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Performance of Gas Compressors Under Very Widely Varying Pressure Ratios

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0.0. PURPOSE OF THE PAPER.

The well known methods for design and computation of gas compressors, that is for determination of power, displacement and for multistage machines - the optimal interstage pressures generally consider that inlet and outlet pressures are constant or vary so little that average values may be used. This paper deals with cases where either or both of these pressures vary over a very wide range.

E.g. when evacuating a vessel to 29" vacuum the pressure ratio varies from 1 to 30; when pumping up a shop air supply system it varies from 1 to 8; and transferring bottled gas from shipping containers to high pressure storage tanks may involve a variation from 1 to 100.

Equations and diagrams will be developed for volumetric efficiency, capacity, power, and work at large pressure variations. For very high pressures, where the perfect gas law would lead to large errors, a more accurate equation of state is introduced, most conveniently via a log p-log v-diagram, such as previously published by the author.

Most results, e.g. concerning temperature, work, power, capacity are applicable also to dynamic machines (jet and turbo); however, all those involving clearance space, such as volumetric efficiency and displacement, are significant only to positive displacement compressors (reciprocating or rotary).

The proposed calculations can easily be programmed for computer, although it may seem to be doubtful whether most of them have to be repeated often enough to make this worthwhile.

0.1. SYMBOLS. (See foot note next page)

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<td>η&lt;sub&gt;ΔV&lt;/sub&gt;</td>
<td>Abbreviation = $(n-1)/n$</td>
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Superscripts

- Prime = re-expansion
- Double Prime = intake vessel
- Triple Prime = discharge vessel
- Asterisk, see Sec.1.1.1.1. Eq. (13b)
- Bar, Transition, Sec.1.2.1.1. Eq. (45)

Subscripts

- A = Atmosphere
- a = Beginning, initial
- ad = Adiabatic
- D = Discharge (receiver)
- f = Final
- g = Grand Total
- H = High Pressure
for a start we use the perfect gas law. This is permissible even for high pressure compressors, at least for the lower stages, because the dimensioning of a cylinder depends on the gas volume at its inlet. E.g. for one of the very popular four-stage 2200 psig compressors the intake to the last stage is about 800 psig, and the error by assuming a perfect gas is about 0.5 o/o for air or 1 o/o for Helium. Real gas behavior will be considered in Sec. 2, if, and as far as necessary (usually above 2000 psig).

We furthermore assume that compression and expansion proceed along polytropics; theoretically this is not even true for perfect gases, but only the end points of the curves are important for our purpose, which always can be correctly determined, even for real gases, by a suitable choice of the exponent "n". For the power input it makes little difference whether the compression line I-II in Fig. 1 is a true polytropic (full line) or deviates somewhat (dash line), and for the volumetric efficiency only the end point IV of the re-expansion line is decisive.

1.0.2. MOMENTARY VALUES.

For easy notation we use the symbols of Eq.'s (1) to (5):

(1) \( \varepsilon = p_2 / p_0 \)

(2) \( \phi = v_2 / v_1 \) ; see foot note

(3) \( \omega = (n-1)/n \)

(4) \( \zeta = \varepsilon^\omega \)

(5) \( \delta = 460 \cdot (\zeta - 1) \)

For polytropic processes

(2a) \( \phi = \varepsilon^\omega \)

(6) \( \Delta V = 144 \cdot P_0 \cdot dV_0 + \zeta \cdot (\zeta - 1) \)

see foot note. And only for perfect gases

(7a) \( t_1 = \zeta \cdot t_0 + \delta \)

(7b) \( \Delta t = t_1 - t_0 = t_0 \cdot (\zeta - 1) + \delta \)

To facilitate numerical computations tables have been prepared, which, however, for lack of space cannot be reproduced here: Table I lists the \( \varepsilon \) values for adiabatic compression of a perfect diatomic gas, i.e., for \( n=1.4 \), thus \( \phi = \varepsilon^{0.7 \zeta} \), \( \zeta = \varepsilon^{3.5 \cdot (n-1)} \), and the specific work

(8) \( \psi = (\zeta - 1)/4.5 \) HP/Acfm; see foot note. The values for other exponents can then be easily found on the auxiliary tables II to V, which for various \( n \) and \( \varepsilon \) give resp. the temperature-rise, work, and volume factors.

\[
(9) \chi = \Delta t_{\text{pol}} / \Delta t_{\text{ad}} = (\zeta - 1)/(\zeta - 1)
\]

\[
(10) \frac{\phi}{\text{pol}} / \phi_{\text{ad}} = \zeta^{n-1}
\]

\[
(11) \frac{\phi}{\text{pol}} / \phi_{\text{ad}} = \zeta^{n-1}
\]

Table II and Eq. (9) are particularly useful, because estimation and/or experimental determination of \( n \) is most conveniently done by temperature; every compressor man has a pretty good idea of the final temperature, and in tests it can easily be ascertained.

