Populating SAE J2735 Message Confidence Values for Traffic Signal Transitions along a Signalized Corridor

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ABSTRACT

The communication between connected vehicles and traffic signal controllers is defined in SAE Surface Vehicle Standard J2735. SAE J2735 defines traffic signal status messages and a series of 16 confidence levels for traffic signal transitions. This paper discusses a statistical method for tabulating traffic signal data by phase and time of day and populating the SAE J2735 messages. Graphical representation of the red-green and green-yellow transitions are presented from six intersections along a 4-mile corridor for five different time of day timing plans. The case study provided illustrates the importance of characterizing the stochastic variation of traffic signals to understand locations, phases, and time of day when traffic indications operate with high predictability, and periods when there are large variations in traffic signal change times. Specific cases, such as low vehicle demand and occasional actuation of pedestrian phases are highlighted as situations that may reduce the predictability of traffic signal change intervals. The results from this study also opens up discussion among transportation professionals on the importance of consistent tabulation of confidence values for both beginning and end of green signal states. We believe this paper will initiate dialog on how to consistently tabulate important data elements transmitted in SAE J2735 and perhaps refine those definitions. The paper concludes by highlighting the importance of traffic engineers and connected vehicle developers to work together to develop shared visions on traffic signal change characteristics so that the in-vehicle use cases and human-machine interface (HMI) meet user expectations.

Keywords: confidence interval, human-machine interface (HMI), connected vehicle (CV), SAE J2735

INTRODUCTION

In recent years, the integration of traffic signals with connected and autonomous vehicles (CAV) has emerged (*1, 2*). A fairly common connected vehicle (CV) application has been the incorporation of near real-time green and speed advisory information in production vehicles. Vehicle-to-Infrastructure (V2I) features often use the Society of Automotive Engineers (SAE) Surface Vehicle Standard J2735 for that interface (*3*). SAE J2735 defines a Dedicated Short-Range Communication (DSRC) Message Set Dictionary. Although designed for DSRC, these messages are often used for V2I communication through the cloud (*4, 5*).

Messages defined in the standard include the Signal Phase and Timing (SPaT) message that describes the intersection state per movement, phase timing, and includes speed advisory details. The contents of SPaT are designed to be generated by a traffic signal controller, sent over the network, and received and interpreted by the vehicle. However, many of the parameters are optional as of the most recent revision of the standard and there are no guidelines as to how they should be populated.

Applications such as eco-driving and dilemma zone protection, require precise information about a signal's phase status. However, unlike fixed-time signals, modern actuated traffic signal controllers adjust to changing traffic conditions based on vehicle and pedestrian sensor actuation, so the start and end of green can vary by many seconds (if not tens of seconds) in each cycle depending on time-of-day conditions and stochastic arrivals of other vehicles (*6, 7*). Although these systems provide very efficient real-time allocation of green time to vehicle and pedestrians, this lack of deterministic operation requires a careful definition of the confidence that a traffic signal indication will or will not change as a vehicle approaches a traffic signal.

STUDY OBJECTIVE AND SCOPE

The literature is sparse on how a traffic signal controller should provide green time advisory for CAV and CV's. SAE J2735 provides for SPAT messages to have an associated confidence code. Few studies have documented methodologies on the estimation of SPaT messages (*8–10*) and there are no clearly defined methodologies for statistically characterizing the temporal distribution of traffic signal phase change times in a cycle. This paper reviews SAE J2735 SPaT definitions and proposes a methodology for populating the confidence codes. The importance of populating these messages with confidence codes is illustrated using real data for an actuated-coordinated traffic signal corridor. The paper also recommends a simple state of box-whisker plots for both red-green and green-yellow transition for traffic engineers and automotive engineers to review so they have a shared vision on how the traffic signal system performs for certain movements throughout different periods of the day.

SAE J2735 DEFINITIONS

The *TimeChangeDetails* data frame contained by SPaT has six parameters: *startTime*, *minEndTime*, *maxEndTime*, *likelyTime*, *confidence*, and *nextTime*. The description for the data frame states:

> *"The core data concept expressed is the time stamp (time mark) at which the related phase will change to the next state. This is often*

> *found in the MinEndTime element, but the other elements may be*
> *needed to convey the full concept when adaptive timing is employed."*

Only the *MinEndTime* is required per the specification. The description of *likelyTime* states:

> *"The element likelyTime is used to convey the most likely time the*
> *phase changes. This occurs between MinEndTime and MaxEndTime*
> *and is only relevant for traffic-actuated control programs. This time*
> *might be calculated out of logged historical values, detected events*
> *(e.g., from inductive loops), or from other sources."*

The companion parameter to *likelyTime* is *confidence*, an enumerated parameter that describes the confidence the controller has of the *likelyTime*, expressed as a percentage. The range of values and corresponding probabilities are listed in Table 1.

TABLE 1 SAE time interval confidence values and probability

Value	Probability
0	21%
1	36%
2	47%
3	56%
4	62%
5	68%
6	73%
7	77%
8	81%
9	85%
10	88%
11	91%
12	94%
13	96%
14	98%
15	100%

TRAFFIC SIGNAL DATA
Common event logging capabilities for traffic signal controllers allow practitioners to store and review signal phasing and vehicle detection events from sensors (*11*). This data can be used for generating cyclic green profiles that describe the distribution of green start and end times over a historic period (*5*). More specifically, the profiles quantify the stochastic variation of green expressed as a percentage that can be translated into prediction confidence as proposed by SAE. Although historical data is used for this paper, all of the methods for processing this high resolution traffic signal data can be performed in real-time in the controller.

