4-1998

The Wave Number Decomposition Approach to the Analysis of Tire Vibration

J. Stuart Bolton
Purdue University, bolton@purdue.edu

Heuk Jin Song

Yoon Ki Kim

THE WAVE NUMBER DECOMPOSITION APPROACH TO THE ANALYSIS OF TIRE VIBRATION

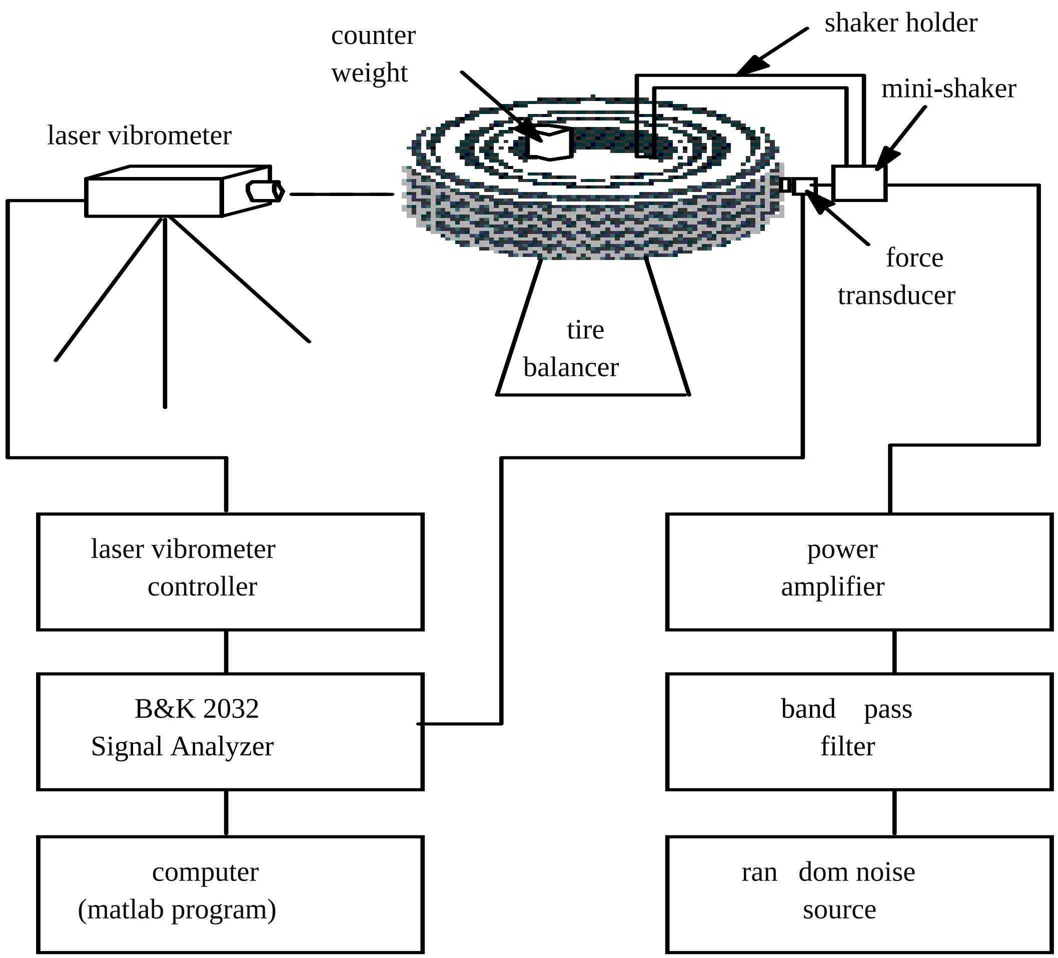
J. Stuart Bolton, Heuk Jin Song, Yoon Ki Kim
Ray W. Herrick Laboratories
School of Mechanical Engineering
Purdue University

Yeon June Kang
Dept. of Mechanical Design and Production Engineering
Seoul National University

