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# An Approximate Method For Determining The Hydraulic Conductivity Of Unsaturated Soils

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TECHNICAL REPORT NO. 11

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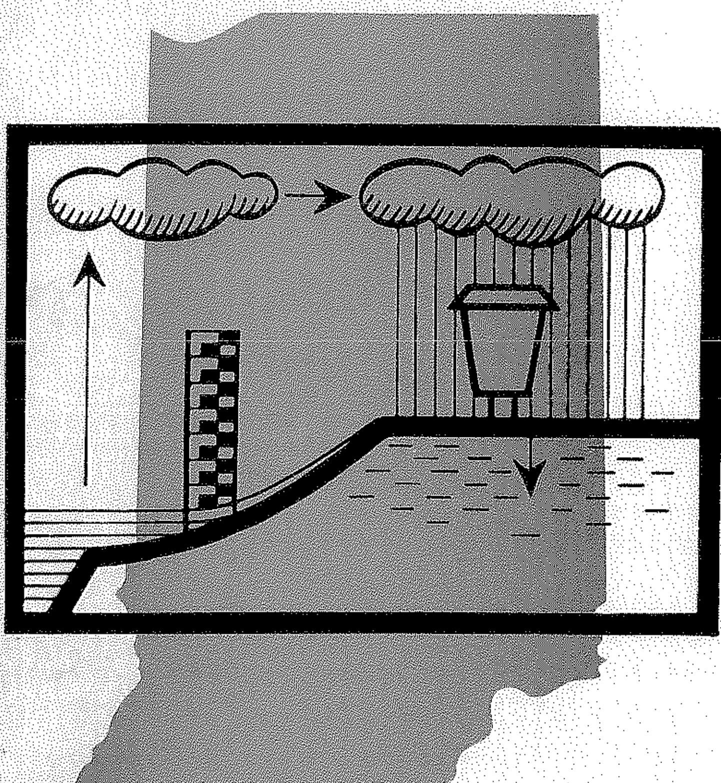


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PURDUE UNIVERSITY
WATER RESOURCES RESEARCH CENTER
LAFAYETTE, INDIANA

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| | |
|---|---|
| A | Water droplet application rate (cm/hr) |
| a | Parameter in Gardner's equation |
| b | Parameter in Gardner's equation (cm) |
| C | Water capacity ($cm^{-1}$) |
| C1 | Water capacity of top layer ($cm^{-1}$) |
| C2 | Water capacity of bottom layer ($cm^{-1}$) |
| D | Soil-water diffusivity, $D = K/C$, ($cm^2/sec$) |
| F | Generalized notation for transport property in Richards' equation ($F(y) = K(h)$ for h-based equation, $F(y) = D(\theta)$ for $\theta$-based equation) |
| f | Infiltration rate (cm/hr) |
| G | Generalized notation for water capacity in Richards' equation ($G(y) = C(h)$ for h-based equation, $G(y) = 1$ for $\theta$-based equation) |
| H | Total hydraulic head (cm) |
| h | Pressure head (cm) |
| $h_i$ | Pressure head at $t = 0$ and at $x = L$ (cm) |
| $h_1$ | Parameter in Gardner's equation (cm) |
| I | Gamma-ray count rate (counts per minute) |
| K | Hydraulic conductivity (cm/hr) -- written as a function of the pressure head, $K = K(h)$, for unsaturated soils |
| $K_s$ | Saturated hydraulic conductivity (cm/hr) |
| $K_{sc}$ | Computed saturated conductivity (cm/hr) |
| K1 | Conductivity of top layer (cm/hr) |
| K2 | Conductivity of bottom layer (cm/hr) |
| L | Length of soil column (cm) |
| M | Number of depth nodes in finite difference scheme |
| N | Number of pore classes |
| n | Subscript denoting node at junction of two layers |
| $p_b$ | Bubbling pressure (cm) |

| | |
|---|---|
| $\overline{q}$ | Water flux (a vector quantity) (cm/hr) |
| $q$, $\overline{q}_x$ | Water flux in the vertical (x) direction (cm/hr) |
| $R$ | Area between measured and predicted influx curves (cm) |
| $t$ | Time (seconds) |
| $\Delta t$ | Time increment (seconds) |
| $t_e$ | Test duration (minutes) |
| $U$ | Product of mass attenuation coefficient and effective thickness ($cm^3/gm$) |
| $x$ | Depth (cm) |
| $\Delta x$ | Depth increment (cm) |
| $y$ | Generalized notation for dependent variable in the Richards equation |
| $\alpha$ | Coefficient in finite difference form of Richards equation |
| $\beta$ | Coefficient in finite difference form of Richards equation |
| $\epsilon$ | Coefficient in finite difference form of Richards equation |
| $\zeta$ | Coefficient in finite difference form of Richards equation |
| $\delta$ | Depth of ponded surface water (cm) |
| $\eta$ | Exponent in equation of Brooks and Corey |
| $\theta$ | Water content (volumetric, $cm^3/cm^3$) |
| $\theta_i$ | Initial water content (volumetric, $cm^3/cm^3$) |
| $\theta_s$ | Saturated water content (volumetric, $cm^3/cm^3$) |
| $\rho_s$ | Bulk density at a point ($gm/cm^3$) |
| $\overline{\rho}_s$ | Average bulk density in a column ($gm/cm^3$) |

# ABSTRACT

Skaggs, Richard Wayne. Ph.D., Purdue University, June 1970. An Approximate Method for Determining the Hydraulic Conductivity Function of Unsaturated Soil. Major Professor: Dr. E. J. Monke.

A method is proposed for determining the hydraulic conductivity function of unsaturated soil. The method is based on the assumption that the conductivity-pressure head relationship, K(h), can be effectively represented by an empirical three-parameter equation. Procedures are given for evaluating the equation parameters from measurements of the soil-water characteristic, the saturated conductivity, and the infiltration rate-time relationship for an initially dry soil. Repeated numerical solutions to the Richards equation for the movement of water in unsaturated soil are used with a search procedure to determine the parameter values giving the minimum difference between measured and calculated infiltration rate-time relationships. Computer programs, written in FORTRAN IV, are developed to apply and evaluate the method.

An evaluation of the proposed method using soil property data obtained from the literature showed that a good approximation of the conductivity data could be obtained for soils in which the conductivity function had the general form of the assumed relationship. For soils having other forms of K(h), the functions obtained still allowed accurate predictions of the infiltration rate-time relationship for a wide range of initial water contents.

The results of an experimental evaluation of the method are reported. The conductivity functions of two artificially packed soils were determined and used to calculate infiltration rate-time relationships for infiltration into soil columns having different initial water contents. The measured and calculated influx curves are compared and discussed. Comparisons are also made between the measured and calculated

wetting front movement for initially dry soils. The effect of errors in the soil-water characteristic on calculated infiltration rate-time relationships is reported. When the conductivity function is defined by the method proposed the effect of such errors is reduced.

Results of the study showed that the proposed method is a promising means of determining the effective hydraulic conductivity function of unsaturated soils. With the exception of the soil-water characteristic, which is necessary if theory is used to characterize infiltration regardless of the means for determining $K(h)$, the measurements required by this method may be made with much less time and effort than is required by conventional procedures.

# INTRODUCTION

The determination of the rate of water infiltration into soil is a necessary step in determining the rate and volume of surface runoff for a given rainfall event. It is consequently basic to the design of reservoirs, flood and erosion control structures, channel improvements, and drainage systems. Furthermore, the characterization of the ability of a soil to infiltrate water is fundamental to the design of irrigation systems, which, according to Mathur (1968), comprise the largest single group of water users in the United States, expending over 40 percent of the total annual water usage. A workable scheme to determine infiltration for the varied initial and boundary conditions existing in nature is necessary to the logical and efficient development of our nation's water resources.

As with the investigation of most complex phenomena, efforts to characterize infiltration have followed varied approaches. Several algebraic equations stemming from both theoretical and empirical origin have been proposed to express the infiltration rate or volume in terms of time or soil water content. While the use of the algebraic equations would provide a convenient method for estimating infiltration rates, the equation parameters often vary sharply due to soil variations, crusting-sealing effects, and nonuniform antecedent water contents. Consequently, the use of algebraic infiltration equations is for most cases impractical since a new set of parameters is needed for each new set of boundary conditions.

The so-called exact approach for characterizing infiltration requires the solution of a partial differential equation describing the movement of fluids in unsaturated porous media. The nonlinear nature of the governing equation has formerly prohibited its solution for most infiltration problems. During the past decade, however, numerical procedures utilizing high speed digital computers have been developed to solve the governing equation for many of the important boundary conditions. Further refinement in numerical techniques and in computing hardware promises to make this approach a practical means for quantifying infiltration on a field scale.

The relationship between the hydraulic conductivity, K, and the pressure head, h, or the water content, θ, is necessary for solving the governing equation. The slow and laborious task of measuring the conductivity function with conventional techniques is a primary reason that the exact approach has not yet gained prominence on a wider scale. This study presents an approximate method for defining the conductivity function of unsaturated soil.

## Objectives

The objectives of the study are as follows:

1. To develop an approximate method for determining the hydraulic conductivity function of unsaturated soils. The necessary measurements should be made more quickly and with less effort than required by conventional methods.
2. To test the validity of the method by using published soil property data.
3. To use the method to experimentally determine the conductivity function for two soils. These experiments will provide information concerning errors and experimental difficulties encountered in making required measurements.
4. To test the validity of the conductivity functions by measuring infiltration into the soils under various boundary conditions. The observed infiltration rates will be compared with rates given by the solution to the governing equation.

# CHAPTER 1

# THEORY OF INFILTRATION

This chapter presents a review of the theory of vertical water infiltration into soil which will serve as a base for the development of a method to define the conductivity function. The treatment here is necessarily brief and the reader is referred to Swartzendruber (1966), Gardner (1967), Childs (1969), and Irrigation of Agricultural Lands edited by Hagan, et al. (1967) for more complete reviews of the subject.

## The Governing Equations

The movement of water in an unsaturated soil may be expressed by the Buckingham-Darcy equation as

$$\overline{q} = K(h)\ \nabla H \tag{1}$$

where $\overline{q}$ is the flux of water transmitted, $K(h)$ is the hydraulic conductivity expressed as a function of the pressure head, $h$, and $H$ is the total hydraulic head. Equation 1 was first proposed by Buckingham (1907) and may be viewed as Darcy's equation generalized for unsaturated flow. Deviations from the Buckingham-Darcy equation have been observed for some soils and are discussed by Swartzendruber (1966); however, the validity of the equation is generally accepted and is assumed in this study.

The hydraulic head, $H$, involves several force fields existing in the soil system and is discussed at length by Day, et al. (1967). Assuming the resistance to movement of the soil air is negligible and its pressure constant, $H$ may be expressed as the sum of the gravitational head, defined by the vertical distance from a datum plane, and the pressure head as

$$H = h - x \tag{2}$$

where x is the distance measured positively downward from the soil surface. The pressure head, h (also referred to as matric potential, capillary potential, soil-water tension, or soil-water suction), may be defined as the pressure of free water in equilibrium with the soil at the point in question. For an unsaturated, wettable soil, h is inherently negative and its magnitude is related in a nonlinear manner to the volumetric soil water content, $\theta$.

The principle of conservation of mass for the soil-water system may be expressed as

$$\frac{\partial \theta}{\partial t} = - \operatorname{div} \bar{q} \tag{3}$$

Considering infiltration as the flow of water in the x direction only, equations 1 and 3 become

$$\bar{q}_x = q = -K(h) \frac{\partial H}{\partial x} \tag{4}$$

and

$$\frac{\partial \theta}{\partial t} = - \frac{\partial q}{\partial x} \tag{5}$$

Then combining equations 2, 4, and 5, an equation governing the flow of water in unsaturated soil may be written

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[K(h) \frac{\partial h}{\partial x}\right] - \frac{\partial K}{\partial x} \tag{6}$$

By writing

$$\frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial h} \frac{\partial h}{\partial t} \tag{7}$$

and defining a soil property

$$C(h) = \frac{\partial \theta}{\partial h} \tag{8}$$

equation 6 may be expressed as

$$C(h)\,\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\,[K(h)\,\frac{\partial h}{\partial x}] - \frac{\partial K}{\partial x} \qquad (9)$$

Equation 6 can also be written with the water content, $\theta$, as the dependent variable. Defining soil water diffusivity as $D(\theta) = K(h)\,\frac{\partial h}{\partial \theta}$, equation 6 may be written

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\,[D(\theta)\,\frac{\partial \theta}{\partial x}] - \frac{\partial K}{\partial x} \qquad (10)$$

Equations 6, 9, and 10 were first derived by Richards (1931) and can logically be referred to as forms of the Richards equation (Swartzendruber, 1969).

Both forms of the Richards equation for flow in the vertical direction contain two soil parameters; the $\theta$-based equation contains $D(\theta)$ and $K(h)$ and the h-based equation contains $C(h)$ and $K(h)$. These parameters are related for unsaturated soil by $D = K/C$. For most soils, all three parameters vary markedly with water content or pressure head. The pronounced nonlinearity of these parameters is a prime source of difficulty in solving the Richards equation subject to boundary conditions pertinent to infiltration.

Advantages can be stated for both the h-based and $\theta$-based equations in describing the movement of water in unsaturated soil. When saturated conditions are reached, the h-based equation reduces to the familiar Laplace equation describing saturated flow. For saturated flow $K(h)$ reaches a constant value, $C(h) = 0$, and the pressure head, h, changes from a negative to a positive quantity. For cases where both saturated and unsaturated flow conditions exist, the solution to the h-based equation will be valid; however, the $\theta$-based equation "blows up" for saturated conditions as $D(\theta)$ tends to infinity for saturation. On the other hand, the $\theta$-based equation is superior for describing unsaturated flow only as the changes in both $\theta$ and D are typically 2 or 3 orders of magnitude less than corresponding changes in h and C. In general, round-off errors in numerical solutions to the $\theta$-based equation are of lesser consequence than for the h-based equation.

Solutions to the Richards Equation

The nonlinearity of the soil properties K, D, and C has prevented the analytical solution of equations 9 and 10 except for a few limited cases (for example, Gardner, 1958). Numerical techniques have been presented, however, for the solution of the equations subject to various boundary conditions of interest. One of the first, and perhaps the best known method of solution, was presented by Philip (1957a) for a homogeneous, deep soil with a uniform initial water content. Philip used transformations of equation 10 to obtain an infinite series solution in powers of $t^{1/2}$. Although it is generally recognized that Philip's solution and the analysis that followed it (Philip, 1957b-f, 1958) have been invaluable to the development of infiltration theory, many important boundary conditions exist for which the solution does not apply.

During the past decade, numerical techniques have been developed which use high speed digital computers to solve finite difference forms of equations 9 and 10. Hanks and Bowers (1962) presented a method to solve equation 9 for infiltration from a ponded surface into a layered soil of arbitrary depth and initial head distribution. Infiltration into layered soils has also been treated by Whisler and Klute (1966) and by Staple and Gupta (1966), who solved explicit finite difference forms of equation 9. Whisler and Klute (1965) used an iterative procedure to solve equation 9 subject to a nonuniform initial water content. Their technique took into account hysteresis, a factor that was also included in the numerical technique of Staple (1966). Techniques for solving equations 9 and 10 subject to rainfall boundary conditions have been presented by Rubin (1966) and Rubin and Steinhardt (1963). The technique given by Rubin (1966) can also be used to account for the effects of a nonzero air entry suction.

Numerical procedures for solving both equations 9 and 10 will be developed in a subsequent chapter. Specific references to the procedures used in the methods cited above will be made at that time. The point to be made here is that methods are available to solve the governing equation for most of the cases of interest for vertical infiltration.

Evaluations of Infiltration Theory

Laboratory Studies

Bodman and Colman (1943) measured the water content distribution in a vertical soil column during infiltration from a ponded water surface. They showed that the water content profile could be divided into the four zones shown schematically in Figure 1. The saturation zone extended from the surface to a maximum depth of approximately 1.5 cm. The transition zone, a region of rapid decrease of soil water content, extended from the zone of saturation for a depth of about 5 cm to the transmission zone, a zone of rather constant water content which lengthens as infiltration proceeds. The wetting zone was defined as a region of rapid change of soil water content culminating in the wet front which is the visible limit of water penetration into the soil.

Philip (1957d) showed that the solution to equation 10 predicts all of the zones except the transition zone. He attributed the presence of a transition zone to a dependence of the soil-water characteristic on depth due to the smaller probability of entrapped air in the soil close

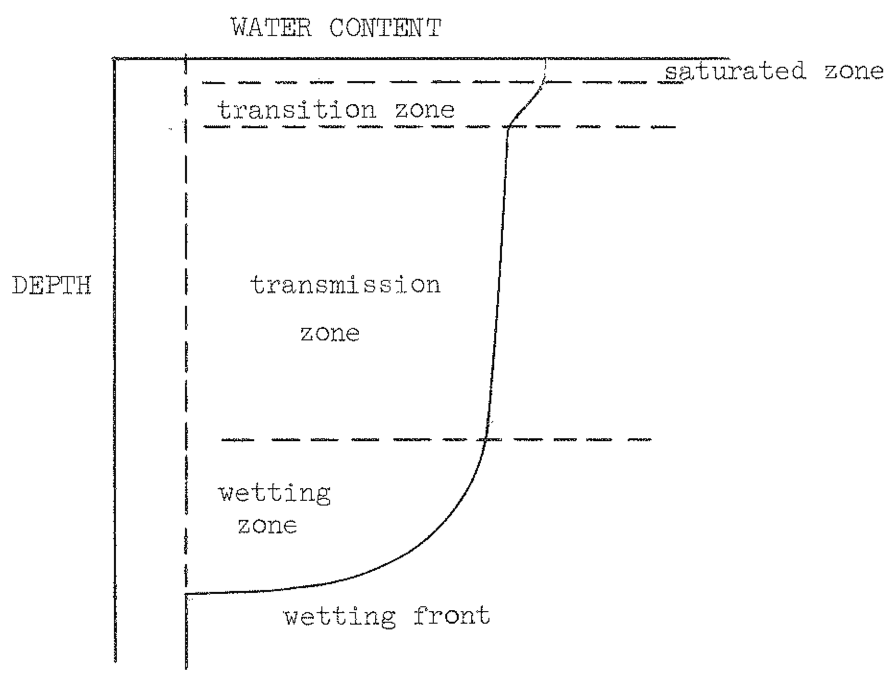


Figure 1. Infiltration Zones of Bodman and Colman (1943).

to the surface. According to Philip's (1958) analysis, the existence of a saturated zone depends on either a ponded surface of greater than zero depth, or the presence of nonzero air entry suction which is exhibited by the water characteristic of many soils. Philip (1957d) concluded that, in a qualitative sense, there was good agreement between the solution to the governing flow equation and the experimental observations of Bodman and Colman (1943).

Youngs (1958) observed infiltration from a ponded water surface into columns of slate dust and glass beads. The results agreed closely with those predicted by the solution of equation 10. Davidson, et al. (1963) studied vertical infiltration into columns of silt and sandy loam soils. They studied the infiltration process for water applied at near saturation (pressure head of -0.2 cm water) and at water contents much less than saturation. For water applied at saturation, the solution to equation 10 agreed very closely with the observed water content distribution. However, for water contents much less than saturation at $x = 0$, the water content profile deviated from the solution to equation 10.

Gupta and Staple (1964) studied infiltration into a packed column of soil from a ponded surface of 0.4 cm depth. The observed water content profiles were similar to those discussed by Bodman and Colman (1943). When the variation of the soil parameters with depth was considered, a transition zone was predicted and good agreement between the solution and the observed results was found. When this variation was not considered, however, considerable discrepancy between the measured and predicted water profiles occurred.

## Field Studies

The bulk of experimental infiltration research has been performed in the laboratory where the soils studied have involved homogeneous columns subjected to boundary conditions in accordance with the assumptions made in the derivation and solution of the governing Richards equation. The utility of the so-called exact approach in characterizing infiltration for practical situations, however, depends on the accuracy of solutions under natural conditions.

Nielson, et al. (1961) compared soil-water profiles which were determined experimentally for Ida and Monona silt loam soils with profiles computed using Philip's solution of equation 10. For the Monona soil, the calculated soil-water profile was in fair agreement with experimental results; however, the calculated profile for Ida had a more advanced, sharper wetting front than the experimental. The investigators noted that the nonuniformity of the soil horizon could have been responsible for the discrepancy in the measured and predicted soil-water profiles. The transport functions, D and K, used in computing the profiles were determined with a drying, or outflow, process on cores taken from the top 30 inches. Whisler and Klute (1965) showed that the predicted water content profiles were dependent on the branch of the hysteresis loop of the soil properties used. Therefore, the discrepancies noted above might have been at least partially due to using parameters obtained by an outflow method to evaluate infiltration, a wetting process.

Green, et al. (1964) used the numerical method of Hanks and Bowers (1962) to calculate infiltration rates for a natural soil. The initial soil-water distribution was measured experimentally and considered in the solution. The $D(\theta)$ and $h(\theta)$ relationships were also determined as a function of depth and the soil was considered as stratified in making the theoretical calculations. After approximately 20 minutes of infiltration, the calculated infiltration rates agreed quite well with rates determined experimentally using a sprinkling infiltrometer. There was considerable disagreement between the calculated and observed rates during the initial stages of the run. Again the flow properties were determined using an outflow process; this was cited by the authors as being a possible reason for the discrepancies.

## Review of Methods to Determine K(h)

The methods for determining the K(h) relationship may be classified as steady state, transient, and computational methods. While many techniques are described in the literature for each of these classes, not all of them are adaptable for determining the imbibition branch of the function. Some that are adaptable are discussed below.

## Steady State Methods

Klute (1965) and Anat, et al. (1965) described similar steady state methods for measuring K(h) based on the defining relationship given in equation 1. Essentially the methods consisted of setting up boundary conditions to obtain steady, one directional flow for adjustable pressure heads. A soil sample was placed in an airtight cavity between two horizontal porous plates through which water flowed into and out of the sample. The average pressure head in the sample was controlled by the air pressure in the cavity. The mean hydraulic gradient between two points in the sample was determined by using tensiometers to measure the difference in pressure head. By measuring the steady state flow rate, q, the conductivity was calculated directly from equation 1. Then the air pressure was changed and the procedure repeated for another value of pressure head.

Although simple in concept, this method has several disadvantages. Since soil conductivities are small in general, particularly at the lower water contents, long times are required to reach steady state. This is especially true for imbibition. Also the conductivity function obtained using the above method represents a "point" determination, or at most a determination for a short soil section. In order to incorporate some of the heterogeneities of natural soils in the conductivity function, it is desirable to make conductivity determinations on rather large soil samples. This can be done by increasing the length of the soil sample but the time required to reach steady state, already prohibitive, would also increase.

Youngs (1964) determined K(h) by measuring the rate of infiltration into soil columns from water supplied at the surface at a negative pressure. Although the effective conductivity function for a rather large sample was determined and the experimental measurements were simple in nature, the method required a separate soil column for each water content. Also several days were required to determine the conductivity for low water contents.

A zone of entrapped air was used by Watson (1967) to speed up the formation of steady state conditions for infiltration into a soil column initially drained to a water table at its base. A large number of tensiometers were used to determine the pressure head and hydraulic gradient distribution at steady state. Then, by measuring the steady state flow rate, q, the conductivity at each point in the column was calculated. While this method is attractive from the standpoint of required time, it is restricted to coarse grained soils and to relatively low pressure heads.

## Transient Methods

The steady state methods discussed above were simple in theory, but, except for the last case, long periods of time were required for the determination of K(h). In general, transient methods require less time, but the experimental measurements and calculations are more difficult. Gardner and Miklich (1962) proposed a method for determining K(h) by introducing water into (or removing water for the drainage cycle) one end of a soil column at a constant flux. The experimental data required were the flux and continuous measurements of the pressure head at two points in the column. Although the method appeared to work well for the drainage case, three weeks were still required to obtain K(h) for the tensiometer range. The method appeared to work well for the drainage case but was not evaluated for the imbibition cycle.

A transient in situ method for determining the hydraulic properties of soils was presented by Van Bavel, et al. (1968). A large number of tensiometers were used in conjunction with neutron equipment to measure the pressure head and water content profiles at specified time intervals Procedures were given to obtain K(h) from these profiles. This method can be used to obtain a rather complete description of a field soil's hydraulic properties, including their variation with depth. The experimental setup, however, is complex and expensive. Also, about ten days are still necessary to determine the conductivities over the tensiometer range of pressure head.

## Methods for Calculating K(h)

In view of the difficulties involved in measuring K(h) directly, methods to compute K(h) from the h(θ) relationship are most worthy of consideration. Millington and Quirk (1960) refined an equation originally derived by Childs and Colis George to compute K(h) from h(θ).

Jackson, et al. (1965) found that a matching factor was needed to make the calculated conductivities agree with measured data. The equation may be written

$$K(h_i) = 314 \frac{K_s}{K_{sc}} \theta^{4/3} \frac{1}{N^2} \sum_{j=i}^{N} [(2j + 1 - 2i) \frac{1}{h_j^2}] \qquad i = 1, 2, \ldots N \quad (11)$$

where the matching factor $K_s/K_{sc}$ is the quotient of the saturated conductivity to the calculated saturated conductivity, N is the number of pore classes obtained by dividing the water content scale of the soil-water characteristic into increments of equal length, and $h_j$ is the mean pressure head of the jth pore class. Kunze, et al. (1968) found that better agreement between predicted and observed conductivities could be obtained by using $\theta$ rather than $\theta^{4/3}$ in equation 11. While good predictions of the conductivity function have been obtained for most of the published investigations of equation 11, discrepancies from the equation have been found for some of the soils investigated in this study.

Brooks and Corey (1964) showed that for -h values greater than the bubbling pressure, the conductivity for the outflow or drainage cycle could be related to the pressure head by the following equation

$$K(h) = K_s \left(\frac{p_b}{-h}\right)^{\eta} \qquad (12)$$

where $K_s$ is the saturated conductivity, $p_b$ is the bubbling pressure, and $\eta$ is related to the pore size distribution and may be determined directly from the soil-water characteristic. Although the validity of equation 12 for the drainage cycle has been well proven by Brooks and Corey (1964) and Laliberte, et al. (1966), no evidence has been presented concerning its validity for imbibition.

# CHAPTER 2

## METHOD FOR DETERMINING K(h)

Evidence has been presented showing that the theoretical approach can be used to quantify infiltration for many of the boundary conditions of interest. The soil properties required as inputs are the imbibition branches of the conductivity function, K(h), and the soil-water characteristic, h(Θ). While results of experiments and solutions to the Richards equation have been in general agreement, instances of significant disagreements for both laboratory and field investigations have been recorded. The lack of reliable measurements of the soil properties has been frequently cited as a reason for these disagreements. A review of the literature showed that present techniques for measuring K(h) are difficult to implement and usually require a prohibitive amount of time to carry out. Consequently, the development of a fast, reliable method for estimating K(h) is essential to theoretical description of infiltration if it is to be used in a practical sense. Furthermore, the method should define an effective conductivity function, one that will compensate for the effects of minor heterogeneities such as worm holes and plant roots which exist in even the most homogeneous soils.

