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J. Stuart Bolton

Purdue University, bolton@purdue.edu

Hyu-Sang Kwon

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Plane Wave Reflection Coefficient Estimation By Use Of Spatial Parametric Signal Modeling

Hyu-Sang Kwon and J. Stuart Bolton
(Herrick Labs., Purdue Univ.)

Purdue University

Herrick Laboratories

INTRODUCTION

- How can we measure the reflection coefficient ?

WAVE NUMBER SPECTRUM

- Plane wave decomposition theory

ZEROth ORDER HANKEL TRANSFORM

- An alternative of Fourier transform
 - Axisymmetric field
 - Corresponds to 2dim. spatial Fourier transform

Non-axisymmetric
field

Non-axisymmetric
wave number spectrum

2 dimensional spatial Fourier transform

Axisymmetric
field

Axisymmetric
wave number spectrum

space domain

wave number
domain

r dependent

k_r dependent

REFLECTION COEFFICIENT

- Procedure

- Zeroth order Hankel transform

– Definition: $\tilde{X}(k_r) = \int_0^\infty r X(r) J_0(k_r r) dr$

– Wave number and angle relation

$$k_r = \sqrt{k_x^2 + k_y^2} \quad \theta = \cos^{-1} \left(\frac{\sqrt{k^2 - k_r^2}}{k} \right)$$

PRONY APPROACH

- **Finite measurement aperture**

- Finite size of anechoic room & material
- Finite number of measuring microphones

- **DHT(Discrete Hankel Transform)**

- Leakage & poor spectral resolution
- Less information

$$\frac{\Delta k_r}{k} = \frac{\lambda}{N\Delta r}$$

- **Parametric spectral estimation**

- Prony model: damped harmonic signal
- High resolution of reflection coefficient
- Less measurement positions
- Model order selection: by using input white noise variance

$$P(r) = \sum_{n=1}^N A_n e^{-\alpha_n r}$$

$$X(z) = H(z)U(z), \quad \rho_w = P_{UU}(z)$$

constant

Ref.: Akaike Information Criterion, etc.

SIMULATION

- **Simulation setup**

Frequency=343Hz
Wavelength=1m
No. of data=50 per line

- Monopole, dipole simulation
- No wall, rigid wall simulation

Reflection coefficients calculation
- Discrete Hankel Transform
- Prony spectral estimation

MONOPOLE CASE

Model order, $N=5$
No. of poles=6

Monopole without reflecting wall

Model order, $N=5$
No. of poles=6

Monopole with rigid reflecting wall

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DIPOLE CASE

Model order, $N=6$
No. of poles=7

Dipole without reflecting wall

Model order, $N=6$
No. of poles=7

Dipole with rigid reflecting wall

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EXPERIMENT

- **Experimental setup**

21 frequencies (1953Hz-4004Hz)
No. of data=120 per line
In a semi-anechoic room

- Reflection coefficient calculation
 - Discrete Hankel transform approach
 - Prony spectral estimation approach

EXPERIMENTAL RESULTS

Model order, $N=18$
No. of poles=19

Frequency=1953Hz, $N=120$

Model order, $N=11$
No. of poles=12

Frequency=1953Hz, $N=50$

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EXPERIMENTAL RESULTS

Model order, $N=14$
No. of poles=15

Frequency=2832Hz, $N=120$

Model order, $N=13$
No. of poles=14

Frequency=2832Hz, $N=50$

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CONCLUSIONS

- **Reflection coefficient measurement**
 - Wave decomposition theory
 - 2 line microphone array
- **Prony approach**
 - Higher spectral resolution than discrete Hankel transform approach
 - Effective with less measurement points
 - Less leakage due to finite aperture
- **Simulation & Experiment**

APPENDIX

- Prony model**

$$P(r) = \sum_{n=1}^N A_n e^{-\alpha_n r} \quad P(i) = \sum_{n=1}^N B_n Z_n^i \quad r = \delta\Delta r, (1+\delta)\Delta r, (2+\delta)\Delta r, \dots$$

$$\alpha_n = -\frac{\log Z_n}{\Delta r}, A_n = B_n e^{\alpha_n \delta \Delta r} \quad |Z_n| < 1$$

$$\begin{aligned} \tilde{P}(k_r) &= \int_0^\infty r P(r) J_0(k_r r) dr \\ &= \sum_{n=1}^N A_n \int_0^\infty r e^{-\alpha_n r} J_0(k_r r) dr \\ &= \sum_{n=1}^N \frac{A_n \alpha_n}{(\alpha_n^2 + k_r^2)^{3/2}} \end{aligned}$$

Prony method:
Calculate A_n, α_n by use of
2N or more data

Reflection coefficient

$$R(k_z) = \frac{-\tilde{P}_1 e^{-jk_z z_2} + \tilde{P}_2 e^{-jk_z z_1}}{\tilde{P}_1 e^{jk_z z_2} - \tilde{P}_2 e^{jk_z z_1}}$$