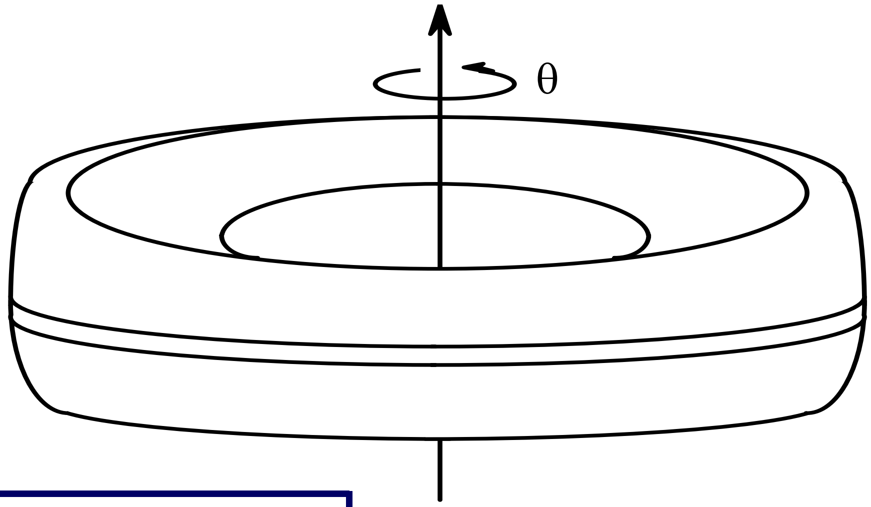
OBJECTIVE

- **Experimental measurement of wave propagation in tires**
- **Characterize tire vibration as wave process**
- **Identification of origin of structure-borne road noise**