### Method Description

Consider the infiltration of water from a shallow ponded surface into a column of soil with a uniform initial water content. The boundary conditions may be written in the form

$$\begin{aligned} h &= 0 \quad (\Theta = \Theta_s) \quad x = 0 \quad t > 0 \\ h &= h_i \quad (\Theta = \Theta_i) \quad x > 0 \quad t = 0 \\ h &= h_i \quad (\Theta = \Theta_i) \quad x = L \quad t \leq t_e \end{aligned} \qquad (13)$$

where $\theta_s$ is the saturated volumetric water content, $h_i$ and $\theta_i$ are the initial pressure head and water content respectively, L is the column length, and $t_e$ is the time during which infiltration is considered. This formulation of the boundary conditions assumes that the initial water content is small so that water movement in advance of the wetting front is negligible.

If it is tentatively assumed that the soil properties, K(h) and C(h) are constant, and that C(h) = C is known, equation 9 may be written

$$\frac{\partial h}{\partial t} = \frac{K}{C} \frac{\partial^2 h}{\partial x^2} \tag{14}$$

The solution to equation 14, now linearized, subject to boundary conditions 13 is given by Schneider (1957) as

$$h = h_i \operatorname{erf} \frac{x}{2\sqrt{Kt/C}} \tag{15}$$

Then

$$\left.\frac{dh}{dx}\right|_{x=0} = \frac{h_i}{\sqrt{\pi K t/C}} \tag{16}$$

and the infiltration rate may be obtained from equations 2, 4, and 16 as

$$f = q\Big|_{x=0} = - \quad KC \frac{h_i}{\sqrt{\pi t}} + K \tag{17}$$

Consequently K can be determined for the linear system by measuring the infiltration rate, f, at two arbitrary values of time, t. Since the infiltration rate is much less difficult to measure than either h or $\theta$, K can be determined from equation 17 with less time and effort than by methods previously discussed.

Since K(h) and C(h) are not constant for soils, but are nonlinear functions of the pressure head, K(h) cannot be explicitly determined from the influx curve as above. However, an analogous method will be proposed to define K(h) from a known C(h) relationship and a measured influx curve.

A critical assumption in the proposed method is that $K(h)$ can be effectively represented by an explicit, functional relationship. Gardner (1958) proposed an empirical equation relating the conductivity to the pressure head. A slightly rearranged form of that equation may be written

$$K(h) = [(h/h_1)^a + b]^{-1} \tag{18}$$

where a, $h_1$, and b are equation parameters which must be determined empirically. Gardner indicated that the conductivity functions of a number of soils were represented quite well by equation 18. King (1965) obtained a nondimensional form of equation 18 by dividing both sides of the equation by the saturated conductivity, $K_s$. He used regression techniques to fit equation 18 to the imbibition conductivity data of four soils and reported excellent agreement between the equation and the measured data.

The procedure given here for approximating $K(h)$ assumes the conductivity function can be effectively represented by equation 18. Under this assumption, the method for approximating $K(h)$ is resolved to procedures for defining the parameters a, $h_1$, and b in equation 18. Other equations for $K(h)$ have been proposed by Rubin, et al. (1964), Brooks and Corey (1964), and King (1965). The general concepts of the method developed here can also be used to define the parameters in these or other equations for $K(h)$.

A schematic plot of the infiltration rate-time relationship that would be measured for a soil column subjected to boundary conditions 13 is shown in Figure 2. Assuming $h(\theta)$ for the soil is known, the Richards equation can be solved subject to boundary conditions 13 by taking trial values of the parameters a, $h_1$, and b in equation 18. The infiltration rate-time relationship obtained from the solution is also shown schematically in Figure 2. Another set of trial values for the equation 18 parameters would result in a second predicted influx curve which might more closely approximate the measured relationship. By varying the parameters in equation 18 and obtaining repeated solutions of the governing equation, values of a, $h_1$, and b can be found which give the

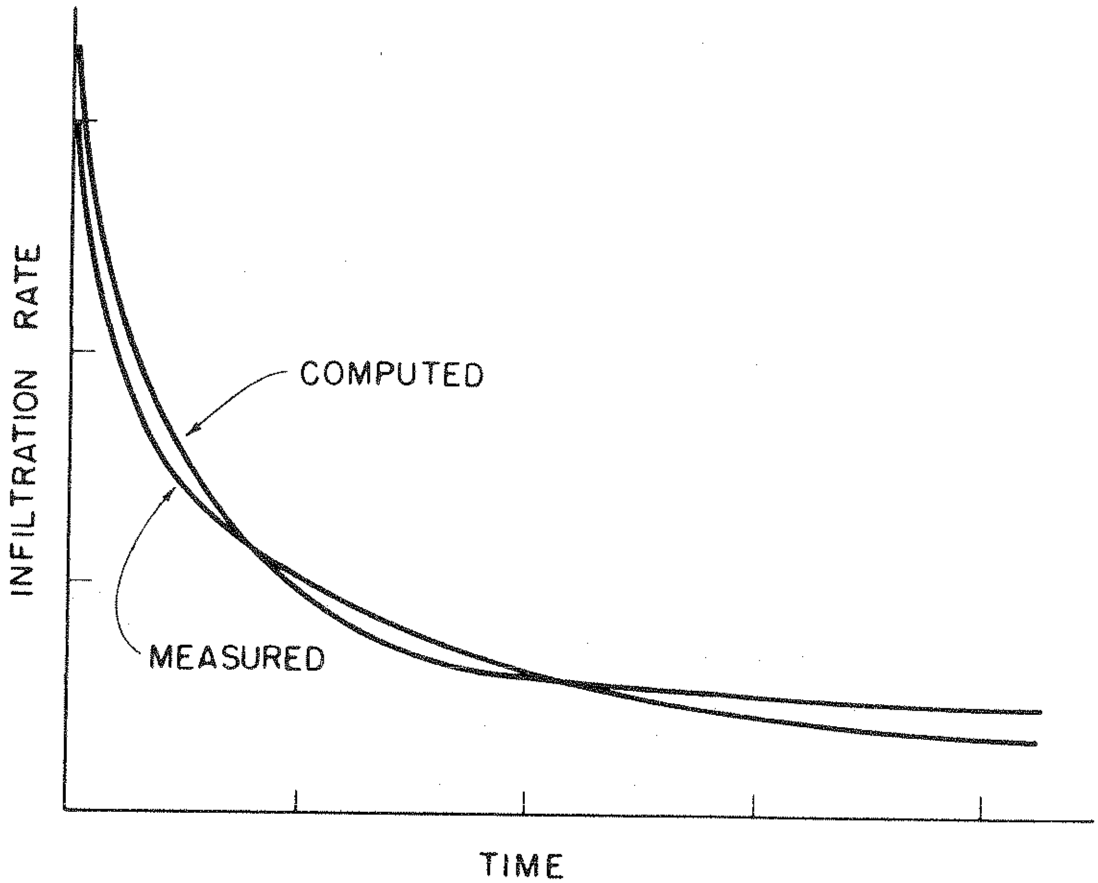


Figure 2. Schematic Plots of Measured and Computed Influx Curves.

minimum difference in the predicted and measured infiltration rate-time curves. This set of parameters can then be used in equation 18 to describe the effective conductivity-head relationship for that soil.

Since substitution of $h = 0$ in equation 18 results in $K(0) = 1/b$, the parameter b can be evaluated independently by measuring $K(0)$. If water is allowed to enter the column until steady state flow is attained, and if the boundary condition at $x = L$ is held at $h = 0$, $K(0)$ can be obtained directly by setting it equal to the steady state flow rate. The procedure of defining the conductivity function is now simplified to the task of finding values for only two parameters, a and $h_1$, in equation 18. $K(0)$ would be less than the saturated conductivity, $K_s$, since air is entrapped during the inflow process. $K_s$ is conventionally determined by wetting under suction to purge the air in the soil matrix.

The agreement between measured and predicted influx curves may be quantified by computing the sum of the areas lying between the two curves. The sum of the areas, denoted as the objective function, R, can be computed numerically by dividing the time axis in small increments, calculating the area between the curves for each increment by the trapezoidal method, and then summing the areas.

Techniques for defining parameters a and $h_1$ in equation 18 will be further developed in an example using Sarpy loam soil. The points plotted in Figure 3 were obtained by solving equation 10 subject to boundary conditions 13 with $\theta_i = 0.06$ ($h_i = -3300$ cm) and $\theta_s = 0.41$. (The numerical method used to solve equation 10 will be described in Chapter 3.) The $\theta$-based equation was solved because the smaller gradients allowed a faster, more efficient solution than was possible for the h-based equation. The soil properties for Sarpy were taken from Hanks and Bowers (1962). The infiltration rate-time curve corresponds to that which would be measured assuming equation 10 is valid, boundary conditions 13 were maintained perfectly, and the soil property measurements were free of experimental error. Treating the computed influx curve as a "measured" relationship, equation 10 was repeatedly solved for various combinations of a and $h_1$ in equation 18 and the objective function, R, determined. The response surface shown in Figure 4, is a unimodal surface with its minimum situated on a sharp, relatively flat ridge. The minimum R value of 0.28 cm occurred at a = 3.1 and $h_i = -1.95$ cm. The parameter b = 717 sec/cm was determined from K(0) in the original data of Hanks and Bowers. The solid line in Figure 3 represents the solution to equation 10 using the conductivity function of equation 18 with the above parameter values.

Various search techniques can be used to reduce the number of trials necessary to define the minimum point on the response surface. A two dimensional search technique of the Pattern or Partan type as discussed by Thompson and Peart (1968) is suitable for determining the position of the minimum point on the surface. Since the response surface has a relatively flat ridge, however, a large number of trials may be required to find the minimum point. On the other hand, the possibility exists that a good fit of the "measured" influx curve can be

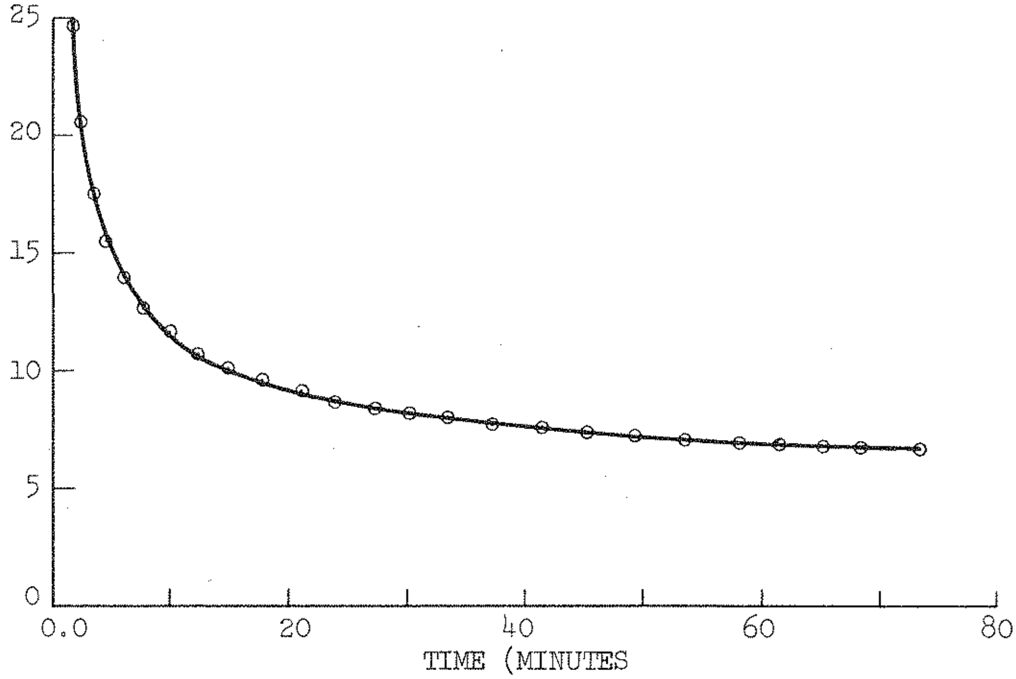


Figure 3. Influx Curves for Dry Sarpy Loam.

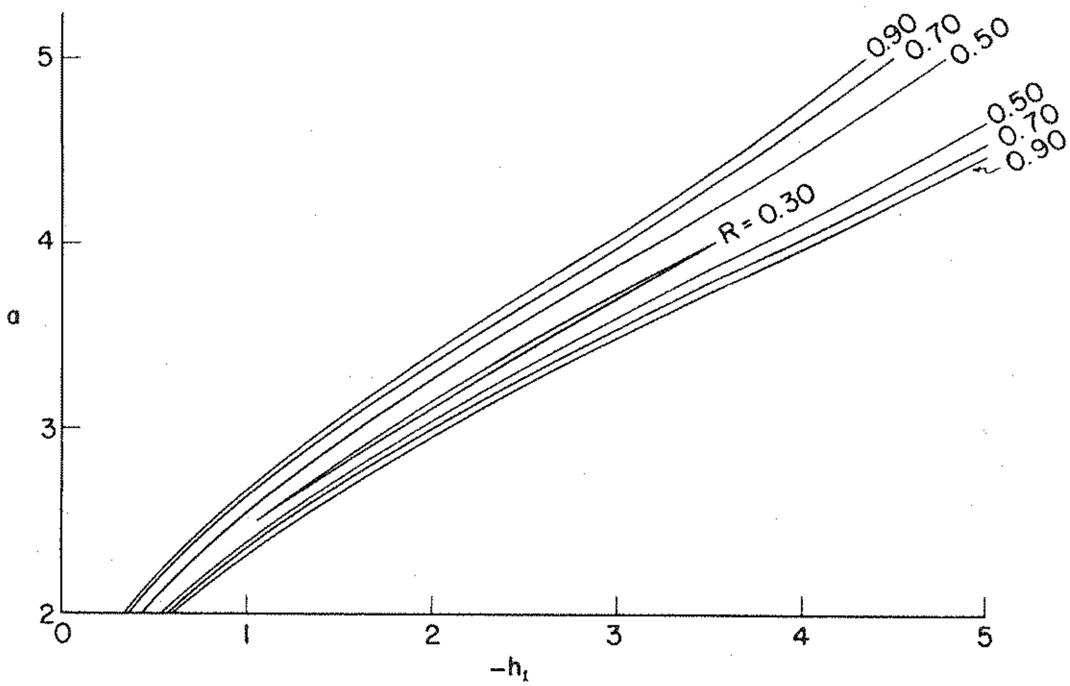


Figure 4. Response Surface for Sarpy Loam.

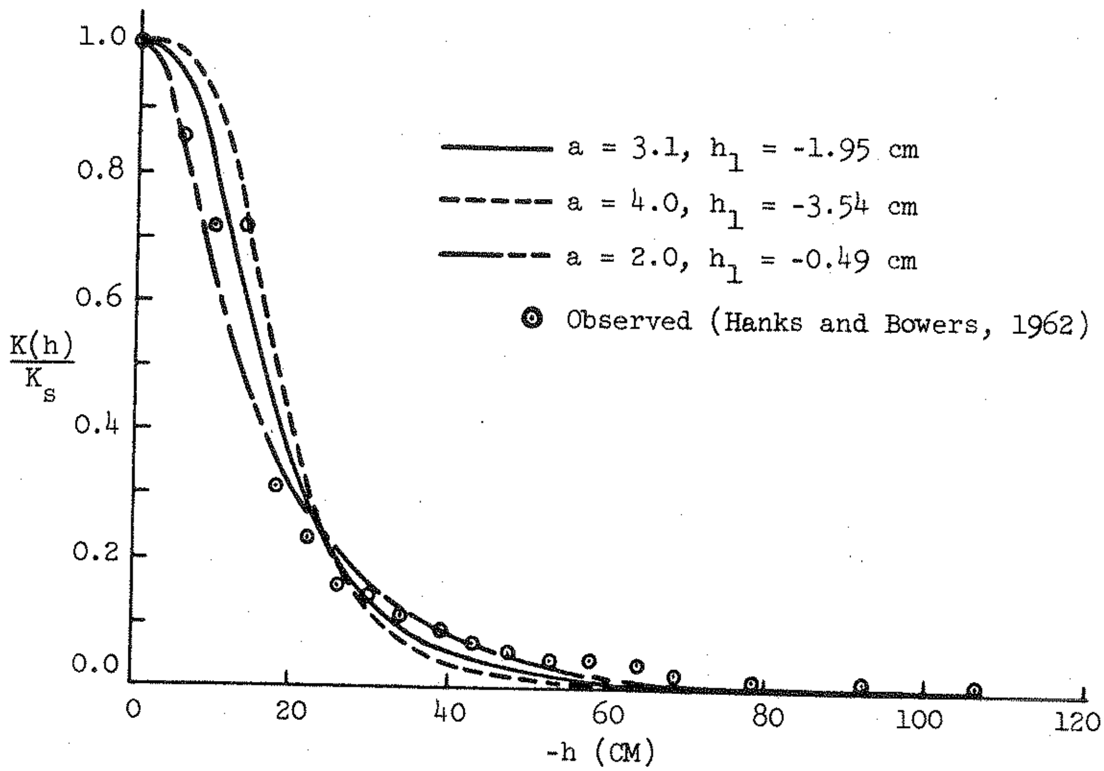


Figure 5. Relative Conductivity Versus Pressure Head for Sarpy Loam.

obtained by estimating one of the parameters a or $h_1$ independently, thereby simplifying the search for the remaining parameter.

Plots of relative conductivity function $K(h)/K_s$, for Sarpy are given in Figure 5. The discrete points represent the conductivity values given by Hanks and Bowers (1962), while the curves represent equation 18 for various values of parameters a and $h_1$. The solid curve is the conductivity function which gave the minimum R value on the response surface with $a = 3.1$ and $h_1 = -1.95$. Two additional values of the parameter a were arbitrarily chosen. Curves are given in Figure 5 corresponding to the ridge peaks for $a = 2$, where $h_1 = -0.49$ cm and $R = 0.34$ cm and for $a = 4.0$ where $h_1 = -3.54$ and $R = 0.30$. Although the solid curve appears to give a better approximation of the measured conductivity data over the entire range, the other two curves are also in fair agreement. This supports the argument that the conductivity function may be adequately described by estimating one of the parameters independently and determining the other by a one-dimensional search procedure.

The parameter a can be estimated independently from the model of Brooks and Corey (1964) given by equation 12. If log K(h) is plotted versus log (-h) for equation 12, the result will be a straight line with a slope of $-\eta$. A plot of log K(h) versus log (-h) for the conductivity function of equation 18 will also yield a straight line with slope -a for large values of h. Consequently, an initial approximation of parameter a in equation 18 can be made by obtaining $\eta$ from $h(\theta)$ by the method given by Brooks and Corey, and setting $a = \eta$.

Preliminary Evaluation of the Method

The method for determining K(h) was based on the critical assumption that the conductivity function could be effectively represented by equation 18. Although evidence has been cited showing that equation 18 is appropriate for many soils, it obviously will not describe the highly nonlinear K(h) exactly for all soils; the same can be said of other equations for K(h). However, by obtaining the equation parameters from an influx curve, an average or effective conductivity function should be defined which will be adequate for characterizing infiltration.

The validity of this assumption was evaluated for four soils whose properties were obtained from the literature. In addition to Sarpy loam, data were obtained from Hanks and Bowers (1962) for the soil properties of Geary silt loam, and from Staple and Gupta (1966) for Castor loam and Grenville silt loam.[1] The properties for each soil were used in the same manner as described for Sarpy to define the infiltration rate-time relationships for low initial water contents. Treating this relationship as the "measured" influx curve, equation 10 was solved subject to boundary conditions 13 for many values of a and $h_1$, and the response surfaces were defined. The response surface for each of the soils was similar to that given for Sarpy in Figure 4. In addition, the exponent, a, was estimated for each soil using the model of Brooks and Corey (1964). The Golden Section Search technique described by Thompson and Peart (1968) was then used to find $h_1$ corresponding to the minimum value

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[1]The data obtained from both sources was $\theta$ - h - D. K was determined from these data by the relationship $K = D \frac{d\theta}{dh}$.

of R at a. The R values obtained in this way were within 1.2 percent of the minimum point on the response surfaces for each of the four soils analyzed.

The influx curves shown in Figure 6 for Castor loam are analogous to those given in Figure 3 for Sarpy. As in Figure 3 the points correspond to a "measured" influx curve while the solid line was obtained using equation 18 for the conductivity function.

The conductivity function defined by the proposed method agreed well with the original data for the Sarpy, Grenville, and Geary soils. However, the form of the conductivity-head relationship for Castor loam deviated from that given by equation 18. This is shown in Figure 7 where the relative conductivity is plotted versus pressure head for the original data and for equation 18.

Figures 8 and 9 compare "measured" and computed influx curves for Sarpy and Castor loams at different initial water contents. As in Figures 3 and 6 the "measured" curves represented by the discrete points were obtained by solving equation 10 subject to boundary conditions 13 with soil properties obtained from the literature. The influx curves defined by the solid and broken lines were also obtained by solving equation 10 subject to 13, but the conductivity function was calculated from equation 18 with the parameter values obtained from the influx relationship for the dry condition. The results obtained for Geary and Grenville soils were very similar to those given in Figures 3, 5, and 8 for Sarpy loam. Although the form of the original conductivity data for Castor loam differed considerably from that given by equation 18, the curves given in Figure 8 show that equation 18 can be used to characterize infiltration for relatively high initial water contents. Thus by defining the parameters in equation 18 by the method described, an effective K(h) relationship can be obtained.

## Discussion

Simulated tests on four separate soils indicate that an effective K(h) relationship can be defined by the proposed method. Based on the results of these tests, a step by step procedure for determining the parameters in equation 18 for a soil column are outlined as follows.

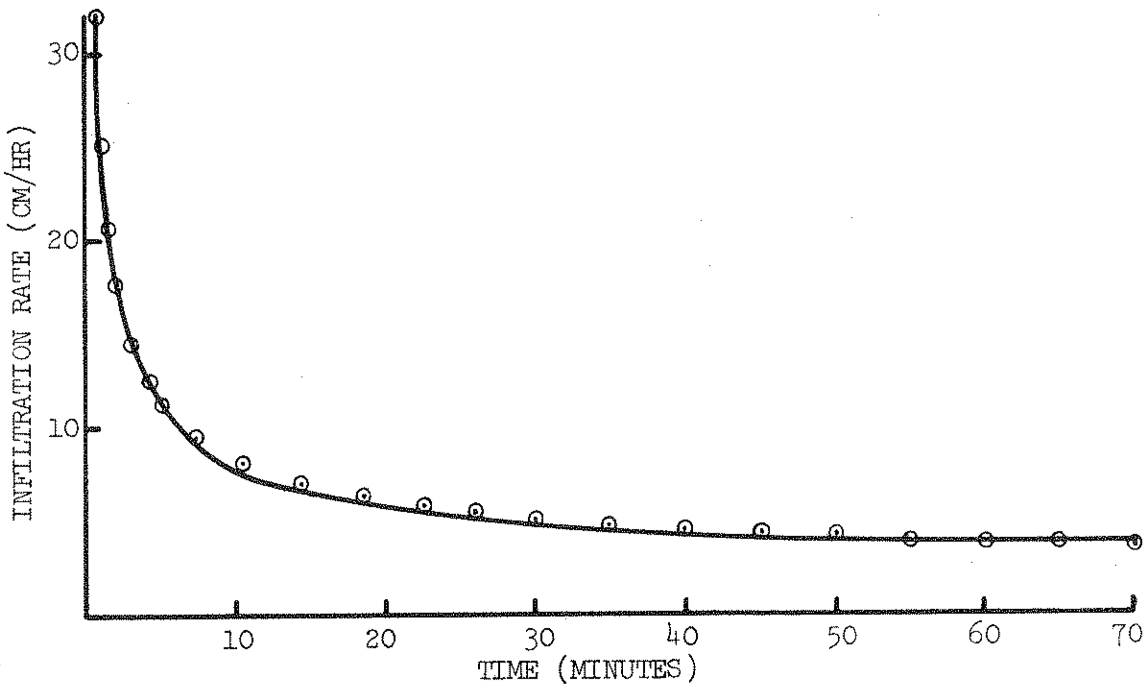


Figure 6. Influx Curves for Dry Castor Loam.

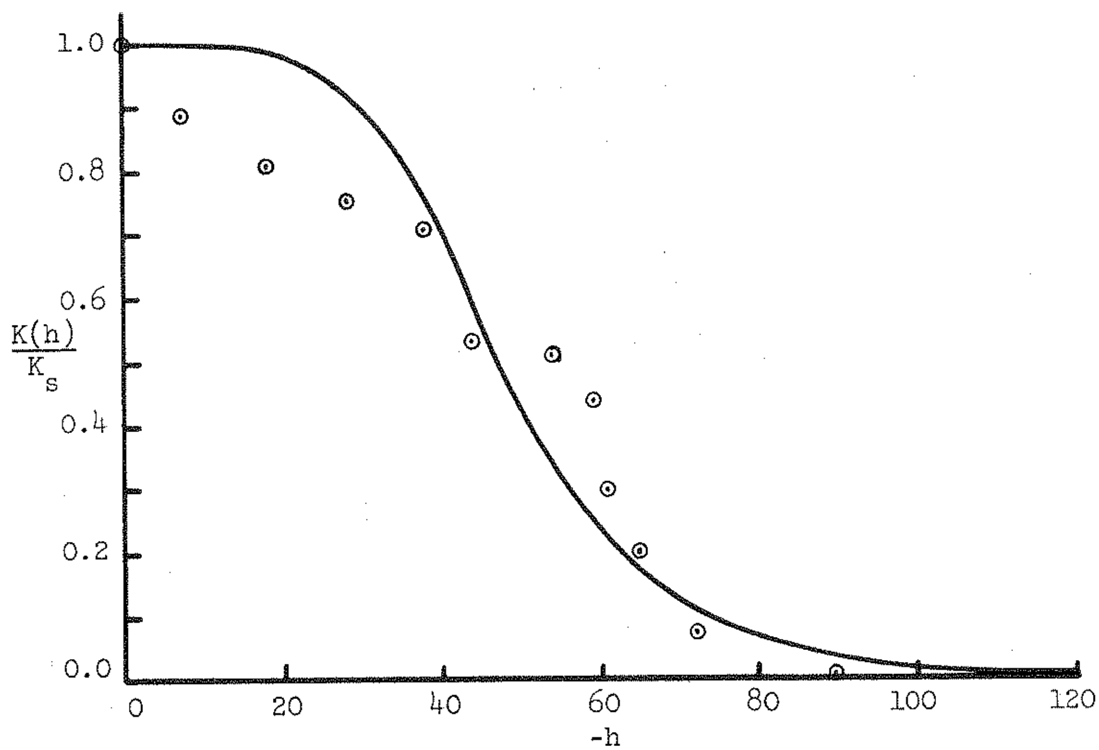


Figure 7. Relative Conductivity Versus Pressure Head for Castor Loam.

