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CAVITY RESONANCE IN FRACTIONAL HP REFRIGERANT COMPRESSORS

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INTRODUCTION

Increased consumer awareness of household appliance noise is causing the manufacturers of these products to take a closer look at their product sound levels. Because refrigerant compressors in home refrigerators, freezers and air conditioners are automatically controlled, their operation must not create high noise levels since the option of turning off the appliance is unacceptable.

Cavity resonance in fractional horsepower refrigerant compressors has been suggested as a potential source of compressor noise [1]*, but little published information is available to estimate its significance in a particular compressor. This paper describes the study of a cavity resonance in a one-third horsepower rotary-vane refrigerant compressor, and provides a basis for determining an approximate natural frequency.

BACKGROUND

It is known that a volume of gas will vibrate when excited by pressure or velocity fluctuations. If an excitation frequency coincides with one of the natural frequencies of the cavity, resonant amplification can result in large cavity pressure fluctuations.

The cavity bounded by the outer surface of a compressor mechanism (Figure 1) and the inner surface of its hermetic shell is an ideal location for the generation of a gas cavity resonance since refrigerant compressors have strong excitations over a wide frequency range. Resonance can cause substantial pressure fluctuation amplitudes at the surface of the shell causing it to vibrate and radiate sound. For regular cavity shapes (ie a cylinder), the natural cavity shapes and mode shapes can be determined by solving the governing equations directly. Solutions for some irregular cavities can often be obtained by numerical methods. Once these mode shapes (potential maps) are known (the nature of the problem dictates the use of a sound pressure/particle velocity description of the sound field external to the shell while applying a velocity potential description of the flow field within the shell), dissipaters such as baffles or fibrous materials can be placed in locations where large velocities exist to dampen resonant amplitudes.

The geometries of the inner boundary of the cavity within most compressors including the one in this study are usually extremely irregular, frequently having passages that connect through the body of the compressor. Hence, the prediction of the exact frequencies and modes of gas vibration is likely to be inaccurate because of the simplifications of the geometry required to carry out a solution. The problem then reduces to the identification of a sound-related phenomenon without an accurate theoretical model to compare with experiment.

DETERMINATION OF UPPER AND LOWER BOUNDS TO THE FIRST TRANSVERSE MODE OF GAS RESONANCE

In order to examine the effect of parameters such as shell diameter and radius ratio of the annulus, the simple problem of determining the first natural frequency of an annulus was solved.

Consider an annular cavity (Figure 2) filled with a gas whose sonic velocity is

* Numbers in brackets refer to listings in the bibliography.
c (a constant). For small gas disturbances the acoustic wave equation is [2]:

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0$$  \hspace{1cm} (1)

The fluid velocity is derived from the potential function:

$$u = - \nabla \phi$$  \hspace{1cm} (2)

and the perturbation pressure:

$$p' = p - p_\infty = \rho_\infty \frac{\partial \phi}{\partial t}$$  \hspace{1cm} (3)

where the \(U_i\) are the velocity components, \(p\) is the instantaneous absolute pressure, \(p_\infty\) and \(\rho_\infty\) are the constant pressure and density values about which the perturbations occur.

Application of the boundary conditions (zero normal particle velocity at the cavity walls) yields the frequency equation (See appendix):

$$\left[ \frac{m^2}{a^2} J_m(k_a) - k_a J_{m+1}(k_a) \right] \left[ \frac{m^2}{b^2} Y_m(k_b) - k_b Y_{m+1}(k_b) \right] - \left[ \frac{m^2}{b^2} J_m(k_b) - k_b J_{m+1}(k_b) \right] \left[ \frac{m^2}{a^2} Y_m(k_a) - k_a Y_{m+1}(k_a) \right] = 0$$  \hspace{1cm} (4)

where \(J_m\) is the Bessel Function of the first kind of integral order \(m\) and \(Y_m\) is the Bessel Function of the second kind of integral order \(m\).

The lowest frequency occurs for the case where \(m = 1\) and results in the transverse or "sloshing" mode of vibration. Equation (4) was evaluated for \(m = 1\), outer radii of 2 to 6 inches and annulus radius ratios of from 0 to 1. The results are shown in Figure 5.