STUDY CORRIDOR

The corridor chosen for this study is a 4-mi section along the US-231 mainline between the signalized intersections of River Road and Cumberland Avenue situated near the Purdue University campus in West Lafayette, Indiana (Figure 1). This is a high-speed corridor that runs north-south with a speed limit of 55mph. This study evaluates the performance of six signalized intersections along this corridor – River Road, Martin Jischke Drive, Airport Road, State St, Lindberg Road and Cumberland Ave. All are four-legged intersections with the exception of Martin Jischke Drive, which is a three-legged intersection with a northbound, southbound and westbound movements. These intersections run on an actuated-coordinated operation across eight timing plans from 6AM to 9PM on weekdays with a median cycle length of 82 seconds, except during morning (07:15-08:15) and evening (16:45-17:45) peak periods, when they operate on a 116 seconds cycle length. River Road through State Street are major traffic entry points to the Purdue campus. Apart from River Rd and Martin Jischke Drive, all other intersections are configured with an oversized pedestrian crossing.
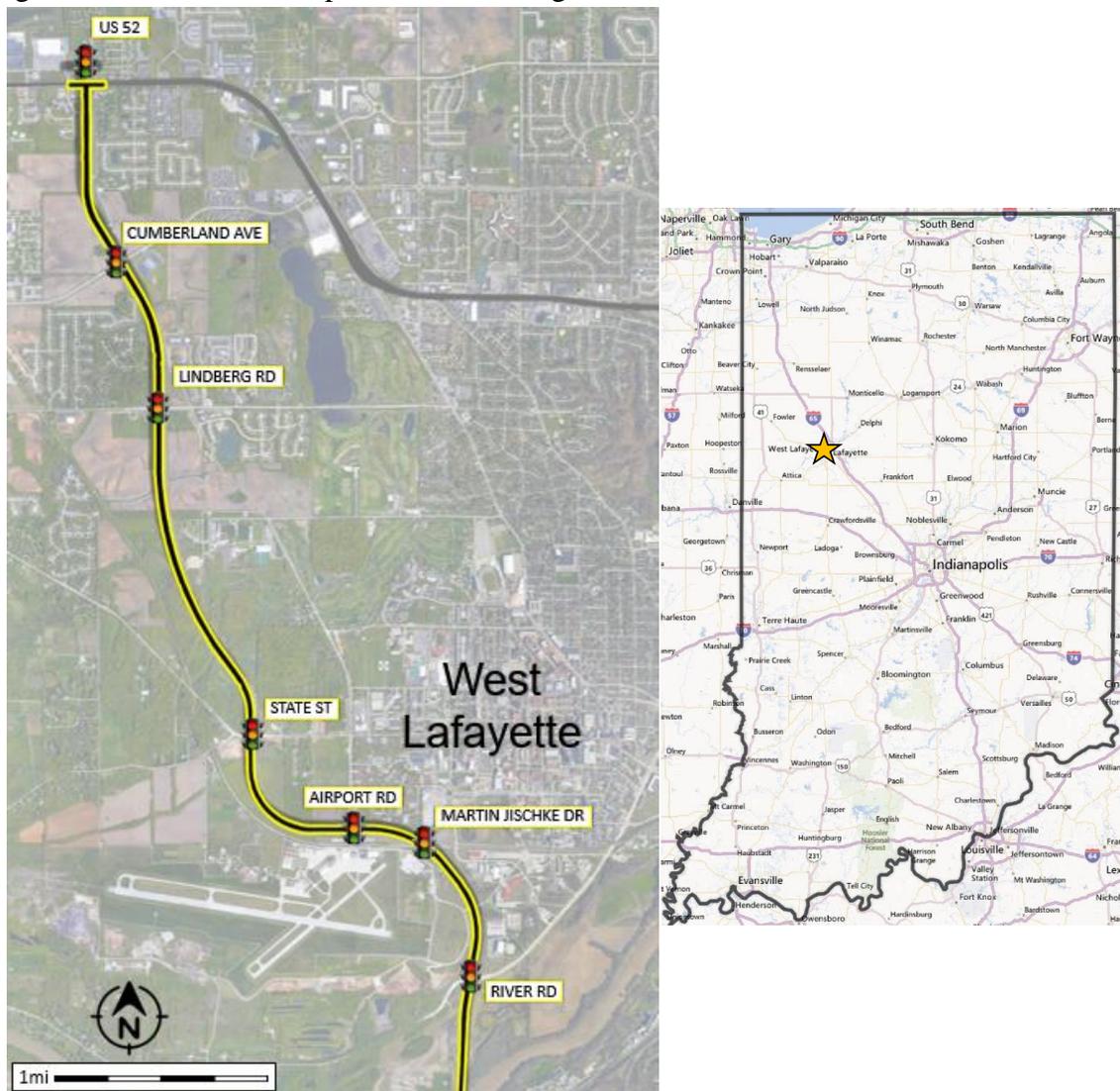

Figure 1 Study intersections along the US 231 corridor in West Lafayette, IN

PROBABILISTIC DISTRIBUTION OF GREEN FOR TRAFFIC SIGNAL PHASES
Fixed and actuated-coordinated systems

Fixed time systems usually operate on set schedules and typically assign a pre-defined length of green time for each movement. In contrast, actuated systems rely on input from sensors such as loop detectors and radars to determine the amount of green time for movements. For modern coordinated systems, the coordinated phases are provided with a dedicated time in the cycle to allow for vehicle progression. When a system is both actuated and coordinated, there is flexibility in both the start of green and end of green time for the coordinated movement. The relative timing of these coordinated phase time windows are then optimized to provide the maximum number of vehicles arriving on green along the arterial (*12*).

A convenient way of visualizing the difference between fixed time traffic signal operation and actuated coordination operation is to examine the cyclic green profile diagram (Table 2). This table show how the range in Beginning of Green (BOG) and End of Green (EOG) varies for both fixed time and actuated-coordinated operation. This table also graphically illustrates the probability of green during a specific time in cycle (TIC) for both a fixed time and actuated-coordinated signal. The X-axis shows the TIC and Y-axis shows the probability of the green for a movement from all the cycles during a time period. As seen, the fixed time system is deterministic, between 37 and 58 seconds as the range of EOG and BOG are both zero. The probability of green is estimated using Equation (1)

$$G_b = \frac{1}{N_C} \sum_{i \bmod C \in b} g_i \tag{1}$$

where G_b is the probability of green for bin b and N_C is the total number of cycles in the analysis period and g_i is the state of green for period i obtained from the high resolution traffic signal data. In this study, a bin size = 0.1s is used. Detailed computation on the probability estimation and their methodologies are well documented in the literature (*5, 6, 13*).

However, for actuated-coordinated operation, the BOG for at least one cycle begins at 18 seconds. This is due to the early return to green for one or more coordinated phases as a result of the preceding phase gapping out (finished serving demand before the end of the allocated split). The probability then increases to 100% at 37 seconds to allow for platoon progression and continues until 58 seconds where the EOG occurs for majority of the cycles. There are few cycles that end later as they rest in green waiting for a call from the non-coordinated movement.