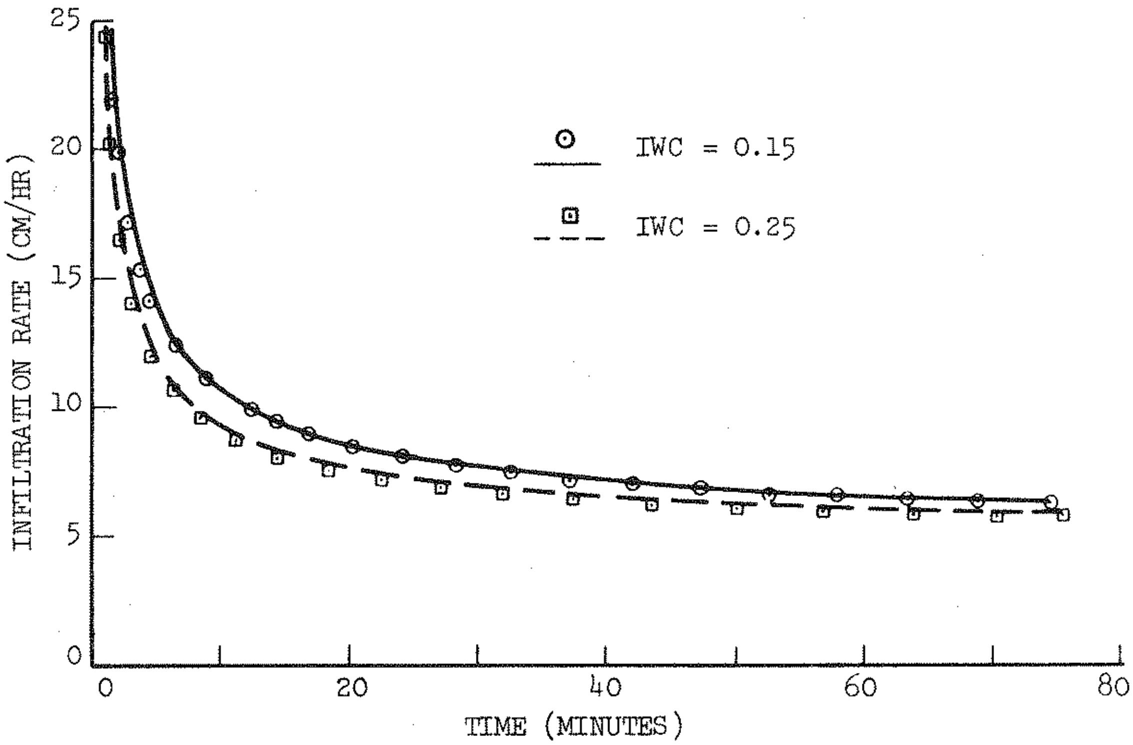


Figure 8. Influx Curves for Sarpy Loam With Initial Water Contents of 0.15 and 0.25.

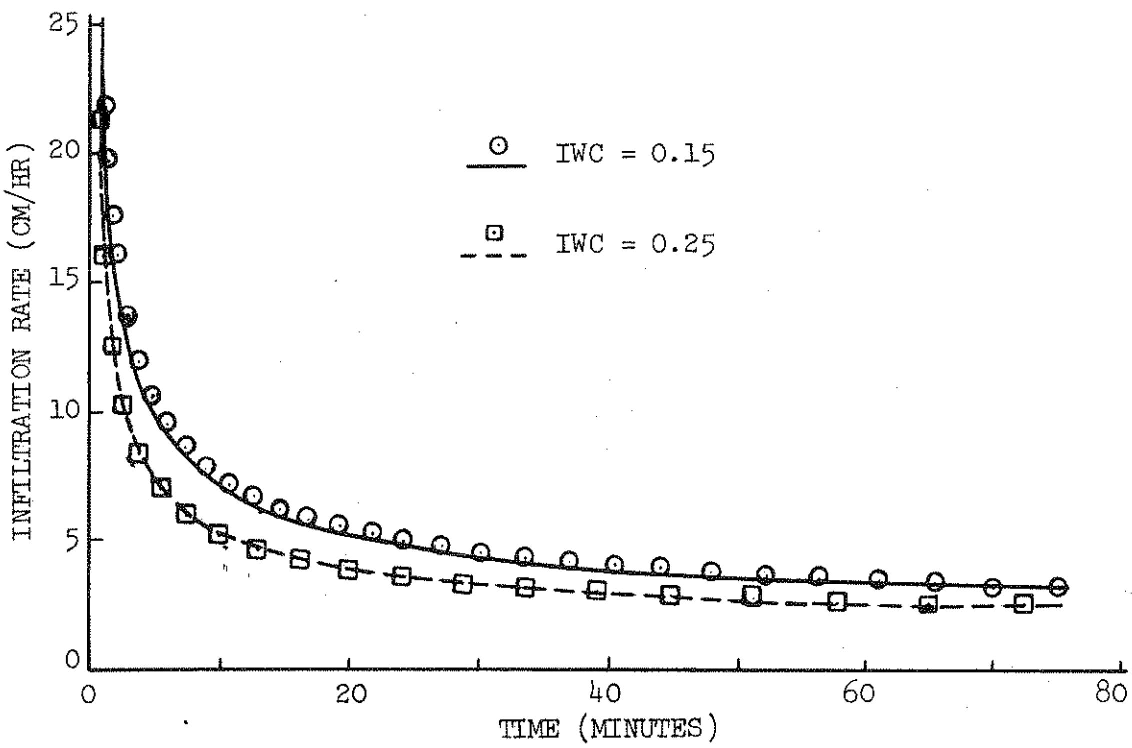


Figure 9. Influx Curves for Castor Loam With Initial Water Contents of 0.15 and 0.25.

1. Determine the imbibition branch of the water characteristic, $h(\theta)$. Methods and equipment described by Tanner and Elrick (1958) or Anat, et al. (1965) can be used to determine this relationship.
2. Measure the influx-time relationship for infiltration into the soil column subject to boundary conditions 13.
3. Determine $K(0)$ by allowing infiltration under the above boundary conditions to be continued until the column is saturated and $h = 0$ at the bottom. Set $b = 1/K(0)$.
4. Estimate the parameter a from the model of Brooks and Corey (1964).
5. Using the Golden section search procedure to make successive estimates of $h_1$, repeatedly solve equation 10 subject to boundary conditions 13 until the objective function, R, is minimized. A listing of the computer program used to accomplish this step is given in Appendix B.

The most obvious drawback to the proposed method is the computer time necessary to repeatedly solve equation 10 in order to determine the parameters in equation 18 corresponding to the minimum of the response surface. By using the independent methods for evaluating the parameters a and b in equation 18, however, a one-dimensional search to determine parameter $h_1$ can be accomplished at reasonable cost. The time required to determine $h_1$ for the soils analyzed in this study was on the order of 2 minutes of central processor time on a CDC 6500 computer. The time could probably be reduced by at least a factor of 2 by refining the computer program. One way to reduce the computation for determining $h_1$ is to use an influx curve for a shorter period of real time. Experimentally, this would correspond to subjecting shorter soil columns to boundary conditions 13. Considering the approximate nature of the proposed method, however, a more reliable $K(h)$ function would be defined if the influx curve is considered for a time, $t_e$, corresponding to the maximum core length available. For field cores, this length would be limited by the depth of a soil layer which could be considered homogeneous.

The major disadvantage in the proposed method is the requirement of the inflow branch of the water characteristic. Available techniques such as the one proposed by Tanner and Elrick (1958) require several days to determine the $h(\theta)$ relationship. However, $h(\theta)$ is a necessary input for characterizing infiltration by solving equations 9 or 10, regardless of the method used to define $K(h)$. The additional measurements needed to determine $K(h)$ by the present method are rather simple in nature and can be made rapidly, particularly in comparison to conventional techniques.

The criteria used to judge the validity of the theoretical approach for characterizing infiltration has commonly been the agreement between predicted and observed water content distributions. In most practical applications, however, the infiltration rate-time relationship is a more important variable. Since the proposed method of defining $K(h)$ is based on predictions of the infiltration rate, the effect of some of the heterogeneities present in natural soils will be lumped into the conductivity function. The effect on infiltration rate predictions of other inconsistencies such as the transition zone observed by Bodman and Colman (1943) will also be lumped into $K(h)$. Consequently, solutions to the Richards equation obtained with this conductivity function could possibly give more accurate predictions of the infiltration rate-time relationship than would predictions based on a $K(h)$ determined by conventional techniques.

Comparison with the Method of Millington and Quirk

The method of Millington and Quirk (1960) was used to compute the $K(h)$ relationship of the four soils investigated above. The saturated conductivity from the original data was used to obtain a matching factor and the number of pore classes, N, was arbitrarily determined by breaking the water content axis of the $h(\theta)$ relationship into increments of 1 percent. The computed and observed relative conductivities of Sarpy and Castor loams are plotted in Figure 10. Although the computed $K(h)$ relationship for Castor has the same general shape as the measured data, it underestimates the conductivities for $-h$ values less than 70 cm. Also the computed conductivities for Sarpy are much lower over the entire h range than the observed.

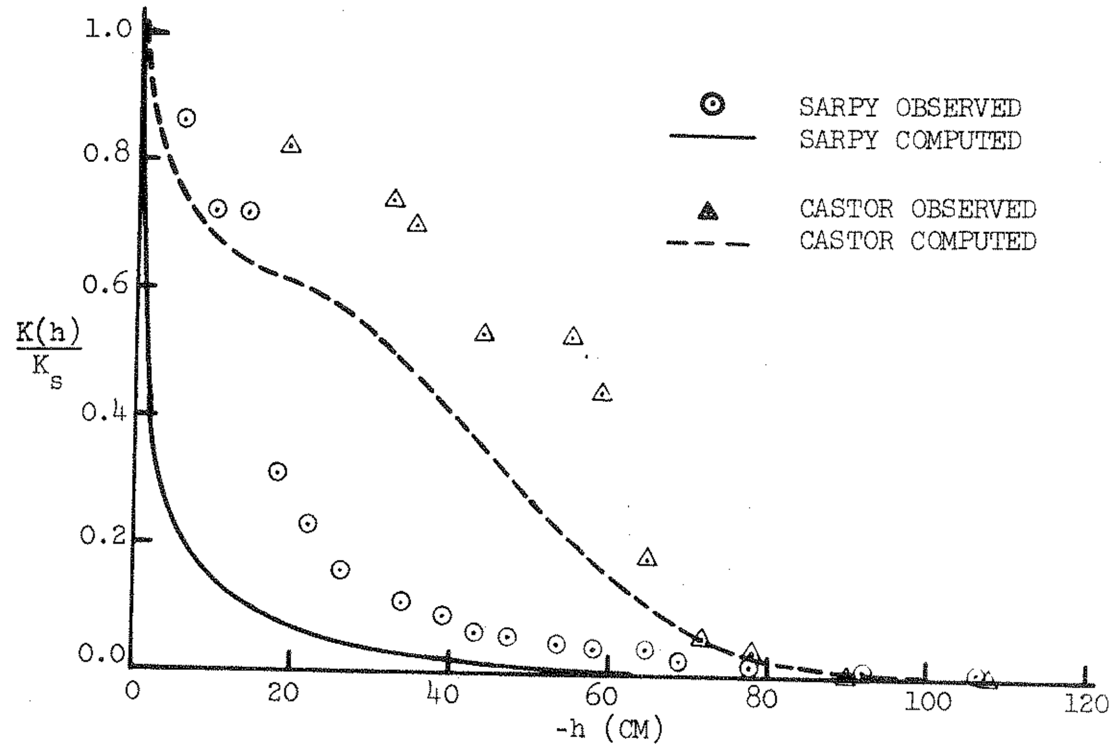


Figure 10. Relative Conductivities for Sarpy and Castor Loams.

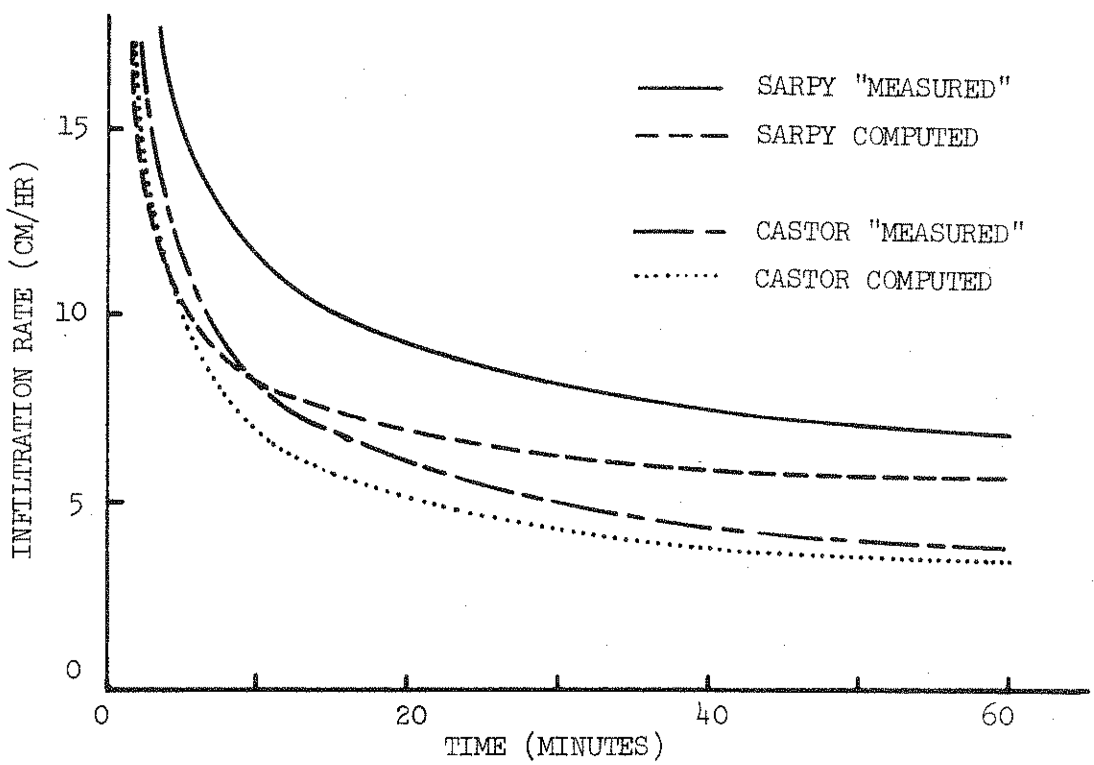


Figure 11. "Measured" and Computed Influx Curves for Sarpy and Castor Loams.

In Figure 11, the "measured" influx curves of Figures 3 and 6 are compared to curves obtained by solving equation 10 with K(h) computed by the method of Millington and Quirk. Of the four soils investigated, the best agreement between the "measured" and computed influx curves was obtained for Castor while the worst agreement was obtained for Sarpy. Kunze, et al. (1968) showed that a somewhat better estimate of the conductivity function could be obtained by using a total 5 to 10 pressure classes compared with 35 used here. However, their results showed little difference in the computed relationship when the number of classes was increased from 8 to 32. This was especially true for the higher water contents where large deviations occurred for the soils investigated above.

A comparison of Figures 10 and 11 with Figures 3, 5, 6, and 7 shows that, for the soils investigated, the proposed method gives a better estimate of the conductivity function than can be obtained by the method of Millington and Quirk. The only additional measurement required by the proposed method is the influx curve for infiltration into an initially dry soil. A technique for measuring the influx curve is described in Chapter 4.

# CHAPTER 3

## NUMERICAL SOLUTION OF THE GOVERNING EQUATION

The method developed to define the conductivity function required repeated solutions to the governing Richards equation. Numerical solutions to the $\theta$-based equation (equation 10) which required less computer time than corresponding solutions to the h-based equation (equation 9) were used in determining the conductivity function. However, a numerical technique to solve the h-based equation was also necessary to test the conductivity function for infiltration into layered columns where both saturated and unsaturated flow can occur. The numerical procedures developed to solve both forms of the Richards equation will be described by considering equations 9 and 10 in generalized notation as

$$G(y)\,\frac{\partial y}{\partial t} = \frac{\partial}{\partial x}\left[F(y)\,\frac{\partial y}{\partial x}\right] - \frac{\partial K}{\partial x} \tag{19}$$

where $y = h$, $G(y) = C(h)$, and $F(y) = K(h)$ for equation 9, and $y = \theta$, $G(y) = 1$, and $F(y) = D(\theta) = K(h)/C(h)$ for equation 10.

Consider the solution of 19 subject to the boundary conditions 13. Taking a time increment of $\Delta t$ and a depth increment of $\Delta x$, the solution domain can be represented schematically by Figure 12.

### Finite Difference Equations

Using a backward difference approximation, $\frac{\partial y}{\partial t}$ may be written

$$\frac{\partial y}{\partial t} = (y_i^j - y_i^{j-1})\,\frac{1}{\Delta t} \tag{20}$$

where i and j denote depth and time, respectively.

A three point central difference approximation of the derivatives with respect to x may be used to express the right side of equation 19 as

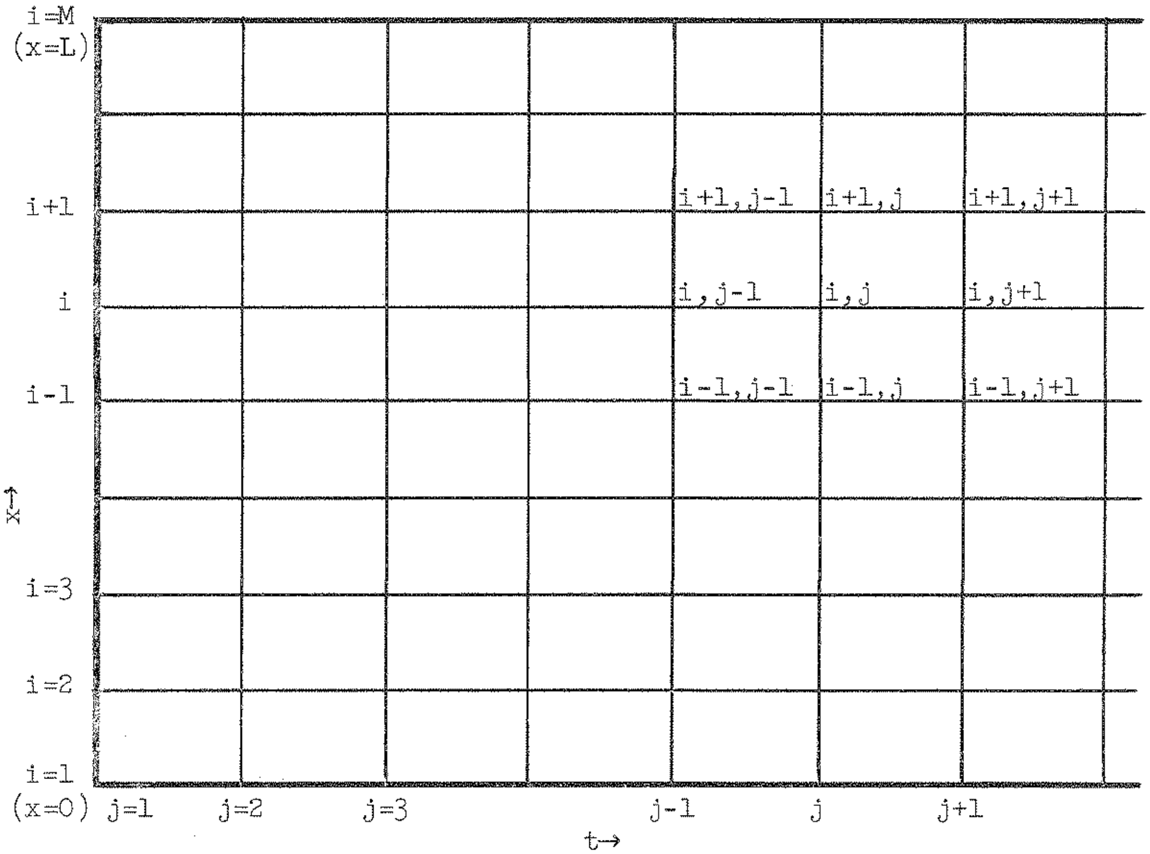


Figure 12. Schematic of Solution Domain.

$$\frac{\partial}{\partial x}\left[F(y)\,\frac{\partial y}{\partial x}\right] - \frac{\partial K}{\partial x} = F_i^j\,(y_{i-1}^j - 2y_i^j + y_{i+1}^j)\,\frac{1}{\Delta x^2}$$

$$+ (y_{i+1}^j - y_{i-1}^j)(F_{i+1}^j - F_{i-1}^j)\,\frac{1}{4\Delta x^2} - (K_{i+1}^j - K_{i-1}^j)\,\frac{1}{2\Delta x} \qquad (21)$$

Then equation 19 may be written in finite difference form as

$$G_i^j\,(y_i^j - y_i^{j-1})\,\frac{1}{\Delta t} = F_i^j\,(y_i^j - 2y_{i-1}^j + y_{i+1}^j)\,\frac{1}{\Delta x^2}$$

$$+ (y_{i+1}^j - y_{i-1}^j)(F_{i+1}^j - F_{i-1}^j)\,\frac{1}{4\Delta x^2} - (K_{i+1}^j - K_{i-1}^j)\,\frac{1}{2\Delta x} \qquad (22)$$

By combining terms, equation 22 can be reduced to

$$\alpha_i^j\, y_{i-1}^j + \beta_i^j\, y_i^j + \epsilon_i^j\, y_{i+1}^j = \zeta_i^j \tag{23}$$

where

$$\alpha_i^j = (4F_i^j + F_{i+1}^j - F_{i-1}^j)\,\frac{1}{4\Delta x^2} \tag{24}$$

$$\beta_i^j = -2F_i^j\,\frac{1}{\Delta x^2} - G_i^j\,\frac{1}{\Delta t} \tag{25}$$

$$\epsilon_i^j = (4F_i^j - F_{i+1}^j + F_{i-1}^j)\,\frac{1}{4\Delta x^2} \tag{26}$$

$$\zeta_i^j = (K_{i+1}^j - K_{i-1}^j)\,\frac{1}{2\Delta x} - G_i^j\, y_i^{j-1}\,\frac{1}{\Delta t} \tag{27}$$

Solution of Finite Difference Equations

The coefficients $\alpha$, $\beta$, $\epsilon$, and $\zeta$ for time step j are expressed in terms of the soil properties for that time and the y values for step j-1. Tentatively assume that these coefficients can be evaluated directly. Then equation 23 can be written for each interior node at time j giving M-2 equations in M unknowns. Two additional equations may be formulated from the boundary conditions at $x = 0$ and $x = L$. The resulting tridiagonal set of equations may be rapidly solved by a technique given by Richtmyer (1957). A solution of the finite difference equations is obtained by first defining the coefficients $\alpha$, $\beta$, $\epsilon$, and $\zeta$ for $j = 2$ from the $y_i^1$ values given by the initial conditions. The tridiagonal set of equations is then solved for the y's at $j = 2$, and the next time step is considered. Thus the solution scheme proceeds from left to right in Figure 12 until a solution has been generated for the length of time desired.

Since $G_i^j$, $F_i^j$, and $K_i^j$ are dependent on $y_i^j$, however, either an iteration process or an extrapolation procedure is required to evaluate the coefficients in equation 23. In the iteration process, values of G, F, and K (soil properties) are based on values of $y_i^{j-1}$, a solution is obtained, and the properties are re-evaluated based on the new $y_i^j$

values. Iteration is continued until the parameters used are functionally consistent with the y's obtained. An iteration process was used by Whisler and Klute (1965) to solve equation 9. Hanks and Bowers (1962) used an extrapolation technique to estimate the soil properties for succeeding time steps. Although this permitted rapid solution of the finite difference equations, the solution is not as reliable as one obtained by iterating. The apparent instability in the early portions of the computed infiltration rate-time curves published by Hanks (1964) and by Green, et al. (1964) are probably due to inadequate estimates of soil property values for succeeding time steps.

The difficulties of evaluating the soil properties for the jth time step can be avoided by using an explicit finite difference form of equation 19 as was used by Staple (1966). In the explicit form, the derivatives with respect to x are written in terms of the soil properties at the (j-1)th time step. A reliable solution can be obtained using this form of the equation, but a small time step is usually needed for convergence and, for most cases, large amounts of computer time are required.

The computer program developed to solve the θ-based equation (INFIL2) was used extensively in determining the conductivity function as described previously. A program was also developed to solve the h-based equation (INFIL3) and was used to predict infiltration for layered soils. Listings of both programs, along with sample input data and computer output, are given in Appendix B.

## Determination of Infiltration Rate

Once the y distribution has been determined for a particular time, the infiltration rate can be obtained from equation 4. Equation 4 written in generalized form is

$$f = q\Big|_{x=0} = F(y)\,\frac{dy}{dx}\Big|_{x=0} + K(h)\Big|_{x=0} \qquad (28)$$

where f represents the infiltration rate. Using a three point forward difference approximation of $\frac{dy}{dx}$, equation 28 may be written

$$f^j = -F_i^j \left(4y_2^j - 3y_1^j - y_3^j\right) \frac{1}{2\Delta x} + K_1^j \qquad (29)$$

After the wetting front has progressed beyond two depth increments, i.e., beyond i = 3, the three point forward difference approximation of $\frac{dy}{dx}$ is more accurate than the two point forward difference form often used. Prior to that time, a better estimate of the infiltration rate can be obtained by using a two point forward difference approximation of

$$\frac{dy}{dt} = -F_i^j \left(y_2^j - y_1^j\right) \frac{1}{\Delta x} + K_1^j \qquad (30)$$

Layered Soils

The computer program developed to solve the h-based equation was written to accommodate a two layered system. For infiltration into a layered soil in which a top layer has a higher conductivity than some lower layer, positive pressures may develop. The result is a situation in which water is flowing under saturated conditions in part of the soil profile and unsaturated conditions in the remainder of the profile. As was noted in the review of theory, the h-based equation reduces to Laplace's equation for regions of saturated flow. Substitution of $G(y) = C(h) = 0$ and $F(y) = K(h) = K_s$ reduces equation 22 to a finite difference form of the Laplace equation for flow in the vertical direction. In solving the finite difference equations for a layered system, the solution was continuously checked at each node to determine if saturation had occurred. When an $h_i^j$ greater than zero was found, saturation was assumed; $C(h) = 0$ and $K(h) = K_s$ were substituted, and the procedure for solving the finite difference equations was continued.

The soil properties K(h) and C(h) are both discontinuous functions at the junction of two layers. Denoting the node situated at the layer junction by the subscript n, the coefficients in equation 23 were defined for i = n by equating the efflux from the top layer to the influx to the bottom layer. This equation was written in finite difference form as

$$K_1(h_n^j)\left[\left(h_n^j - h_{n-1}^j\right) \frac{1}{\Delta x} + 1\right] = K_2(h_n^j)\left[\left(h_{n+1}^j - h_n^j\right) \frac{1}{\Delta x} + 1\right] \qquad (31)$$

where $K_1(h)$ and $K_2(h)$ denote the conductivity functions of the top and bottom layers, respectively.

