8-2000

Wave Number Domain Representation of Tire Vibration

J. Stuart Bolton
Purdue University, bolton@purdue.edu

Yong-Joe Kim

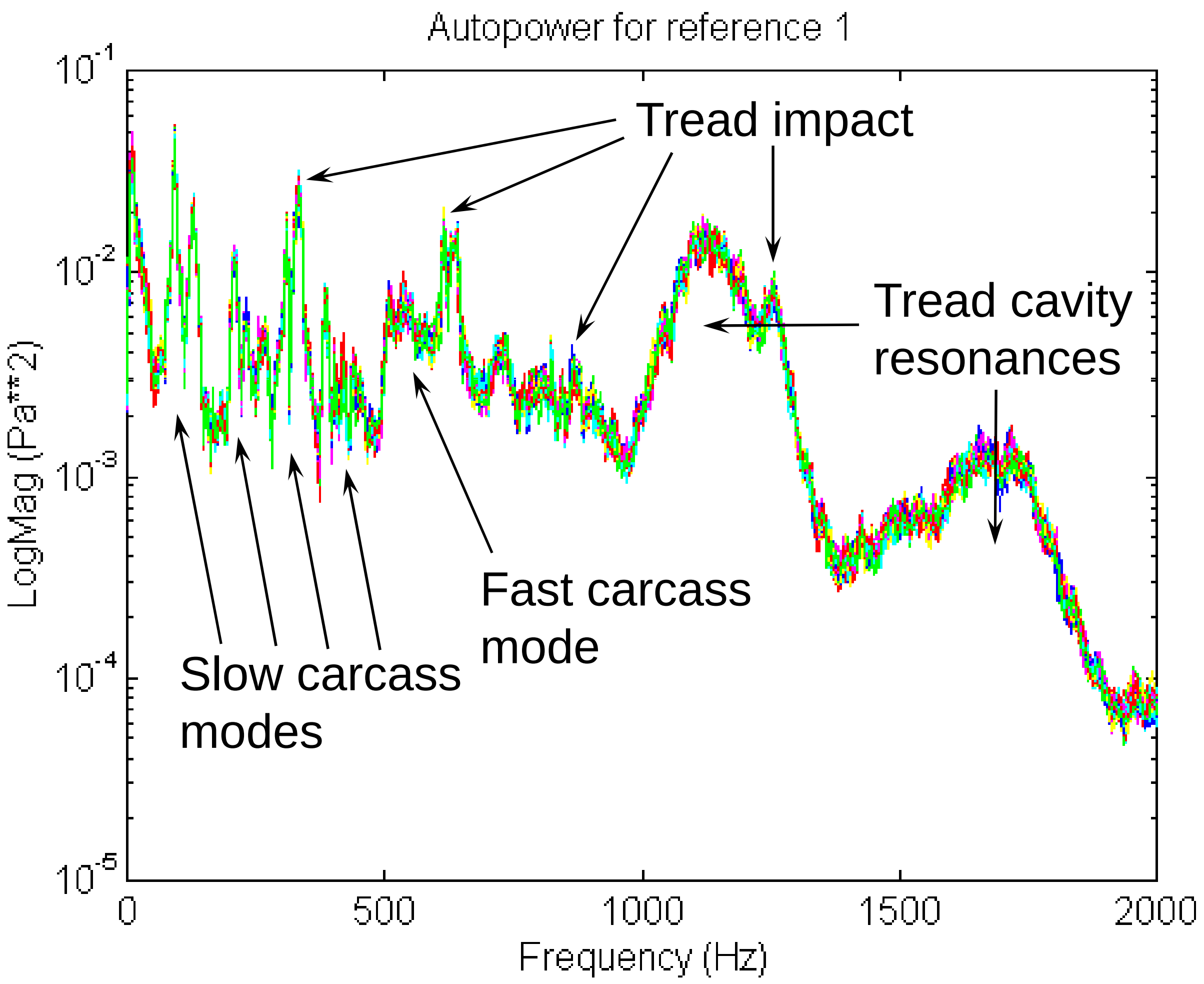
Bolton, J. Stuart and Kim, Yong-Joe, "Wave Number Domain Representation of Tire Vibration" (2000). *Publications of the Ray W. Herrick Laboratories.* Paper 14.
http://docs.lib.purdue.edu/herrick/14