If the actual compressor gas vibration is substantially transverse, and this is probably the case except at places where there are abrupt changes in diameter of the inner boundary, the end points of each curve of Figure 5 represent upper and lower bounds to the lowest transverse resonance in cylindrical compressor cavities (Figure 6).

The actual compressor cavity, however, is not a true annulus. In most cases the volume consists of a thin hollow cylindrical shell enclosing a solid cylindrical pump assembly (Figure 1) and therefore contains an annular volume bounded by two cylindrical volumes (cylinder-annulus-cylinder). The resonant frequency associated with the annular part of the cavity is given by Figure 5 entering with the proper \(b/a\) ratio and housing radius. The resonant frequency associated with the cylindrical volumes at the ends is given also by Figure 5 for the case where \(b/a = 0\). Clearly then, the resonant frequency of the compressor gas cavity will fall between these frequencies since the time for a pressure wave to travel around the circumference will be shorter in the case of a cylinder-annulus-cylinder combination than a true annulus because part of the wave travels the shorter path across the diameter at the ends of the housing. In similar fashion the time for a pressure wave to travel across a diameter will be longer in the case of a cylinder-annulus-cylinder than for a true cylinder because part of the wave must traverse the longer annular path. Thus the actual compressor cavity transverse resonant frequency lies between the cases for \(b/a = 0\) and \(b/a = 1\).

It should be noted that this restricts the application of this analysis to cavities whose boundaries are such that every horizontal plane cross section approximates an annulus.

**EXPERIMENTS**

Compressor sound measurements were made in an anechoic chamber whose inside dimensions are a 12 feet cube having a cut off frequency of 100 Hz. A one inch Bruel and Kjaer condenser microphone was employed at a distance of 18 inches from the compressor shell. Narrowband instrumentation included a Bruel and Kjaer microphone amplifier a Spectral Dynamics Dynamic Analyzer, a Spectral Dynamics Sweep Oscillator, a Hewlett-Packard Log Converter and an Electro Instruments X-Y Recorder. A Bruel and Kjaer pistonphone was used for calibration. Sufficiently slow filter sweep rates were employed to insure an accurate reproduction of the spectra. A more complete description of the experimental setup as well as an analysis of the repeatability of compressor sound spectra are given in reference [3].

A typical narrowband sound spectrum for a fractional horsepower rotary vane compressor from a distance of 1.5 feet is shown in Figure 3. Sound measurements in this case were made when the gas temperature within the shell cavity was at a level producing the highest amplitude for the harmonic at 460 Hz. Further testing revealed the sensitivity of the 460 Hz tone to temperature shown in Figure 2. This temperature sensitivity, together with two-lobed sound directivity pattern prompted an investigation into the possibility that amplification of the 460 Hz peak (4th harmonic of pumping frequency) is caused by a transverse resonance of the gas within the shell cavity.
The compressor motor cooling system operates by passing high pressure discharge gas through a heat exchanger, then returning it to the compressor at a lower temperature to receive part of the pump heat. Prior to discharge, the pulsating gas passes through a discharge muffler section to reduce pressure oscillation amplitudes. Since the pulsating gas is returned to the shell cavity, a test was devised to determine if this discharge return gas upon entering the shell cavity contained pressure fluctuations of such an amplitude to excite the shell to generate a sound field comparable to that which is actually produced by the compressor. Additionally, information regarding the frequency dependence of the sound generated by the return gas was desired.

A test scheme utilizing two identical compressors was devised to excite the cavity to resonance. The discharge gas from one compressor operating at normal conditions (running) was fed to the discharge return fitting of an unpowered second compressor (static). Sound measurements were made at the static compressor location. This setup provided excitation, in the form of discharge gas pulsations, to the compressor cavity and eliminated other sources of compressor shell excitation. The two compressors were separated by a sufficient distance in an anechoic chamber so that the sound pressure levels from the running compressor were reduced 15 dB at the static compressor location. A microphone distance of 12 inches was used for the static compressor during these tests. Since the primary purpose of these tests was to study the 460 Hz tone, the running compressor was oriented so that its two-lobed directional pattern was perpendicular to the line connecting the centers of the compressors. This reduced further the effect of the running compressor noise on the static compressor measurements.