Solution Parameters and a Shortcut Procedure

In solving the finite difference form of the governing equation, it was necessary to select values for $\Delta x$, $\Delta t$, and an iteration parameter denoted as CRIT in the computer programs given in Appendix B. CRIT was defined for the θ-based solution as the minimum allowable absolute difference between the values of $\theta_i^j$ assumed to define the soil properties and the $\theta_i^j$ values finally obtained. For the h-based equation, CRIT was defined as the minimum allowable absolute value of the ratio $(h_i^j - hl_i^j)/h_i^j$ where $hl_i^j$ was the value of $h_i^j$ assumed in evaluating the soil properties. Solutions for four different soils showed that the computed influx curves remained essentially constant for values of CRIT less than 0.001 for the θ-based equation and 0.01 for the h-based equation. These values were assumed for CRIT in all subsequent solutions. In most cases 2 or 3 iterations were sufficient to reduce the above quantities to values less than CRIT for all i's at a given j.

The effect of the size of $\Delta x$ was determined by solving both the θ-based and h-based equations for different values of $\Delta x$. Figure 13 shows the effect of different $\Delta x$ values on the influx curves predicted by the solution to the θ-based equation for infiltration into Sarpy loam subject to boundary conditions 13. The solution for $\Delta x = 5$ cm demonstrates instability caused by choosing a value of $\Delta x$ that was too large. The predicted influx curves for $\Delta x$ values of 0.25, 0.5, and 1.0 cm, however, were in good agreement. The smaller $\Delta x$ values tended to produce slightly higher infiltration rates. However, the maximum difference between predicted rates for $\Delta x = 1$ cm and $\Delta x = .25$ cm was less than 2.5 percent while the computational time was increased by a factor of 5.6 for the smaller increment. Based on these and similar results for other soils, $\Delta x = 1$ cm was selected for use in the computer solution to the θ-based equation. A similar analysis for the solution of the h-based equation resulted in the selection of $\Delta x = 0.5$ cm.

A variable increment, $\Delta t$, was used. Since the gradients for both water content and soil properties are normally steeper during the initial

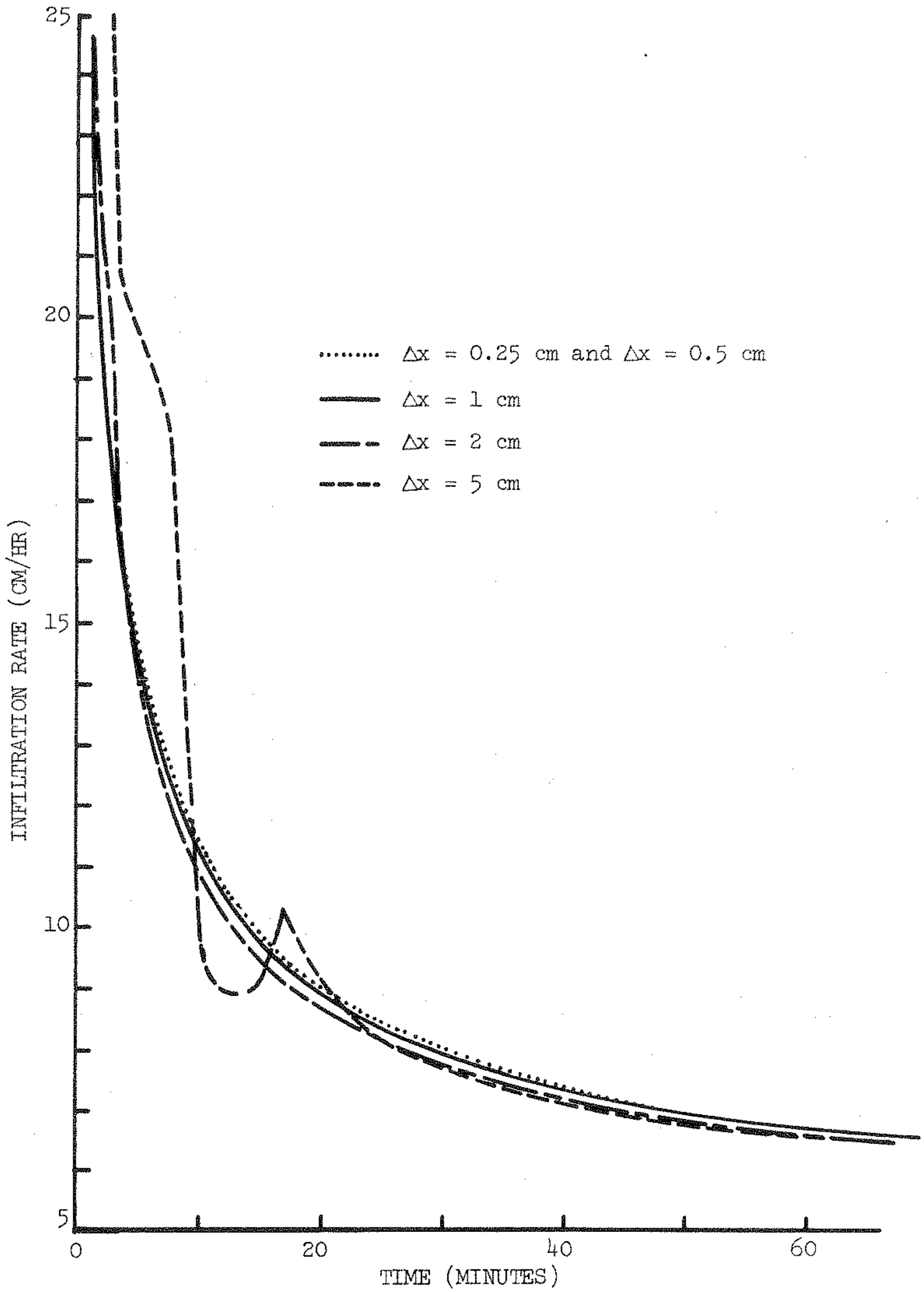


Figure 13. Effect of $\Delta x$ on the Computed Influx Curve for Dry Sarpy Loam.

stages, a time increment of increasing length could be used in the finite difference equations without sacrificing accuracy of solution. Variable time increments have also been used by Hanks and Bowers (1962), and Whisler and Klute (1965). Prior to evaluating the coefficients in equation 22 for each succeeding time step, $\Delta t$ was increased by a value that was dependent on the number of iterations required in the previous time step. The length of the increase was $(1 - 0.4I)$ seconds, where I was the number of iterations in the previous time step.

Table 1 presents the computer times required for solutions with fixed and variable $\Delta t$ for two soils with and without a shortcut procedure. The values given represent seconds of CDC 6500 central processor time required to solve the $\theta$-based form of the Richards equation subject to boundary conditions 13. The initial water content was 0.10 and the duration of infiltration ($t_e$) was 60 minutes for both Sarpy and Castor loams. For the variable time increments, $\Delta t = 1.0$ sec was assumed at $t = 0$, and was increased as previously discussed. In order to obtain a stable solution at small times, the fixed $\Delta t$ was restricted to 5.0 seconds. The computer times required for the fixed $\Delta t$ were approximately 3.0 and 4.5 times greater than the variable $\Delta t$ for Sarpy and Castor loams, respectively. There was essentially no difference in the computed influx curves for fixed and variable $\Delta t$.

Table 1. Effect of Fixed and Variable Time Increments and a Shortcut Procedure on Computer Solution Times (Seconds) for Sarpy and Castor Loams.

| | Sarpy Loam | | Castor Loam | |
|---|---|---|---|---|
| Shortcut | Fixed t | Variable t | Fixed t | Variable t |
| No | 93.0 | 33.0 | 89.6 | 21.0 |
| Yes | 42.0 | 13.7 | 39.0 | 8.2 |

Since the procedure described in the previous chapter for determining K(h) required many solutions to the governing equation, it was imperative to minimize the required computer time. For infiltration into soils at low initial water contents, the conductivity and consequently the water movement at all points in advance of the wetting front are negligible. Therefore, nothing is accomplished by solving for $\theta$ values far in advance of the wetting front, as these values remain essentially constant until the wetting front approaches. As a result of trial solutions with different column lengths, the procedure given earlier was altered for dry soils. The solution commenced by considering the column depth to be L/4 which, in effect, changed the boundary conditions $\theta = \theta_i$ at $x = L$ to $\theta = \theta_i$ at $x = L/4$. The value of $\theta$ at the node $i = \frac{M}{4} - 5$ was then checked after each time step. When this value had increased by 1 percent, the length of the column considered was increased by $5\Delta x$ and the check point advanced by a like amount. Thus, the length of column considered was increased as the wetting front moved down. Extensive comparisons between solutions obtained using this procedure and solutions obtained by considering the entire column length showed no difference in either the predicted water content profiles or the influx curves. As shown in Table 1, use of the shortcut procedure reduced the computer time to less than one-half the time otherwise required.

Evaluation of the Numerical Method

Tests were conducted to determine the reliability of the computer programs developed to implement the numerical method given above. The computer programs were used to solve equation 19 for several different cases and comparisons were made with solutions obtained by independent methods.

For constant K and C, analytical methods were used to solve equation 19 subject to boundary conditions 13 for both uniform and layered soils. The solution for a uniform soil was given in equations 15 and 17. Influx curves computed by equation 17, INFIL2, and INFIL3 for $K = 0.0001$ cm/sec and $C = 0.001\ cm^{-1}$ were in almost exact agreement. For time greater than 0.75 minutes the difference in the infiltration rates computed by the three methods was less than 1 percent. Influx curves for a layered soil with $K_1 = 0.0001$ cm/sec, $K_2 = 0.00001$ cm/sec, and $C_1 = C_2 = 0.001\ cm^{-1}$ were computed from an analytical solution by Carslaw and Jager (1959) and from INFIL3. Although the agreement between infiltration rates was not as good as for uniform soils, the difference was less than 4 percent for times greater than 0.5 minutes.

The agreement between the analytical and numerical solutions for constant soil properties showed that the procedures programmed to set up and solve the finite difference equations were basically correct. However, conclusions should not be drawn concerning the ability of the computer programs to handle nonlinear soil properties. The agreement found for constant soil properties represents necessary but not sufficient evidence of the validity of the computer solutions.

Figures 14 and 15 present solutions for infiltration into Sarpy loam subject to the boundary conditions 13 with initial water contents of 0.15 and 0.25. Influx curves predicted by INFIL2 and INFIL3 are compared with relationships predicted from Philip's (1957a) solution to equation 10. The first three terms in Philip's series expression for infiltration rate were used in calculating the influx curves. For an initial water content of 0.25, the three solutions agreed almost exactly. The infiltration rates predicted by INFIL3 for an initial water content of 0.15, however, were slightly lower than those predicted by the other two methods. This deviation could have been caused by round-off errors due to steep gradients of h and K occurring at the lower initial water contents.

From the evidence presented, it was concluded that reliable solutions to the $\theta$-based governing equation can be obtained from INFIL2. A direct test of INFIL3 for infiltration into a layered soil with variable K and C was not made. The solutions given in Figures 14 and 15 show that INFIL3 can be used to characterize infiltration into uniform soil profiles. In addition, acceptable solutions were obtained for layered systems with constant K and C. On this basis it was concluded that INFIL3 would probably provide valid solutions for infiltration into layered soils.

Although INFIL3 appeared to give accurate solutions, it was difficult to apply for infiltration into a layered soil with a low initial water content. Due to the high gradients of h, large amounts of computer time were required for convergence. Convergence was especially difficult to obtain when the wetting front approached the junction of the two layers.

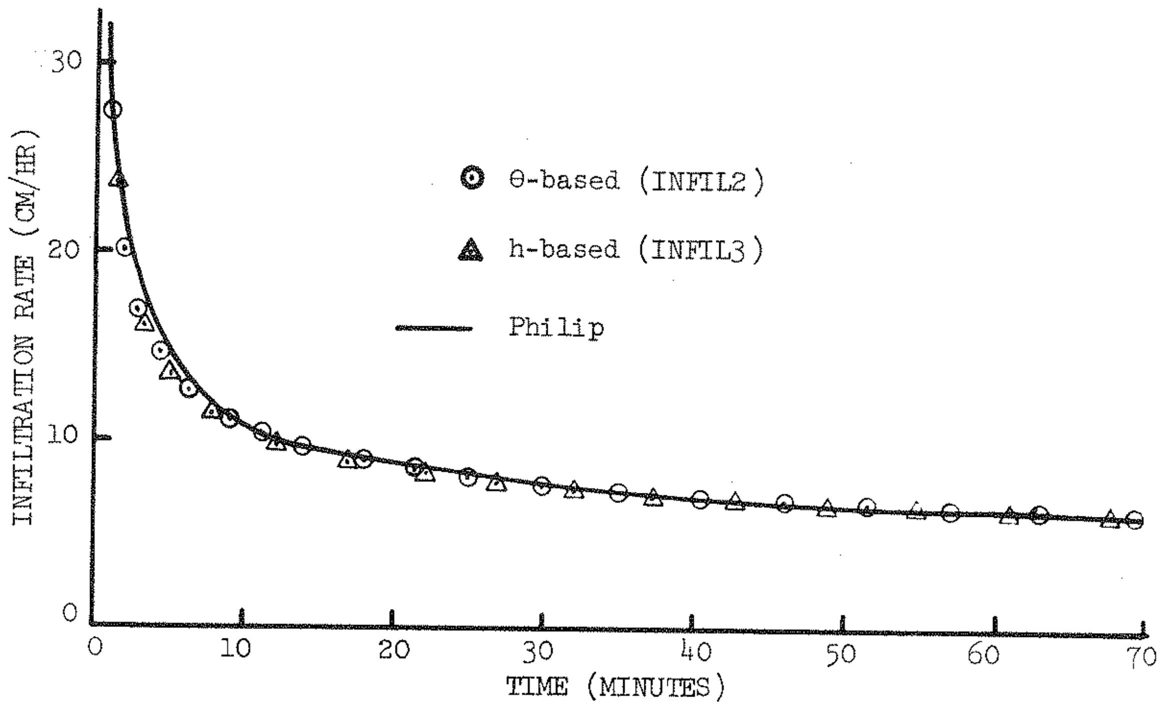


Figure 14. Comparison of Influx Curves Computed by Three Numerical Methods for Sarpy Loam with 0.15 Initial Water Content.

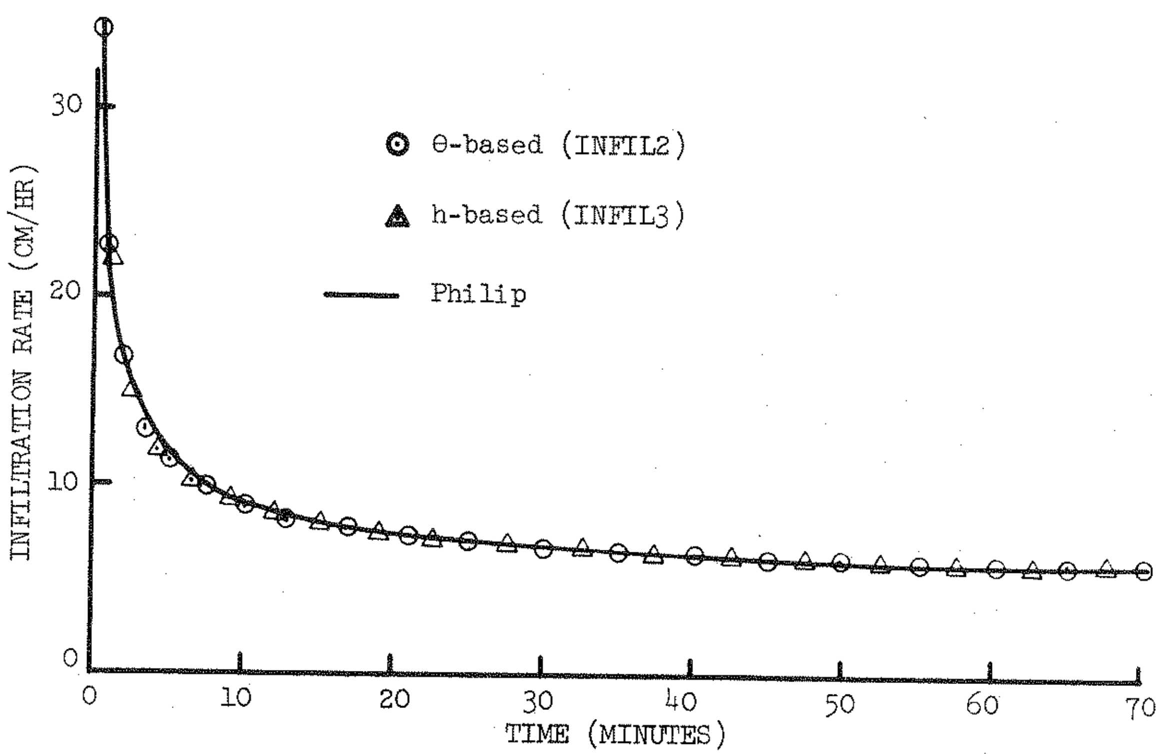


Figure 15. Comparison of Influx Curves Computed by Three Numerical Methods for Sarpy Loam with 0.25 Initial Water Content.

# CHAPTER 4

# EXPERIMENTS

Experiments were conducted to test the reliability of the method given in Chapter 2 for determining K(h). The method was developed to define an effective conductivity function, one that could be used to compute the infiltration rate-time relationship by solving equations 9 or 10 for pertinent boundary conditions. Therefore, the experiments were designed to test the method based on predictions of the infiltration rate-time relationships for various boundary conditions.

## Experimental Design

Experiments were conducted on two soils. One was an artificial soil which was made up of a mixture of a graded silica sand and 10 percent by weight of No. 270 ground silica. The other was a natural sandy loam of till origin. Mechanical analyses of the soils are given in Table 2.

Table 2. Mechanical Analysis of Soils Used in Experiments

| Size (microns) | Textural Classification | Percent by Weight | |
|---|---|---|---|
| | | Sand Mixture | Sandy Loam |
| 0-2 | Clay | ---- | 14.3 |
| 2-50 | Silt | 6.2 | 21.8 |
| 50-100 | Sand | 16.9 | 13.9 |
| 100-250 | " | 65.9 | 27.6 |
| 250-500 | " | 10.7 | 16.5 |
| 500-1000 | " | 0.2 | 5.6 |
| 1000-2000 | " | 0.0 | 0.0 |

The following experiments were conducted on each soil:

1. The imbibition branch of the soil-water characteristic, $h(\theta)$, was determined with a pressure plate apparatus of the type used by Tanner and Elrick (1958). This experiment was replicated three times for the sand mixture and twice for the sandy loam.
2. Influx curves were measured for infiltration into packed columns of initially dry soil with boundary conditions given by equation 13. $K(0)$ was determined by measuring the flow rate at steady state. Then the $K(h)$ relationship was determined by the method given in Chapter 2. Two replications were made for each soil.
3. Soil columns were prepared with 3 different initial water content distributions. Influx curves were measured for infiltration from a shallow ponded surface into the soil columns. These influx curves were compared to relationships obtained by solving equation 10 using $K(h)$ determined above. Two replications of each initial water content distribution were made.
4. Influx curves were measured for infiltration into layered soils. Measurements were made for columns made up of 6 inches of sand over 18 inches of sandy loam and 12 inches of sand over 12 inches of sandy loam. Measurements were also made for sandy loam over sand with the same layer dimensions.

## Experimental Equipment and Procedure

### Soil-Water Characteristic

The imbibition branch of the soil-water characteristic was determined with a commercially available volumetric pressure plate apparatus and hysteresis attachment.[1] A schematic of one of the four pressure plate units used is given in Figure 16.

Prior to beginning a test, the pressure plate was saturated by soaking in de-aired water for a period of 1 to 2 weeks. A test was initiated by adjusting the water level in the ballast tube to the

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[1] Soilmoisture Equipment Company, P. O. Box 30025, Santa Barbara, California.

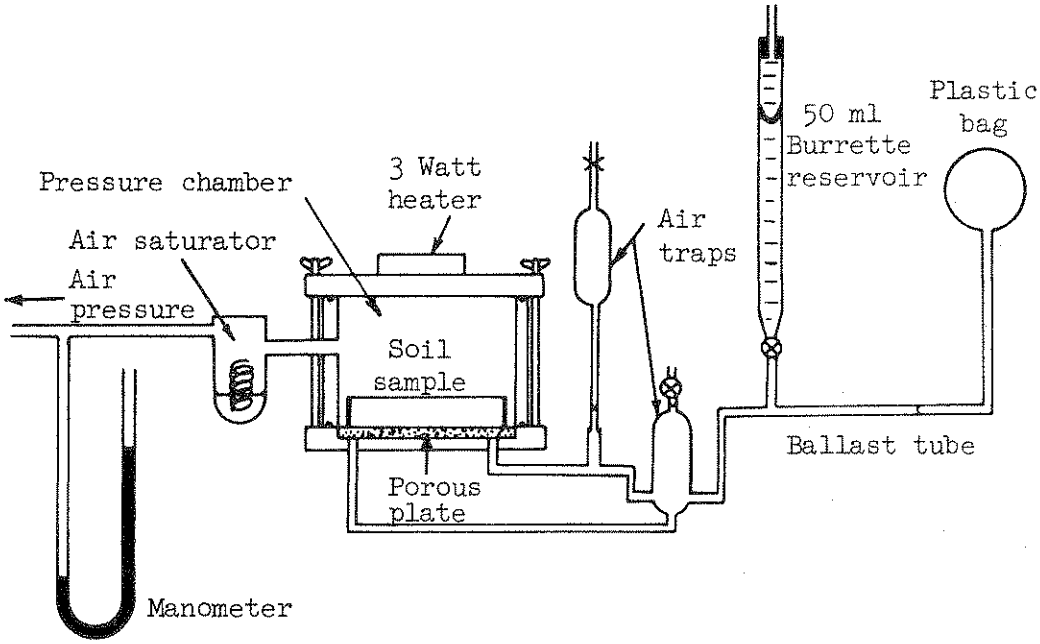



Figure 16. Schematic of Pressure Plate Apparatus.

reference mark, making sure that all air bubbles were removed from the water supply lines and from the grooves under the porous plate. Then a soil sample was packed to the required density in a 4 inch I.D., 1 inch deep Lucite ring which rested directly on the porous plate. A picture of the disassembled apparatus containing a packed soil sample is shown in Figure 17.

After the sample was packed, the apparatus was sealed and the pressure maintained at atmospheric until approximately 3 cc of water had been taken up. This insured good contact between the soil sample and the water menisci in the porous plate. Then the pressure in the chamber was raised to two atmospheres which was approximately the bubbling pressure of the porous plates and, hence, the maximum pressure that could be applied to the sample. Pressure was maintained by bottled compressed air and regulated by a set of precision regulators.[2] Pressures were measured with a water manometer for pressures less than 150 cm of water and with a mercury manometer for greater pressures. A constant head was maintained on the low pressure side of the porous plate by introducing

[2] Ibid.

water from a horizontal ballast tube. The level of the ballast tube was set at the center of the soil sample.

The water level in the reservoir was read daily. When the intake rate had decreased to 0.1 cc per day, the pressure was reduced to the next lowest step. A sample set of data is given in Appendix C.

Techniques described by Tanner and Elrick (1958) were used to reduce errors in inflow measurements. Condensation on the inside walls of the pressure chambers was prevented by maintaining a small temperature gradient with a 3-watt heater placed on the top of each apparatus. Air bubbles were removed from beneath the porous plate by clamping off the water supply reservoir and circulating water between the two air traps. To prevent a net water loss from the chamber by escaping air or from back diffusion, the air from the regulator was passed through an air saturator. The end of the ballast tube was sealed with a plastic bag. For the experiments on sand, an empty unit was cycled through the same pressure steps as the other 3 replications. The water loss was to be used as a correction for the other tests; however, the loss was negligible and the check was not made for the sandy loam. An overall view of the four pressure plate units and associated equipment is given in Figure 18.

## Column Packing

The soil was packed into 24-inch cylindrical columns which were constructed from Lucite tubing with 0.25 inch wall thickness. Two columns had inside diameters of 3.5 inches and were segmented at 6 and 12 inch depths for packing layered soils. A third column had an inside diameter of 4.25 inches. The bottom end of each column was fixed to a 6 x 7 inch Plexiglas base. The base was constructed so that a constant head could be maintained at the end of the column for saturated conditions. A schematic of a soil column is included in Figure 19.

The sand mixture used in these experiments was obtained by thoroughly mixing graded sand and ground silica in a concrete mixer. The sandy loam soil was collected from the field, air dried in the laboratory, and screened through an ASTM No. 30 sieve (0.590 mm sieve openings).

An extension of known volume was attached to the top of an empty column as a first step in packing a soil column. Then a mass of soil

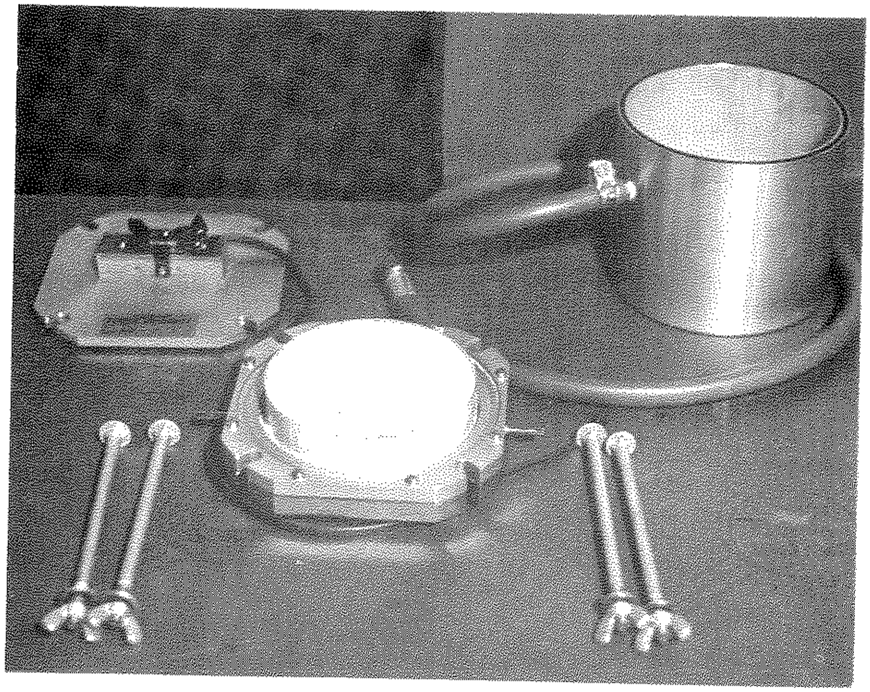


Figure 17. Soil Sample in Disassembled Pressure Plate Unit.