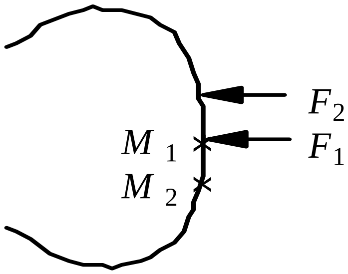
EXPERIMENTAL SETUP



TIRE GEOMETRY

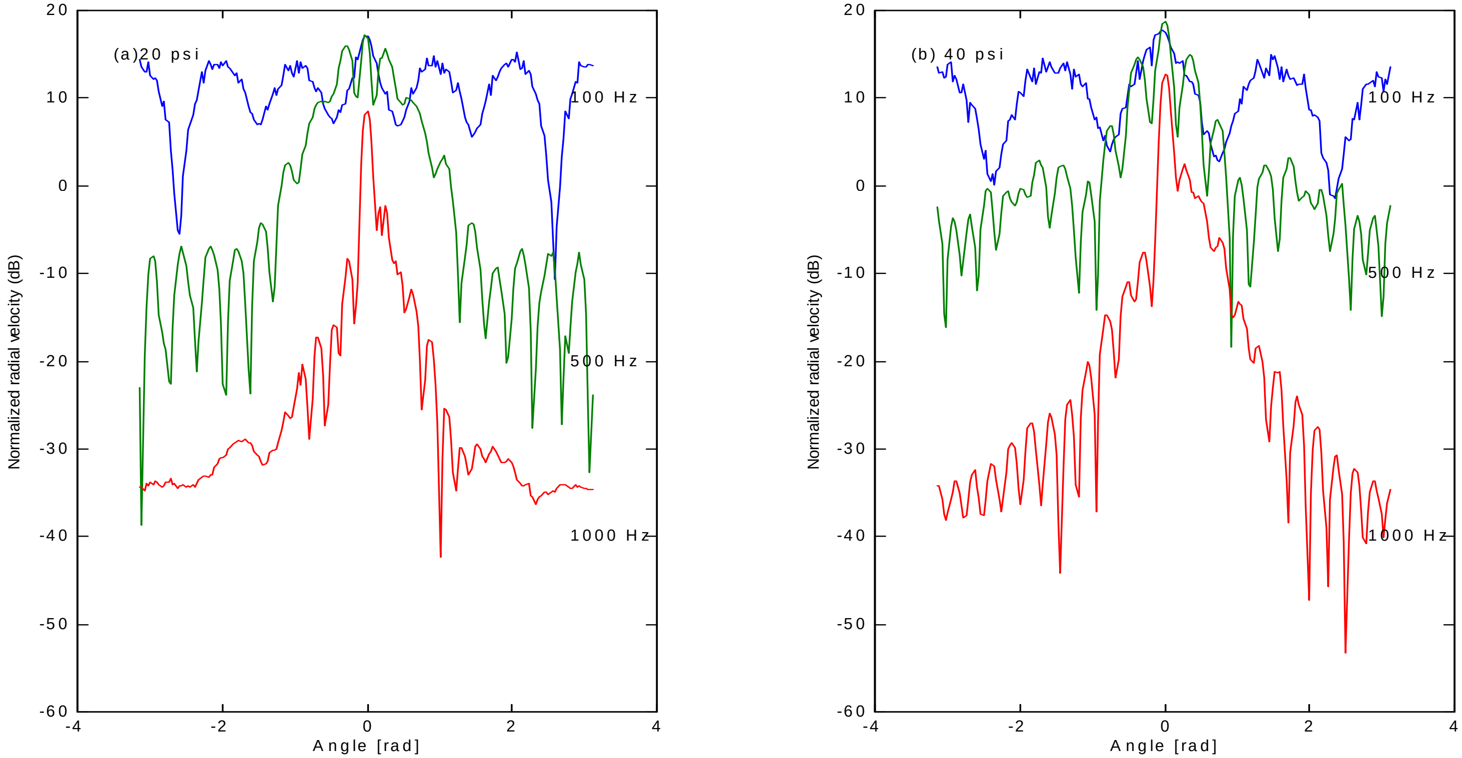


Firestone P215/70R 14 M+S
Inflation: 20 psi and 40 psi
Internal Gas: Air
CO_2
He

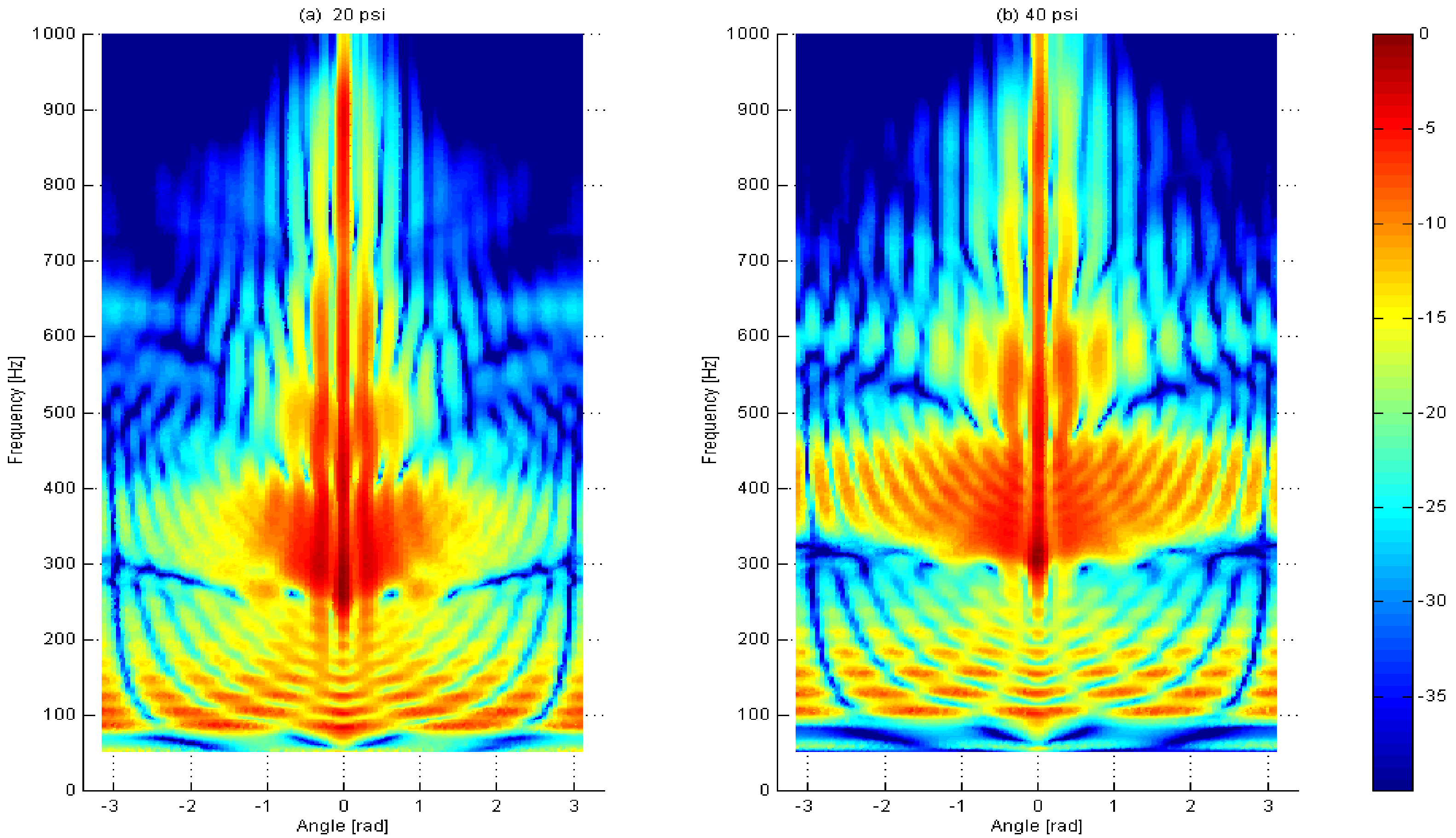


cross-section

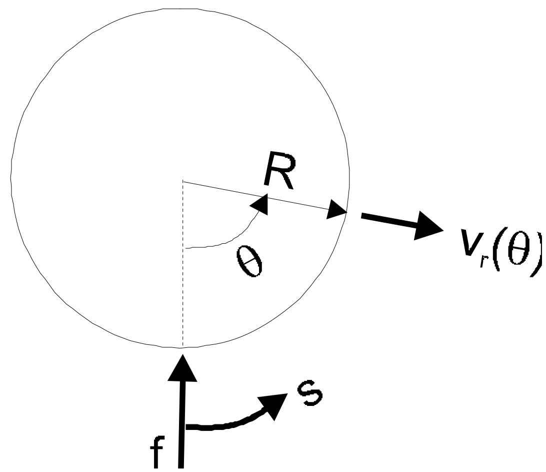
SPATIAL DATA:
SELECTED FREQUENCIES



RADIAL TREADBAND VELOCITY



CIRCUMFERENTIAL WAVE NUMBER DECOMPOSITION

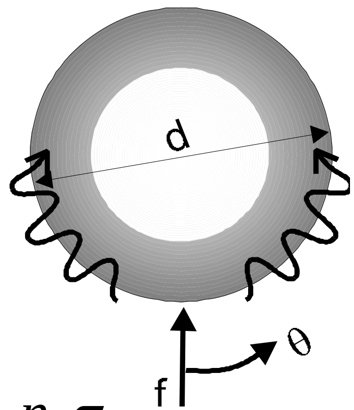


- **At each frequency, represent radial velocity as**