1.1. SINGLE STAGE OPERATION.

1.1.1. Pumping Down.

1.1.1.1. Capacity.

See Fig. 1. The compressor works from a decreasing intake to a constant discharge pressure; the cylinder volume is small compared to the vessel. Consider the end of the intake period; with each stroke the gas mass \( dV/(v+dv) \) is removed; at the beginning the vessel contained the gas mass \( 8/v \), at the end \( 8/(v+dv) \); thus

\[
(13) \frac{dV}{dV + dv} = \frac{8}{v} - \frac{8}{v+dv}
\]

\[
(13a) \frac{dV}{dV} = \frac{1}{n} \cdot \frac{dp}{p}
\]

\[
(13b) V_f = \frac{B}{\text{pol} \cdot \ln \frac{P_f}{p_0}} \frac{B}{\text{ad} \cdot \ln \frac{P_f}{p_0}}
\]

\( V_f \) is the capacity (not displacement) required to pump down the receiver volume \( 8 \) from \( P_0 \) to \( P_f \). We designate the ratio \( p_0 / P_f \) by \( \varepsilon \), that is the ratio of two pressures at the same location at different times, whereas \( \varepsilon \) (without asterisk) is always the ratio at the same time, but at different locations.

If we pump very fast, there will be no opportunity for heat exchange, and compression and expansion within a vessel will be practically adiabatic. \( n=\varepsilon \); however, the pumping will in most cases take considerable time, and in as much as the heat capacity of the vessel plus possible liquid or solid filling usually by far exceeds that of the gas, the temperature will vary only slightly, even if the vessel is insulated, thus \( n \approx 1 \).

If the temperature at the end of the pumping is lower, it will rise again, when the compressor is stopped; thus, if we specify the initial volume in \( \zeta \) is compared to the end volume, not vice versa; thus for compression both \( \varepsilon \) and \( \phi > 1 \).

Factors 144 and 4.5 in (6) and (8) appear due to inconsistent units used; such factors will be omitted in the following, where consistent units are taken for granted. The unit Acfm (ambient cft) is that quantity of gas, which would occupy 1 cft at 14.7 psia and the prevailing temperature, whereas a soft (standard cft) refers to a standard temperature.
the vacuum after temperature equalization, we must first pump down to a lower pressure $p_1$. It will be

$$T_1/T_0 = e^{-\omega}$$

and the required capacity will be

$$v_1 = 8 \cdot \ln \frac{\xi}{\xi_f}$$

which differs from eq. (13b) by the factor $n''$.

1.1.1.2. Work.

Substituting (13a) into (6), considering $P_0 = P_1E$ and integrating, we get

$$W = \frac{P_1 \cdot B \cdot \ln \frac{T_1}{T_0}}{n''} - \frac{1}{n''} \cdot \frac{1}{\xi} \cdot \ln \frac{\xi}{\xi_f}$$

which for the most frequent case $\xi_a = 1$ simplifies to

$$W = 8 \cdot \frac{P_1}{\xi_f} \cdot \frac{n}{n''} \cdot \left[ \frac{1}{\xi} + \frac{1}{n''} - \frac{1}{\xi_f} \right]$$

Strictly this is only correct for $n''n''$, the error, however, is in practical cases only small; a rough rule of thumb is $P < 1$ if $\xi \cdot n'' \cdot (n''-n') < 0.03$.

1.1.1.3. Displacement.

Computation of the required displacement from eq. (13a), by estimating an average value for the volumetric efficiency $\lambda$, is not satisfactory for wide variations of $\xi$. The "conventional volumetric efficiency", as e, q. taken from an indicator chart, is

$$\eta = 1 - c \cdot (\phi-1) = 1 + c - c \cdot \phi$$

The real volumetric efficiency $\lambda$ is considerably less due to leakage and heat exchange between the incoming gas and the cylinder; this can be considered by multiplying the $(\phi-1)$-term with a factor $\beta$ (perhaps 1.2); many designers prefer to deduct a constant, thus making $\lambda = \eta - \phi$. In order to satisfy everybody's taste, we combine both methods, viz.