TABLE 2 Comparison of probabilistic distribution of green for fixed and actuated coordinated systems

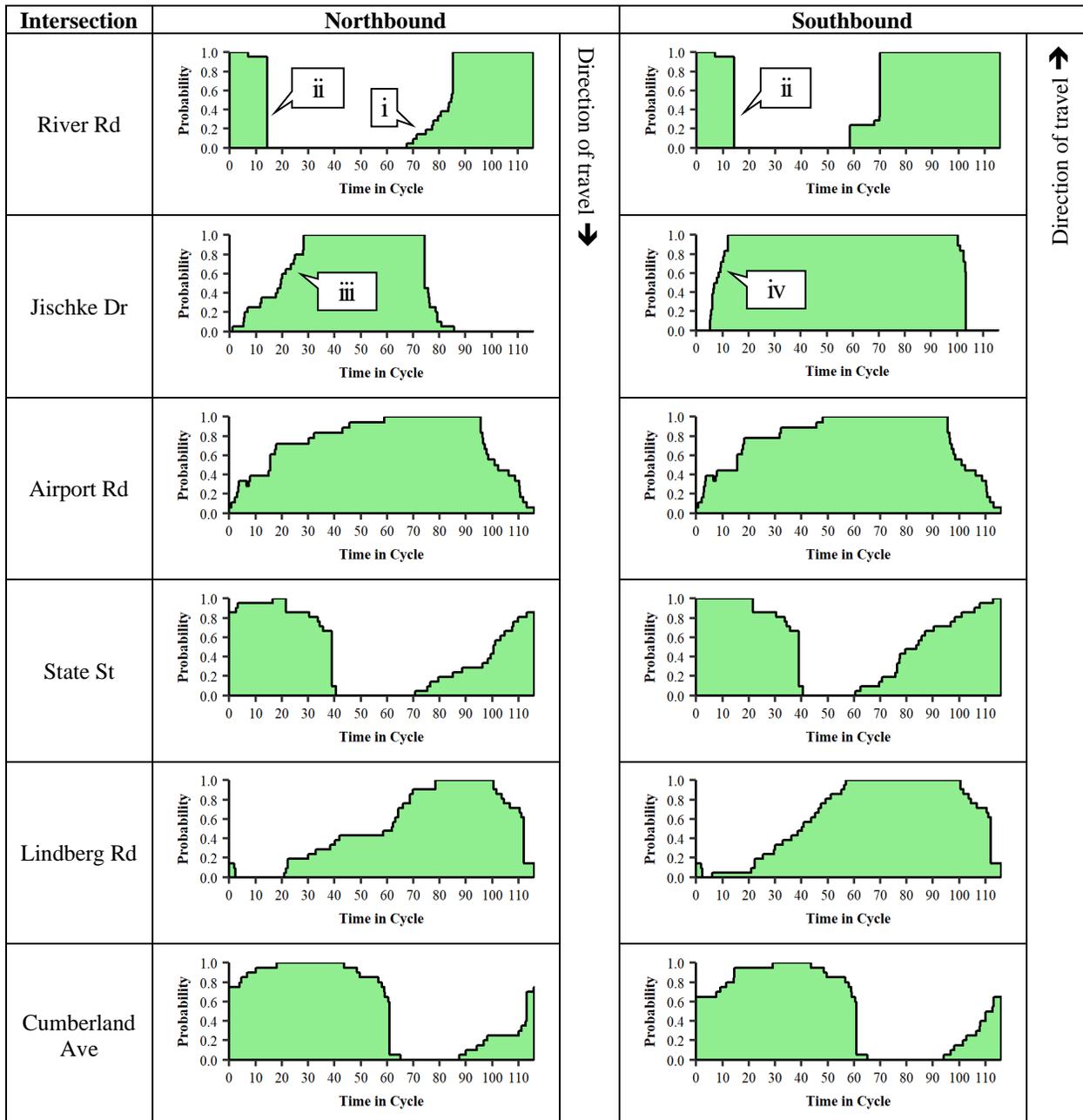|	Fixed time	Actuated coordinated
Cyclic green profile		
Deterministic window	37 - 58 seconds TIC	37 - 58 seconds TIC
BOG Window	37 seconds TIC	17 – 37 seconds TIC
BOG Range	0	20 seconds
EOG Window	58 seconds TIC	58 – 3 seconds TIC
EOG Range	0	27 seconds

Actuated coordinated variations along a corridor
The probability distributions can vary both temporally with respect to different time of day plans and spatially along a corridor. Actuated-coordinated systems provide maximum benefits when operated along a corridor with close to moderately spaced intersections. The cycle splits for the coordinated movements along the corridor are usually offset to allow for platoon progression (*14*). Although the traffic volume on the coordinated movement may remain quite similar throughout the corridor, there are other factors that can affect the traffic controller behavior at an intersection. For example, intersections with low volume on side street movements will see more green rests whereas intersections with oversized pedestrian calls can break the coordination. Other factors like intersection geometry and land-use can also affect the performance. As a result, the BOG and EOG along the corridor can also vary significantly.

Table 3 compares the probabilistic distribution of green for the six intersections, by the two coordinated movements, along the study corridor during the 07:15-08:15 signal timing plan. The BOG for River Rd intersection is fairly stochastic for northbound (callout i), however the EOG for both directions is relatively more deterministic (callout ii). This intersection is one of the major entry points for the peak traffic coming into the Purdue campus and the sharp EOG are a result of the continuous demand in the non-coordinated phase during this morning peak. For Martin Jischke Drive, there is some stochastic variation on northbound (callout iii) compared to southbound (callout iv). The intersection at Martin Jischke Drive is 3-legged where the northbound through movement gets stopped for the conflicting southbound left with re-service, which could explain the stochasticity associated with the northbound movement. The intersection at Airport Rd is very interesting with no sharp BOG and EOG, which is due to the oversized pedestrian calls used by students crossing the campus that breaks the coordination. State St, Lindberg Rd and Cumberland Ave also saw fairly stochastic BOG, possibly due to the early return to green because of the low demand on the side streets. Visual inspection of Table 3 shows

that EOGs are more deterministic than BOGs during this morning peak period. Although one would expect the traffic to be fairly saturated during the peak hours, it is quite interesting to see the BOG and EOG vary dramatically across these intersections.

TABLE 3 Green probabilistic distributions for through movements along the corridor during the 07:15 – 08:15 plan

Intersection	Northbound	Southbound
River Rd		
Jischke Dr		
Airport Rd		
State St		
Lindberg Rd		
Cumberland Ave		

Direction of travel ↓ (Northbound)   Direction of travel ↑ (Southbound)

QUANTIFYING BEGINNING AND END OF GREEN VARIATIONS

As discussed earlier, characterizing the expected transition times, as well as confidence, is important for many CV applications. Figure 2 illustrates how the BOG and EOG stochastic variation can be characterized for a specific movement using a box-and-whisker plot.