$$v_r(\theta) = \sum_{n=-\infty}^{\infty} a_n e^{-in\theta} = \sum_{n=-\infty}^{\infty} a_n e^{-ik_{\theta_n} s} \quad \textit{where} \quad k_{\theta_n} = 2\, n / R$$

- **Each component,** $a_n e^{-ik_{\theta_n} s}$ **represents a circumferentially propagating disturbance**

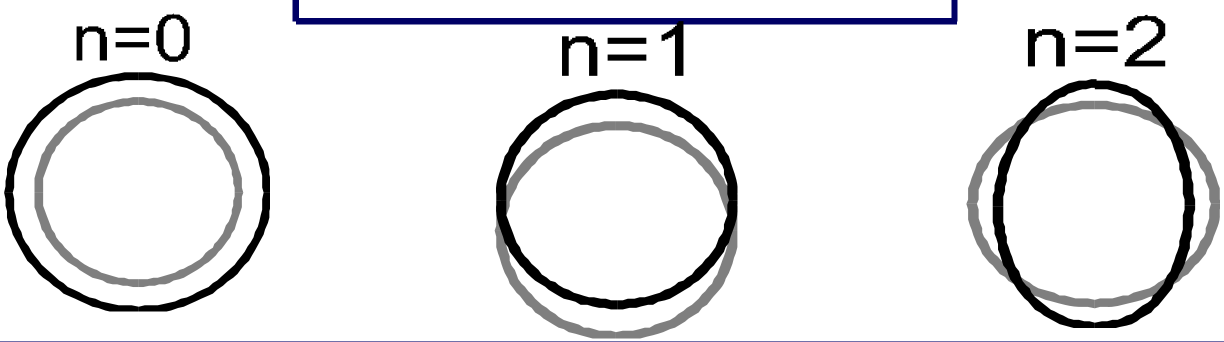
CREATION OF CIRCUMFERENTIAL MODES



When $k_\theta \pi d = 2 n \pi$ (i.e., $k_\theta = \frac{2n}{d}$)

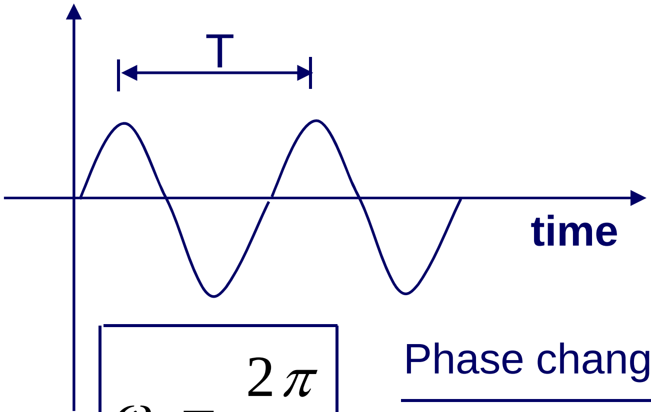
Propagating waves interfere to create standing waves

Circumferential modes

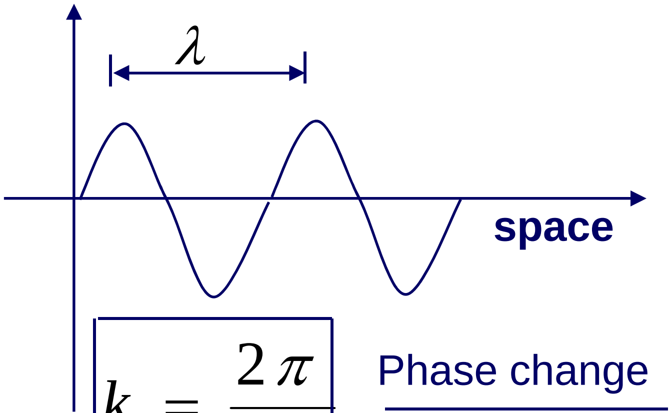


etc.