$$\lambda = 1 - \beta \cdot c \cdot (\phi-1) - \phi$$

Introducing $m = \beta \cdot c$ and $u = 1 + m - \phi$ we get

$$\lambda = u - m \cdot \phi$$

with the definition

$$(\lambda) = dv/ds$$

Substituting into eq. (13a), and integrating from a to $f$, we arrive at

$$S_f = 8 \cdot \frac{1}{n''} \cdot \frac{n'}{u} \cdot \ln \left( \frac{u \cdot m - 1}{u - m} \phi \right)$$

which for $P_a = P_1$, thus $\xi_a = \phi$, $\xi_a = \phi$ and

$$\lambda_a = u - m \cdot \phi$$

$$S_f = 8 \cdot \frac{1}{n''} \cdot \frac{n'}{u} \cdot \ln \left( \frac{u \cdot m - 1}{u - m} \phi \right)$$

This enables us to compute the total displacement required from the values at the final vacuum alone.

1.1.1.4. Attainable Vacuum.

Setting $\lambda = 0$, we obtain from (18c)

$$S_f = 8 \cdot \frac{1}{n''} \cdot \frac{n'}{u} \cdot \ln \left( \frac{u \cdot m - 1}{u - m} \phi \right)$$

This can be integrated by developing into a series, which however for large $\xi$ and low $n'$ converges very slowly.

1.1.1.5. Power.

Consider $L = dv/dz$, and $ds/dz = const$, we find from eq. (5a), (18c), and (18d) as condition for $L_{max}$ (see Table VII)

$$T/n + m \cdot \xi_p \cdot \phi/u = 1$$

The old rule of thumb, to lay out the drive of single-stage vacuum pumps for $\xi = 3$ is a rough approximation; with increasing $c$ the maximum power shifts toward lower $\xi$ and decreases.

1.1.2. Pumping Up.

We pump from constant inlet conditions, $P_1$, $T_0$ (usually atmospheric pressure, ambient temperature) against varying pressure $p_1$. In most cases the gas will be cooled back to its original temperature $T_0 = T_0$ in an aftercooler. Under operating method "A" in Fig. 4 is wide open and the receiver pressure controlled by valve $z$: the work is represented by I-II-III-IV in Fig. 3; under method "B" valve $A$ throttles the flow to maintain the cooler pressure $p_1 = p_1$ always at the maximum $= p_2$; the work represented by I-II-III-V is considerably greater; this operation is still often applied to squeeze out water.

1.1.2.1. Capacity.

1.1.2.1.1. With Cooling.

If we could avoid temperature rise in the receiver, we would have

$$\triangle V_0 = \frac{\xi_a}{\xi_f}$$

Actually the gas heats up due to its compression, is mixed with the cold gas from the cooler, and exchanges heat with the receiver material and the outside; therefore the receiver material and the outside, therefore

$$\triangle V_0 = \frac{1}{n''} \cdot \left( \frac{\xi_a - \xi_f}{\xi_f} \right) \frac{\xi_f}{\xi_f}$$

For large $\xi$, with $T_0 \approx T'' = \theta''$, from which $n''$ can be found by observation. For perfect gases this remains valid also for operation "B", as throttling does not change the enthalpy.

1.1.2.1.2. Without Cooling.

If the gas is to be used hot and, therefore, not cooled, eq. (7) and (2a) apply, wherein $n''$ must refer to the entire system, i.e. compression plus heat transfer in pipes and receiver, thus

$$S_f = 8 \cdot \frac{1}{n''} \cdot \frac{n'}{u} \cdot \ln \left( \frac{u \cdot m - 1}{u - m} \phi \right)$$

Operation "A" is out of question as no moisture can be removed.

1.1.2.2. Displacement.

From eq. (18c) and (2a) we get

$$S_f = 8 \cdot \frac{1}{n''} \cdot \frac{n'}{u} \cdot \ln \left( \frac{u \cdot m - 1}{u - m} \phi \right)$$

which can be integrated by developing into a series, which however for large $\xi$ and low $n'$ converges very slowly.

1.1.2.3. Work.

We need only consider the case $\xi_a = 1$, as work from "a" to "f" can be found as difference "1" to "f" minus "1" to "a". For operation "A" we get
For convenience we write
\[ P_{s_1} = \frac{P_0 \cdot n^{(2n-1)}}{n^{(2n-1)}} \]
whereas for "A"
\[ (30) \frac{\dot{W}}{\dot{W}_A} = \frac{P_0}{n^{(2n-1)}} \cdot (\varepsilon_f-1) \cdot (\varepsilon_f-1) \]
Table VI shows the specific work \( w = \frac{\dot{w}^n}{\dot{w}^n_0} \) and the ratio \( \dot{W}/\dot{W}_A \), \( \dot{f} \)

As can be seen \( \dot{W}/\dot{W}_A \) is roughly near 0.6.