WAVE NUMBER DOMAIN REPRESENTATION OF TIRE VIBRATION

J. Stuart Bolton and Yong-Joe Kim

Ray W. Herrick Laboratories
School of Mechanical Engineering
Purdue University

TIRE NOISE SOURCE MECHANISMS



Tread impact

Tread cavity resonances

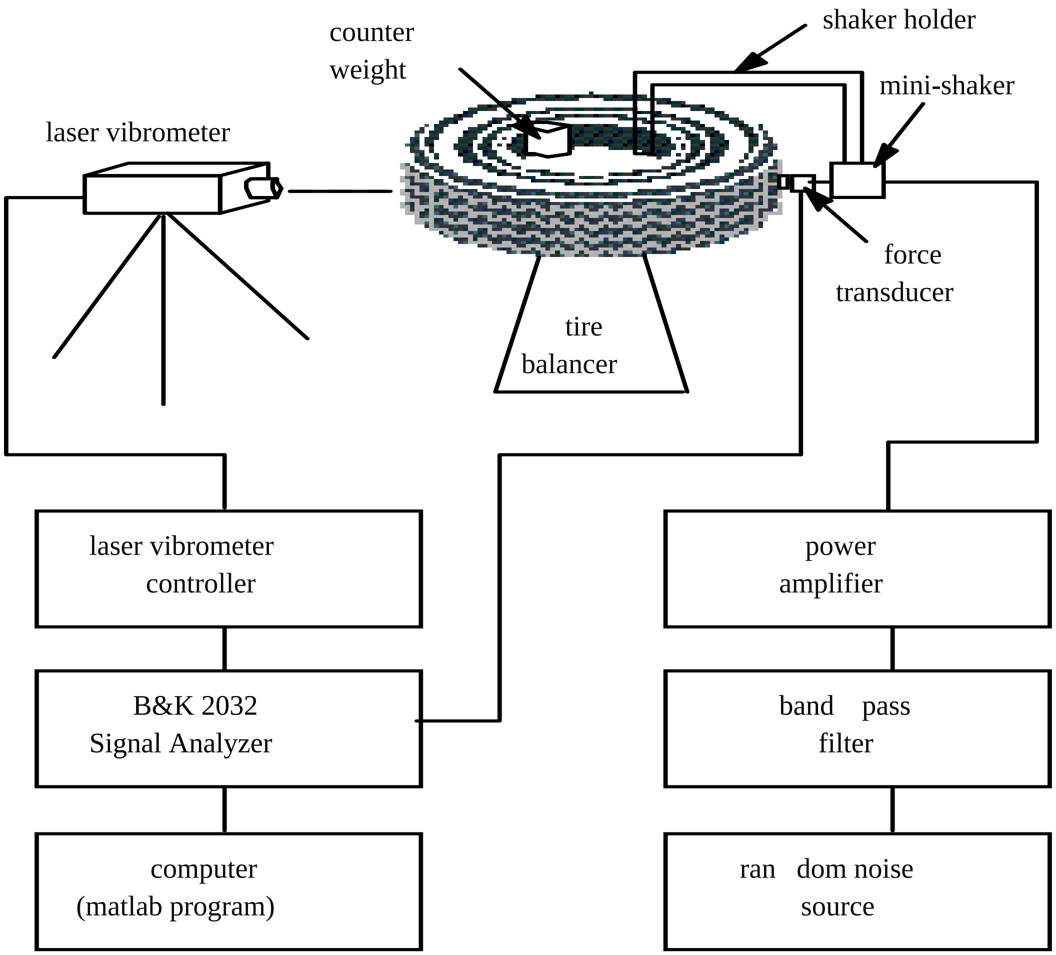
Fast carcass mode

Slow carcass modes

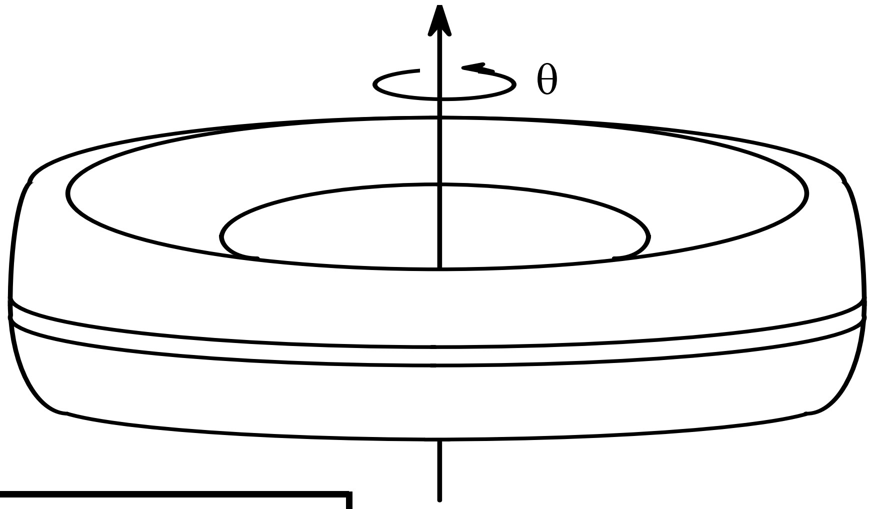
OBJECTIVE

- Experimental measurement of wave propagation in tires
- Characterize tire vibration as wave process
- Identification of origin of structure-borne road noise

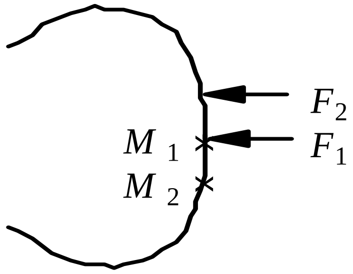
EXPERIMENTAL SETUP



TIRE GEOMETRY

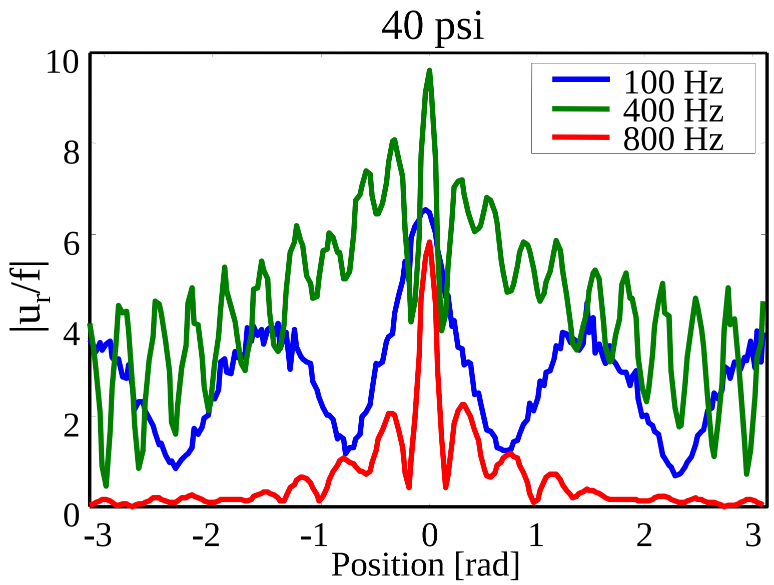


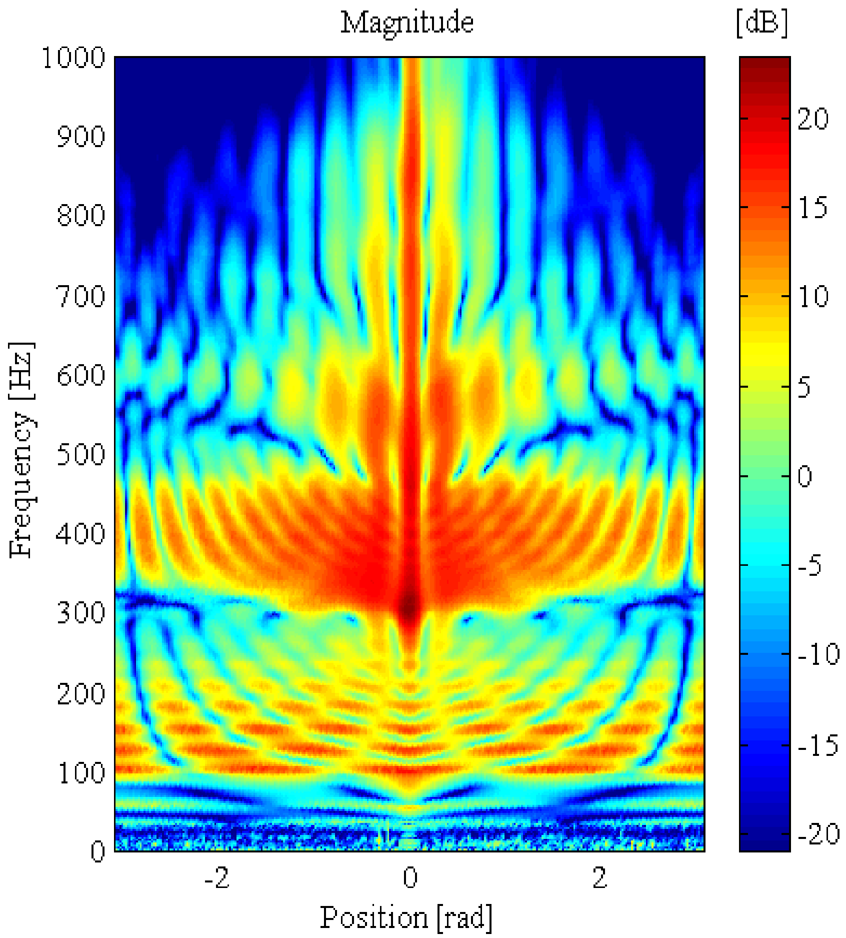
Firestone P215/70R 14 M+S
Inflation: 20 psi and 40 psi
Internal Gas: Air
CO_2
He