The BOG range provides an indication of how early a cycle can start with respect to the deterministic window (Pr(Green) =1). In this study, the BOG range is defined as the TIC difference between the earliest BOG and the start of the deterministic window. In Figure 2, the BOG occurs as early as 54 seconds into the cycle, whereas the deterministic window begins around 68 seconds into the cycle, resulting in a BOG range of 14 seconds. This is represented by the range of the whiskers. The ranges of the box correspond to the BOG range where the probability of green is between 0.25 (callout i) and 0.75 (callout ii). Similarly, the EOG range is defined as the TIC difference between the end of the deterministic window and latest EOG. The box ends also correspond to the EOG range between green probabilities of 0.75 (callout iv) and 0.25 (callout iii). In cases where the latest EOG could not be estimated (see Airport Rd northbound in Table 3), the TIC between the end and beginning of the deterministic window with the minimum probability was assumed to be the latest EOG.

These box and whisker plots provide a simple and quick assessment of the overall performance. The range between the box represents the inter-quartile (IQR) variation or slope of the corresponding BOG or EOG range. When both IQR and range are low, the probabilistic distributions will be more deterministic. CV applications that require accurate signal state estimations should target such cases.
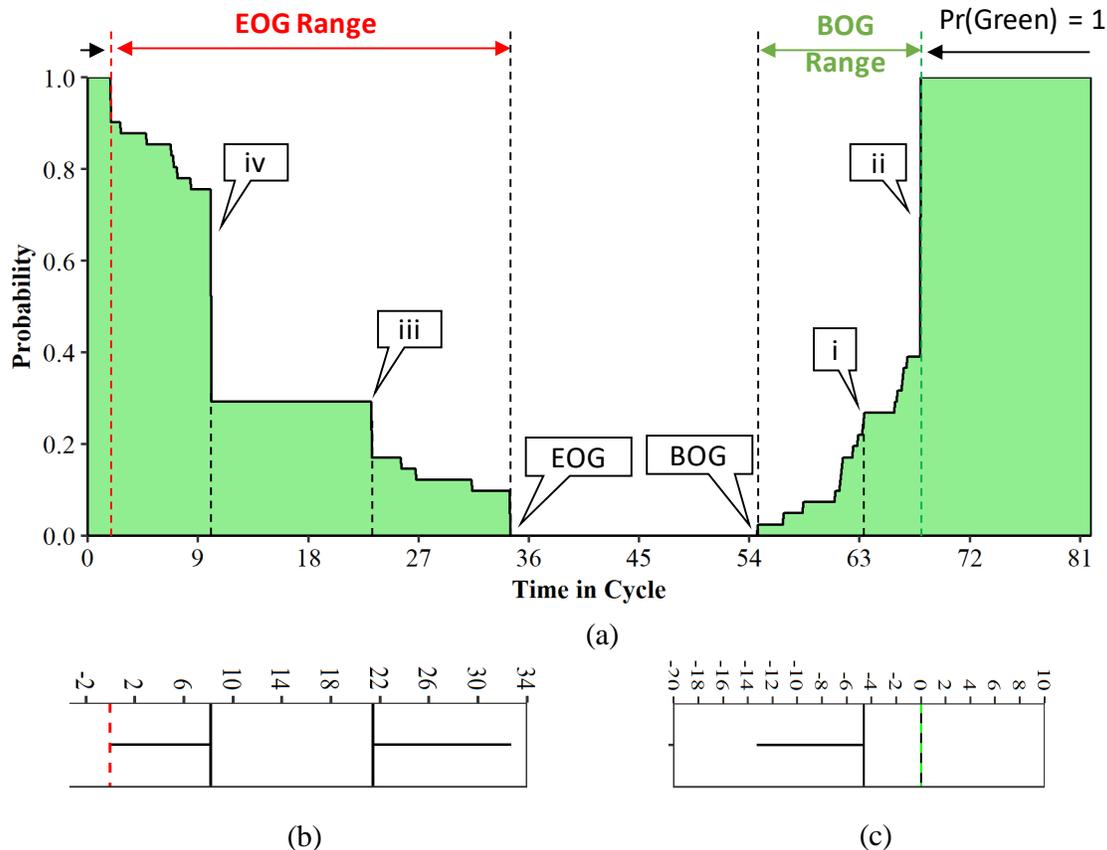

Figure 2 Example statistical characterization of BOG and EOG with box and whisker plots. (a) shows the green distribution, (b) shows the corresponding EOG box and whisker plot and (c) shows the corresponding BOG box and whisker plot

Figure 3 illustrates the box-and-whisker plot for the green probabilistic distributions corresponding to the distributions in Table 3. As described earlier, the stark contrast between the fairly stochastic BOG at River Rd (callout i) and the highly deterministic EOG (callout ii) in the northbound direction is well captured. Looking at the EOG for both the directions at River Rd, the onset of yellow can happen anywhere between 0 to10 seconds from the end of the deterministic green window (callout ii). However, at Airport Rd there are some splits that could cause the yellow interval to begin at least 20 seconds after the deterministic window (callout v). Moreover, there is considerable slope variation across the EOG indicated by the high IQR, which makes the predictions highly challenging.

Airport Rd through Cumberland Ave are the most challenging intersections with high ranges and IQR, which could be due to the early return to green and oversized pedestrian calls that break the coordination. Martin Jischke Drive is the most reliable with the highly deterministic EOG and BOG (callout iv), except for the northbound BOG (callout iii). EOGs are also found to be more deterministic than BOGs during this morning peak period.