Directivity patterns of the 460 Hz tone for the static unit were measured for two different discharge gas insertion locations as shown in Figure 7. The two-lobed pattern, characteristic of shell translation, is perpendicular to the shell axis and indicates shell motion in a direction coincident with the location of the excitation. Therefore, the directivity pattern of the running compressor reveals the location of the excitation source.

Earlier, it was shown that the wave number for a given compressor geometry transverse resonance is a constant, $k$:

$$k = \frac{2\pi f c}{c} \tag{5}$$

where $f$ is the first transverse mode resonant frequency of the cavity (Hz) and $c$ the sonic velocity (ft./sec.). For a given $k$, $f$ is proportional to $c$.

As the gas in the static compressor heats up, its sonic velocity increases causing an increase in the cavity resonant frequency. The development of this resonance is seen in Figure 8 which is a series of eight portions of the static compressor sound spectrum from 300 Hz to 600 Hz showing the amplification of the fourth harmonic with temperature. At the beginning of the sequence, the resonant frequency is 5 to 10 Hz below the fourth harmonic and amplifies the broadband noise. As the temperature increases, the resonant frequency moves through the fourth harmonic.

Two cavity shapes whose resonant frequencies were determined from equation (4) were tested. The first, a six inch diameter empty compressor shell was put into the static compressor location and the recorded sound spectrum is shown in Figure 9 (upper spectrum) compared with a complete compressor as the static unit (lower spectrum). The filtering effect of the broadband noise due to resonance takes place at 540 to 550 Hz compared with 530 Hz predicted by theory. The discrepancy is probably due to temperature differences within the cavity. Secondly, a four inch pipe was brazed into the empty shell so that an annular cavity was formed with a b/a of 2/3. The predicted natural frequency of 345 Hz was in close agreement with the measured value of 340 Hz. 340 Hz coincides with the frequency of the third harmonic of the discharge pulsation, therefore, as the temperature was increased, amplification of this harmonic took place in the same manner as the fourth harmonic in the compressor.

It is to be expected that any change in sonic velocity will produce a corresponding change in the transverse cavity resonant frequency. To confirm this, Refrigerant 22 ($c = 575$ ft./sec.) was substituted for Refrigerant 12 ($c = 450$ ft./sec.) normally used and variations of the fourth and fifth harmonic amplitudes with temperature were examined. As predicted by Equation (5) the resonant frequency had shifted to 575 Hz which coincided with the fifth harmonic of the discharge pulsations and again the fourth harmonic temperature sensitivity had disappeared.

CONCLUSIONS

The following conclusions are drawn concerning the identification of cavity resonances:

1. Except in the case where strong broadband excitation exists, cavity resonance in compressors will be of consequence only if it coincides with
a pumping frequency harmonic.

2. Slower compressors with higher harmonic densities have a higher probability of exhibiting the effects of a cavity resonance. This is offset, of course, by the fact that the higher harmonic numbers are likely to have lower amplitudes. (i.e. if the pumping frequency were 29 Hz, then 464 Hz corresponds to the 16th harmonic.)

3. Other modes, if strongly excited, will be present. Temperature test data have also indicated the possibility of a higher mode resonance at 1750 Hz. At this frequency, compressor running speed has a greater effect on the coincidence of the harmonic and the resonance. The level of the corresponding harmonic (15th) was not very high and the amplification shown in the data did not bring it to a seriously high sound level.