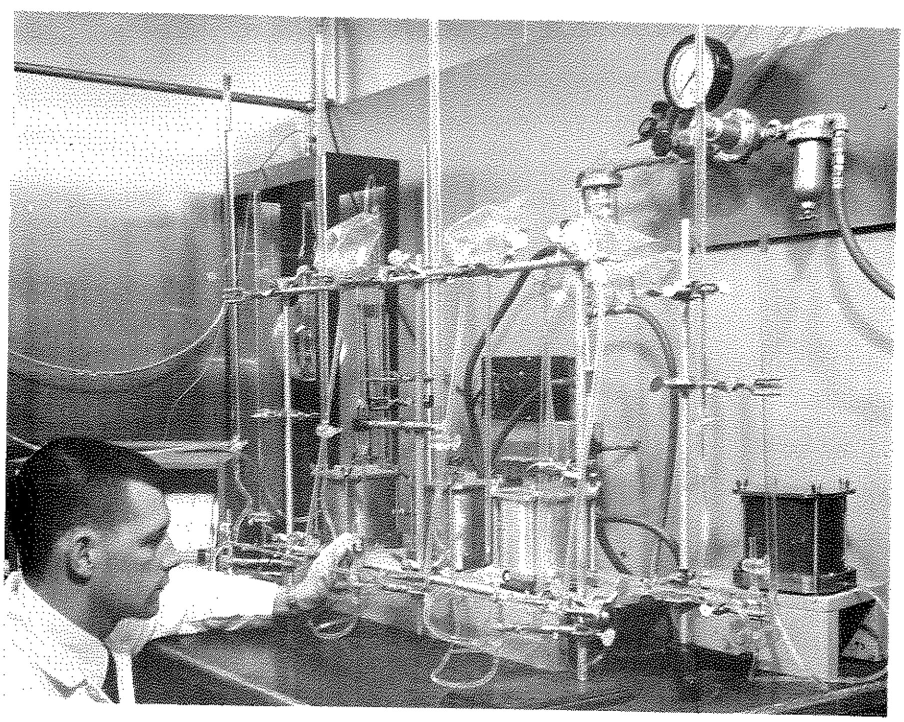


Figure 18. Experimental Setup of Pressure Plate Apparatus and Associated Equipment.

necessary to give the desired mean bulk density was packed into the extended column. The soil was poured into the empty column by funneling it through a 6 mm glass tube initially extended to the bottom end of the column. The column was filled evenly by pouring soil into the funnel so that the tube was always full while slowly raising the tube and moving it laterally to distribute the soil. When the extended column was filled, a small vibrator was run up and down the sides. The end of the glass tube was positioned so that soil fed into the top of the extension as the soil in the column compacted. When the required mass of soil had been packed into the extended column, the extension was removed and the density distribution determined. If the density at any point in the top 20 inches of the column varied from the mean by more than 3 percent, the column was rejected and another packed.

The procedure used for packing layered columns was essentially the same as outlined above. First a full column of the soil used for the bottom layer was packed. Then the top section was removed and emptied and the top soil layer was packed directly onto the bottom layer.

## Determining Density and Initial Water Content Distribution

A gamma ray attenuation device was constructed to measure the bulk density and initial water content distribution of each soil column. The density and water content were measured at 1 inch increments over the length of each soil column. The gamma attenuation equipment, and associated procedures are described in Appendix A.

## Initially Wet Soil Columns

Soil columns with different initial water content distributions were prepared by dripping water onto the packed columns at slow rates. Rubin, et al. (1964) showed that a uniform water content could be obtained in a homogeneous soil column by supplying water to the surface at a rate less than the saturated conductivity. Theoretically, the resulting water content would be the value of $\theta$ for which $K(\theta) = A$, where A is the application rate. Application rates of 0.1 to 0.2 cm/hr were applied in an effort to get varying levels of initial water contents. However, $K(\theta)$ in this range was steep for both soils, and there was not much difference in the levels of water content obtained. Different initial water content distributions were finally obtained by wetting the columns to different depths.

The procedure for wetting the soil columns was begun after the dry density distribution had been measured. A short extension was fixed to the top of the column and filled with soil to prevent packing and sealing of the column surface due to the impact of water droplets. Deaired water was dripped onto the surface of the column at a constant rate from a hypodermic needle which was fed by a siphon from a Mariotte flask.

When the wetting front had reached the desired depth, water application was stopped, and the column extension removed. The column was immediately placed in the gamma attenuation apparatus and the water content distribution was determined as discussed in Appendix A. Then the column was removed from the gamma apparatus and preparations were made to apply a ponded condition to its surface. Finally a ponded surface boundary condition was applied to the wet column and the infiltration rate-time relationship was measured. About 1-1.5 hours normally elapsed from the time the water droplet application was terminated until ponded surface conditions were established. Since the initial water contents were low, there was relatively little movement of water during this interim period; the maximum amount that the wetting advanced was less than 0.5 inches.

## Ponded Water Application

The basic experimental determination required in this investigation was the measurement of influx curves for infiltration into soil columns from shallow ponded surface conditions. A schematic diagram of the experimental setup is given in Figure 19.

Water was applied to the soil surface by an applicator which was constructed of Plexiglas and was similar in many respects to the one described by Swartzendruber, et al. (1968). Water entered the soil column through the applicator plate which consisted of a 0.5 inch Plexiglas plate perforated with 0.079 inch diameter holes. The holes were drilled 0.25 inches apart in parallel rows spaced on 0.25 inch

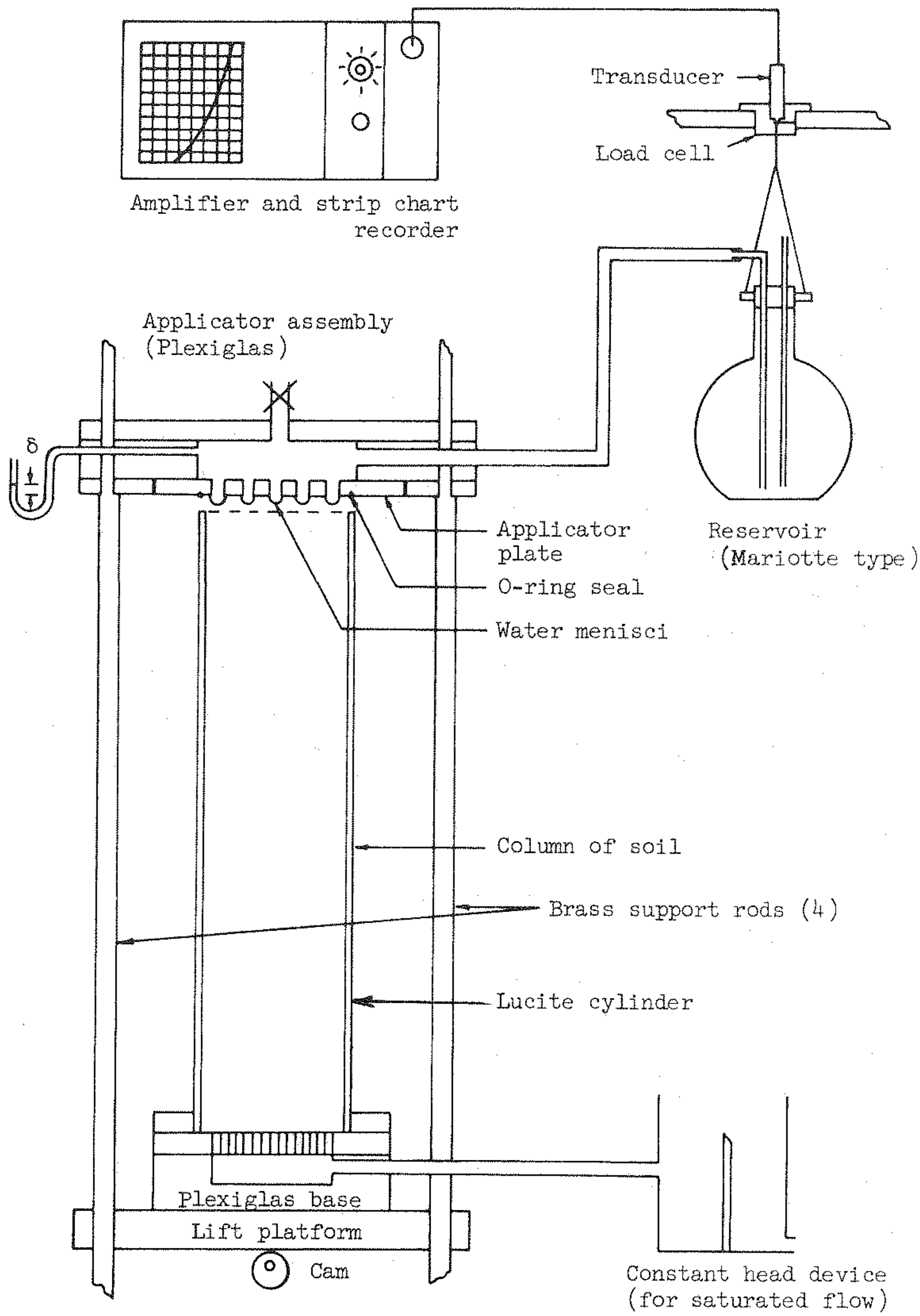



Figure 19. Schematic of Soil Column and Water Application Assembly.

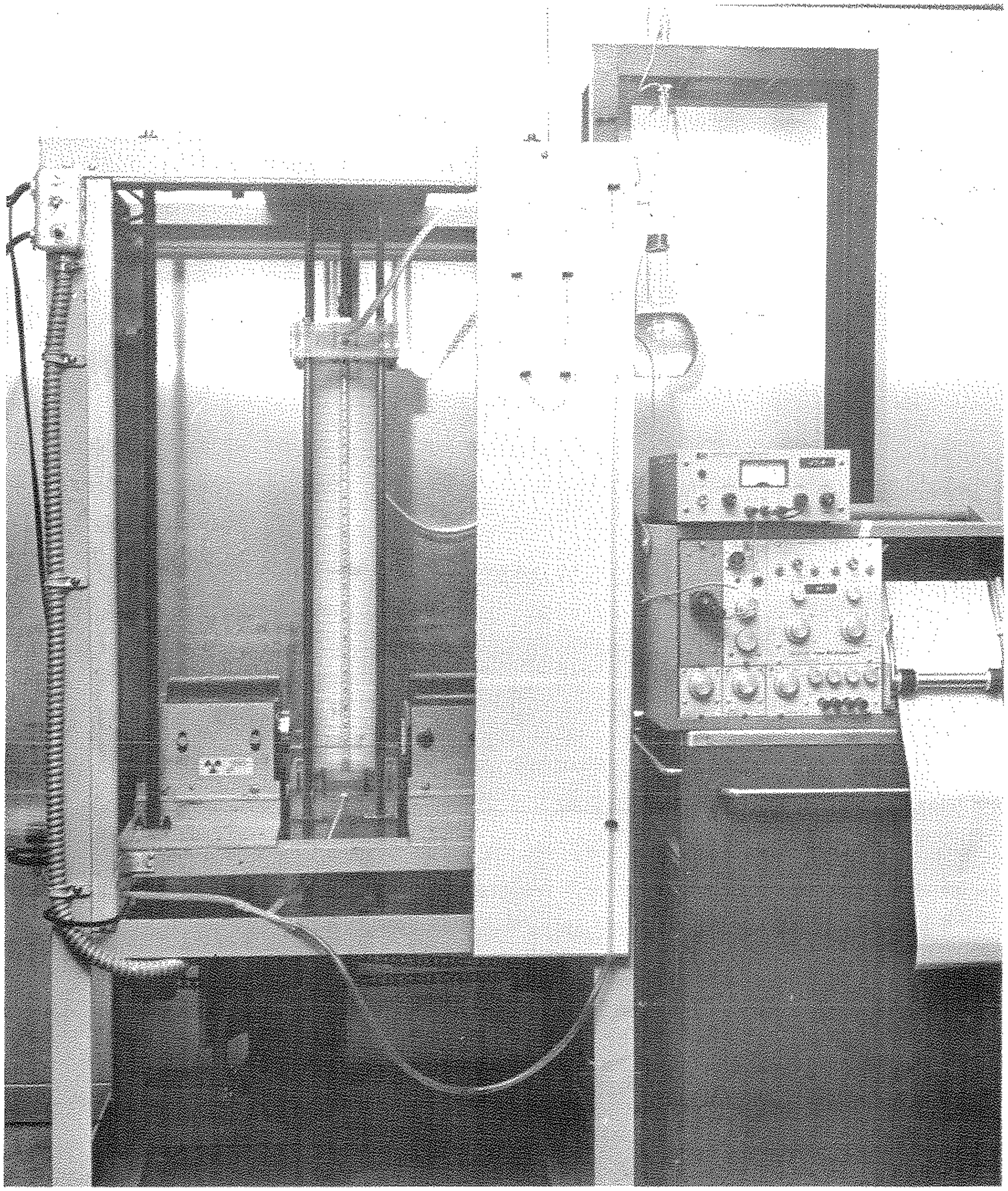


Figure 20. Equipment for Applying Water and Measuring the Rate of Infiltration into Soil Columns.

# CHAPTER 5

## RESULTS AND DISCUSSION

Experiments were run on columns of the sand mixture and sandy loam and on stratified columns made up of layers of both soils. The conductivity functions for both soils were determined by the method given in Chapter 2. The $K(h)$ functions were then used to predict infiltration rate-time relationships for infiltration into soils with different initial water contents and into layered soils. The computed and observed influx curves were compared to determine the reliability of the $K(h)$ function for characterizing infiltration. A summary of the tests is given in Table 3. The bulk density and initial water content distributions for each test are given in Appendix A.

### Experiments on Dry Soils

The infiltration rate-time relationships for two replications of infiltration into initially dry columns of sand mixture (tests 1 and 2) are given by the discrete points in Figure 21. Similar results for the sandy loam soil (tests 15 and 16) are given in Figure 22. The two columns of sand mixture had $K(0)$ values of 2.62 and 2.67 cm/hr, respectively. A mean value of 2.65 cm/hr was assumed for calculating $b = 1385$ sec/cm in equation 18. Values of $K(0)$ for tests on dry sandy loam were 2.03 and 2.18 cm/hr, respectively, giving a mean value of 2.10 cm/hr and $b = 1715$ sec/cm.

Soil water characteristic data collected with the pressure plate apparatus are given by the points plotted in Figures 23 and 24 for the sand mixture and sandy loam, respectively. The solid curves represent visual fits to the $h(\theta)$ data and were used to estimate the exponent, a, in equation 18. The values obtained by the model of Brooks and Corey (1965) were $a = 6.5$ for the sand mixture and $a = 3.9$ for the sandy loam.

Table 3. Summary of Tests on Soil Columns.

| Test No.[1] | Soil | Average Bulk Density (gm/cc) | Initial Wet[2] Front Depth (in) |
|---|---|---|---|
| 1 | Sand Mixture (SM) | 1.71 | Dry |
| 2 | " " | 1.69 | Dry |
| 5 | " " | 1.70 | 12.5 |
| 6 | " " | 1.70 | 12.5 |
| 7 | " " | 1.71 | 8.5 |
| 8 | " " | 1.70 | 8.0 |
| 9 | " " | 1.71 | 17.0 |
| 10 | " " | 1.71 | 17.0 |
| 11 | " " | 1.71 | 18.0 |
| 15 | Sandy Loam (SL) | 1.41 | Dry |
| 16 | " " | 1.41 | Dry |
| 17 | " " | 1.42 | 14.5 |
| 18 | " " | 1.41 | 16.0 |
| 19 | " " | 1.41 | 12.5 |
| 20 | " " | 1.41 | 12.5 |
| 21 | " " | 1.42 | 8.5 |
| 22 | " " | 1.41 | 8.5 |
| 23 | 6 in SL/18 in SM | 1.41/1.68 | Dry |
| 24 | 12 in SL/12 in SM | 1.41/1.68 | Dry |
| 25 | 6 in SM/18 in SL | 1.68/1.41 | Dry |
| 26 | 12 in SM/12 in SL | 1.67/1.41 | Dry |

[1] Test Nos. 3, 4, and 12-14 were preliminary tests and were not considered in the analysis of results.

[2] The bulk density and initial water content distribution for each test is given in Appendix A.

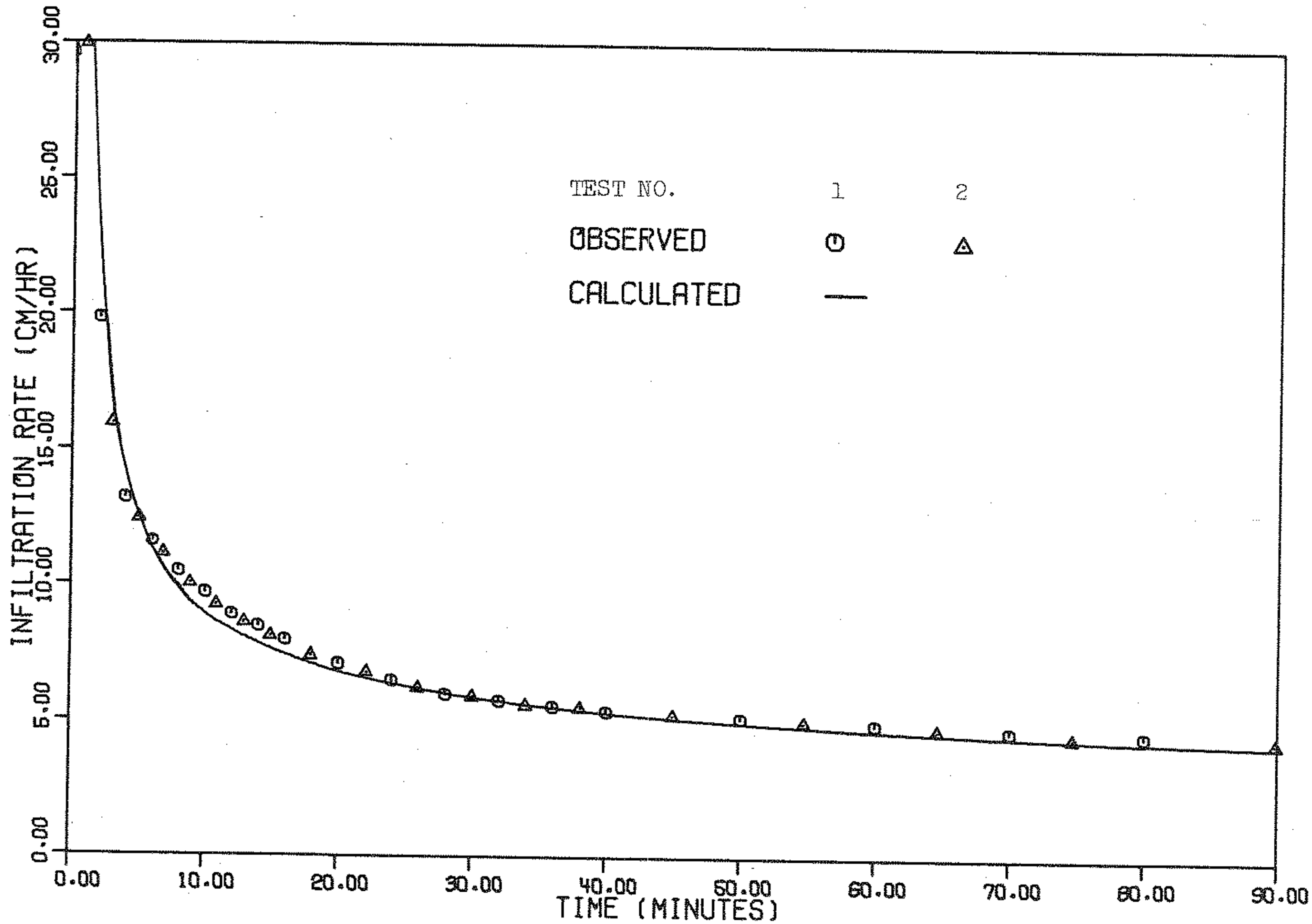



Figure 21. Observed and Calculated Influx Curves for Dry Sand Mixture.

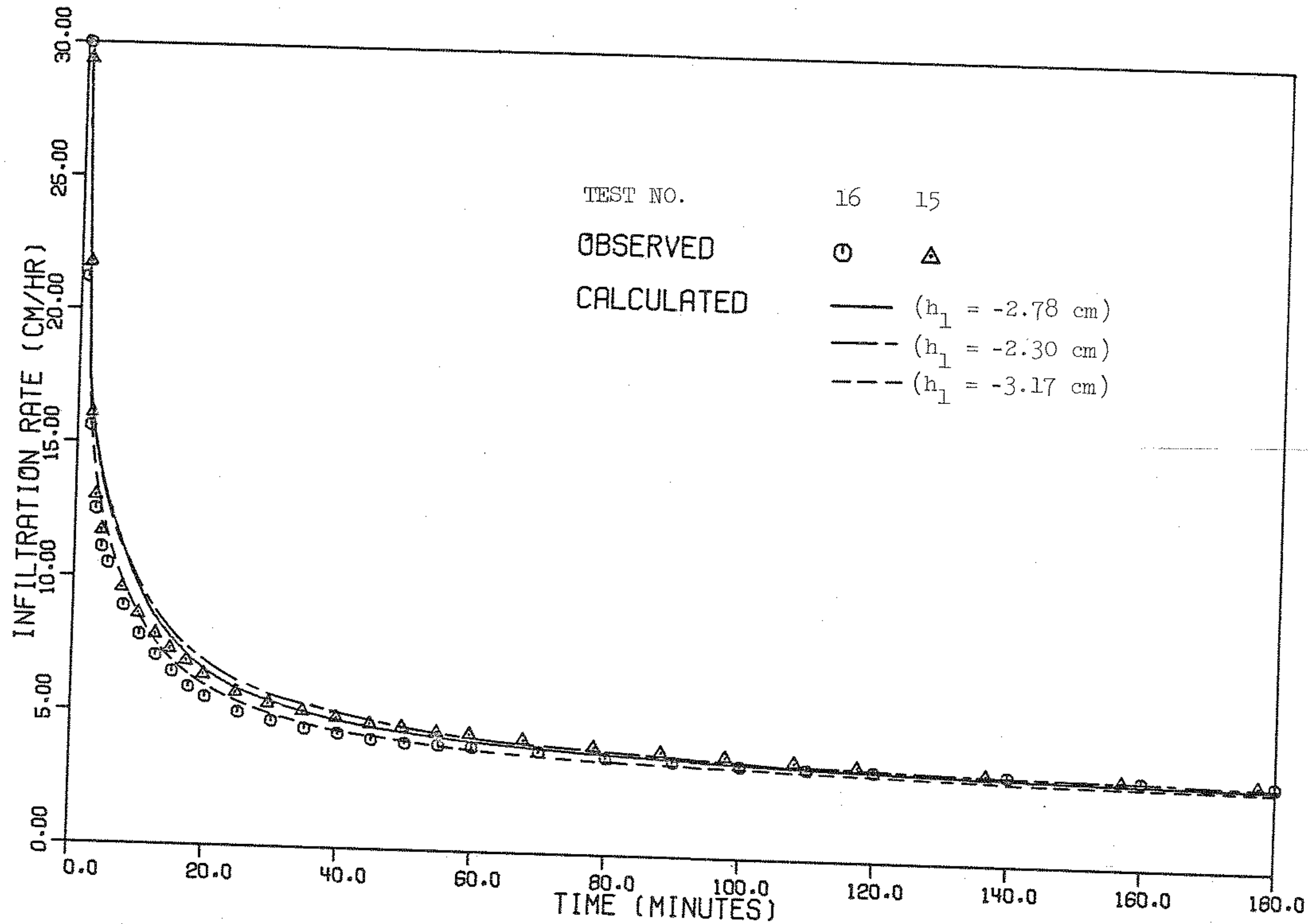



Figure 22. Observed and Calculated Influx Curves for Dry Sandy Loam.

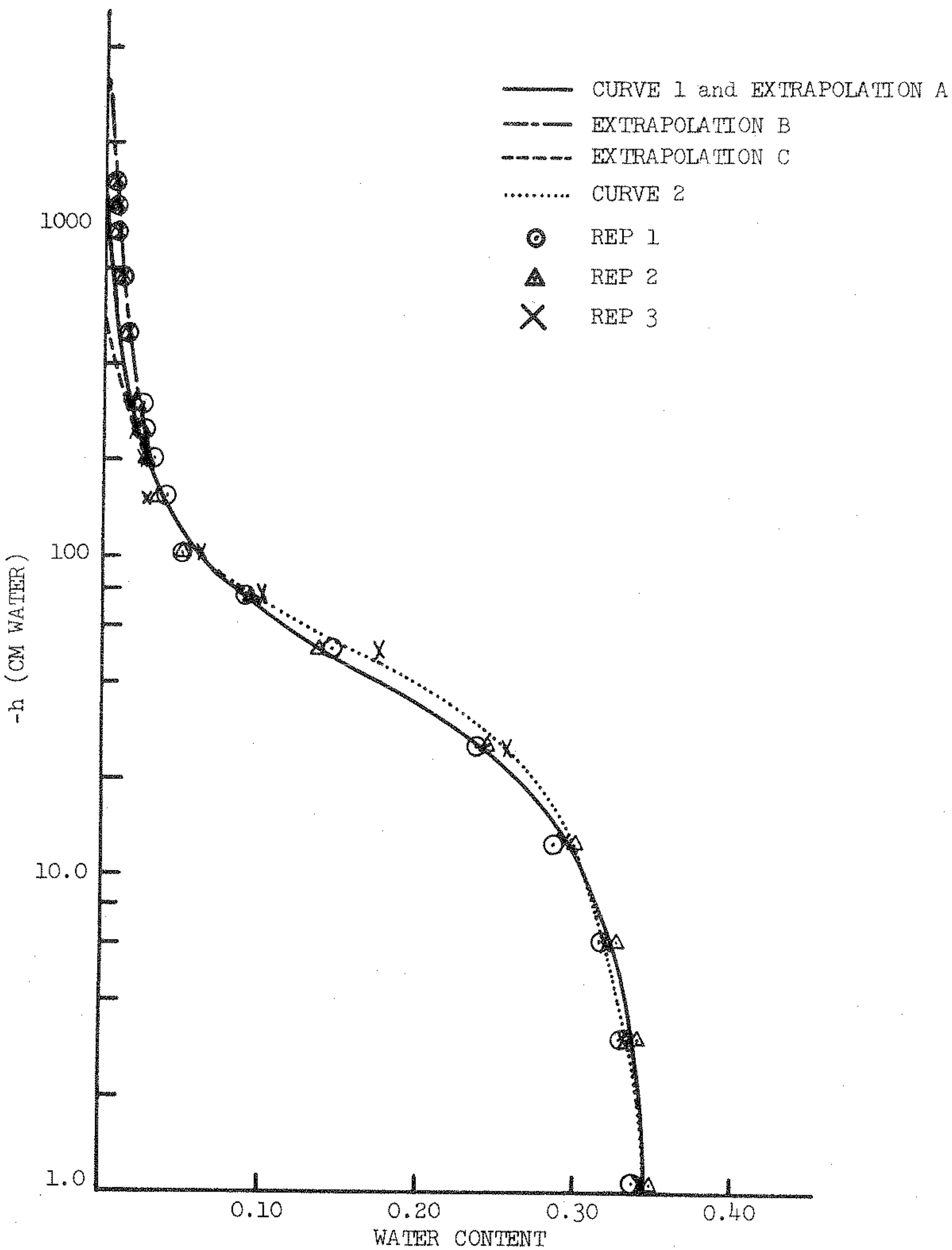



Figure 23. Soil-Water Characteristic for Sand Mixture.