(a) BOG – Northbound

(b) EOG – Northbound
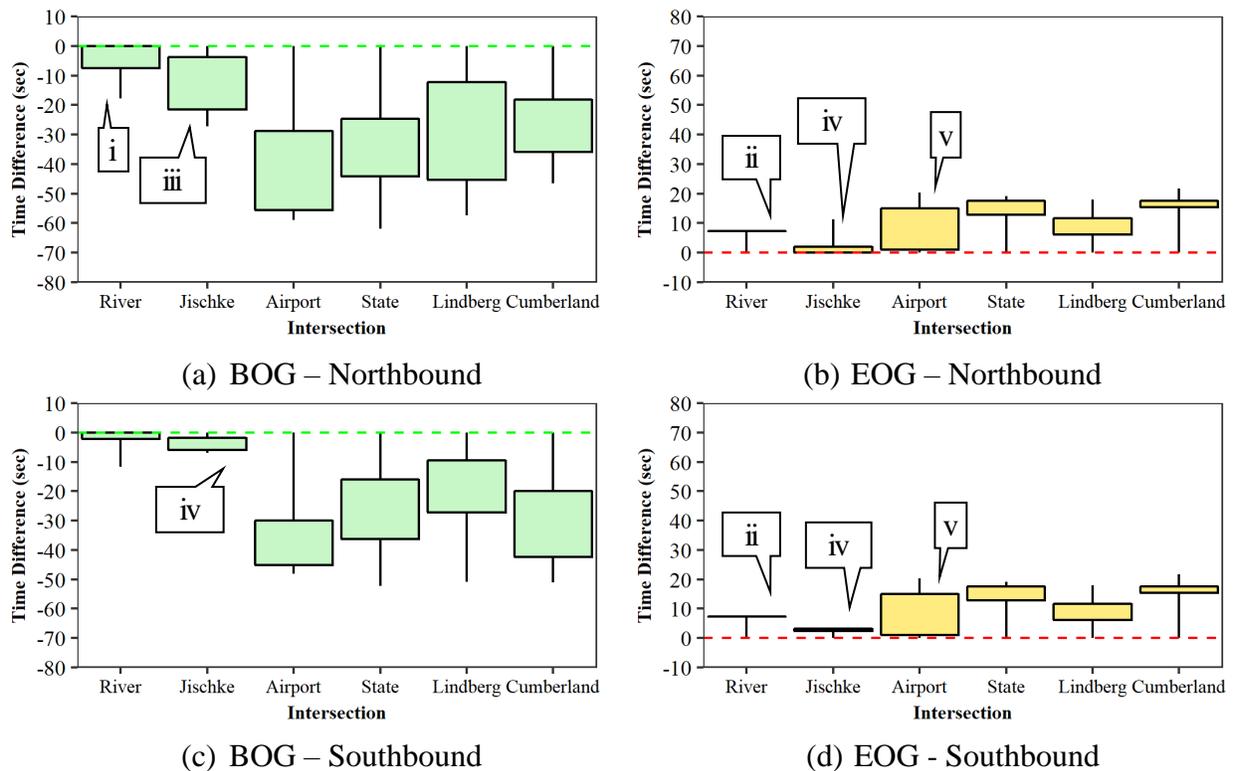
(c) BOG – Southbound

(d) EOG - Southbound

Figure 3 Box and whisker plots for BOG and EOG, by direction for traffic signal timing Plan #2 (Weekdays 07:15-08:15) along corridor shown in Table 3

QUANTIFYING BEGINNING AND END OF GREEN VARIATIONS BY TIME OF DAY
Traffic behavior at intersections also vary significantly by time of the day. Figure 4 illustrates the box-and-whisker plots for BOG during select timing plans of the day (programmed in the controller), by direction along the study corridor. During the morning (08:15-09:00) and evening (17:45-18:30) peak periods, the BOG for River Rd is highly deterministic (callout i), except during the morning peak on the southbound direction (callout ii). The southbound direction was

also found to have high stochasticity compared to the northbound direction during the peak periods. The midday off-peak period (09:00-15:00) had considerably high stochasticity across all intersections compared to the peak periods. At Airport Rd, the splits could start as early as 60 seconds from the start of the deterministic window (callout iii). One possible reason is the oversized pedestrian call at this intersection frequently used by students on campus. The longer duration (6 hours) compared to the other shorter time of day plans could also add to the stochasticity. The evening off-peak period (18:30-21:00) also resembled the midday period with high stochasticity (callout iv).

Figure 5 illustrates the corresponding EOG plots for the four timing plans. Overall, the EOG periods were more deterministic compared to the BOG in Figure 4. Of particular interest is the high range and very low IQR at State St during the evening peak (17:45-18:30) illustrated by callout i. This shows that the onset of yellow occurred for most of the cycles within 10 seconds from the end of the deterministic window. However, there are very few cycles that remained in green rest causing the EOG to occur very late. Moreover, this is a very short period (45 minutes) where the sample cycles might be low to provide an unbiased estimate.

It is interesting to note that River Rd and Martin Jischke Drive, the only two intersections without a pedestrian crossing, have much tighter statistical BOG distributions. In addition, the early return to green at the other intersections could also have contributed to the high variation. However, during EOG these two intersections recorded the highest stochasticity, likely due to the varying traffic conditions at different times of the day.