1.1.2.4. Attainable Pressure.
The maximum pressure from Eq. (20a) with \( \lambda = 0 \) has no practical significance, as it will be way over the safety limit.

1.1.2.5. Power.
Similar to Sec. 1.1.1.5 we find the condition for maximum power
\[ (33a) \eta = \frac{c}{n} - \frac{2 \varepsilon_c}{n(n-1)} = \frac{d}{m} \]

1.1.3. Gas Transfer.
Gas is transferred from the intake vessel, where the pressure steadily decreases, to the discharge vessel (receiver), where the pressure increases. Only operation "A" need be considered, as "B" would be identical with constant discharge pressure, Sec. 1.1.1.

1.1.3.1. Capacity.
As required capacity is independent of volumetric efficiency, it can be calculated from Eq. (13b) with \( \delta \) referring to the intake vessel.

1.1.3.2. Work.
For convenience we write
\[ (34) \int = \frac{P_0 \cdot \dot{w}}{n^{(2n-1)}} \cdot T_0 \]

\[ (35) \delta p_2 = -\int \cdot dp_2 \]
with the boundary condition that at the beginning the vessel pressures were resp. \( P_0 \) and \( P_{a_0} \), we obtain

\[ (35d) \delta p_2 = -P_{a_0} \cdot \frac{1 + \varepsilon_2}{(1 + \varepsilon_2)} \cdot (\varepsilon_2^2 \cdot d \varepsilon) \]
from which with Eq. (6a), (13a) and certain admissible simplifications
\[ (36) \delta \dot{W} = \frac{B_2}{m} \cdot \frac{P_{a_0}}{n^{(2n-1)}}, \quad \frac{1 + \varepsilon_2}{(1 + \varepsilon_2)^2} \cdot d \varepsilon \]
which can be integrated only in finite steps. Note that even for the same initial pressures the work depends on the ratio \( B_2/BD \).

1.1.3.3. Displacement.
From the above equations and (19) we obtain
\[ (35a) \delta p_2 = \frac{d \varepsilon_2}{(1 + \varepsilon_2)^2} \quad (37) \delta S = B_2, \quad \frac{1 + \varepsilon_2}{(1 + \varepsilon_2)^2} \cdot d \varepsilon \]
which also can be integrated only in steps.

1.1.3.4. Attainable Pressures.
Finding \( \varepsilon_{\text{max}} \) from (20a) we get from (35c)
\[ (38) P_{s_{\text{max}}} = \frac{P_{s_0}}{n^{(2n-1)}} \cdot \frac{\varepsilon_{\text{max}}}{n^{(2n-1)}} \]
\[ (39) P_{s_{\text{min}}} = \frac{P_{s_0}}{n^{(2n-1)}} \cdot \frac{\varepsilon_{\text{max}}}{n^{(2n-1)}} \]

1.1.3.5. Power.
According to the initial conditions and the ratio \( \gamma \) the power may go through a relative maximum, which can be found from the condition
\[ (40) n-1 - \frac{n \varepsilon_2}{(1 + \varepsilon_2)^2} = 0 \]
The drive, however, must satisfy the absolute maximum, which may occur at any combination of pressures within the specified limits, namely when either (a) the highest admissible pressure prevails at the discharge side and \( \varepsilon \) has the value from Eq. (24), or (b) the suction pressure is highest and \( \varepsilon \) reaches the value of Eq. (33) or (33a), if this is within the limits, if neither (a) nor (b) is within the limits, maximum power is required, when both pressures are at their maximum.

1.2. STAGING.
1.2.1. Two Stages.
We neglect the pressure drop through the intercoolers, as it is low for well-designed machines, thus \( P_1 = P_{10} \). etc. The intermediate pressure is determined by the displacement ratio, as \( P_{10} \) must assume that value, which makes \( V_0 \) small enough to be swallowed by the second stage. Thus,

\[ (41) \frac{P_{10}}{V_0} = \frac{S_{10}}{V_0} \]

\[ (42) \frac{P_1}{V_0} = \frac{S_{10}}{V_0} \]

Aside from a few special cases \( S_1/S_0 \) remains constant. We set
\[ \frac{S_1}{S_0} = \frac{T_{10}}{T_0} \]

Substituting (18c) and (43) into (42) furnishes the important equation
\[ \frac{P_1}{V_0} = \frac{S_{10}}{V_0} \]

For given suction and discharge pressures, \( P_{0} \) and \( P_{2} \), estimate \( P_1 \) and thus \( \varepsilon \). From (44), find the corresponding \( \varepsilon_H \) and \( P_2 = \frac{P_1}{\varepsilon H} \); if necessary, repeat with a corrected value for \( \varepsilon \).