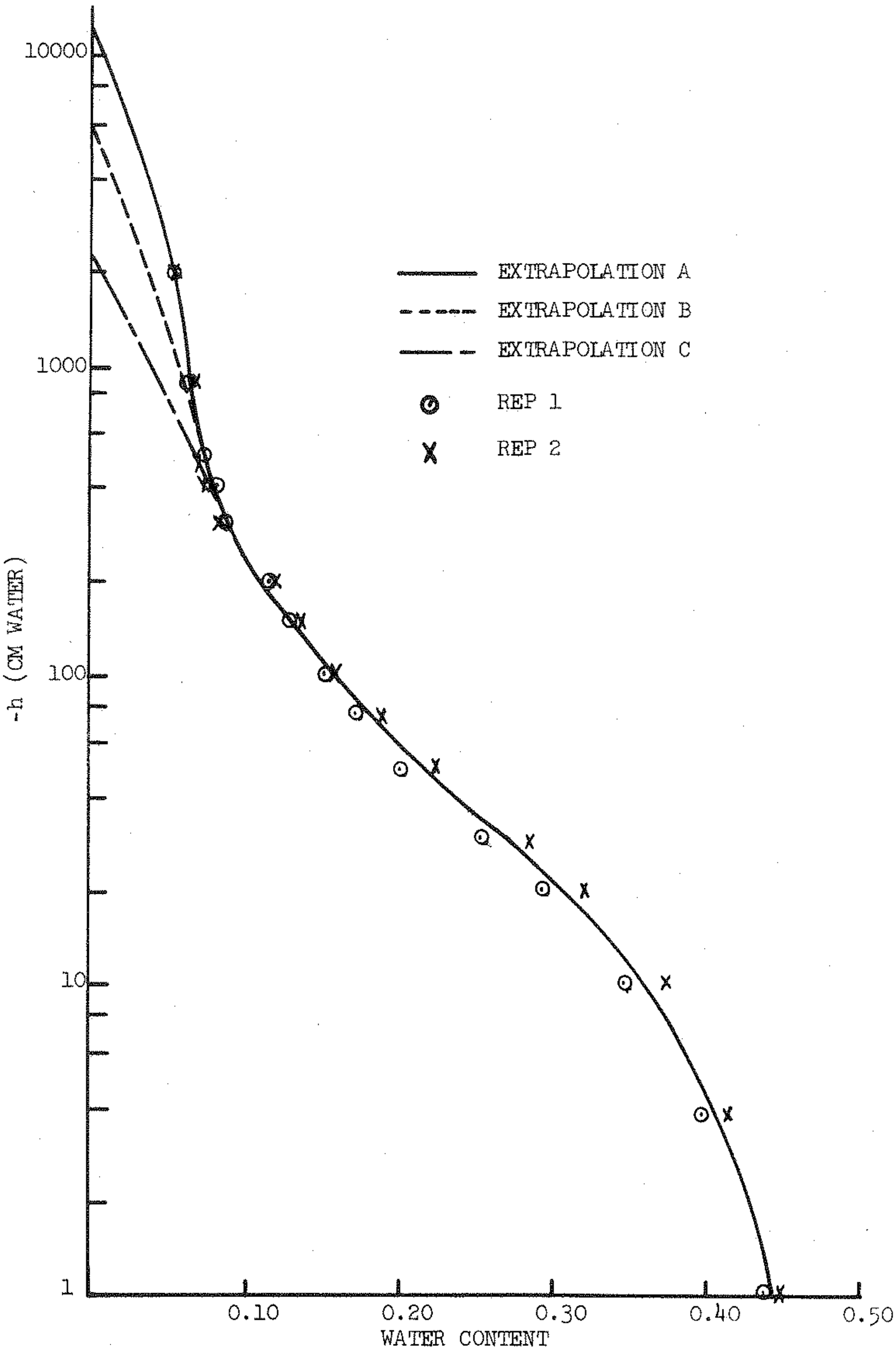



Figure 24. Soil-Water Characteristic for Sandy Loam.

The method described in Chapter 2 was used to determine $h_1$ in equation 18. The sand mixture had $h_1$ values of -7.85 and -7.92 cm for tests 1 and 2, respectively. A mean value of -7.88 cm was used in subsequent calculations. The area between the computed and observed influx curves was R = 0.85 cm for both tests. The solid curve plotted in Figure 21 was obtained by using the conductivity function given by equation 18 with the above parameter values to solve the θ-based Richards equation subject to boundary conditions 13. The difference in the influx curves calculated for $h_1 = -7.85$ cm and $h_1 = -7.92$ cm was not perceptible when they were plotted to the scale of Figure 21.

For the sandy loam, $h_1$ values of -3.17 and -2.30 cm were obtained for tests 15 and 16, respectively. The broken curves given in Figure 22 were calculated using these $h_1$ values in equation 18. The area between the computed and observed influx curves was R = 0.82 cm for test 15 and R = 0.95 cm for test 16. The solid curve given in Figure 22 was calculated using a mean value of $h_1 = -2.78$ cm.

In general, there was close agreement between the calculated and measured influx curves for both the sand and sandy loam soils. The agreement was not as good as that obtained for the simulated tests in Chapter 2 (Figures 3 and 6); however, the curves shown above were subject to experimental error and soil variations while the corresponding results in Chapter 2 were defined strictly from theory.

## Effect of h(θ) Errors on K(h)

The most likely source of experimental error in the proposed method for determining the conductivity function probably occurs in the experimental evaluation of h(θ). Measurements are particularly susceptible to error for low water contents where the total amount of inflow is small and the resulting h(θ) relationship very steep. Since the water content at the largest value of -h was still greater than zero for both soils, it was necessary to extrapolate the h(θ) relationships to θ = 0. Three possible extrapolations for each soil are denoted by A, B, and C in Figures 15 and 16. The consequences of errors made in the measurement and/or extrapolation of h(θ) at low water contents were determined by solving the governing equation using the h(θ) relationship with each extrapolation. The infiltration rate-time relationships are tabulated in Table 4. The results show that the solutions were not sensitive to the nature of $h(\theta)$ at low water contents. The cause of this insensitivity was the extremely low conductivity values for this range in water contents. Since the maximum difference in the three solutions was less than 0.5 percent for both soils, errors in $h(\theta)$ at low water contents were judged negligible and extrapolation A was assumed for subsequent calculations.

Table 4. Infiltration Rates for Different Extrapolations of $h(\theta)$.

| Time (minutes) | Infiltration Rate (cm/hr) Sand Mixture | | | Sandy Loam | | |
|---|---|---|---|---|---|---|
| | A | B | C | A | B | C |
| 0.5 | 54.38 | 54.23 | 54.41 | 48.87 | 48.86 | 48.85 |
| 1.0 | 32.35 | 32.60 | 32.35 | 29.10 | 29.10 | 29.09 |
| 2.0 | 20.93 | 20.94 | 20.93 | 15.54 | 15.53 | 15.52 |
| 3.0 | 16.98 | 17.06 | 16.98 | 14.14 | 14.13 | 14.10 |
| 5.0 | 12.68 | 12.69 | 12.70 | 12.74 | 12.74 | 12.73 |
| 7.5 | 10.23 | 10.22 | 10.21 | 10.98 | 10.98 | 10.99 |
| 10.0 | 8.93 | 8.93 | 8.92 | 9.55 | 9.56 | 9.55 |
| 15.0 | 7.61 | 7.61 | 7.61 | 7.69 | 7.69 | 7.69 |
| 20.0 | 6.81 | 6.81 | 6.81 | 6.58 | 6.57 | 6.57 |
| 30.0 | 5.88 | 5.88 | 5.88 | 5.41 | 5.40 | 5.43 |
| 40.0 | 5.34 | 5.35 | 5.35 | 4.79 | 4.78 | 4.79 |
| 50.0 | 4.97 | 4.97 | 4.97 | 4.40 | 4.40 | 4.41 |
| 60.0 | 4.71 | 4.71 | 4.71 | 4.13 | 4.12 | 4.12 |

The nature of $h(\theta)$ at higher water contents, in contrast to that for low $\theta$, is critical to the solution of the governing Richards equation (Hanks and Bowers, 1963). The effect of $h(\theta)$ at high water contents on the $K(h)$ relationship defined by the proposed method was investigated by using curve 2 in Figure 23 to determine $K(h)$ for the sand mixture. Curve 2 represents one of the three replications made in measuring $h(\theta)$ for this soil. The parameters in equation 18 were $a = 6.5$ and

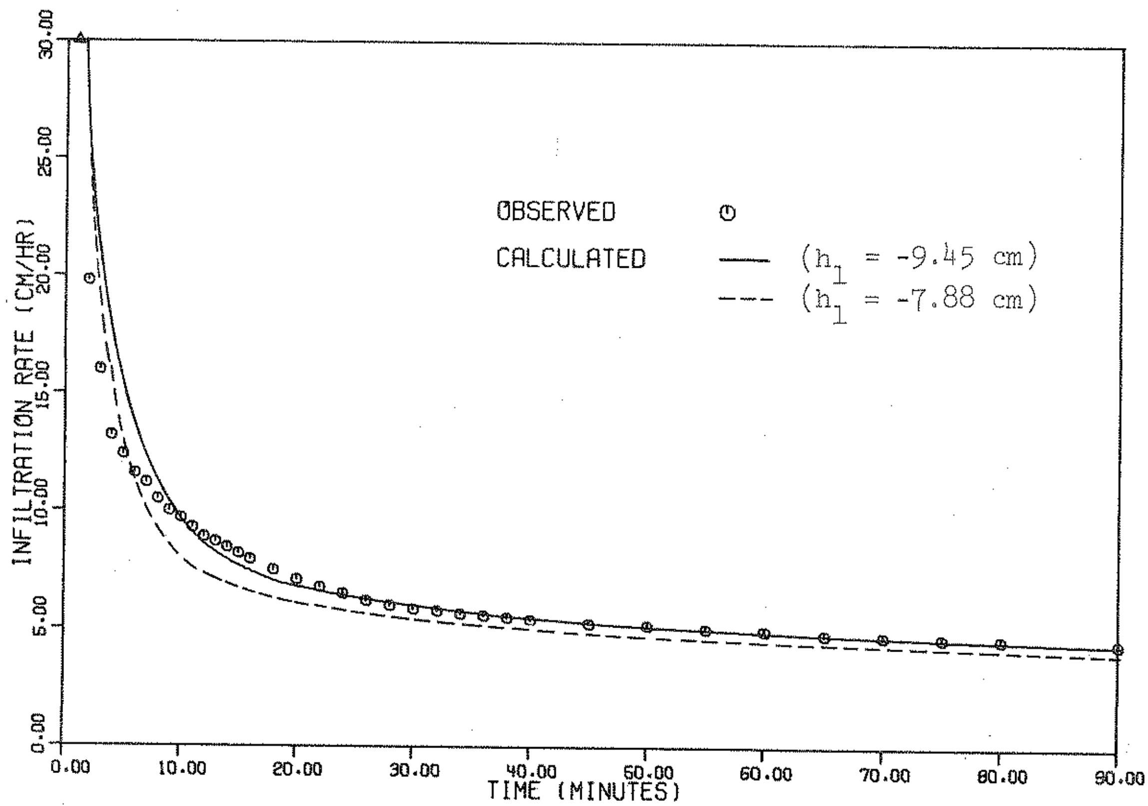



Figure 25. Effect of Errors in $h(\theta)$ at High Water Contents on $K(h)$ and Calculated Influx Curves for Sand Mixture.

$h_1$ = -9.45 cm with an objective function of R = 0.91 cm. (These values compare to a = 6.5, $h_1$ = -7.88 cm, and R = 0.85 cm for curve 1.) The resulting influx relationship is given by the solid curve in Figure 25. Although the total area between the observed and calculated influx curves was only slightly larger than the corresponding area in Figure 21, the deviations at small times were relatively large, while there was almost exact agreement for times greater than 25 minutes.

The consequences of an error in the $h(\theta)$ relationship at high water contents would have been more serious if $K(h)$ had been determined by one of the conventional methods which was independent of $h(\theta)$. This can be shown by assuming that the true conductivity function is given by equation 18 with a = 6.5, $h_1$ = -7.88 cm, and b = 1385 sec/cm. The computed influx curve using this $K(h)$ and $h(\theta)$ as given by curve 2 is represented by the broken curve in Figure 25. The objective function for this curve was 1.36 cm with deviations occurring over the entire test duration of

90 minutes. Although the effects of deviations in $h(\theta)$ for the sandy loam were not as pronounced as for the sand mixture, the results were similar in a qualitative manner to those given in Figure 25.

Based on this evidence it appears that the proposed method for determining $K(h)$ partially compensates for errors made in determining $h(\theta)$. Infiltration predictions with the resulting $h(\theta)$ and $K(h)$ relationships are not as accurate as predictions based on some improved estimate of $h(\theta)$, but more accurate than for cases where $K(h)$ and $h(\theta)$ are determined independently with the same magnitude of error.

## Movement of the Wetting Front

The position of the wetting front as a function of time, as determined from time-lapse photographs, is given in Figures 26 and 27 for the sand mixture and sandy loam, respectively. Gamma ray attenuation measurements on initially wet columns showed that the visually perceptible limits of water content were approximately 7 percent for the sand mixture and 10 percent for the sandy loam. Assuming these values for $\theta$ at the wetting front, its progress was determined from the solution to the Richards equation for each soil as shown in Figures 26 and 27. The computed curves given in Figure 27 for the sandy loam also show the effect of the value of $h_1$ on the predicted wetting front movement. Since the computed wetting front was very steep for both soils, the measurement of the visually perceptible limit of water content was not critical to the determination of the wetting front position. The close agreement between the measured and calculated wetting front depths supports the validity of the conductivity functions obtained by the proposed method.

## Boundary Condition at $x = 0$

As noted in Chapter 4 the boundary condition at $x = 0$ was maintained at $h = \delta \approx 0.75$ cm for $t > 0$. The solutions of the $\theta$-based Richards equation obtained in determining $K(h)$, however, assumed $h = 0$ at $x = 0$, $t > 0$. The error introduced by this inconsistency was evaluated by solving the h-based equation subject to boundary conditions 13 but with $h = \delta$ at $x = 0$. INFIL3 was used to obtain solutions for infiltration into the sand mixture with a uniform initial head distribution

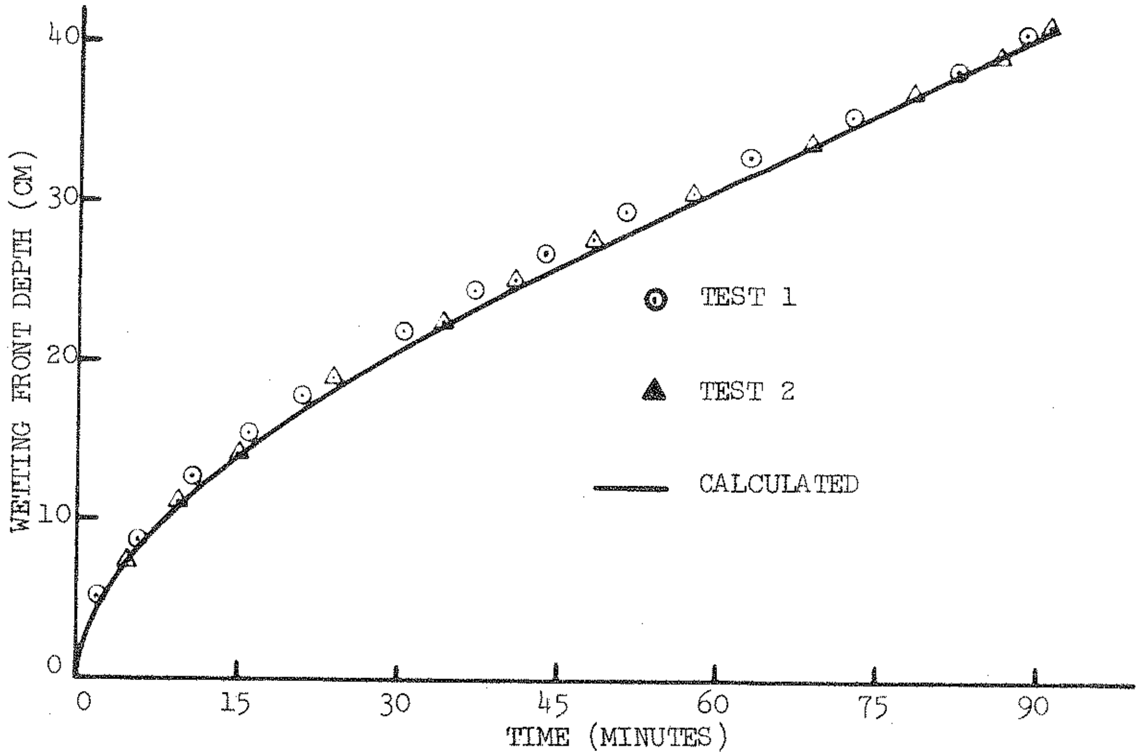



Figure 26. Wetting Front Movement for Sand Mixture.

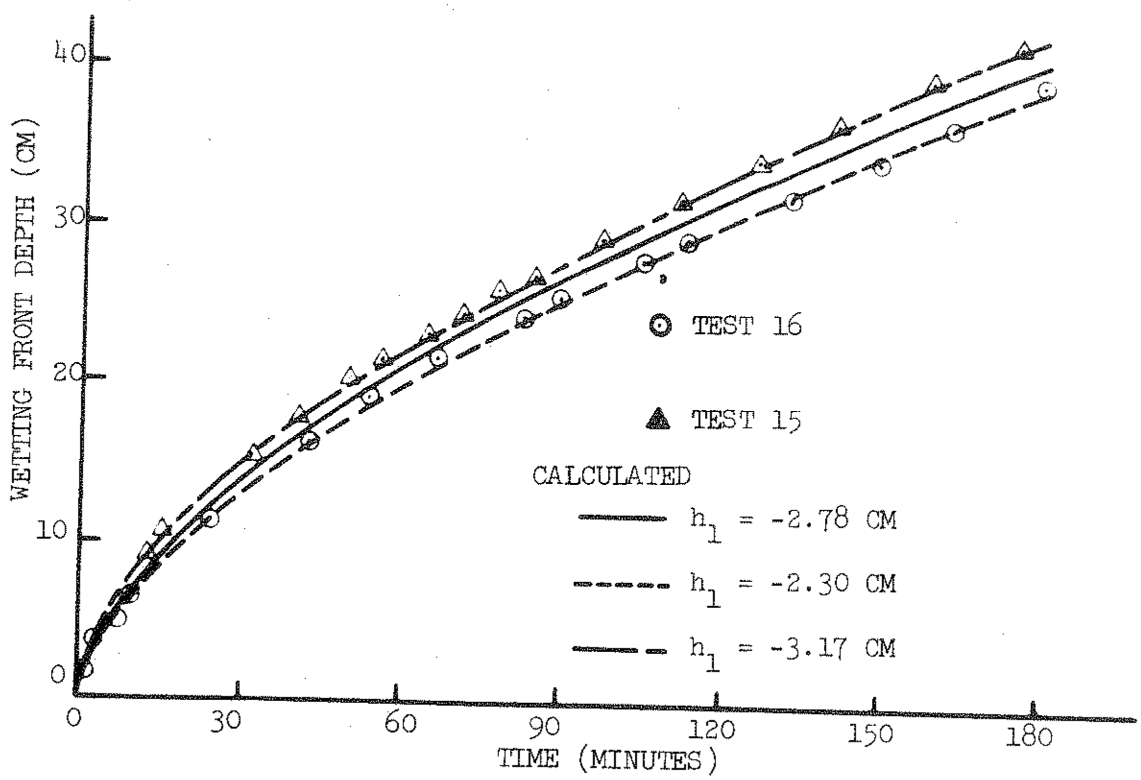



Figure 27. Wetting Front Movement for Sandy Loam.

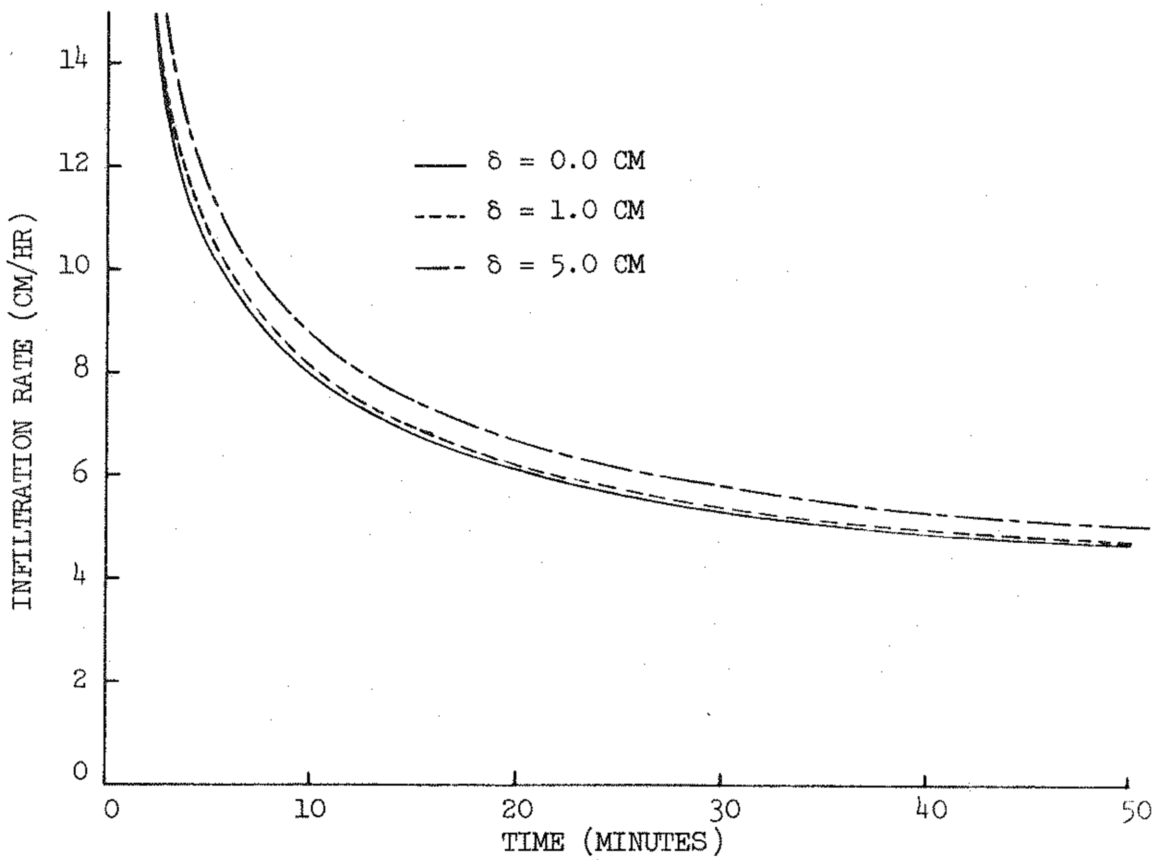



Figure 28. Effect of Ponded Water Depth on Calculated Influx Curves for Sand Mixture.

of -70.0 cm (corresponding to a uniform initial water content of 0.10). The calculated influx curves for $\delta$ values of 0, 1, and 5 cm are given in Figure 28. The difference in calculated infiltration rates for $\delta = 0$ and $\delta = 1$ cm was always less than 2 percent. Since the magnitude of the pressure head gradients increase with decreasing initial water content, the effect of $\delta$ on the solution for an initially dry sand mixture would have been even less than that given in Figure 28. Similar results were obtained for the sandy loam soil. In view of these results, the errors introduced in the $K(h)$ function due to $\delta$ values of approximately 0.75 cm were considered negligible.

Experiments on Wet Soils

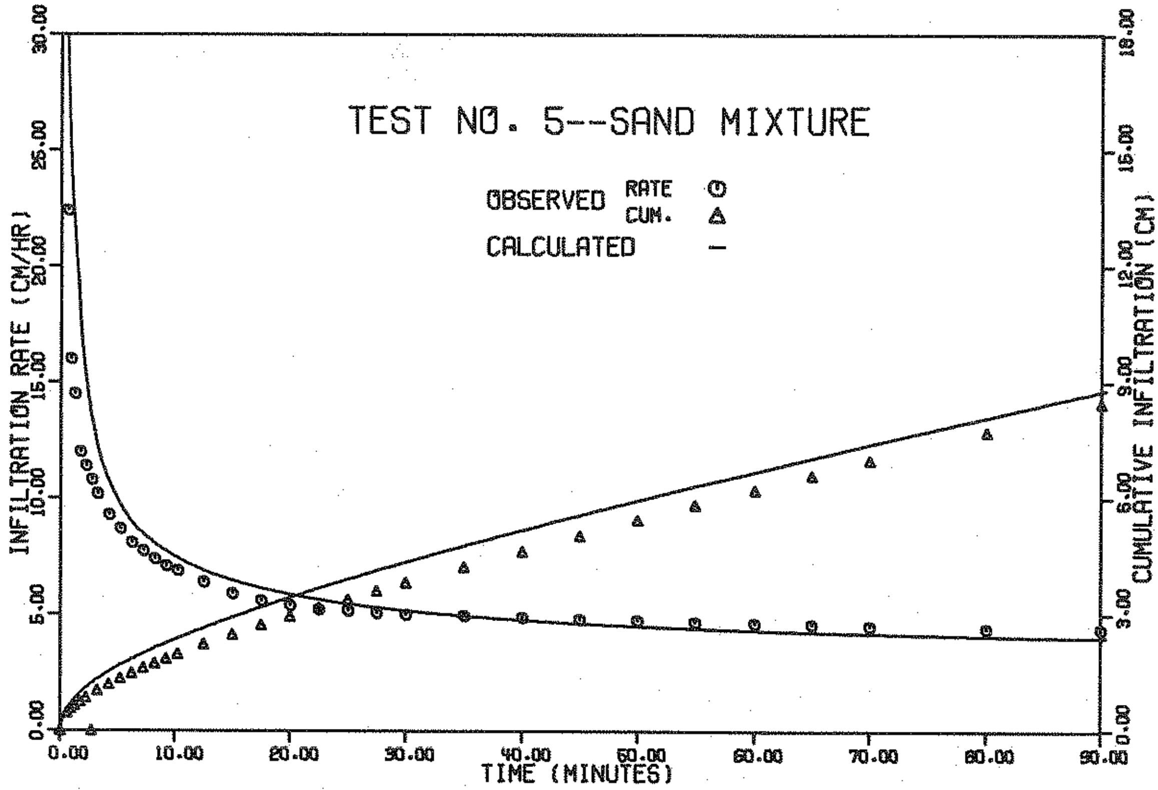
Sand Mixture

Observed and calculated influx and cumulative volume relationships for infiltration into initially wet columns of sand mixture are given in Figure 29 (tests 5-11). The calculated infiltration rates, with the lone exception of test 8, were greater than the observed rates for almost all times during the tests. In general, the deviations were rather large during the early portions of the tests, but diminished to negligible magnitudes for times greater than 25-30 minutes. Both the predicted and observed infiltration rates were much lower than for dry initial conditions; consequently, the areas between the curves were of about the same magnitude as for the dry tests 1 and 2. This is shown in Table 5 where the total area between the calculated and observed influx curves and the value of K(0) measured for each test on the sand mixture are given.