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APPENDIX

Derivation of Frequency Equation for Transverse Resonance in an Annular Cavity

For small gas disturbances, the acoustic wave equation:

\[ \nabla^2 \phi - \frac{1}{C^2} \frac{\partial^2 \phi}{\partial t^2} = 0 \]  
(a-1)

holds where the fluid velocity is derived from the potential:

\[ \mathbf{u} = - \nabla \phi \]  
(a-2)

and the perturbation pressure:

\[ p' = p - p_0 = \rho_0 \frac{\partial \phi}{\partial t} \]  
(a-3)

where \( u_i \) are the velocity components, the instantaneous absolute pressure, \( p \) and \( p_0 \), \( \rho_0 \) are the constant pressure and density about which the perturbations occur.

In cylindrical coordinates \( r, \theta \) and \( z \) (a-1) becomes:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} - \frac{1}{C^2} \frac{\partial^2 \phi}{\partial t^2} = 0 \]  
(a-4)

If it is assumed that the time variation is harmonic:

\[ \phi = R(r, \theta, z) e^{i\omega t} \]  
(a-5)

substitution of (a-5) into (a-4) and recalling that:

\[ c = \lambda f \]  
(a-6)

\( c = \) sonic velocity \( \lambda = \) wave length \( f = \) frequency

yields:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} + k^2 \phi = 0 \]  
(a-7)

where \( k \) is the wave number:

\[ k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} \]  
(a-8)

If it is assumed:

\[ \Phi(r, \theta, z) = R(r) \Theta(\theta) Z(z) \]  
(a-9)

and substitute into (a-7) After some manipulation:

\[ \Phi = [C J_m(\beta r) + D Y_m(\nu r)] \cos \frac{m \pi z}{l} \cos \Theta \]  
(a-10)

provided:

\[ k^2 = \beta^2 + m^2 \frac{\pi^2}{l^2} \]  
(a-11)

where \( C \) and \( D \) are constants. For the transverse vibration mode: \( m' = 0 \)

\[ \Phi = [C J_m(k \Gamma r) + D Y_m(k \nu r)] \cos \Theta \]  
(a-12)

In an annular cavity with rigid walls, the normal velocity vanishes at the
boundaries

\[ \frac{\partial \Phi}{\partial r} \bigg|_{r=a} = 0, \quad \frac{\partial \Phi}{\partial r} \bigg|_{r=b} = 0 \]  
(a-13)

substituting (a-12) into (a-13):

\[ \begin{align*}
\left[ \frac{m}{a} J_m(k_a) - k J_{m+1}(k_a) \right] C + \left[ \frac{m}{a} Y_m(k_a) - k Y_{m+1}(k_a) \right] D &= 0 \\
\left[ \frac{m}{b} J_m(k_b) - k J_{m+1}(k_b) \right] C + \left[ \frac{m}{b} Y_m(k_b) - k Y_{m+1}(k_b) \right] D &= 0
\end{align*} \]  
(a-14)

For a solution to exist, the determinant of the coefficients C and D must vanish yielding the frequency equation:

\[ \begin{align*}
\left[ \frac{m}{a} J_m(k_a) - k J_{m+1}(k_a) \right] \left[ \frac{m}{b} Y_m(k_b) - k Y_{m+1}(k_b) \right] - \\
\left[ \frac{m}{b} J_m(k_b) - k J_{m+1}(k_b) \right] \left[ \frac{m}{a} Y_m(k_a) - k Y_{m+1}(k_a) \right] &= 0
\end{align*} \]  
(a-15)

where \( J_m \) is the Bessel Function of integral order and \( Y_m \) is the Bessel Function of the second kind of integral order.

**BIBLIOGRAPHY**


Fig. 1 Rotary compressor

Fig. 2 Annular cavity

Fig. 3 Cavity resonance frequency for first transverse mode in an annulus

Fig. 4 Bounds of first transverse resonance in cylindrical compressors

Fig. 5 Free field, 5-Hz bandwidth narrow-band, compressor sound spectrum at 15 in. with 460-Hz component (fourth harmonic) amplified by cavity resonance

Fig. 6 Fourth harmonic sound amplitude variation with resonant cavity gas temperature
Fig. 7 Resonant frequency directivity patterns at 12 in.

Fig. 8 Coincidence of fourth harmonic with cavity resonance frequency as a function of compressor discharge temperature

Fig. 9 Free field sound spectra showing transverse resonance in two cavities