1.2.1.1. Pumping Up.
Starting from equalized pressures \( P_0 = P_1 = P_0 \), \( P_1 \) will first rise and the gas will be pushed through the second stage without work being performed there; the first stage works as per Sec. 1.1.2. until \( P_1 \) reaches the value determined by the displacement ratio, from then on \( P_1 \) will remain almost constant, while \( P_0 \) continues rising; the first stage works, thus, as per Sec. 1.0.2. and the second as per 1.1.2.

The values at the transition from single to two-stage operation may be designated by a bar, \( \varepsilon_L \) etc., and can be found by setting \( \varepsilon_H \) in (44) = 0; the condition is, in the most convenient notation
\[ \frac{U_L}{m_L} = \frac{U_H}{m_H} \]

1.2.1.2. Pumping Down.
During the first period \( P_0 = P_1 \) drops, while \( P_1 \) is approximately constant \( = P_0 \), until
The pressure of the first stage increases during the first period from 0 to
\[
L_1 = \frac{\alpha}{L_2} \left( \frac{dS}{dz} \right)^2 \left( \frac{p_2}{L_1} \right) \cdot \left( \frac{L_1 - 1}{\gamma L} \right),
\]
whereby it may pass through a maximum. The work of the first stage during the second period, from Eq.'s (6a) and (13a) is
\[
dW_1 = \left( \beta_S/n^m \right) \cdot dP_0 \cdot \left( \frac{L_1 - 1}{\gamma L} \right),
\]
which can be integrated as
\[
\tau_1 = \text{const} = \frac{1}{\gamma L}.
\]
The power of the second stage may also go through a maximum. For sizing the drive the peak power of the first stage must be compared to the sum of peak power of the second stage plus the simultaneous power of the first, which will be
\[
J_1 \cdot \left( \frac{dSL}{dz} \right)^2 \cdot p_2 \cdot \left( \frac{L_1 - 1}{\gamma L} \right) \cdot \frac{1}{\gamma L}.
\]

1.2.1.3. Transfer.
An analogous method can be used for varying pressures on both sides. A more exact method for greatly varying \( \xi \) will be discussed later. If at the beginning \( \xi_L > \xi_1 \), the first period will be suppressed.

1.2.2. MANY STAGES.
The interstage pressures in multistage compressors are essentially determined by the displacement ratio; as any operator knows, the pressure remains constant regardless of the discharge pressure, as long as the inlet pressure (e.g. atmospheric) does not change, and essential deviations from the "normal" pressures are a sure sign of impending trouble. It is, therefore, rather obvious, how to extend the methods discussed above to more than two stages, but lack of space prohibits spelling out the details here. Multistaging is naturally most often used for high pressures, in the end stages of which deviations from the perfect gas law must be considered, which will be discussed next.

2. REAL GASES.
2.0. GENERAL CONSIDERATIONS.
All our processes may exactly enough be considered polytropic; this remains valid for real gases, provided only we use a correct value for the exponent. Most conveniently a log p-log v-diagram is used, in which the exponent can easily be determined by the slope of the lines. A skeleton of such a diagram, previously published by the author, and still exact enough for our purpose, is given as Fig. 5 [11]. We exclude cases of very low temperature, liquefaction and evaporation. Thus, in our range of moderate to high temperatures and high pressures all polytropes are steeper than for a perfect gas; also the isothermes have an exponent greater than unity. For estimation it is often convenient to set
\[
\xi_{pol} = \xi \cdot (1 - \xi_{pol}),
\]
which will be
\[
\xi_{pol} = \xi \cdot (1 - \xi_{pol}).
\]

Fortunately moderate errors in estimating will not cause great errors in our computations; subsequent checks are more easily done by temperature observations. For real gases the term \( t \) does not represent the temperature ratio, even if the correct n-value, viz. \( \log \xi/\log \xi \), is used; however, \( t \) is still useful for computing.