Table 5. Values of R and K(0) for Tests on Sand Mixture

| Test No. | R(cm) | K(0)(cm/hr) |
|---|---|---|
| 1 | 0.85 | 2.68 |
| 2 | 0.85 | 2.63 |
| 5 | 0.88 | 2.60 |
| 6 | 0.94 | 2.43 |
| 7 | 0.72 | 2.61 |
| 8 | 1.36 | 2.70 |
| 9 | 0.70 | 2.52 |
| 10 | 0.67 | 2.40 |
| 11 | 0.89 | 2.39 |

The fact that the solution to the Richards equation consistently overestimated the infiltration rate is potentially more significant than the magnitudes of the deviations themselves. A possible implication of this tendency is that the K(h) function for the sand mixture cannot be adequately represented by equation 18, and hence the $h_1$ value



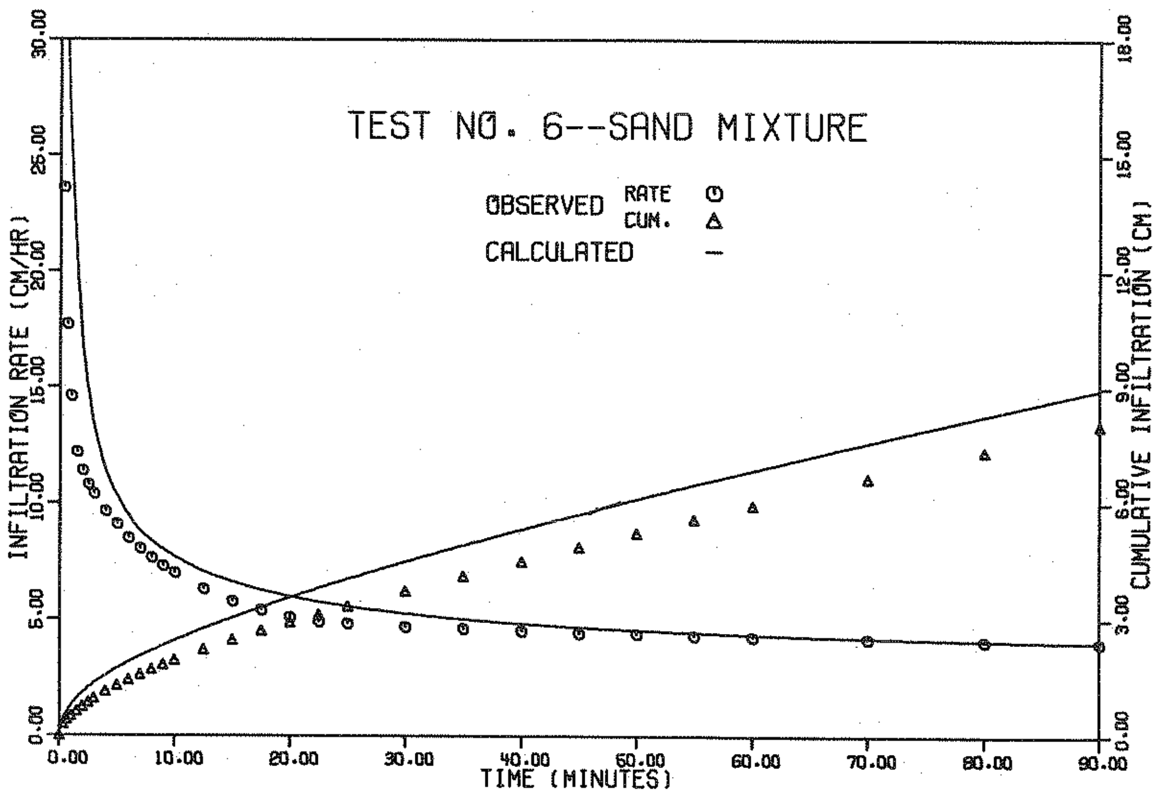
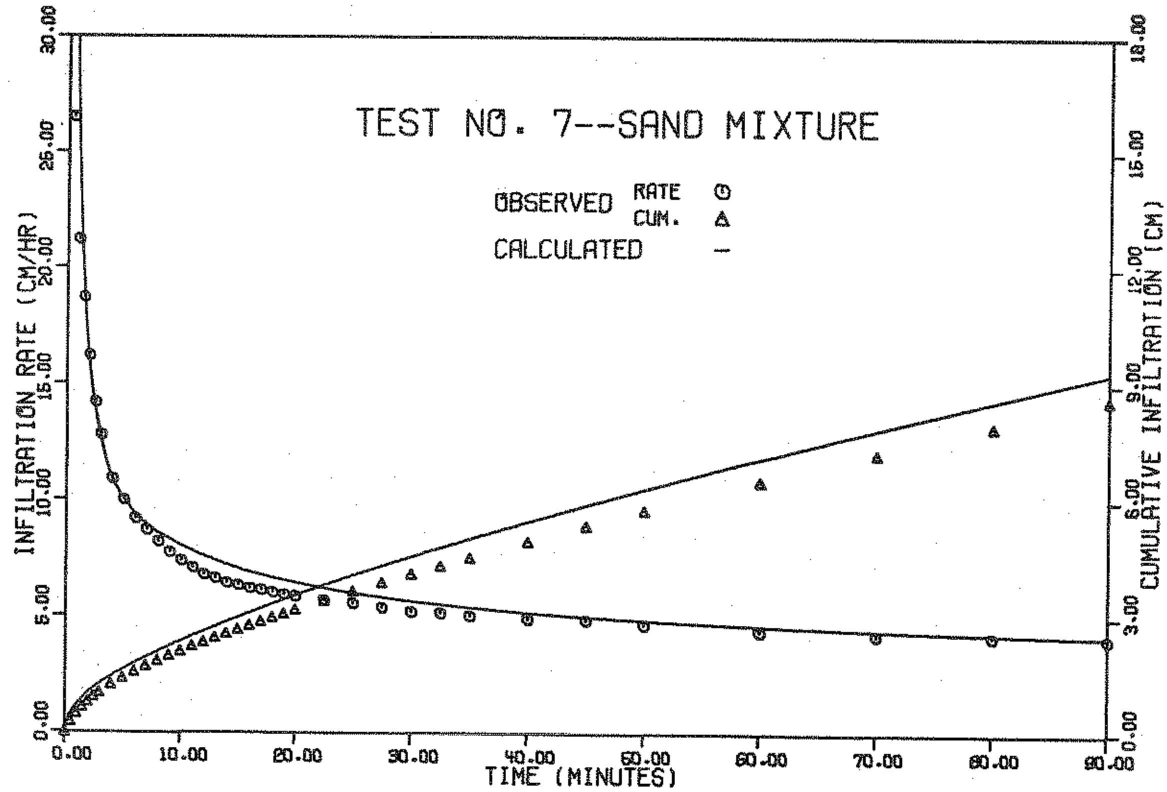




Figure 29. Observed and Calculated Infiltration Into Initially Wet Columns of Sand Mixture.



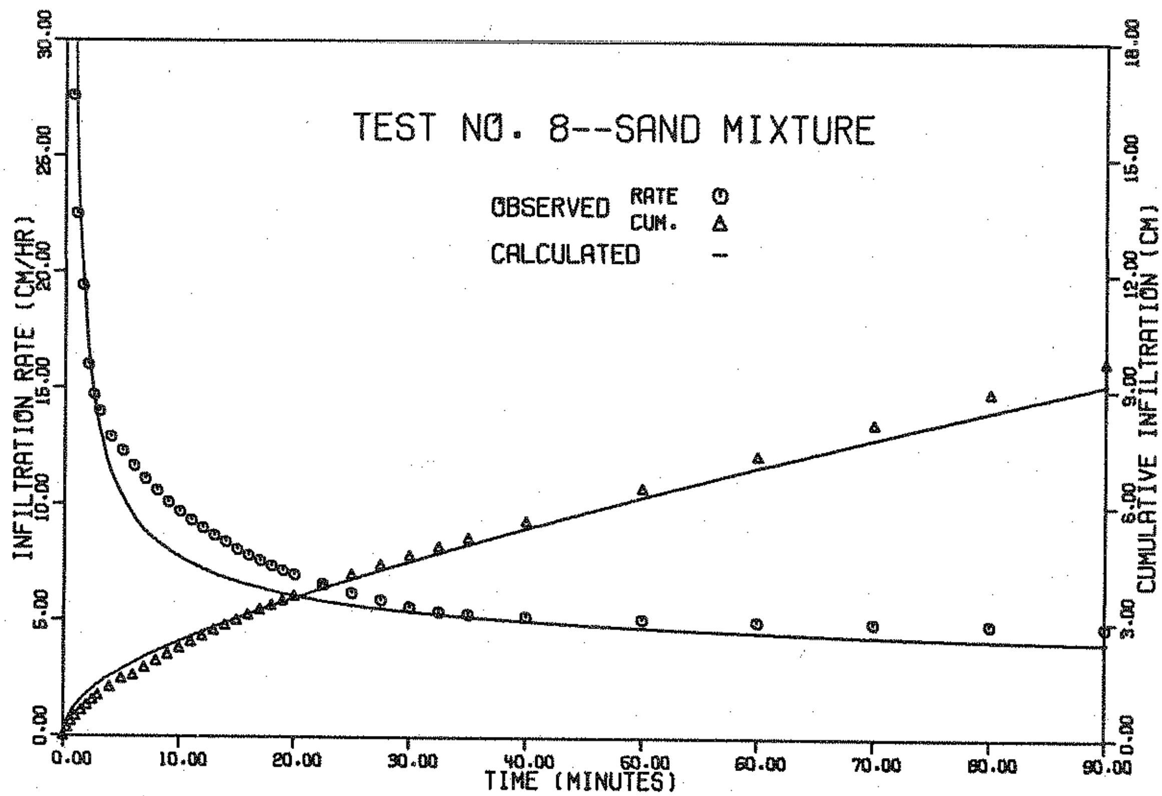
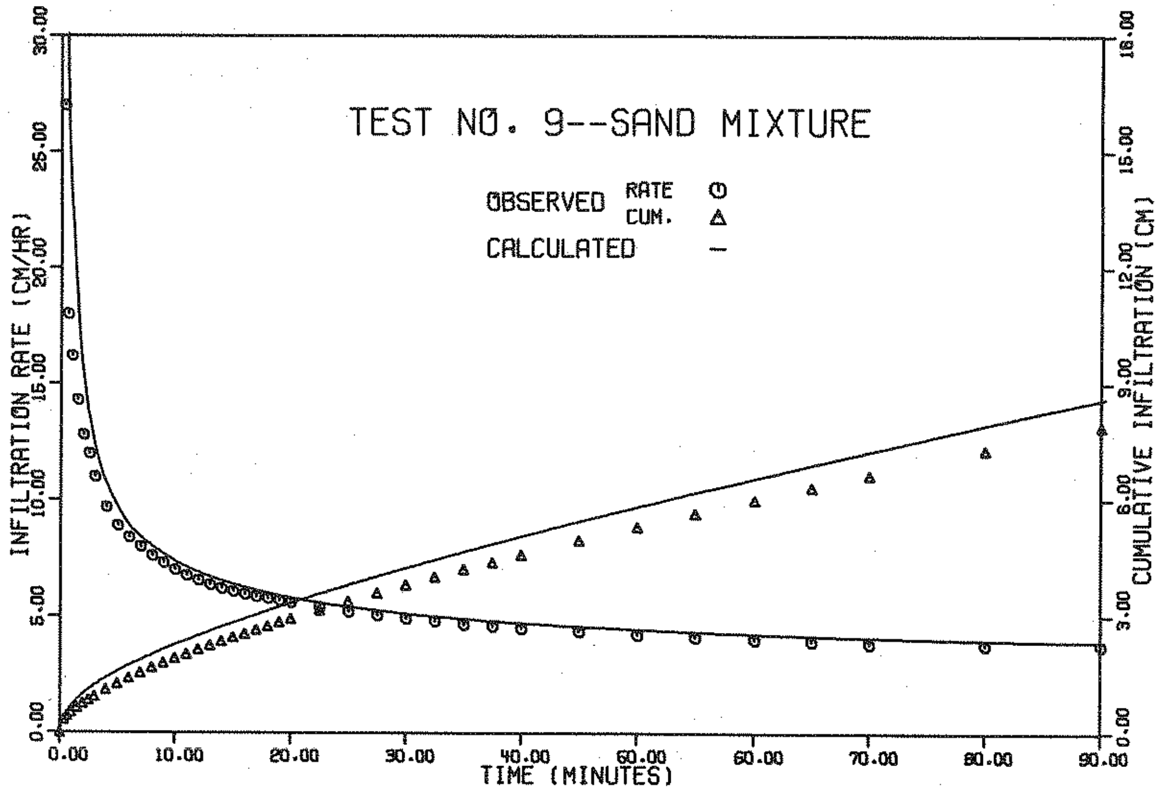
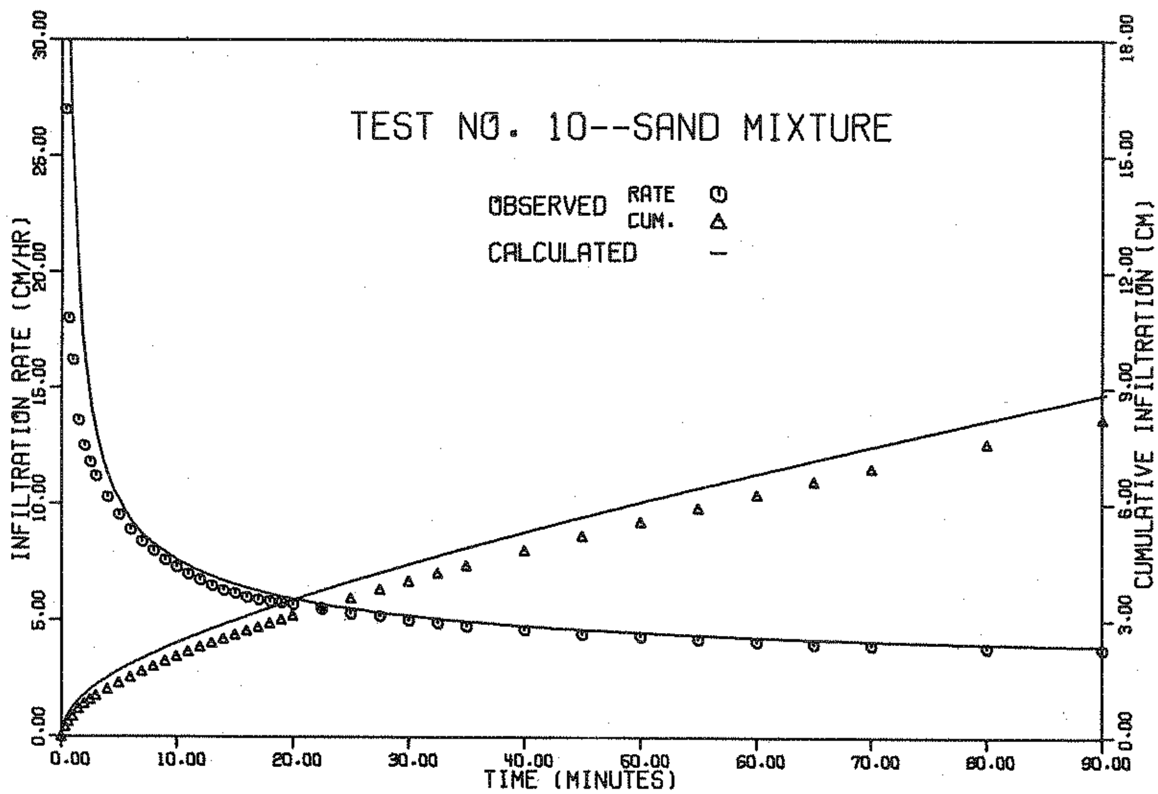




Figure 29, cont.

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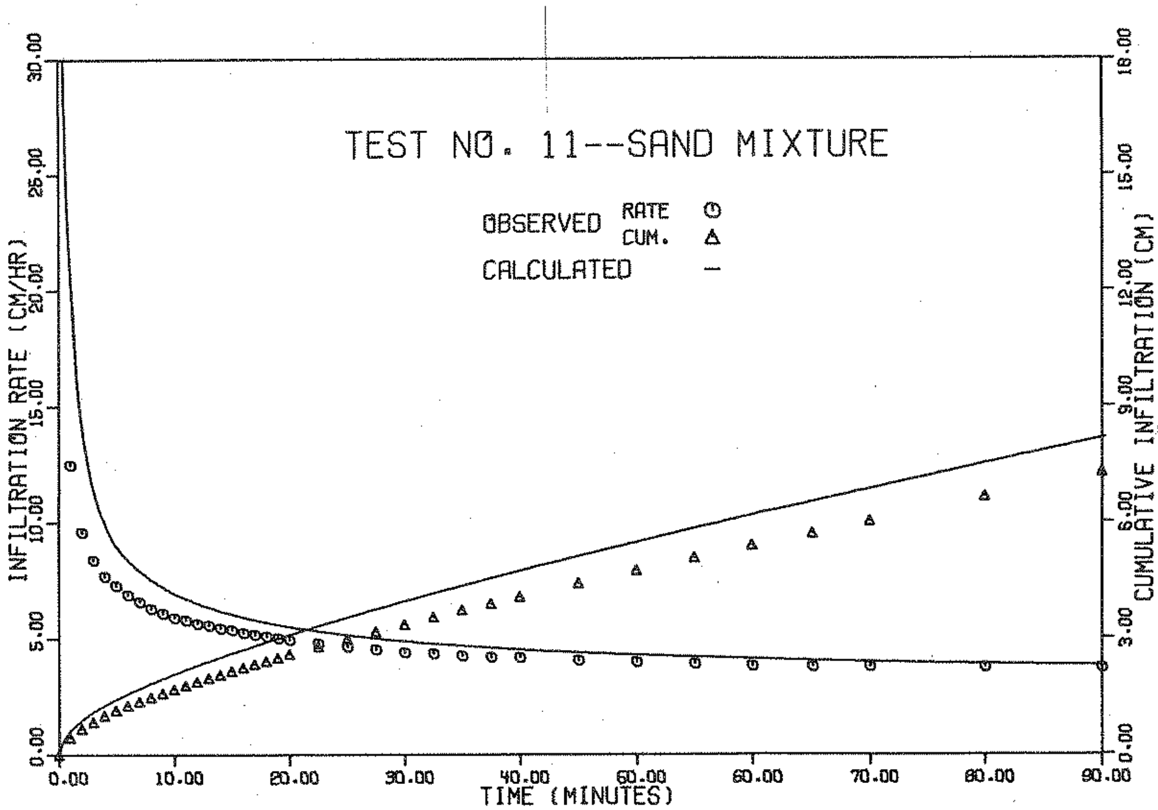



Figure 29, cont.

obtained for dry conditions would not be valid for predicting infiltration for higher initial water contents. In order to determine whether $h_1$ was dependent on the initial water content, the method given in Chapter 2 was used to obtain $h_1$ values for each of the tests on sand mixture. These values are given in Table 6.

An analysis of variance did not show significant difference in the $h_1$ values at the 95 percent confidence level. Due to differences in bulk density and initial water content distributions, the tests were not replicated in an exact sense. However, considering the large difference between the $h_1$ values for tests 7 and 8, which had very similar bulk density and initial water content distributions, the results of the analysis of variance probably would not have been changed if true replications had been possible.

An analysis of variance on the K(0) values given in Table 5 failed to show significant difference at the 95 percent confidence level. However, there was a tendency for K(0) to be lower at the higher initial water contents, and a larger data base might have shown significant difference. In general, the tests showing the larger deviations in Figure 29 had the lower K(0) values.

Table 6. Values of $h_1$ for Tests on Sand Mixture.[1]

| Rep | Approximate Depth of Wetting Front | | | |
|---|---|---|---|---|
| | Dry | 8 inches | 12 inches | 16 inches |
| 1 | -7.85(1) | -6.72(7) | -6.85(5) | -6.45(9) |
| 2 | -7.79(2) | -10.45(8) | -6.09(6) | -6.68(10) |
| | | | | -6.05(11) |

[1]The test number is given in parentheses.

There was no apparent relationship between the deviations noted above and the density distributions which are given in Appendix A. The deviations in infiltration rates shown in Figure 29 could have been partially caused by translocation of fine particles during the initial application of water at slow rates. Accumulation of the particles at a given section of the column could have formed a layer of low permeability resulting in reduced inflow rates. Swartzendruber, et al. (1968), however, showed that fine particle translocation in a mixture of banding sand and ground silica accounted for only slight changes in the bulk density at the entrance end of the column. Neither was there evidence of fine particle translocation in the gamma attenuation measurements of the initial water content distribution.

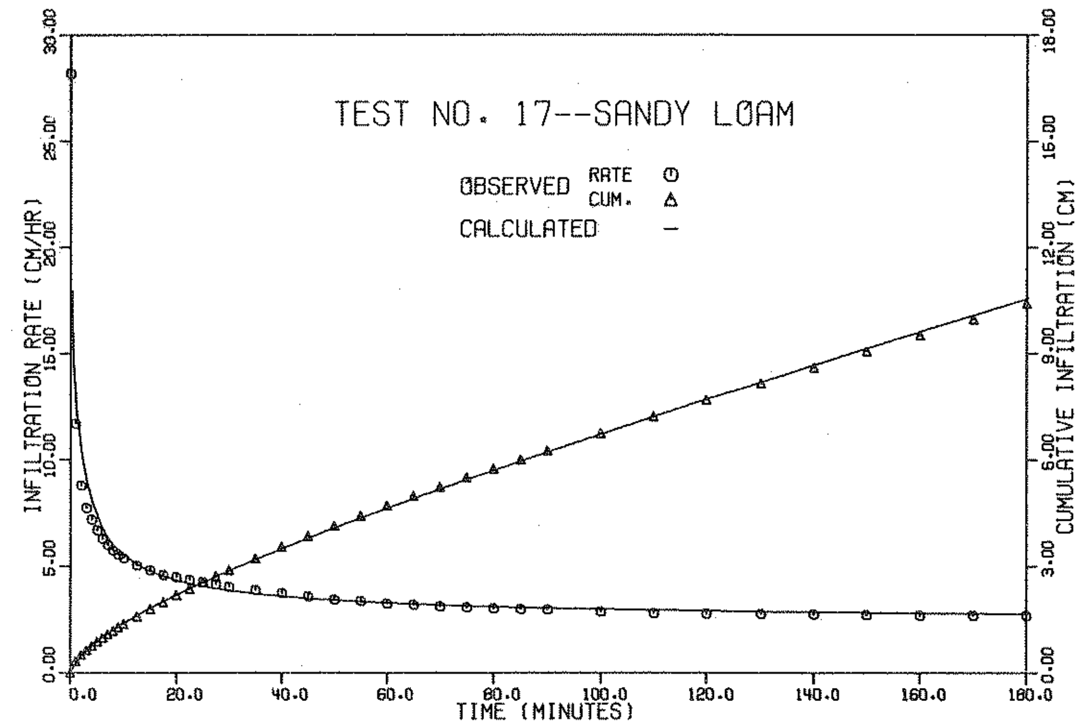
Perhaps a more likely cause of lower than expected inflow rates for the initially wet columns was the possible disturbance of the column surface. As described in Chapter 4, the procedure to obtain initially wet columns utilized a partially filled extension at the top of the column. When the desired initial water content had been obtained, the extension was carefully removed and the surface of the column leveled with a straight edge. Although the sand mixture was of single grain structure and there was no visual evidence of compaction, there still exists the possibility that the surface was slightly compacted causing the subsequent inflow rates to be reduced.

In spite of the deviations above, the experimental results for initially wet columns of sand mixture provide evidence supporting the validity of the conductivity function obtained by the proposed method. Although infiltration rates were consistently overestimated for wet columns, the area between the observed and computed influx curves was still of about the same magnitude as the minimum obtained for dry soil. The accuracy of the predictions given in Figure 29 was judged to be acceptable in view of alternative methods for determining the conductivity function and for evaluating infiltration.

## Sandy Loam

Observed and calculated influx and cumulative volume relationships for infiltration into initially wet columns of sandy loam are given in Figure 30 (tests 17-22). The calculated relationships given by the solid curves in Figure 30 were obtained by using the mean values of the parameters in equation 18 which were determined from tests on dry sandy loam. Influx curves for each test were also calculated with $h_1$ values of -3.17 and -2.30 cm from tests 15 and 16, respectively. Examples of these solutions are shown by the broken curves for tests 20 and 22. The area between the calculated and observed influx curves and the value of $K(0)$ measured for each test are given in Table 7.