(a) TOD #3 (08:15-09:00) – Northbound

(b) TOD #3 (08:15-09:00) – Southbound

(c) TOD #4 (09:00-15:00) – Northbound

(d) TOD #4 (09:00-15:00) – Southbound

(e) TOD #7 (17:45-18:30) – Northbound

(f) TOD #7 (17:45-18:30) – Southbound

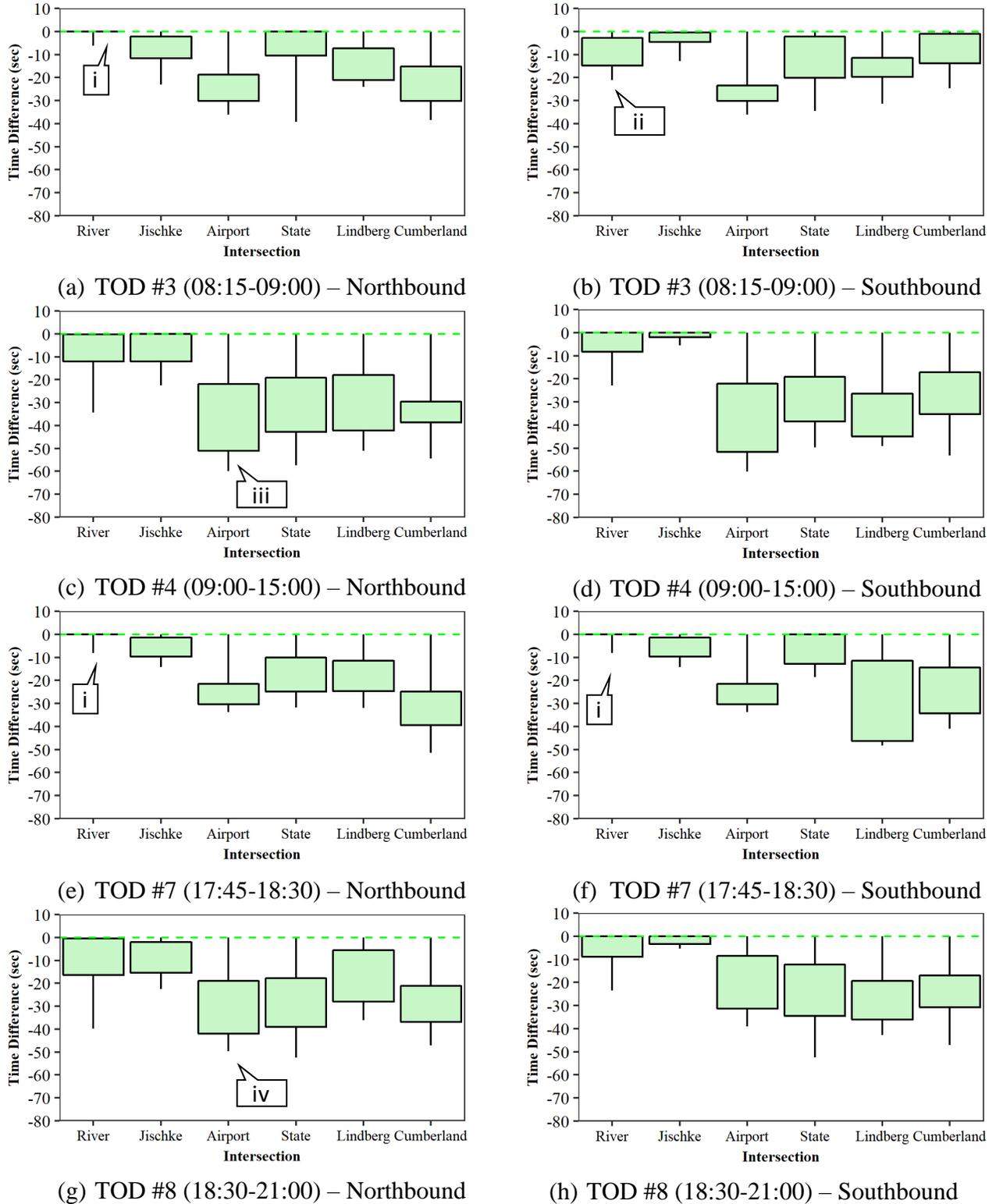(g) TOD #8 (18:30-21:00) – Northbound

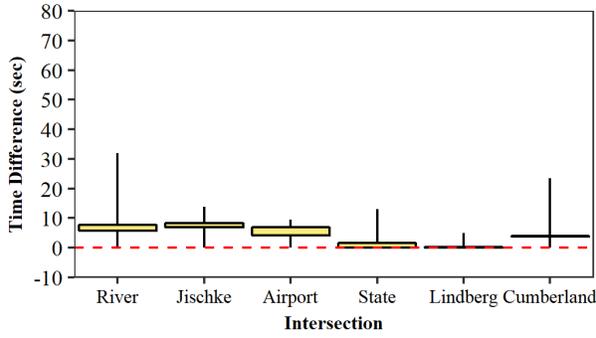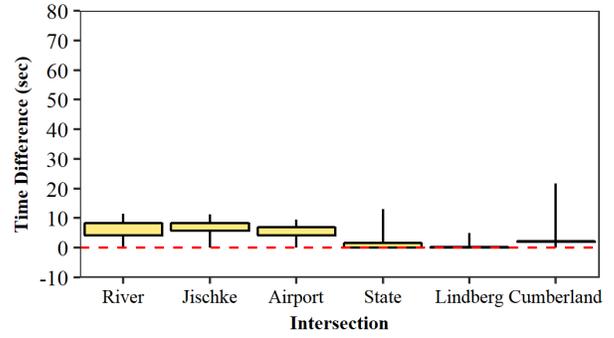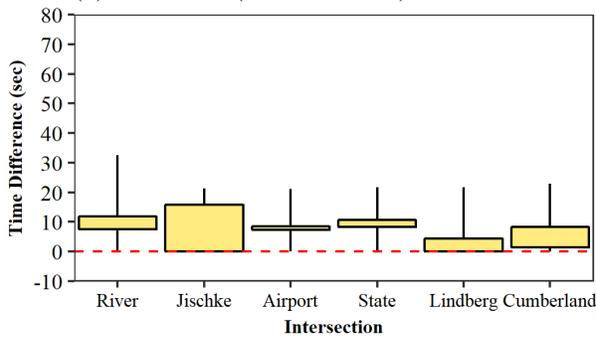(h) TOD #8 (18:30-21:00) – Southbound

Figure 4 Box and whisker plots for BOG, by direction along the corridor for select traffic signal timing weekday plans
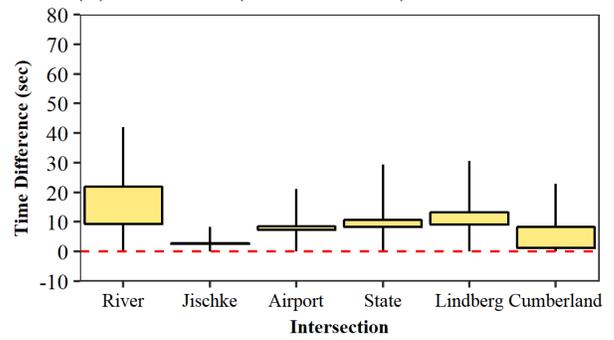
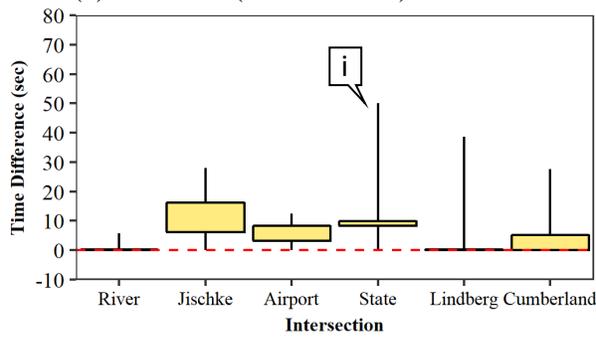(a) TOD #3 (08:15-09:00) – Northbound