2.1. PRESSURE VESSELS.
2.1.0. GENERAL CONDITIONS.
At the temperatures of interest here a high pressure vessel contains less gas than figured with the perfect gas law; a multiplier, sometimes called "compressibility factor" often facilitates calculations. It is listed in Table VIII and defined by
\[
M = \frac{P_A}{v_A},
\]
where \( P_A = 14.7 \text{ psia} \). The pressures prevailing in two vessels, between which gas is transferred, are, of course, independent of the type of compressor and its efficiency, but greatly depend on the temperatures. No general equation for the pressure changes during expansion and compression can be given, as they not only depend on the gas properties, but also on the heat exchange with vessel and pipe lines, and the latter again on the available time, area, shap and surface, specific heat and conductance of the wall material, and the heat transfer to the ambient. With very fast pumping the process within each vessel will be almost adiabatic, with very slow pumping almost isothermic, and in between we have to make an educated guess.

2.1.1. INTAKE VESSEL.
E.g. supply of gas from steel cylinders can be investigated with the diagram: Starting from the initial state go along a polytropic to the final pressure, find so the gas remaining, which deducted from the initial quantity gives the amount taken out; see Fig. 6. A \( \xi \)-value of 0.3 means heavy frost on the cylinder valve, \( \xi = 0.15 \) a light frost.

2.1.2. DISCHARGE VESSEL.
In an analogous manner we find the relation between pressure rise and gas pumped into a receiver, as shown in Fig. 7. Both diagrams are drawn for air with 5400R initial temperature.

2.1.3. TRANSFER BETWEEN VESSELS.
Figures 6 and 9 show four typical cases, how gas supplied in steel cylinders with 2200 psig is transferred to a receiver
with 20000 psi maximum pressure. In case I the receiver — perhaps a large storage tank for nitrogen — has a volume 5 times the volume of the group of steel cylinders connected simultaneously to the intake manifold; the supply cylinders are to be pumped down to 300 psig, further pump-down is not economical in most cases; the receiver is empty at the beginning and the gas is bypassed until pressures are equalized, then the compressor started; almost 23 groups of cylinders will be necessary to completely fill the receiver.

In case II the volume ratio is 0.216, perhaps a small laboratory tank, just the size to take the contents of one supply cylinder. In case III the tank is not emptied completely, but only down to the supply pressure of 2200 psig, the volume ratio must then be 0.289. Case IV illustrates a ratio of 0.567.

2.2.0. GENERAL.

Eq. (6) remains valid with correct choice of n. Each stage must be treated as in Sec. 1.2, Eq. (43) must be replaced by

\[(\frac{S_L \cdot M_L \cdot T_0}{S_H \cdot M_H \cdot T_0})\]

Strictly speaking the use of the factor \(10^7\) is not correct any more, but in as much as both temperatures differ very little, the error is negligible.

2.2.1. BEHAVIOR IN EACH STAGE.

2.2.1.1. Pumping Down.

With correct \(n^\star\)-values most equations remain valid, temperatures, however, must be taken from the diagram, if interesting. Eq. (16) and (19) remain valid, but integration with average values of the exponents involves a small error.

2.2.1.2. Pumping Up.

Capacity cannot be computed any more by Eq. (25a); calculate the gas supplied as per Sec. 2.1.2, and multiply by the constant density at the inlet. Eq. (29) gives a very good approximation, but the displacement can only be computed in steps, as will be exemplified in Sec. 2.2.2.3.3.

2.2.1.3. Transfer.

Eq. (35) cannot be used any more, find pressures according to 2.2.3. Eq. (36) must be modified as follows

\[\omega = 8S(L - 1) \cdot \frac{dp}{M_L \cdot \omega} \]

2.2.2. SEVERAL STAGES.

2.2.2.0. General Arrangement.

A numerical example may illustrate the situation. Gas is supplied in steel cylinders with 2200 psig, which are to be pumped down to 300 psig, and the gas delivered to a receiver with 15000 psig; we select four stages with the pressure ratios of 2.89, 2.7, 2.6, and 2.5 resp.

2.2.2.1. Uncontrolled Admission.

In the seemingly simplest case I no valves, as shown in Fig. 10 are provided; the gas flows directly to the first stage inlet. If then with the lowest intake pressure the stage pressures are to be as indicated in line 1b of Table IX, we obviously must provide for the pressures shown on line 1a, i.e. the individual stages must be built for the pressures marked with asterisks, and it is clear that introduction of the full intake pressure into the large first stage causes excessive cost and extremely high peak power.

2.2.2.2. Controlled Admission.

With arrangement II supply lines are provided to the higher stages,either fully open or closed by automatic valves "A", depending on the upstream pressure. As long as \(\rho_0\) is higher than the desired intake pressure a particular stage the "A" valves remain closed and gas is admitted only to the next higher stage; check valves "R" prevent back flow. The design pressures, also marked by asterisks, are considerably lower than for I.