WAVE NUMBER DECOMPOSITION



$$\omega = \frac{2\pi}{T}$$ $\frac{\text{Phase change}}{\text{Unit time}}$



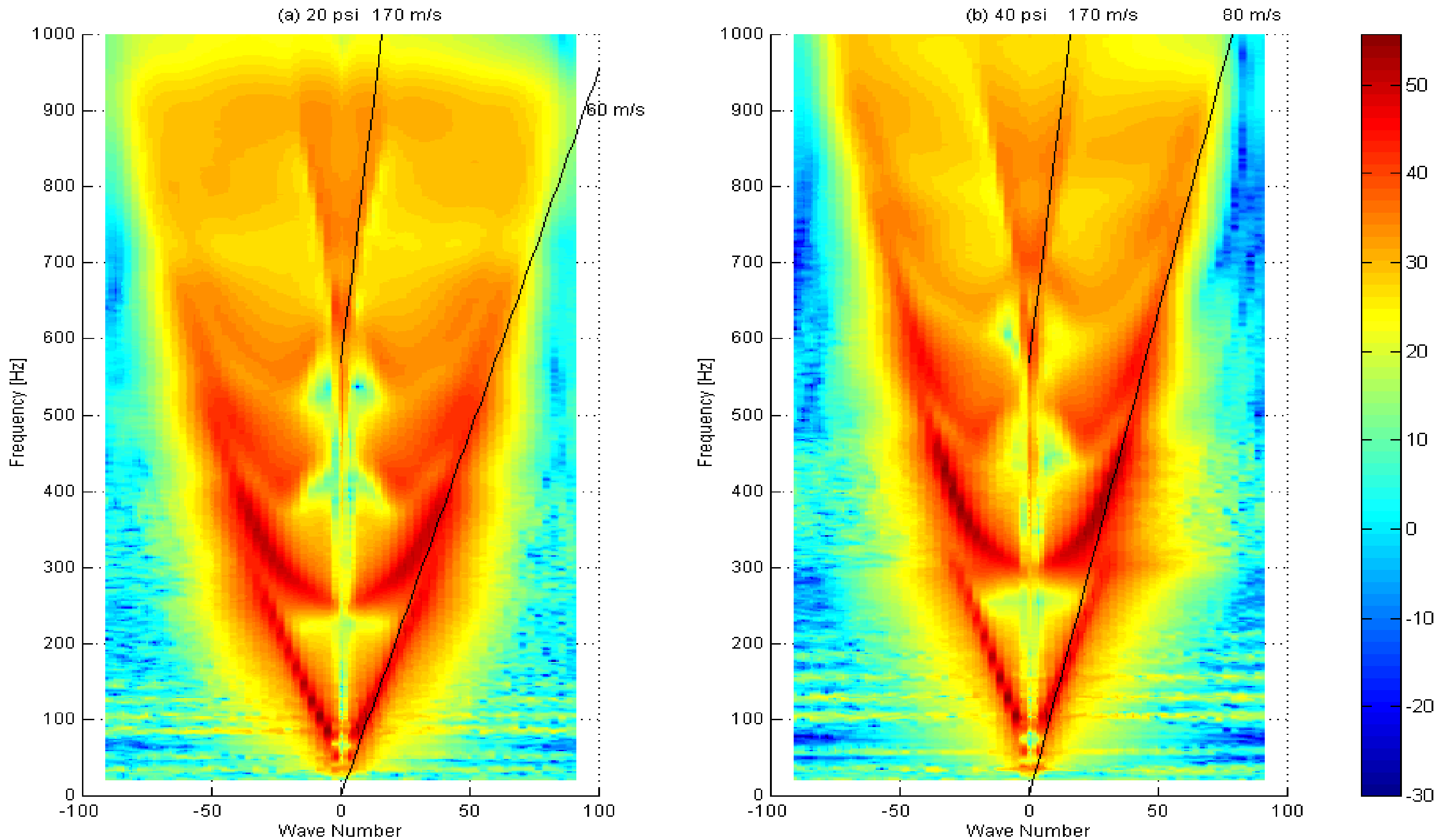
$$k = \frac{2\pi}{\lambda}$$ $\frac{\text{Phase change}}{\text{Unit length}}$

Phase speed: Since $\lambda = c_p T \quad \Rightarrow \quad c_p = \dfrac{\omega}{k}$