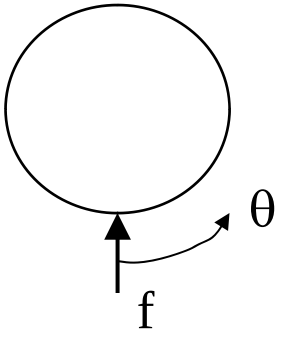


cross-section

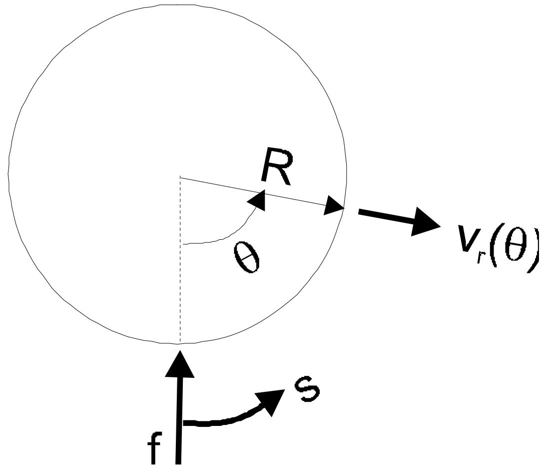
RADIAL TREADBAND VELOCITY







CIRCUMFERENTIAL WAVE NUMBER DECOMPOSITION

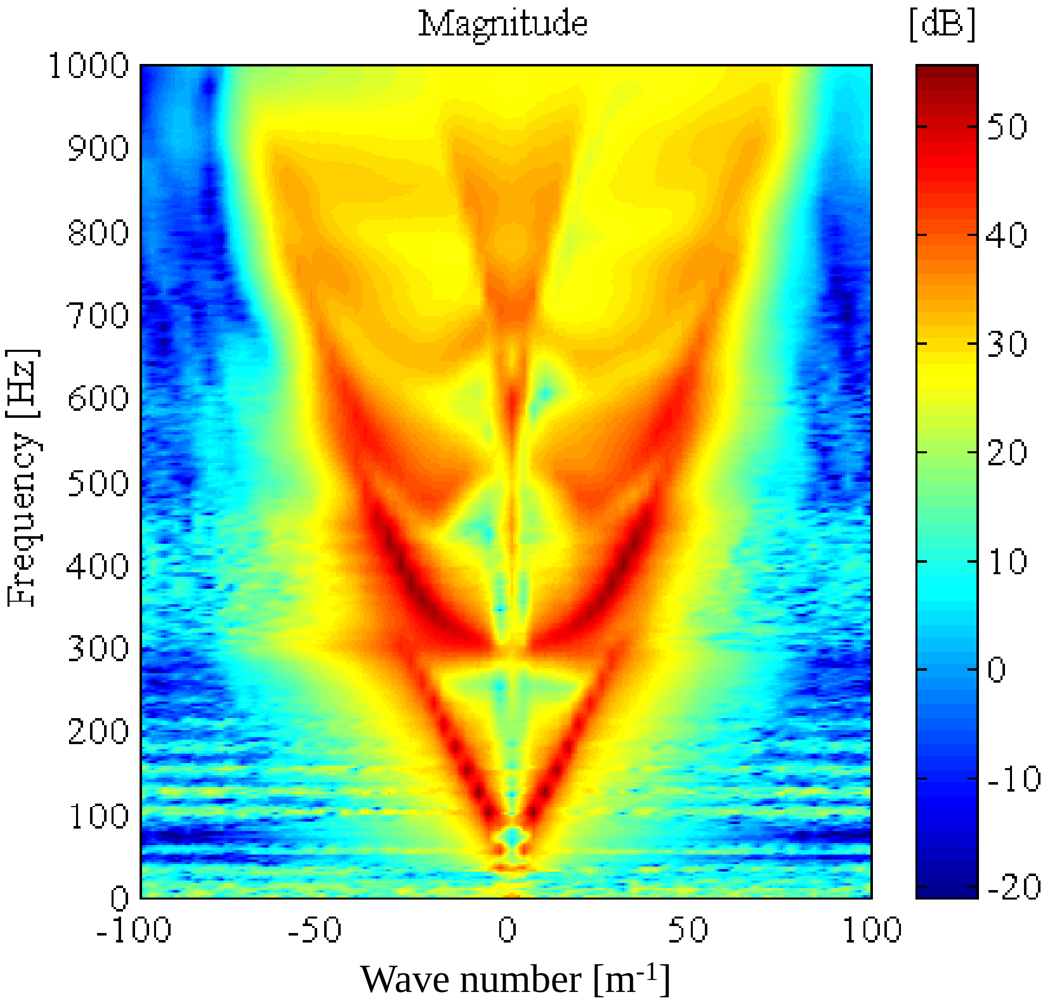


- At each frequency, represent radial velocity as

$$v_r(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{-in\theta} = \sum_{n=-\infty}^{\infty} a_n e^{-ik_{\theta n} s} \quad \textit{where} \qquad k_{\theta\, n} = {}^{2\ n}\!/_{R}$$

- Each component, $a_n e^{-ik_{\theta n} s}$ represents a circumferentially propagating disturbance