In case III automatic reducing valves "M" keep the downstream pressure constantly at the design value, resulting in still lower design pressures.

For equal displacement case I requires by far the greatest weight, construction cost, and largest motor, but also has the greatest capacity; case III is considerably less expensive, but also has a much smaller capacity, whereas case II lies in between. In case IV the gas is always throttled down and fed into the first stage; this results in least cost for compressor and motor and control, has the same capacity and motor size as III, but is far inferior as to total work required, due to the high throttling losses.

For an exact analysis the different periods must be considered, during which eache stage (1) idles, or (2)pumps between constant pressures, or (3)pumps with constant ratio, or (4) from constant inlet against increasing outlet, or (5) from decreasing inlet against constant outlet, or (6) between decreasing inlet and increasing outlet pressures. The first 5 cases can be dealt with as per Sec. 1.1.1, and 2.1; if condition (6) prevails only in one stage one can proceed as per Sec. 2.2.1.3, however, if it prevails simultaneously in several stages, as in case I the matter becomes too complicated for the formulation of generally valid equations. Fortunately this mode of operation is feasible only for small compressors with not more than two stages; we can, therefore, limit our considerations to this special case.
Because of the good cooling we may set
T_{10} = T_0, thus
(43b) \phi = 30^\circ \sin \frac{S_L}{S_H}
Eq. (44) and
(42a) \phi_L = \frac{S_L}{S_H} \cdot \phi \cdot \frac{\phi_L}{\phi_H} \text{ remains valid, but}
replaces (42). Because of the short duration the re-expansion may be presumed adiabatic, n' = k, also at high the discharge time is very short, thus T_{III} \approx T_{II}, see Fig. 1.
Thus
(51a) \frac{T_{IV}}{T_1} = \exp \frac{n-k}{n+k}

The exponent is very closely \(-0.08\), so for \(k\) between 2 and 6 \(\frac{T_{IV}}{T_1}\) may be taken as 0.9 exactly enough for our purpose.

Which intermediate pressure is desirable, and which ratio \(S_L/S_H\) must be chosen to produce it can be approached from two different points of view:
1.) One can endeavor to minimize the work, particularly at the peak pressures, for which the solution is
(52a) \phi_L = \frac{\phi_L}{\phi_T} \cdot \frac{\phi_L}{\phi_H} \cdot \frac{n_L - n_T}{n_L - n_H}

2.) On the other hand, one can endeavor to reduce the cost of compressor and drive, for well known reasons the strokes are preferably alike, thus for best utilization of the structure the dead center forces should be alike, or the condition for minimum peak forces is
(53c) \phi_L = \frac{S_L}{S_H}

Either solution can be found by trial and error (details omitted for lack of space), but for a high pressure compressor they are essentially different (whereas for perfect gases they can be satisfied simultaneously with \(\phi_L = \phi_H = \sqrt{\phi_T}\), as is well known).

2.2.2.3.2. Algorithm.

We may now compute the intermediate pressure of a two-stage compressor for given intake and discharge pressures, and from this all desired parameters such as volumetric efficiency, work, power, temperatures for each operating condition.

Select an array of \(\phi_L\) and \(p_M\) values, figure for each pair \(\phi_L\) and \(p_M\), and draw a diagram with ordinates \(p_M\) and \(p_H\) and parameters \(p_L\) and \(\phi_L\). Auxiliary diagrams will then yield round values for other parameters, which can be transferred to the \(p_L\) - \(p_T\) diagram. This leads to the following algorithm, which, if repetitive enough, can easily be computerized. See overlay.

2.2.2.3.3. Numerical Example.

Design a small nitrogen compressor for laboratory use: single-acting, vertical cylinders, \(180^\circ\) displacement of first stage \(0.15\) cfm, mean piston speed not over \(100\) fpm; intake from \(2200\) to \(300\) psig, discharge pressure up to \(19000\) psig, high pressure received \(0.4\) cfm geometrical volume; manifold connections for \(3\) steel cylinders provided.
Consideration II, Minimizing Construction Cost: Again by trial and error we find that a peak of only 6900 psi satisfies Eq. (53a), resulting in $p_0 = 555 = 9/16"$ and almost equal load for the two stages. A look at table X shows that this second alternative is by far preferable, as the cost of the machine is at least 30% lower. The capacity at 7% higher, although this must be obtained by a 7% increase in peak power.