Group speed: $c_g = \dfrac{d\omega}{dk}$

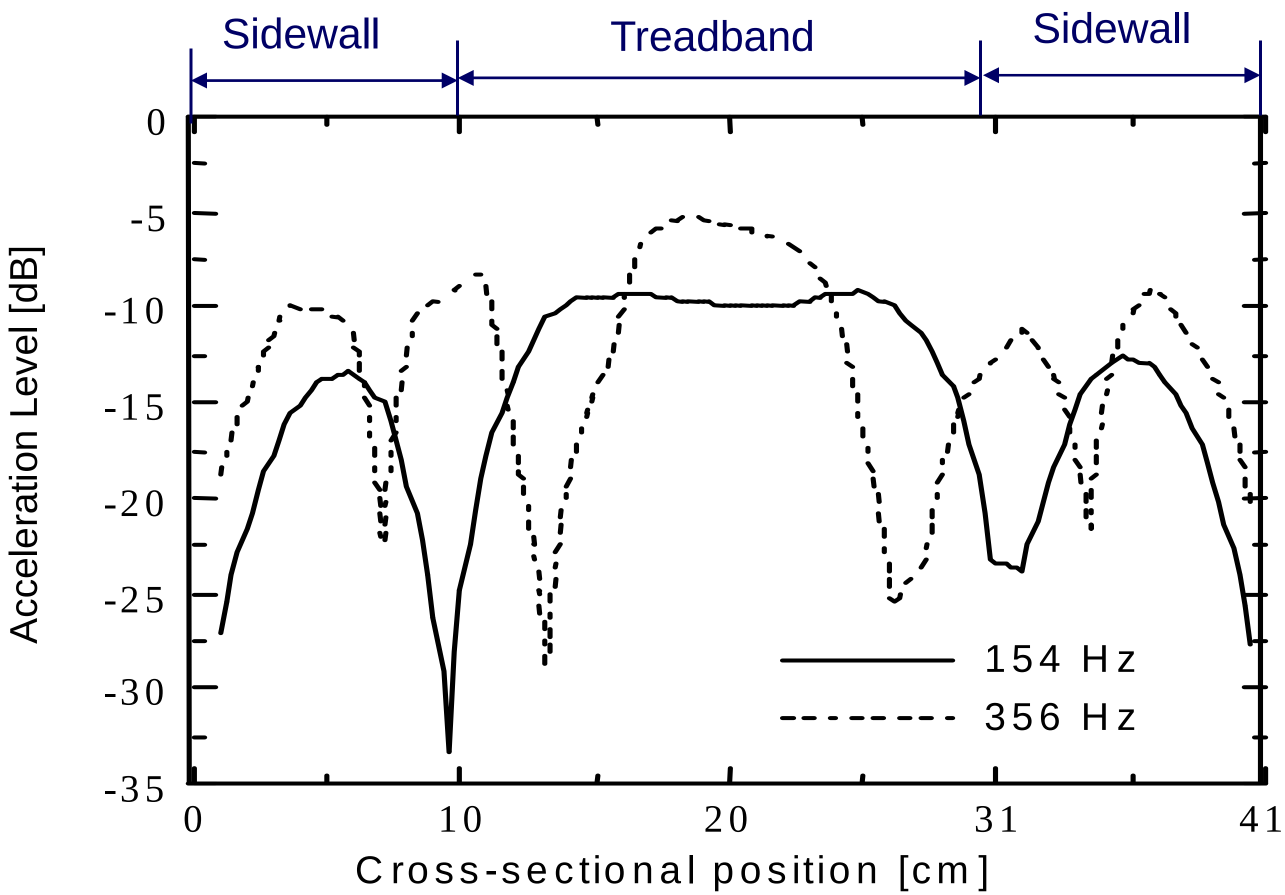
- Wave number gives information about wave propagation speed and attenuation