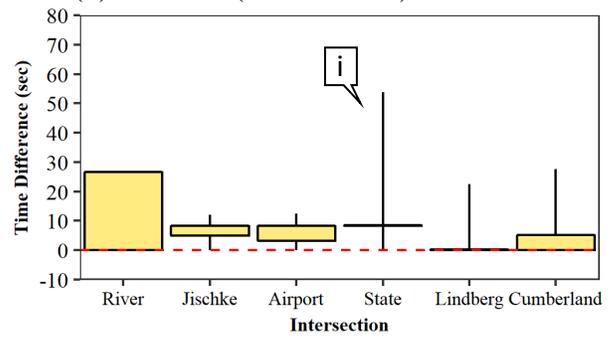(b) TOD #3 (08:15-09:00) – Southbound

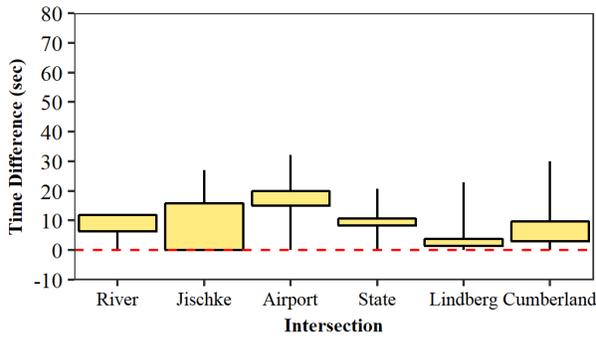(c) TOD #4 (09:00-15:00) – Northbound

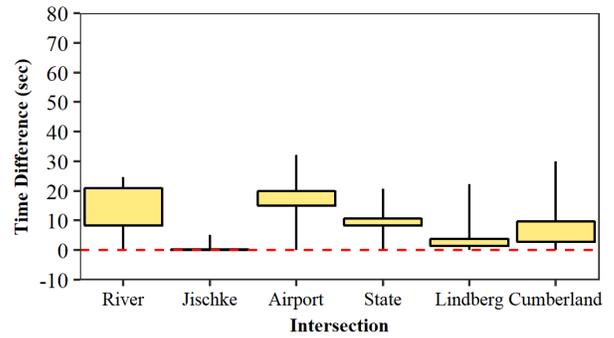(d) TOD #4 (09:00-15:00) – Southbound

(e) TOD #7 (17:45-18:30) – Northbound

(f) TOD #7 (17:45-18:30) – Southbound

(g) TOD #8 (18:30-21:00) – Northbound

(h) TOD #8 (18:30-21:00) – Southbound

Figure 5 Box and whisker plots for EOG, by direction along the corridor for select traffic signal timing weekday plans

**POPULATION OF SAE J2735 TIME INTERVAL CONFIDENCE VALUES**

The cyclic green profiles discussed in this study are critical to understanding how to populate the SAE J2735 time interval confidence values. Figure 6a shows an example cyclic green profile with the SAE J2735 confidence values on the secondary Y-axis, matching their corresponding probabilities (Table 1) on the main Y-axis. For any TIC, the corresponding probability can be mapped to the confidence values. The cyclic profiles are also capable of estimating the confidence intervals with respect to BOG and EOG.

In Figure 6a, the BOG period between 3 and 9 seconds have green probabilities from 0.21 to 0.26 which fall under the corresponding confidence value of 1 (callout i). Between 9 and 13 seconds confidence value can vary from 2 to3 (callout ii) and between 13 to16 seconds it can vary from 3 to7 (callout iii). The value then rises up to 15 during the deterministic period between 16 and 29 seconds (callout iv), after which it starts falling down indicating the EOG period. From 29 to 36 seconds in the cycle, the confidence value drops from 15 to 8 (callout v) and down to 3 at 39 seconds.

For stochastic distributions, the estimated values of the various probabilities will match the expected SAE J2735 confidence values. For example, the expected confidence values at 50%, 75% and 100% probabilities are 3, 7 and 15 (Table 1). For the stochastic BOG in Figure 6a, the estimated confidence values from the secondary Y-axis match the expected values. In contrast, for the fairly deterministic distribution in Figure 6b, the estimated confidence value for any probability above 6% during BOG is 15 (callout vii).

In both the above cases, the maximum value of 15 is possible irrespective of the nature of the distribution. However, there are cases when confidence value of 15 will not be achieved due to maximum probability being less than 1. This often occurs when oversized pedestrian phasing is used and frequently activated. Figure 6c shows an example of such a distribution with maximum probability around 0.92 (callout viii), which corresponds to a confidence value of 12.