2.2.2.3.3. Computation.

With the dimensions now fixed numerical values can be inserted in our equations, which simplifies some of them considerably. The result for our machine is shown in diagram 11. As long as the intake pressure exceeds the receiver pressure, the gas is, of course, bypassed; that is the range left of the line $E_T = 1$ is to be disregarded; between this line and the dashed curve, which was found from Eq. (45a), $E_H$ remains = 1, the compression takes place only in the first stage.

In Fig. 12,13, and 15 parameters for round values of $p_0$, $n$, and $w_0$ were found and then transferred to diagrams 14 and 16, on which also the curves for the receiver pressures were superimposed, computed as per Sec. 2.1.3. and shown as cases II and III on Fig. 9. Remember, they refer to 1 c ft of intake vessel volume. These curves are divided into 9 sections, each of which 14 A ft (of the total 126) are transferred; by dividing 14 by the $Q$-value, taken at the midpoint (marked by a small circle) we obtain the necessary displacement, see Fig. 14; for greater accuracy a correction factor may be applied, considering the fact that the gas is warming up from the supply cylinder to the compressor. Multiplying again the displacement with the specific work $w_0$ (see Fig. 16) yields the work for the first section. Finally adding up the 9 sections we arrive at the total displacement and work required for the transfer of the gas contained in 1 c ft of intake vessel under the given conditions. The result is the following. For case II:

The volume of our receiver plus aftercooler plus HP-pipe lines is 0.41 c ft, the displacement of our compressor is 0.15 c ft. Thus, the steel cylinders connected must have a volume of 0.41/0.216 = 1.9 c ft. The total displacement required for 1 c ft intake vessel was found by the computed displacement to be 2.158 c ft, consequently we need a displacement of 2.158 - 1.9 = 4.1 c ft, or the time required for the transfer is 4.1/0.15 = 27 min; the work was found to 520000 ft-lbs per c ft intake vessel, the total will, thus, be 990000 ft-lbs = 0.558 H.P. = 0.37 kWH. The peak power from Fig 16 can be read as abt. 290000 ft-lbs/c ft = 290000 x 0.15 = 44000 ft-lbs/min = 1.34 H.P.

For case III the result is: Volume of connected cylinders = 1.37 c ft, total displacement 2.6 c ft, time 17.7 min, work 0.32 H.P., peak power 1.45 H.P. These figures, of course refer to indicated power; brake power is considerably higher, as no good mechanical efficiency can be expected.

The power could be kept rather low, if we always could make the highest discharge pressure coincide with the lowest intake pressure and vice versa; in practice, however, pressures may occasionally be high on both sides simultaneously, and we must design for a peak load of 840000 ft-lbs per c ft displacement or 3.8 H.P. (see Fig. 16).

On the diagrams presented thus far, extends from 2200 down to 300 psi, as generally the steel cylinders need not be exhausted farther. If, however, the gas is valuable and the supplier will not give credit for the returned remainder, it may be economical to transfer gas from a group of almost empty cylinders into one of them, when the compressor is not needed otherwise, or to pump the gas from the high pressure vessel, e.g. an autoclave back into the cylinders. Thus, the intake pressures must be obtainable; they refer to indicated power; brake power will not exceed 300 psi, diagrams 17 and 18 are provided. As no great accuracy is required it seemed permissible to use the perfect gas law. It turns out that in this range the curves for $E_T$, $p_0$, and $w_0$ are practically straight lines.

Case V is the reverse of case II, the gas being returned from a 0.41 c ft receiver into 1.9 c ft cylinders; the pressures will equalize at abt. 1770 psi; it will then take 5.6 min to pump the receiver down to 300 psi, and fill the cylinders to 2100; pumping down to 150 psi would require additional 3.4 min.

In case VI 3 cylinders are pumped down from 500 to 70 psi and the gas transferred to a fourth one, in which the pressure will rise to abt. 900 psi; if all cylinders are standard size of abt. 1.2 c ft, this will take 51 min, and the question is whether the 68 A ft saved are worth the effort; Helium probably yes, nitrogen probably no.

Work and power need not be investigated as they are very low and neither important for the design nor for the economy; however, diagram 16 shows that for case V the indicated power will not exceed 0.5 H.P.

3. LITERATURE.


Table VIII. Multiplier (Compressibility Factor) see Eq. (49)

Table VI. Abbreviated Pumping-up Work
Explanation in Text.

Table X. Alternative Design Approaches

Table IX. Comparison of Controls

Table VII. Attainable Pressure Ratios and Peak Power.