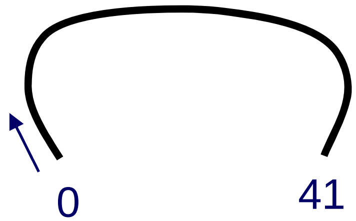
WAVE NUMBER DECOMPOSITION



Radial Treadband Velocity

RADIAL ACCELERATION LEVEL





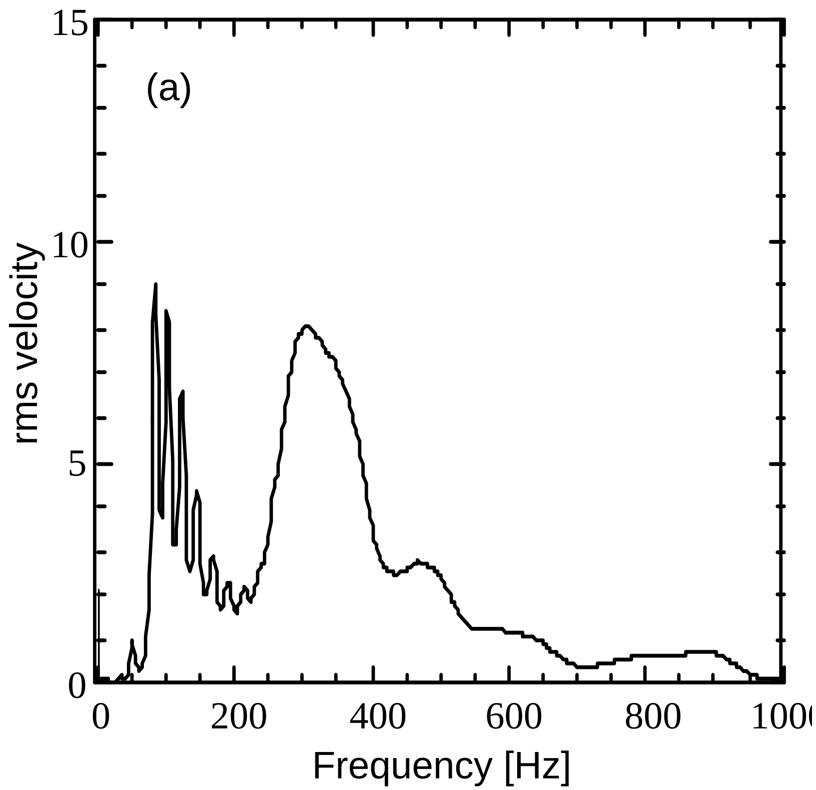
154 Hz: m=3

356 Hz: m=5

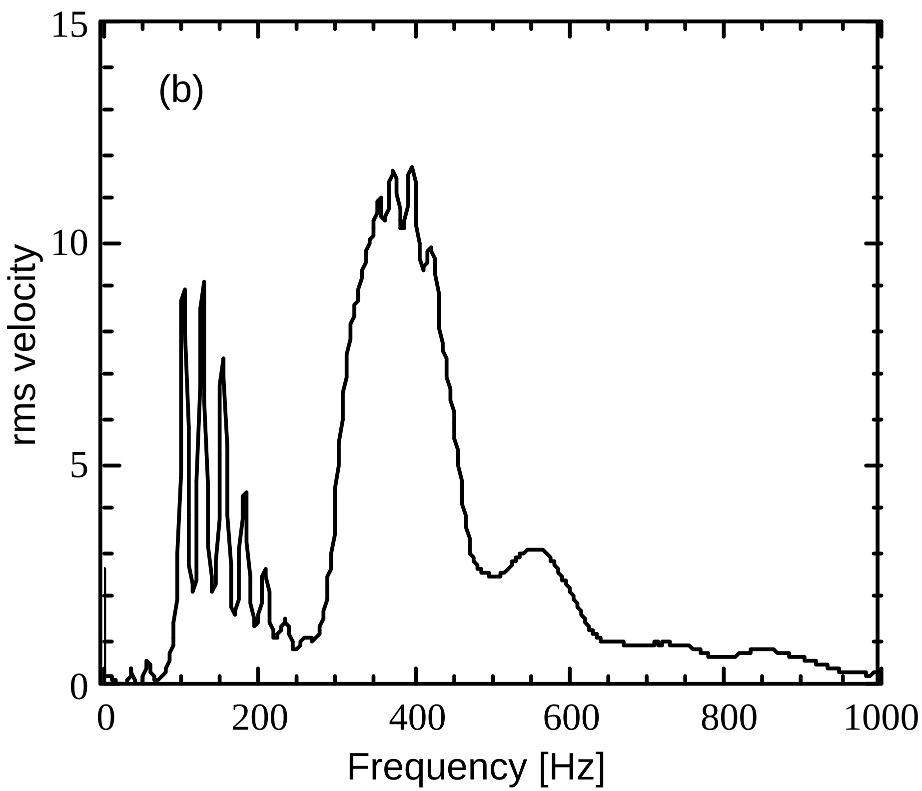
40 psi inflation pressure

RMS NORMALIZED RADIAL VELOCITY

Circumferentially Averaged



20 psi

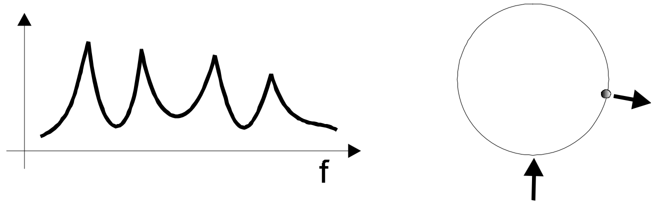


40 psi

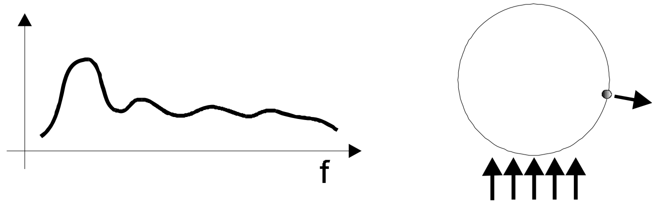
CARCASS VIBRATION

Total vibration is the result of

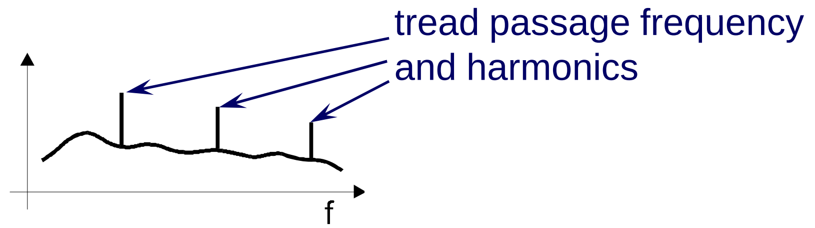
- **Point transfer function**



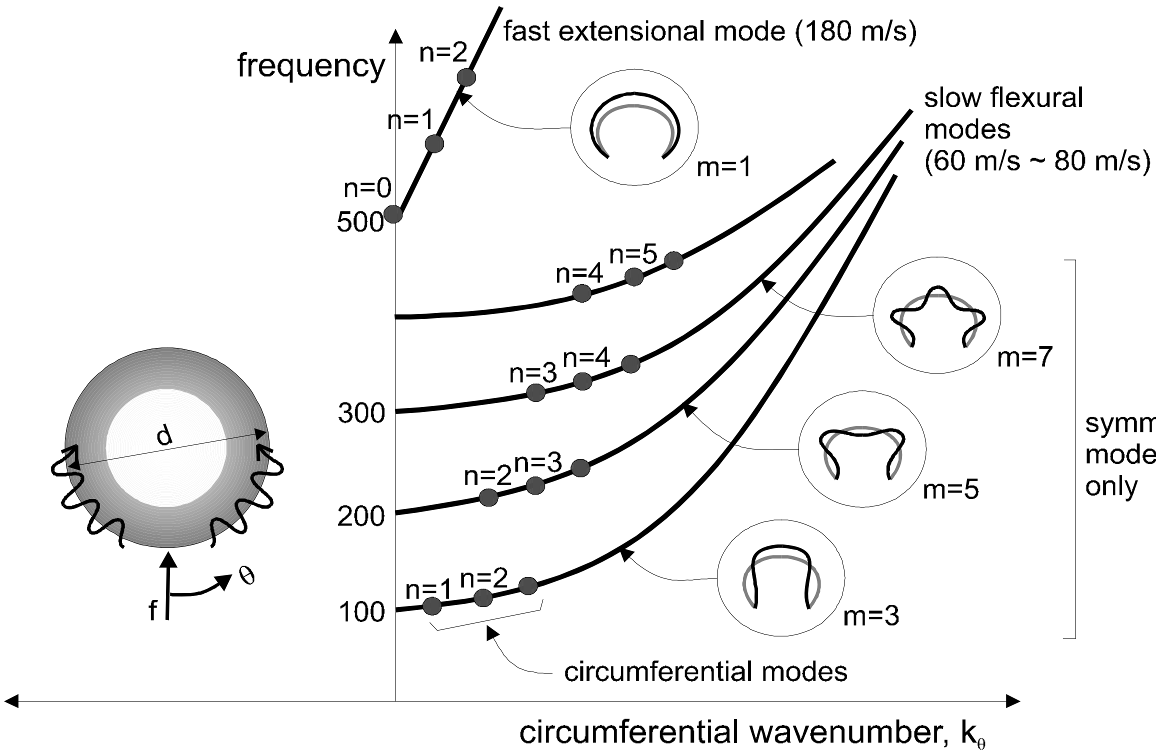
- **Smoothed by spatial distribution of force**

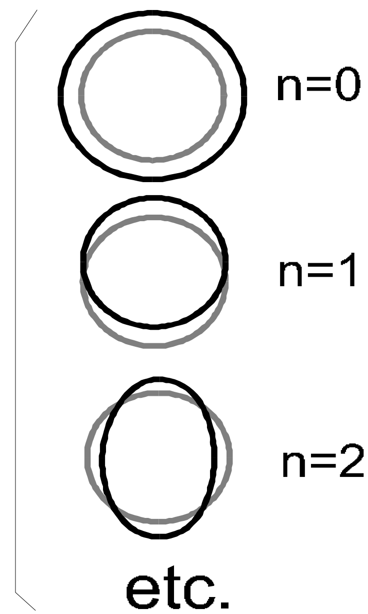


- **Multiplied by forcing function**



TIRE CARCASS DISPERSION CHARACTERISTICS - Static





- Tire carcass vibration is composed of superposition of “waveguide modes”
- Circumferential modes occur when $k_\theta \pi d = 2n\pi$

CONCLUSIONS

- Vibrational response of passenger car tire controlled by six propagating waves below 1000 Hz.
- Circumferential modes occur at 700 Hz and below.
- Maximum of two waves contribute strongly at any single frequency
- Each propagational mode associated with particular cross-sectional mode shape.
- Overall vibration of tire peaks near cut on of each wave type.
- Structure-borne interior noise possibly controlled by n=1 modes which deliver net force to hub.

CONCLUSIONS

- **Radial velocity of tire carcass can be conveniently expressed as**

$$u_r = \sum_{m=1}^{N} A_m \Psi_m(s_c, \omega) e^{\pm i k_{\theta_m} s}$$

- **Simpler to express tire vibration in terms of propagating wave type than in conventional modal form.**
- **Convenient as input to boundary element prediction of sound radiation.**