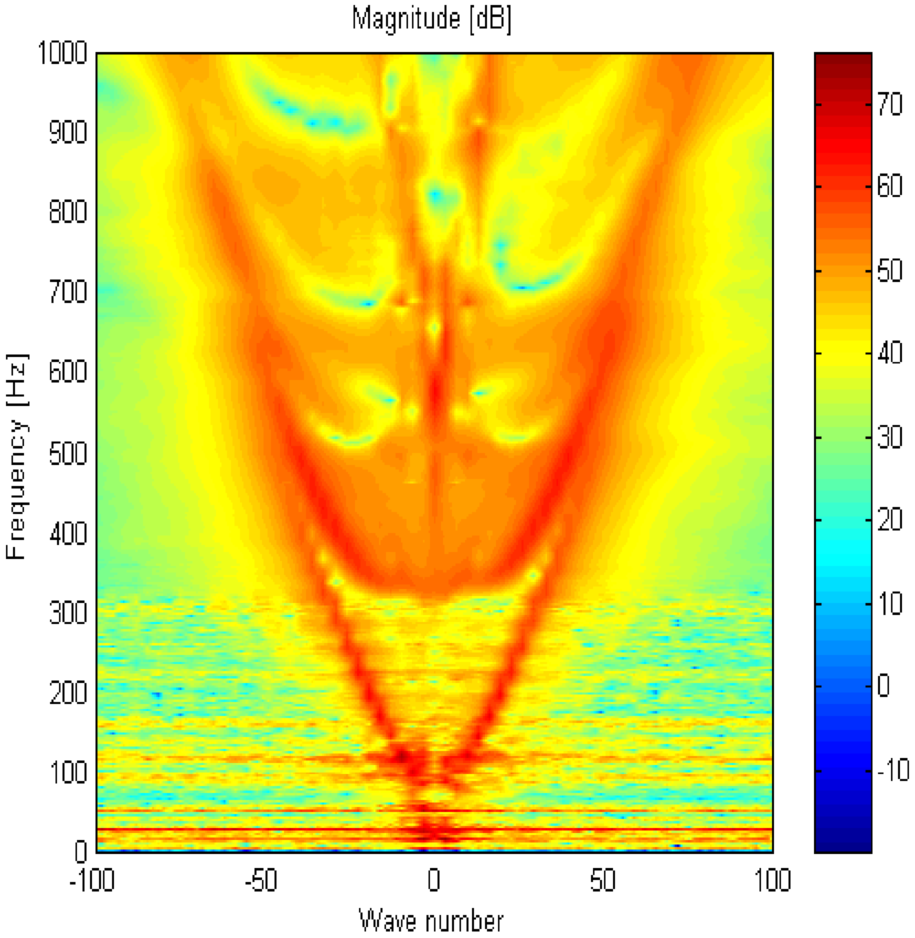
WAVE NUMBER SPECTRUM



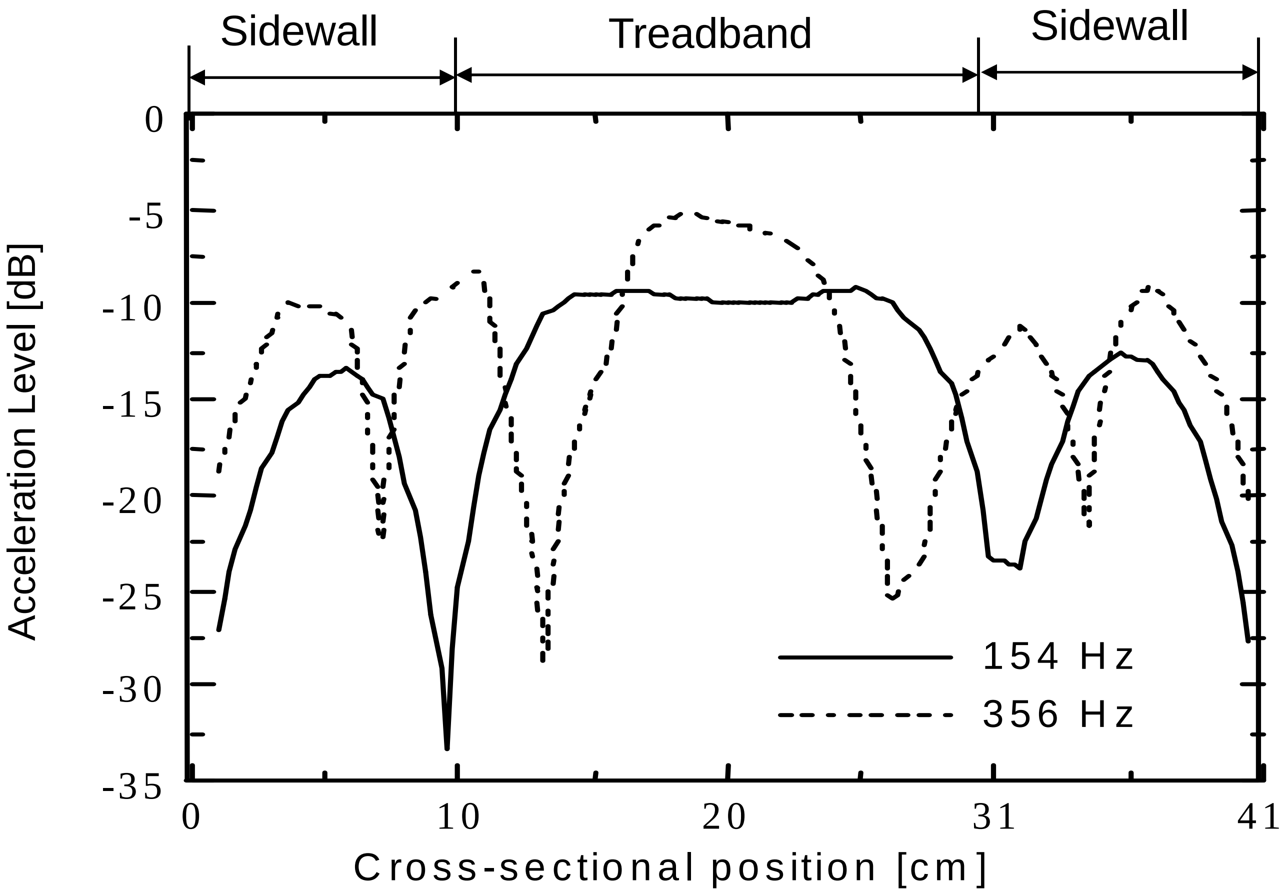
RESULT FOR SLICK TIRE

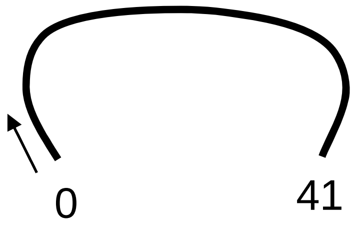
Tire size: 205/70R14

Inflation pressure: 30 psi



RADIAL ACCELERATION LEVEL



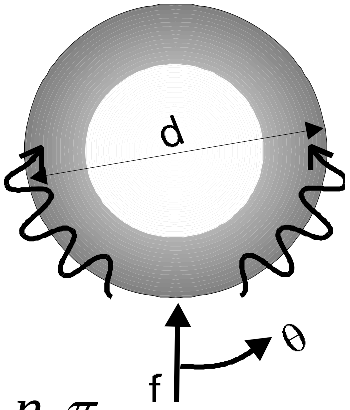


154 Hz: m=3

356 Hz: m=5

40 psi inflation pressure

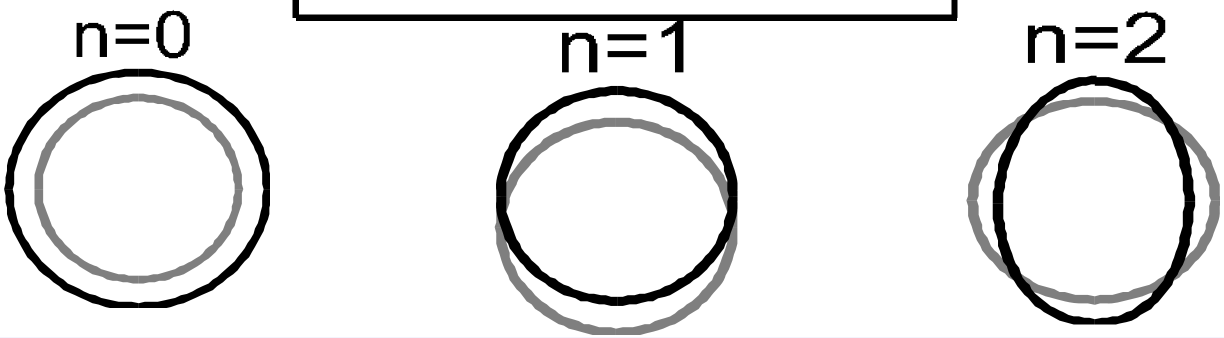
CREATION OF CIRCUMFERENTIAL MODES



When $k_\theta \pi d = 2n\pi$ **(i.e.,** $k_\theta = \frac{2n}{d}$ **)**

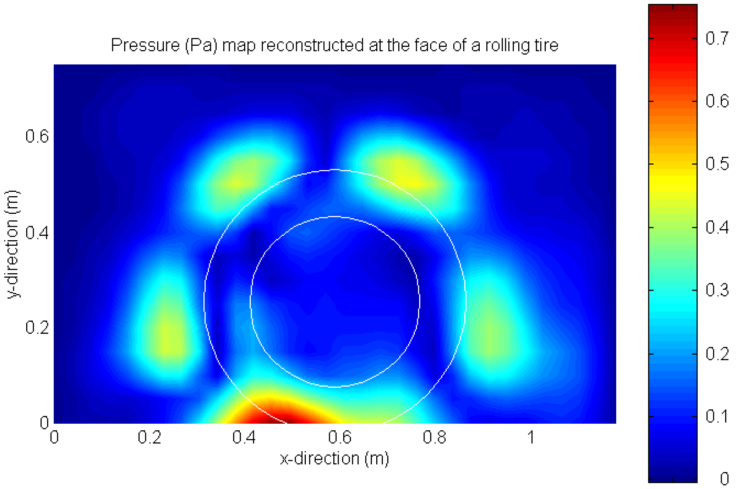
Propagating waves interfere to create standing waves

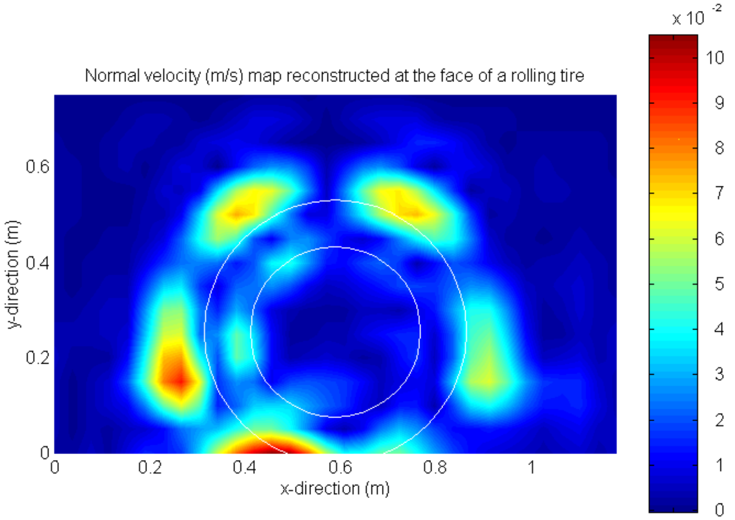
Circumferential modes

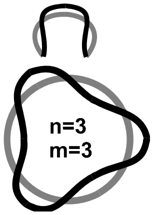


etc.

