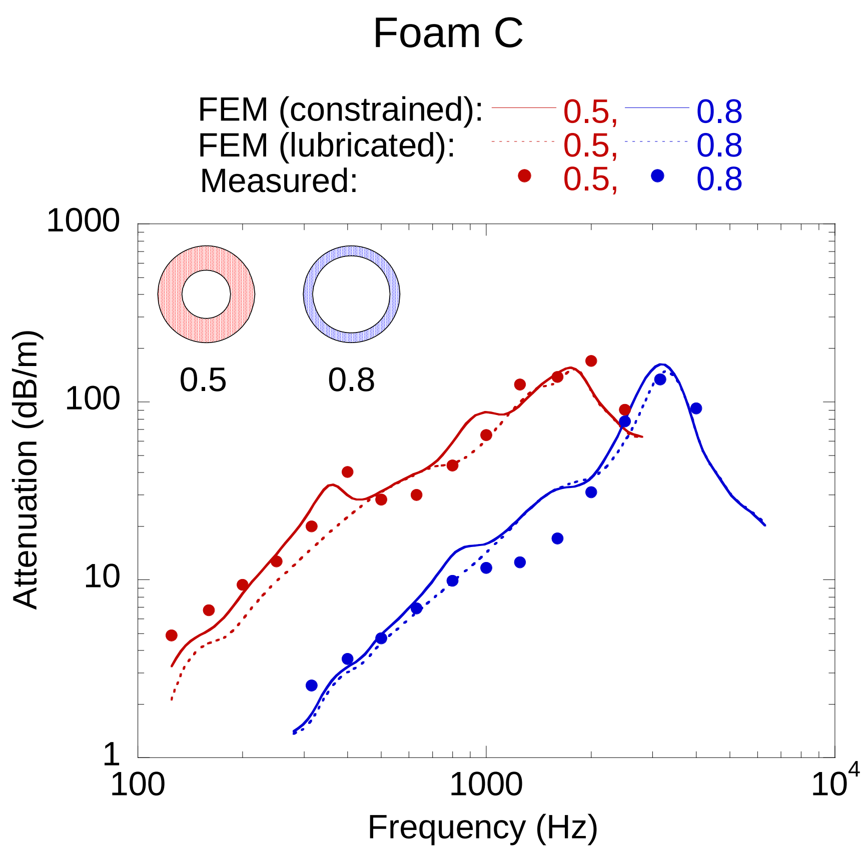
- Bulk Reacting Vs. Locally Reacting Liner





- Effect of Boundary Condition





CONCLUSION

- The AXISYMMETRICAL FOAM FINITE ELEMENTS has been formulated and validated for its accuracy and efficiency.
- It has many applications such as sound transmission and attenuation in axisymmetric configurations.
- Constraining the circumference of the foam plugs decreased the transmission loss at high frequencies.
- Finite termination impedance had rippling effect on sound transmission loss at low frequencies.
- Thicker liner does not always guarantee high sound attenuation.
- Locally reacting assumption is valid for some limited cases.