Table 4 and Table 5 compares the expected confidence values at 50%, 75% and 100% probabilities with the estimated values from the cyclic green profiles for both BOG and EOG across the intersections during the 07:15-08:15 period (Figure 3). The highly deterministic distribution for River Rd (except BOG northbound) is evident from the estimated confidence value of 13 to15 compared to the expected value of 3. In all cases, the maximum estimated value for 100% probability was never less than 15. However, due to a relatively coarse SAE J2735 confidence interval scale (0 to15), the 50th and 75th percentiles have varying confidence scores since there are some sharp "jumps" in the both BOG and EOG distributions due to the nature of the discrete event logic in actuated-coordinated traffic signal controllers.
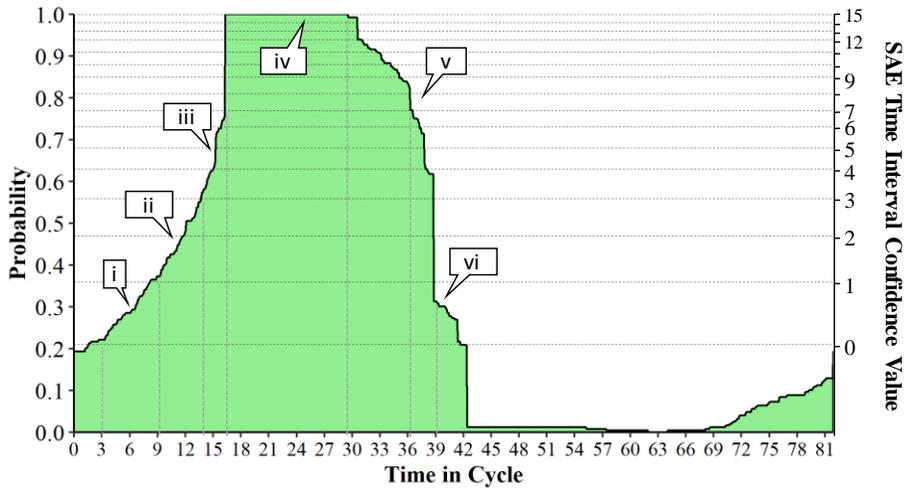
(a) Highly stochastic
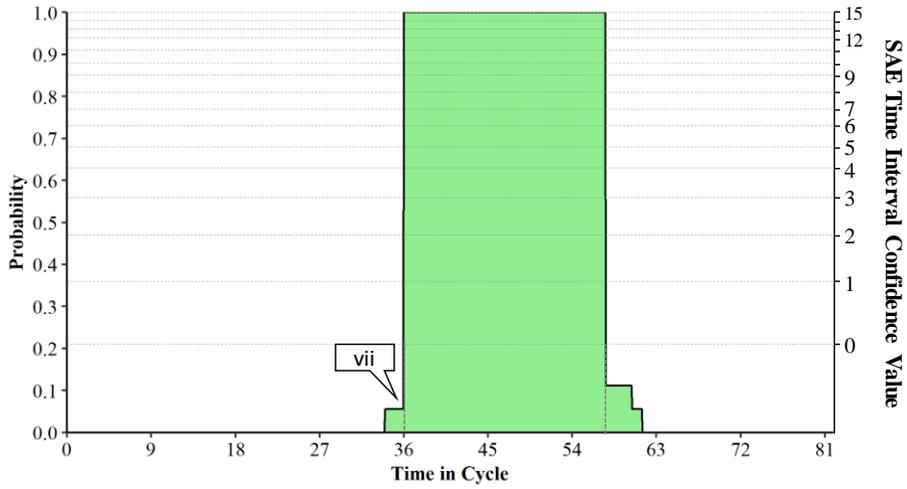
(b) Fairly deterministic
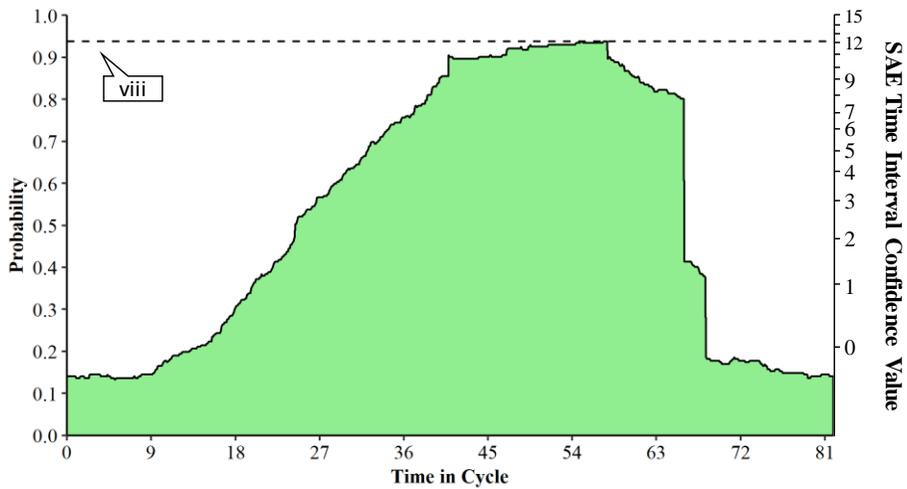
(c) Max probability less than 1

Figure 6 SAE J2735 confidence time intervals projected onto probabilistic distributions for sample intersection

TABLE 4 SAE J2735 confidence interval values for BOG during 07:15-08:15

Direction	Northbound			Southbound		
Probability	50%	75%	100%	50%	75%	100%
Expected Values	*3*	*7*	*15*	*3*	*7*	*15*
Intersection						
River Rd	3	15	15	15	15	15
Jischke Dr	3	7	15	3	8	15
Airport Rd	3	8	15	3	8	15
State St	3	7	15	3	7	15
Lindberg Rd	3	7	15	3	7	15
Cumberland Ave	6	7	15	3	7	15

TABLE 5 SAE J2735 confidence interval values for EOG during 07:15-08:15

Direction	Northbound			Southbound		
Probability	50%	75%	100%	50%	75%	100%
Expected Values	*3*	*7*	*15*	*3*	*7*	*15*
Intersection						
River Rd	13	13	15	13	13	15
Jischke Dr	15	15	15	4	9	15
Airport Rd	3	8	15	3	8	15
State St	5	7	15	5	7	15
Lindberg Rd	4	7	15	4	7	15
Cumberland Ave	3	7	15	3	7	15

SUMMARY

This paper presented a methodology for tabulating the statistical variation of both BOG and EOG events by phase and time of day and populating the confidence interval values for SAE J2735 SPaT messages.

Graphical representation of the red-green and green-yellow transitions are presented from six intersections along a 4-mile corridor for five different time of day timing plans. The case study provided illustrates the importance of characterizing the stochastic variation of traffic signals to understand locations, phases, and time of day when traffic indications operate with high predictability, and periods when there are large variations in traffic signal change times.

The box and whisker plot visualizations (Figure 4) discussed in this study is a valuable metric that provides a quick assessment on the overall performance of the various intersections along a corridor. These plots provide a mechanism for identifying time of day and specific phases at specific intersection that have either tight or highly dispersed statistical distributions for BOG and/or EOG. The charts can be a very useful tool for agencies and automotive partners to develop a shared vision of how a traffic signal system will operate as they develop CV applications that interact with traffic signals.

OPPORTUNITIES FOR FURTHER CLARIFICATION OF SAE J2735
This paper also discusses the authors' interpretation and details regarding the population of the SAE J2735 time interval confidence values from the cyclic green profile distributions generated using historic data. The methodology and framework discussed in this study will enable vendors and other stakeholders to populate the *likelyTime* parameter derived from the *TimeIntervalConfidence.*

The results from this study also opens up for discussion among the transportation professionals on the importance of having confidence values for both BOG and EOG. SAE J2735 defines the *TimeIntervalConfidence* as "*the statistical confidence for the predicted time of signal group state change.*" However, it does not specify if the status change is for end of the current state or beginning of next state. As seen in this study, the beginning and ending of a phase can have different confidence values (Table 4 and Table 5). Other studies have also emphasized that it might be worthwhile providing two estimates of the residual time (*9*). Connected vehicle applications such as green light advisory and eco-driving require an accurate estimation of the traffic signal status for both BOG and EOG. Currently, with just one parameter "*likelyTime*", it might not be possible for applications to estimate the change in signal status for both BOG and EOG. We believe this paper will initiate dialog on how to consistently interpret and evolve the SAE J2735 SPaT definitions.

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AUTHOR CONTRIBUTIONS
The authors confirm contribution to the paper as follows: study conception and design: Darcy M. Bullock; data collection: Howell Li, Jijo K. Mathew; analysis and interpretation of results: Jijo K. Mathew, Howell Li, Darcy M. Bullock; draft manuscript preparation: Jijo K. Mathew, Howell Li, Darcy M. Bullock. All authors reviewed the results and approved the final version of the manuscript.

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