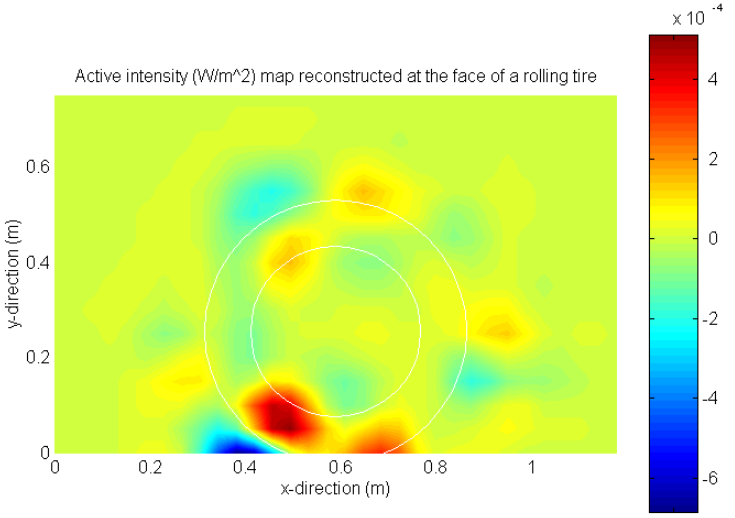
SLOW CARCASS MODE (128 Hz)

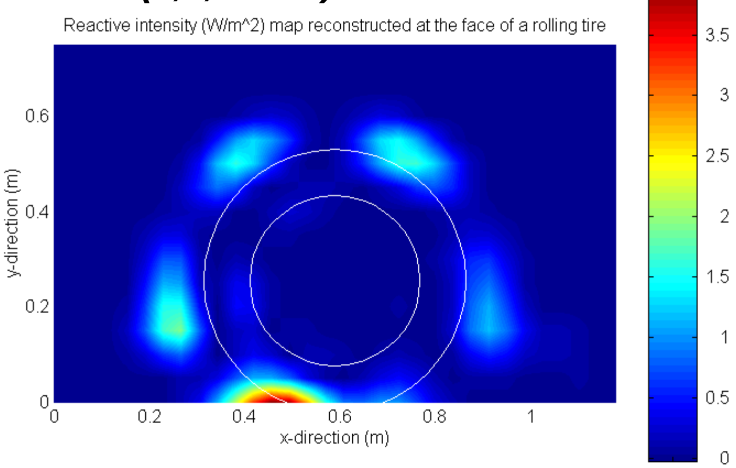

Pressure (Pa) map reconstructed at the face of a rolling tire


Normal velocity (m/s) map reconstructed at the face of a rolling tire

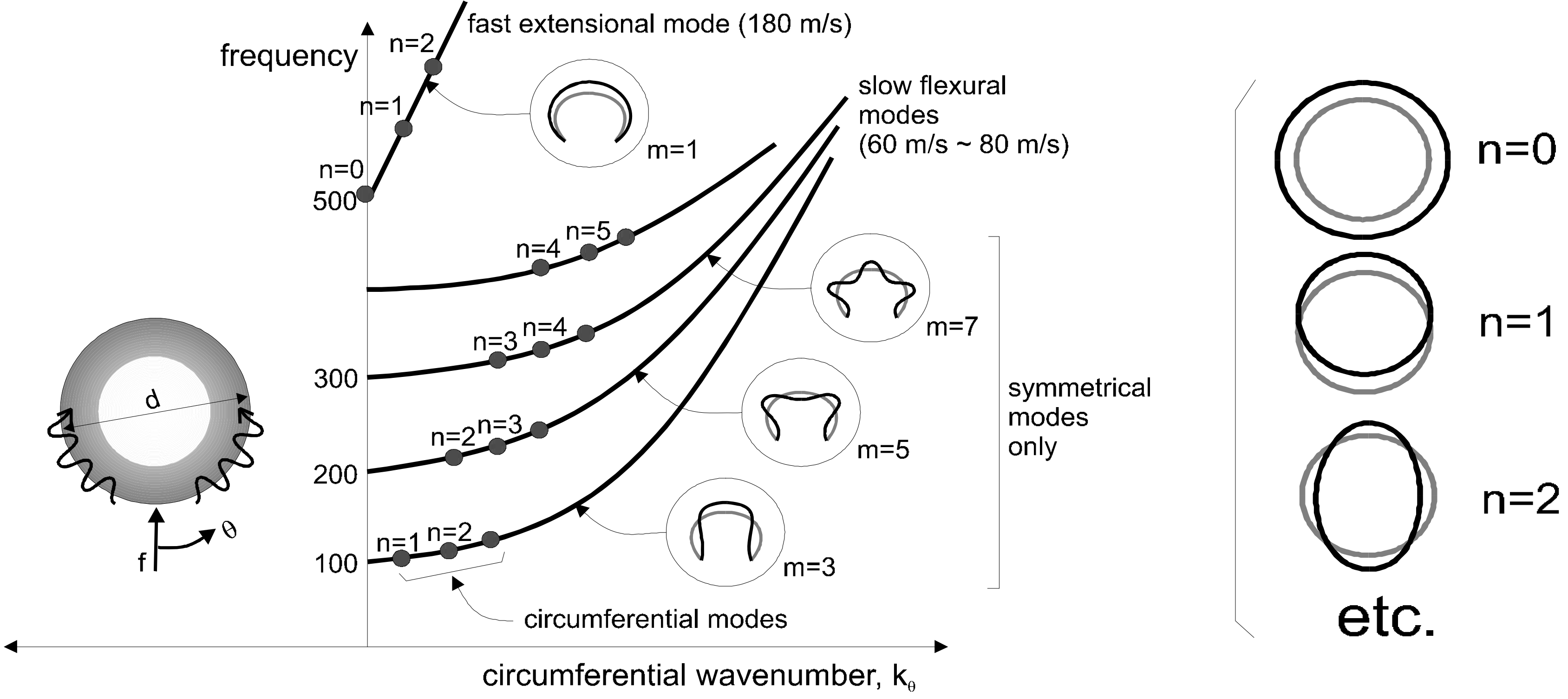


3 refs. (1,2, and 4)


Active intensity (W/m^2) map reconstructed at the face of a rolling tire


Reactive intensity (W/m^2) map reconstructed at the face of a rolling tire

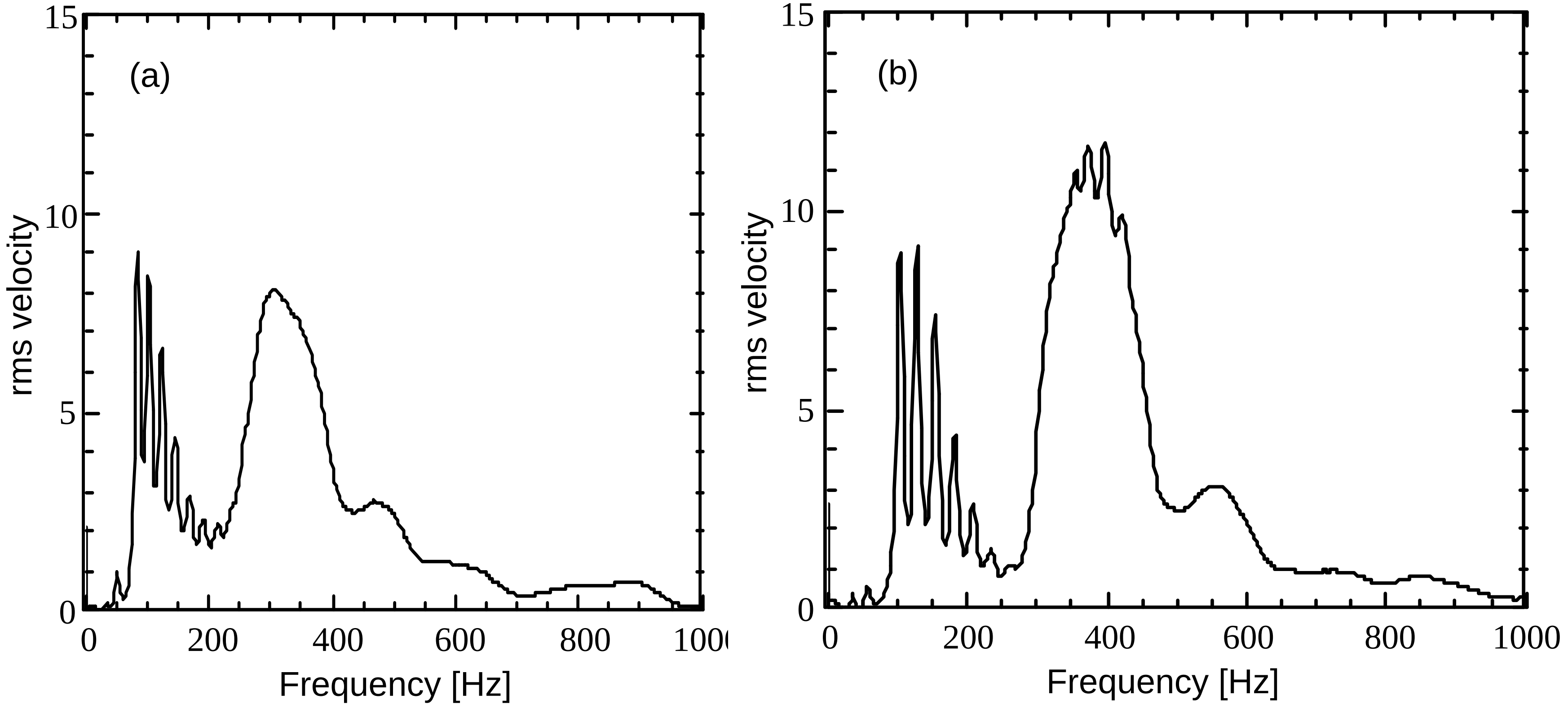
TIRE CARCASS DISPERSION CHARACTERISTICS - Static



- Tire carcass vibration is composed of superposition of “waveguide modes”
- Circumferential modes occur when $k_\theta \pi d = 2n\pi$

RMS NORMALIZED RADIAL VELOCITY

Circumferentially Averaged

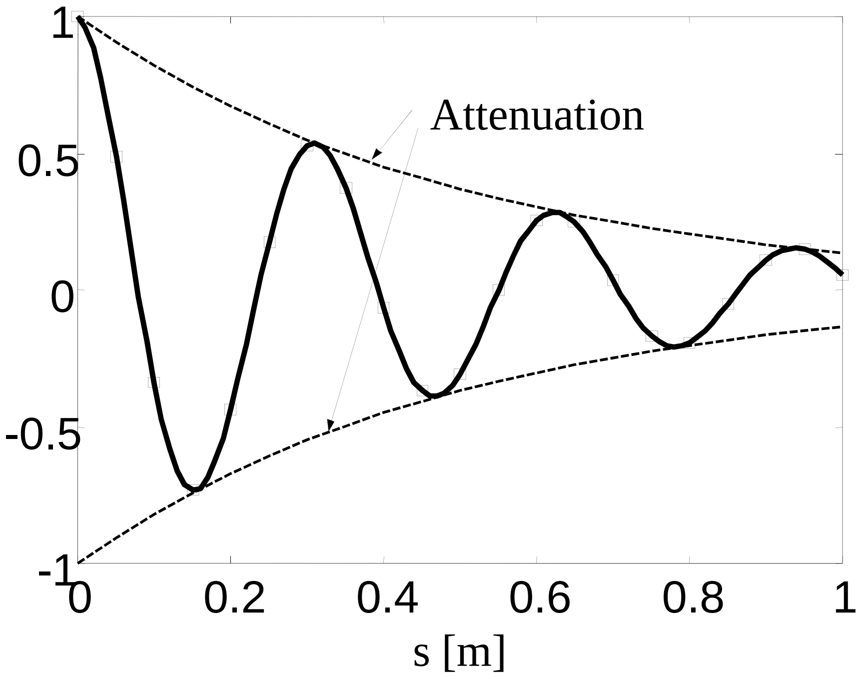


20 psi

40 psi

PROPAGATING WAVE REPRESENTATION

$$u_r = \sum_{m=1}^{N} A_m \Psi_m(s_c, \omega)\exp(\pm i k_{\theta m} s)$$



- $k_{\theta m}$ is complex

  $k_{\theta m} = \beta - i\alpha$

- Phase speed

  $c_p = \omega/\beta$

- Group speed

  $c_g = d\omega/d\beta$

- Attenuation per wavelength

  $\alpha\lambda = 2\pi\alpha/\beta$

PRONY METHOD

- To fit measured data using a finite exponential series

$$u_r(n\Delta s) = \sum_{m=1}^{M} a_m \exp(-ik_{\theta m} n\Delta s)$$

where,

$u_{\mathrm{r}}(n\Delta s)$: the measured data at $s = n\Delta s$

M : model order

$k_{\theta m}$: mth mode's complex wave number

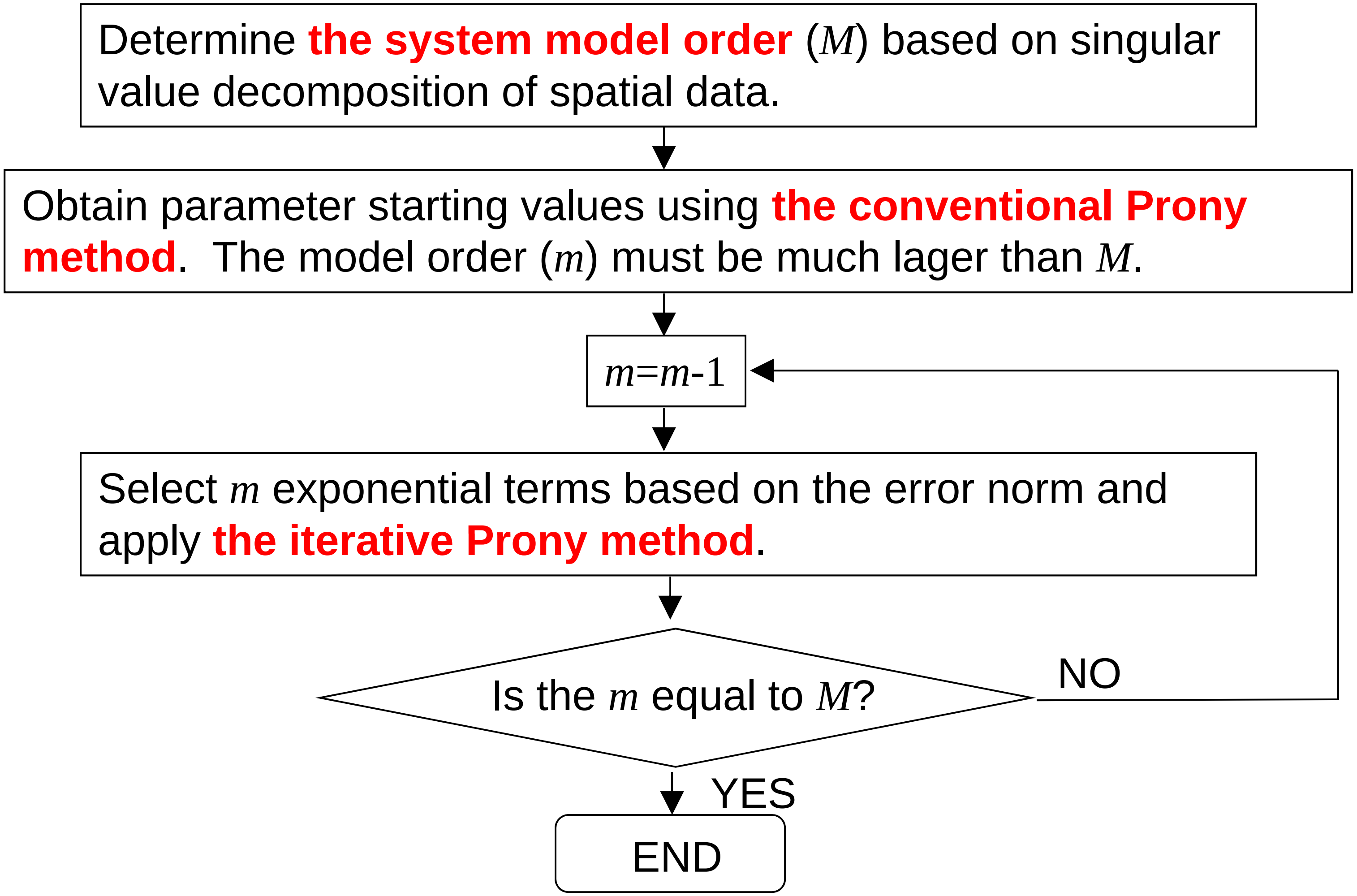
a_m : mth mode's complex amplitude

- Iterative Prony method (definition of error norm)

$$e_n = u_r(n\Delta s) - \sum_{m=1}^{M} a_m \exp(-ik_{\theta m} n\Delta s)$$

$$E(a_m, k_{\theta m}) = \sum_{n=1}^{N} e_n^* e_n$$

PRONY ALGORITHM



Determine **the system model order** (M) based on singular value decomposition of spatial data.

Obtain parameter starting values using **the conventional Prony method**. The model order (m) must be much lager than M.

$m=m-1$

Select m exponential terms based on the error norm and apply **the iterative Prony method**.

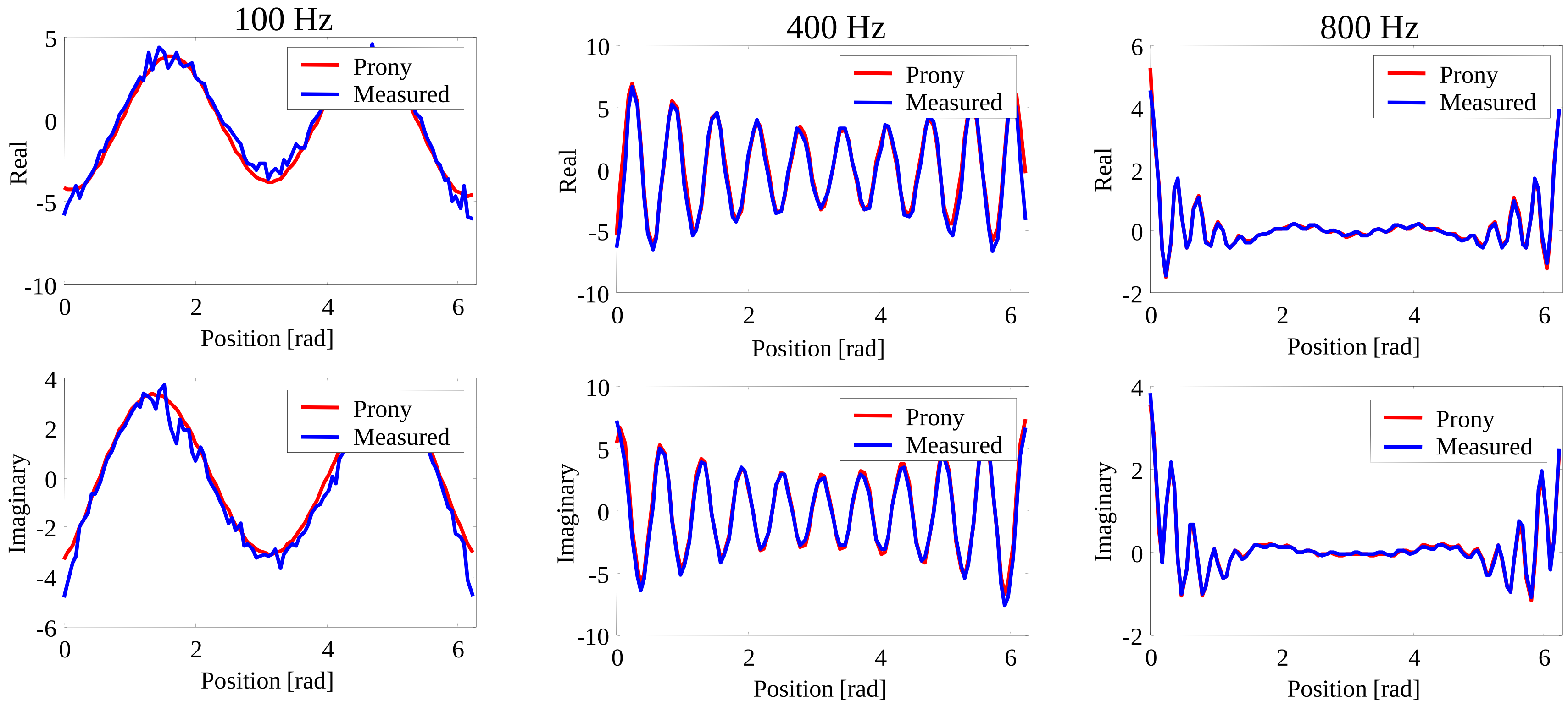
Is the m equal to M?

NO

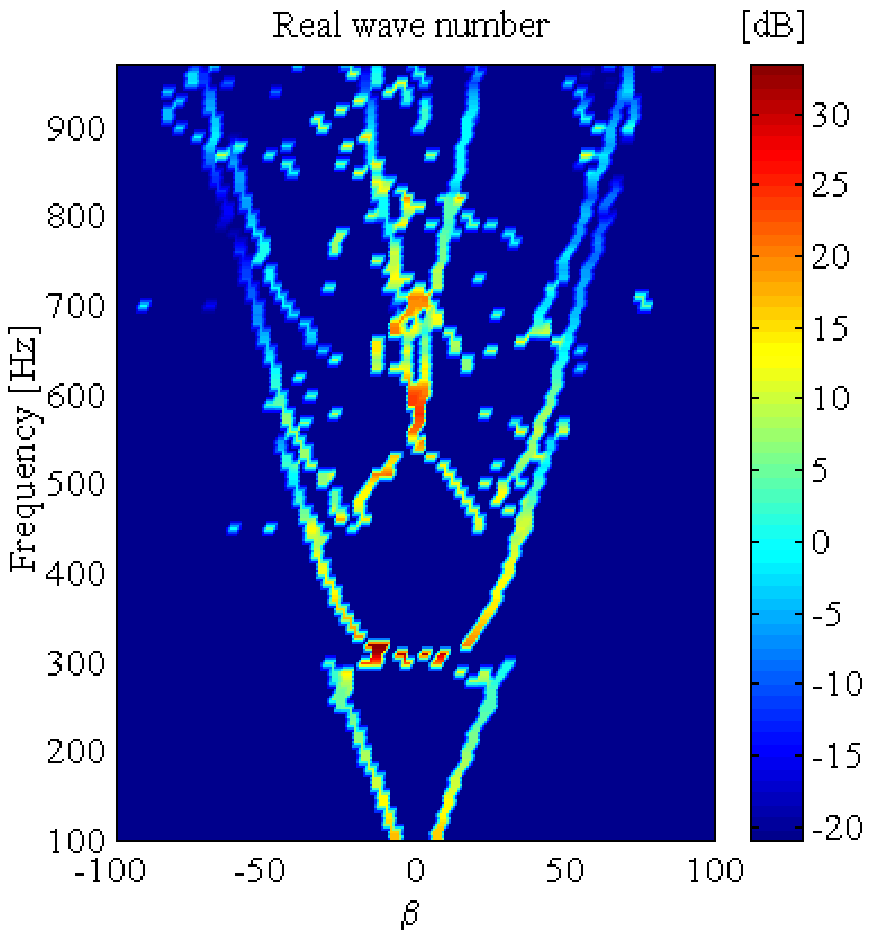
YES

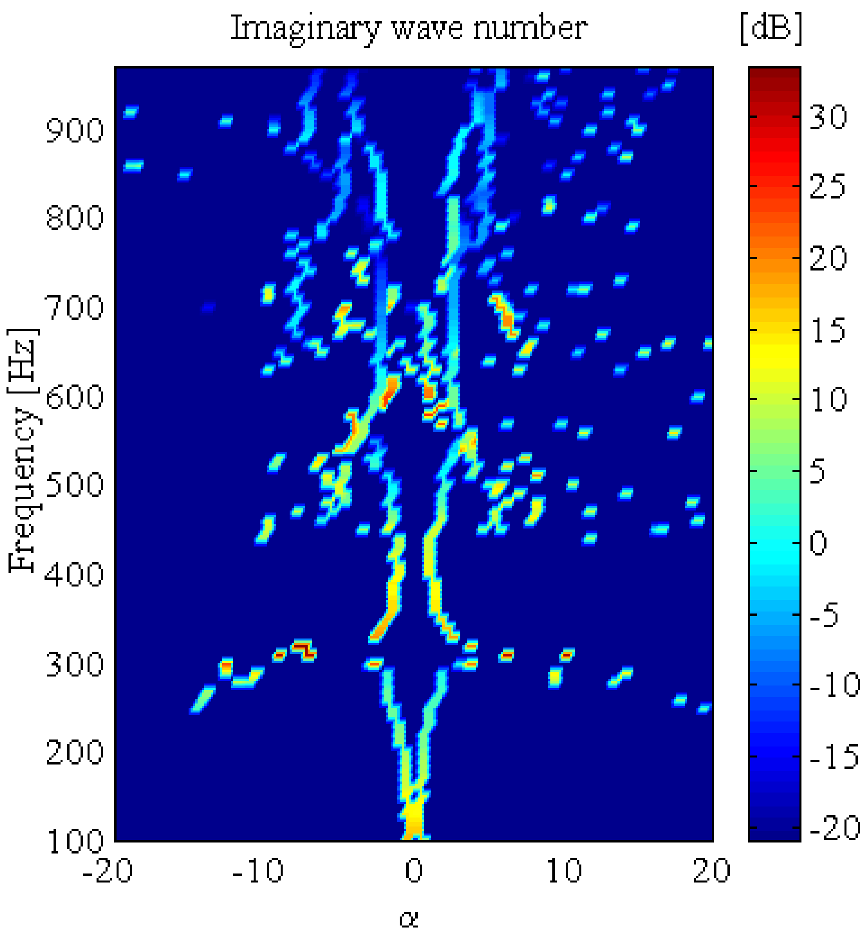
END

PRONY SERIES IDENTIFICATION

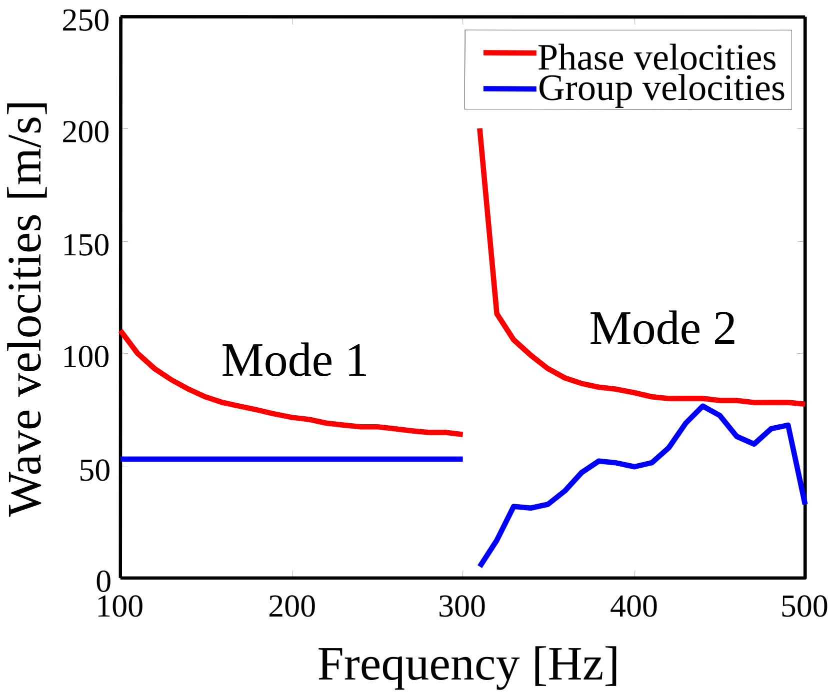


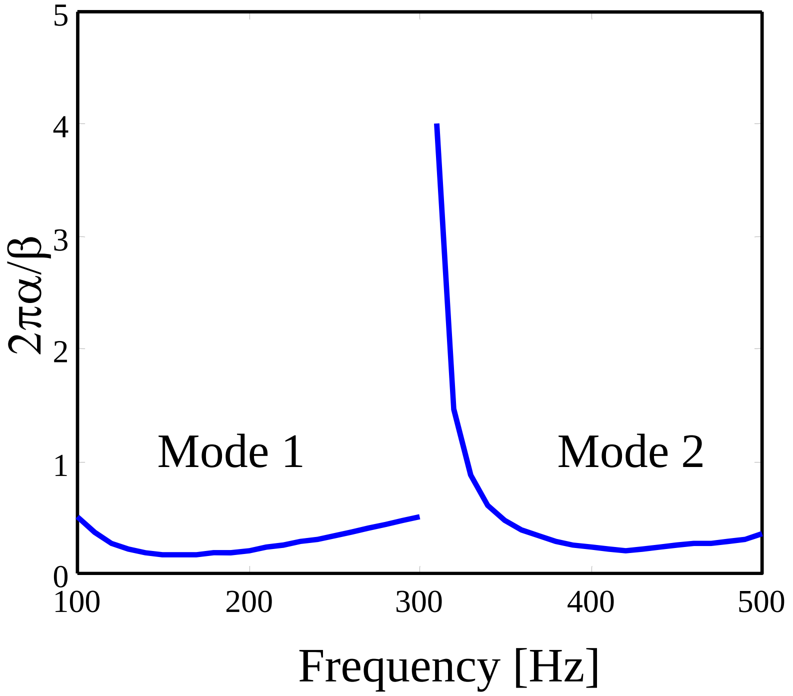
COMPLEX DISPERSION RELATIONS


Real wave number


Imaginary wave number

WAVE VELOCITIES AND ATTENUATIONS




Spatial attenuation per wavelength

CONCLUSIONS

- Vibrational response of passenger car tire controlled by six propagating waves below 1000 Hz.
- Circumferential modes occur at 700 Hz and below.
- Maximum of two waves contribute strongly at any single frequency
- Each propagational mode associated with particular cross-sectional mode shape.
- Overall vibration of tire peaks near cut on of each wave type.
- Structure-borne interior noise possibly controlled by n=1 modes which deliver net force to hub.

CONCLUSIONS

- Radial velocity of tire carcass can be conveniently expressed as

$$u_r = \sum_{m=1}^{N} A_m \Psi_m (s_c , \omega) e^{\pm i k_{\theta_m} s}$$

- Simpler to express tire vibration in terms of propagating wave type than in conventional modal form.
- Convenient as input to boundary element prediction of sound radiation.
- Propagation characteristics of modes can be identified using spatial Prony series.