The data presented in Figure 30 and Table 7 shows general good agreement between calculated and observed infiltration relationships for initially wet columns of the sandy loam soil. The agreements for tests 17, 20, and 22 were actually better than for tests on the dry columns from which $h_1$ was determined. The observed influx and cumulative volume relationships for test 21 were lower than the calculated for times greater than 15 minutes. However, the area between the observed and computed influx curves was still of about the same magnitude as the corresponding area for the dry tests. The cause of the lower infiltration rates for test 21 was also reflected in the value of $K(0)$ which was 12.5 percent lower than the mean for the dry condition. Packing irregularities which might have accounted for the reduced infiltration rates were not detected in measurements of bulk density. Compaction of the column surface prior to establishing the ponded boundary



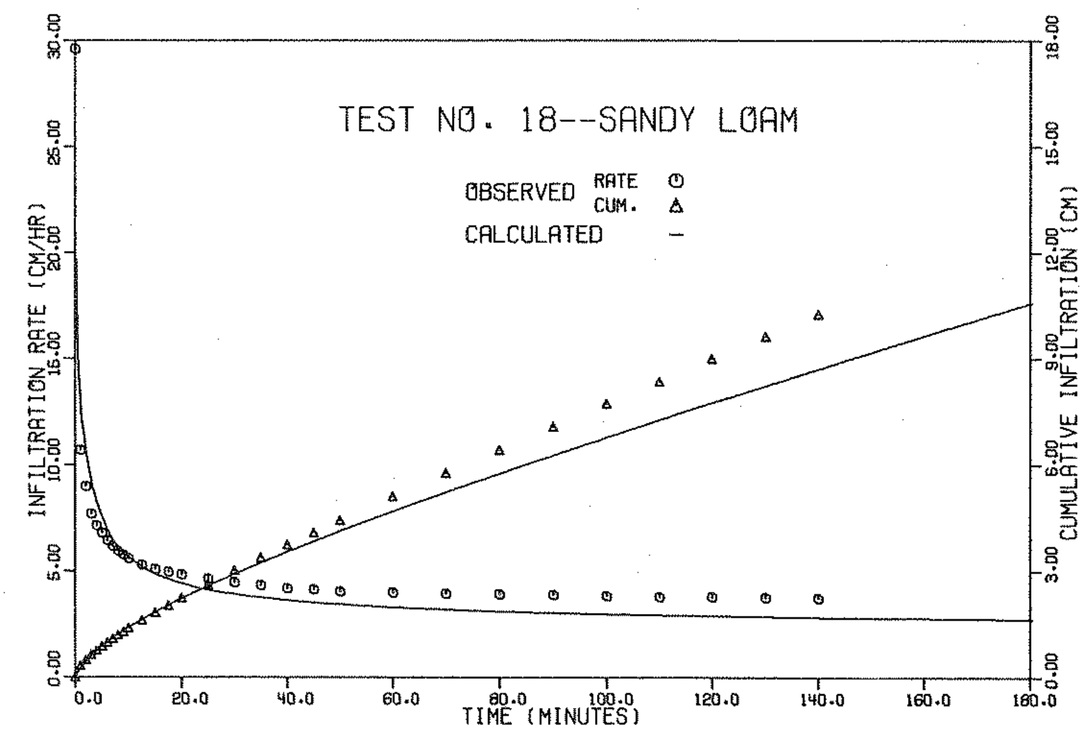
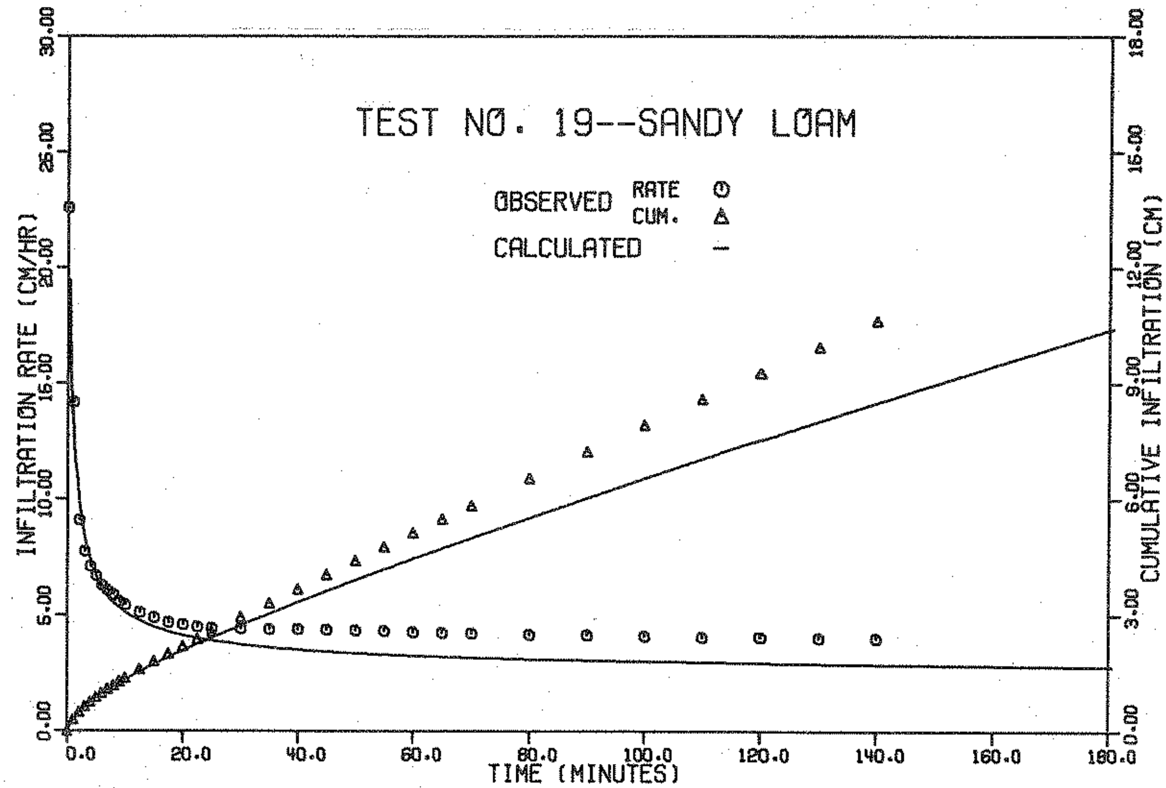




Figure 30. Observed and Calculated Infiltration Into Initially Wet Columns of Sandy Loam Soil.



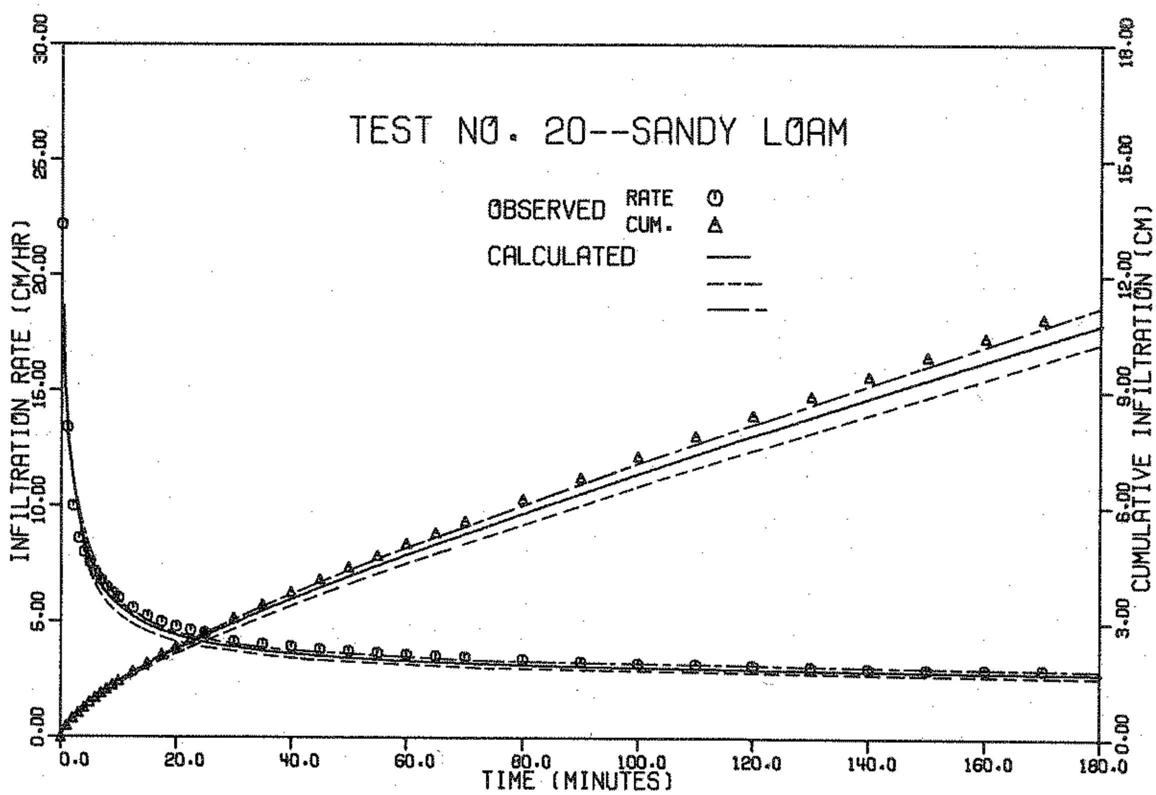
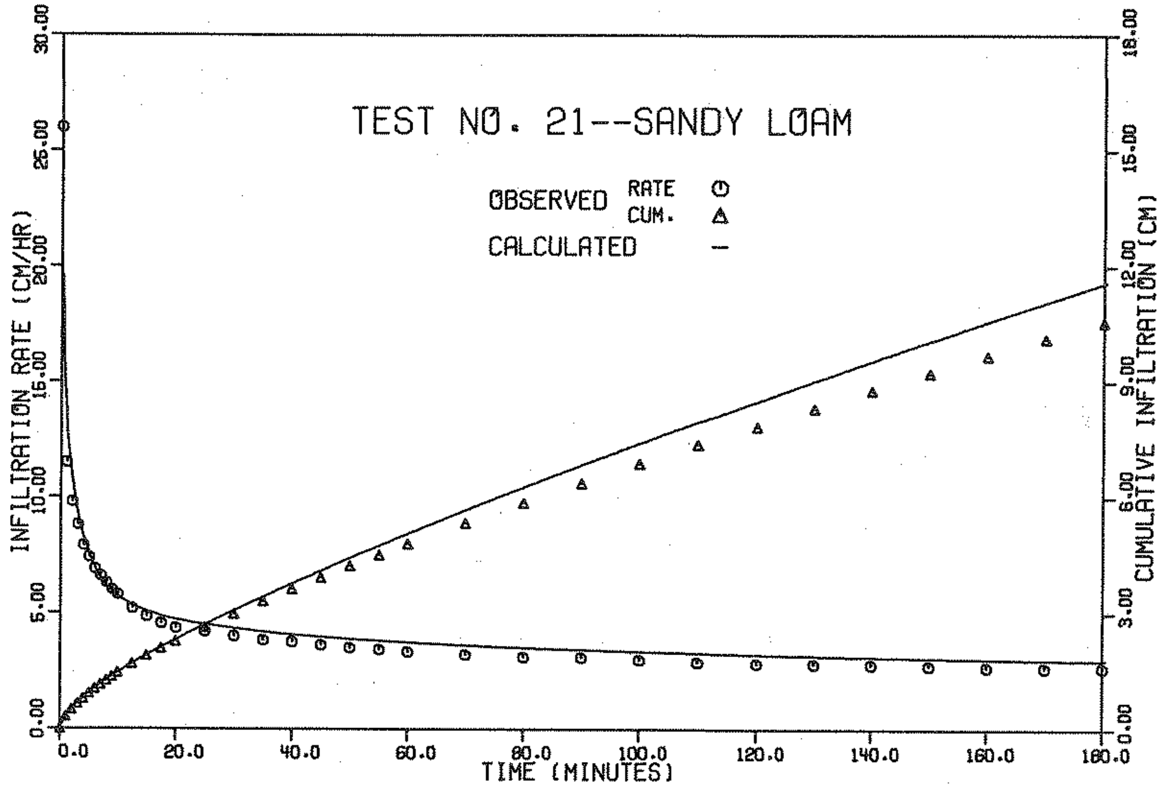




Figure 30, cont.



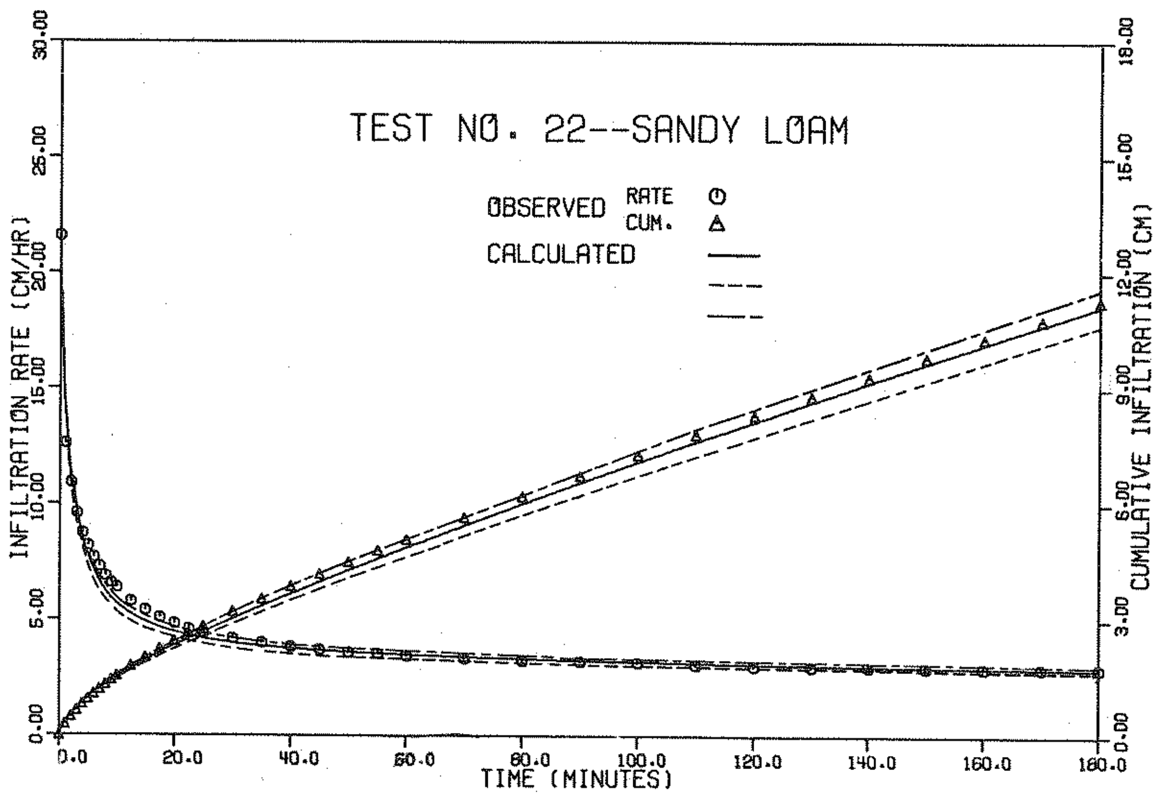




Figure 30, cont.

Table 7. Values of R and K(0) for Tests on Sandy Loam.

| Test No. | R (cm) | | | K(0) (cm/hr) |
|---|---|---|---|---|
| | $h_1$ = -2.78 | $h_1$ = -2.30 | $h_1$ = -3.17 | |
| 15 | 0.98 | ---- | 0.82 | 2.03 |
| 16 | 1.13 | 0.95 | ---- | 2.18 |
| 17 | 0.45 | 0.40 | 0.56 | 2.08 |
| 18 | 1.74 | 2.01 | 1.55 | 2.81 |
| 19 | 2.12 | 2.40 | 1.77 | 2.93 |
| 20 | 0.69 | 1.11 | 0.39 | 2.21 |
| 21 | 1.10 | 0.89 | 1.56 | 1.84 |
| 22 | 0.39 | 0.80 | 0.54 | 2.15 |

condition could have caused reduced infiltration rates. Although the wet sandy loam compacted more readily than the sand mixture, the partially filled extension broke off cleanly and surface compaction was avoided.

Tests 18 and 19 had considerably higher infiltration rates than predicted. The discrepancies between the predicted and observed results occurred for times greater than about 10 minutes. The discrepancies were also reflected in the K(0) values which were about 35 percent higher than the mean value for dry soils. The cause of the high infiltration rates is not known. In preliminary tests the sandy loam was passed through an ASTM No. 20 sieve (0.840 mm openings) and packed in a column with a procedure similar to that given in Chapter 4. However, as the soil was poured into the empty column, the larger particles tended to roll to the column walls forming a circumferential path of low resistance to flow. Extremely high infiltration rates resulted. Subsequently, a smaller sieve size was used (0.590 mm) and the procedure for packing columns was altered so that the larger particles were more evenly distributed throughout the column. In view of consistent results obtained for the other 6 tests on sandy loam, the high infiltration rates observed for tests 18 and 19 were probably due to nonuniformly packed soil columns.

With the exception of tests 18 and 19, the results obtained for infiltration into initially wet columns of the sandy loam supported the validity of the conductivity function obtained by the proposed method.

Layered Soils

Observed and computed influx curves for the tests conducted on layered soils are given in Figures 31-34. The computed influx curves were difficult to obtain. Since the soils were dry, the initial pressure head $h_i$ in boundary conditions 13 was so low that the solution procedure of INFIL3 failed to converge for small times. This difficulty was circumvented by assuming the soil was uniform and using INFIL2 to obtain the initial portion of the solution. The validity of this assumption has already been discussed in connection with the shortcut procedure given in Chapter 3. This procedure was used until the wetting front had advanced to within 2 cm of the layer junction.

The pressure head distribution from INFIL2 was then used as the initial pressure head distribution in INFIL3 to obtain the remainder of the solution for layered soils. Although a relatively large amount of computer time was required, this procedure worked well for layered soils made up of Sarpy loam and Geary silt loam with an initial pressure head of -1000 cm. However, this procedure failed to give a converging solution for the layered soils of the present study. The failure was apparently due to the very steep K(h) function of the sand mixture.

The remainder of the solution was therefore obtained with INFIL3 by assuming an initial pressure head of -200 cm. Although this was more than a tenfold decrease in the magnitude of $h_i$, the corresponding initial water contents were still low, 0.025 for the sand mixture and 0.11 for the sandy loam. However, when the wetting front had passed the layer junction, the computed infiltration rates were slightly lower than would have been obtained if the complete solution for the initially dry condition had been possible. Solutions were obtained for infiltration into uniform soils at the above initial water contents and the infiltration rates compared to those computed for dry initial conditions. The difference in the two solutions was then used as a basis for adjusting the computed infiltration rates for the layered soil.

Since the maximum adjustment was less than 5 percent, the above procedure apparently gave a good estimate of the true solution.

The results for the layered soil made up of 6 inches of sandy loam over 18 inches of sand mixture (test 23) are given in Figure 31. Corresponding results are given in Figure 32 for a column made up of 12 inches of sandy loam over 12 inches of sand mixture (test 24). The calculated infiltration rates for both layered soils are in almost exact agreement with the solution for a uniform sandy loam. This was consistent with solutions obtained by Hanks and Bowers (1962) for a layered soil in which the top layer had a smaller saturated conductivity than the bottom layer.

During the early portions of both tests 23 and 24, the calculated infiltration rates were considerably higher than the observed. However, the differences were of about the magnitude as for the uniform, initially dry sandy loam shown in Figure 22. Good agreement between the computed and observed infiltration rates at times greater than 40 minutes was obtained for test 24. This again corresponded to the results given in Figure 22. For test 23 the computed infiltration rates were at least 10 percent higher than the observed for the entire test duration. The areas between the computed and observed influx curves were 1.53 and 1.18 cm for tests 23 and 24, respectively.

The computed and observed times of arrival of the wetting front at the layer junction were 32 and 43 minutes, respectively, for test 23, and 121 and 146 minutes, respectively, for test 24. The density distributions recorded in Appendix A gave no indication of packing non-uniformities that would account for the differences in the observed and computed results given above. However, there was an error in packing the sand mixture sections of all the layered columns. Due to a calculation error, the sand mixture sections were packed to a mean bulk density of 1.68 gm/cc (the desired bulk density was 1.71 gm/cc). Since the infiltration was essentially controlled by the top layer of sandy loam, however, the error in the mean bulk density of the sand mixture had no apparent effect on the observed infiltration rates for test 23 and 24.

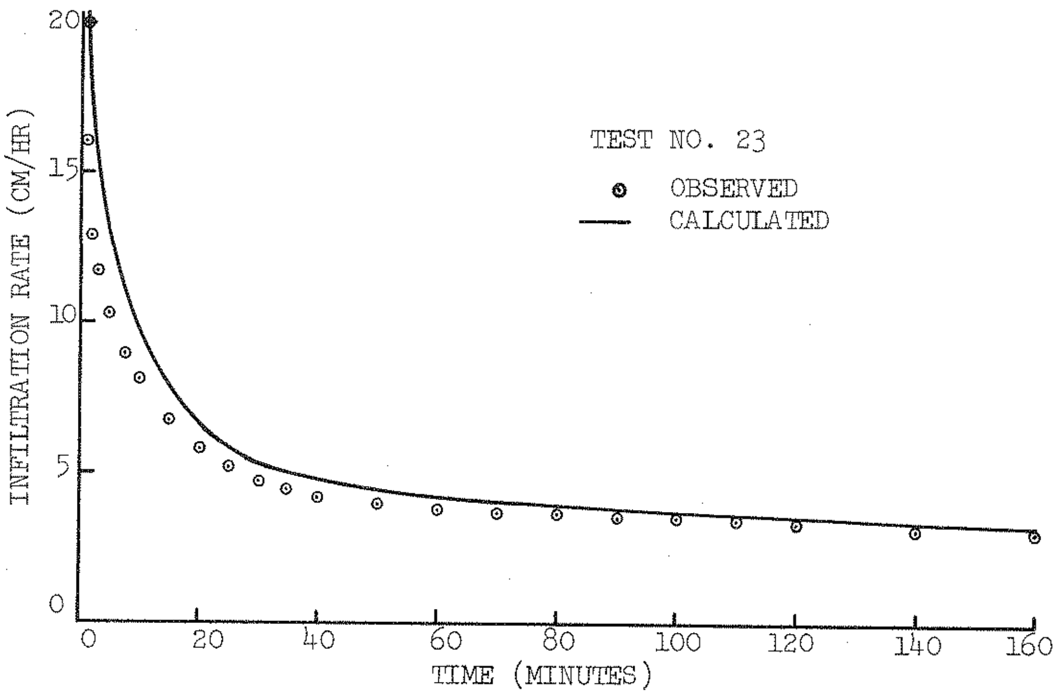



Figure 31. Influx Curves for a Layered Column of 6 Inches Sandy Loam Over 18 Inches Sand Mixture.

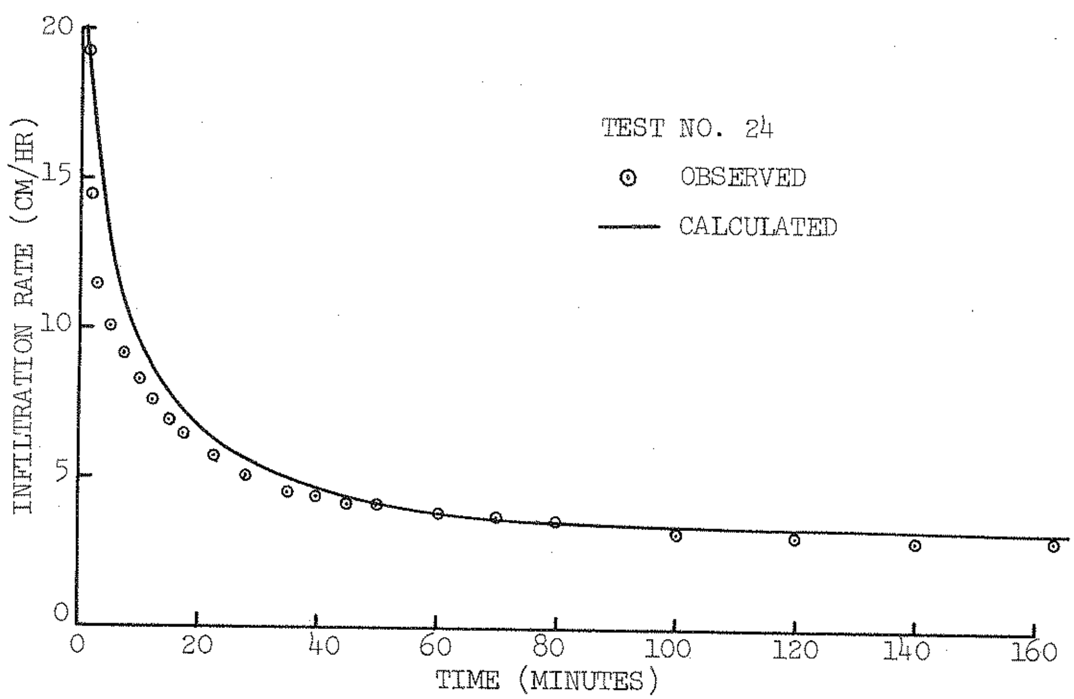



Figure 32. Influx Curves for a Layered Column of 12 Inches Sandy Loam Over 12 Inches Sand Mixture.

The results for tests 25 and 26, for layered columns having sand mixture top layers of 6 and 12 inch depths, respectively, are given in Figures 33 and 34. The observed infiltration rates were higher than the predicted during the initial stages of both tests. These deviations can be at least partially attributed to the error made in packing the sand mixture layer to a smaller bulk density than that for which the K(h) function had been determined. The observed and computed times of arrival of the wetting front at the layer junction were, respectively, 15 and 18 minutes for test 25, and 55 and 51 minutes for test 26.

The computer solutions were unstable for about 5 minutes after the wetting front reached the layer junction. The calculated infiltration rates during that period were nearly constant and, for test 26, were actually higher than the calculated rates for the uniform sand mixture. Then the calculated infiltration rates decreased rapidly and approached the predicted influx curve for a uniform sandy loam. For purposes of comparison the computed influx curves for uniform sand mixture and sandy loam soils are also given in Figures 33 and 34.

The calculated rates for test 25 were in good agreement with the observed for times greater than about 40 minutes. For test 26 the observed infiltration rates approached the calculated rates after the wetting front reached the junction of the two layers. However, there was still a difference of about 10 percent at the end of the 160 minute test. The total areas between the computed and observed influx curves were 1.83 cm for test 25 and 2.15 cm for test 26. These values are considerably larger than the 0.85 cm obtained for uniform columns of dry sand mixture. The differences were due to the high observed infiltration rates in the initial stages of the tests and to longer test durations for the layered soils.

As shown by the broken curves in Figures 33 and 34, there was not much difference in the infiltration rate-time relationships for uniform columns of the two soils investigated. Therefore, the experiments on layered soils did not provide a very severe test of the conductivity functions obtained. The results given for tests 23 and 24 (Figures 31 and 32) indicate that the K(h) functions obtained for the two soils can be used to give reasonably accurate predictions of influx curves for

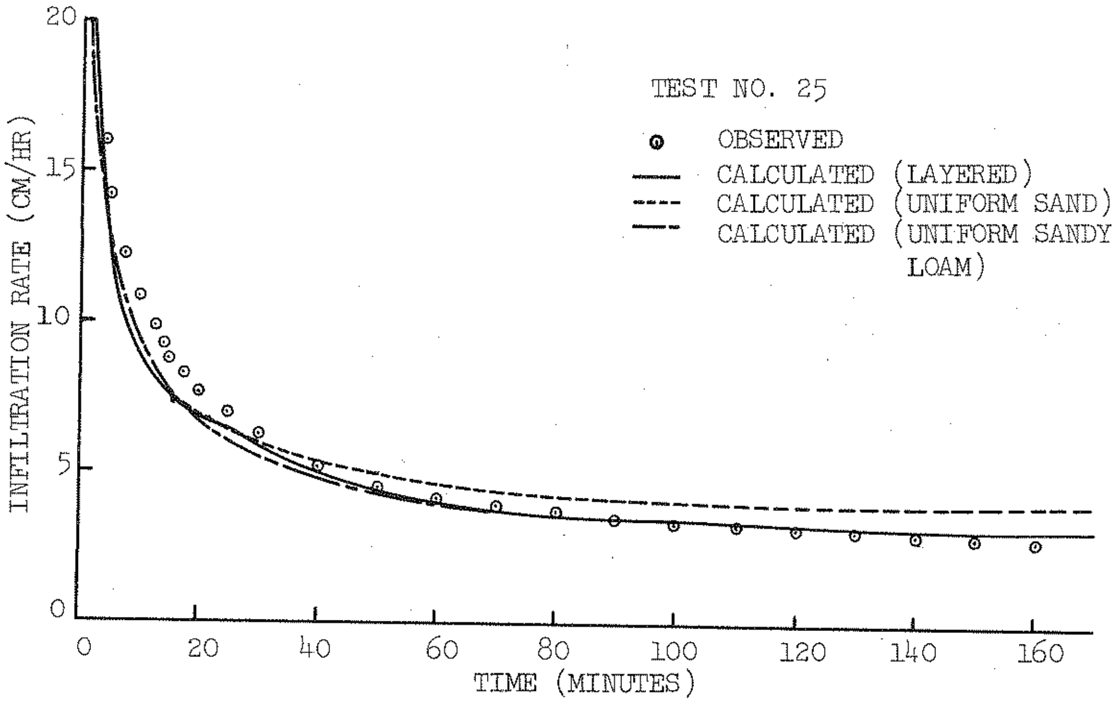



Figure 33. Influx Curves for a Layered Column of 6 Inches Sand Mixture Over 18 Inches Sandy Loam.

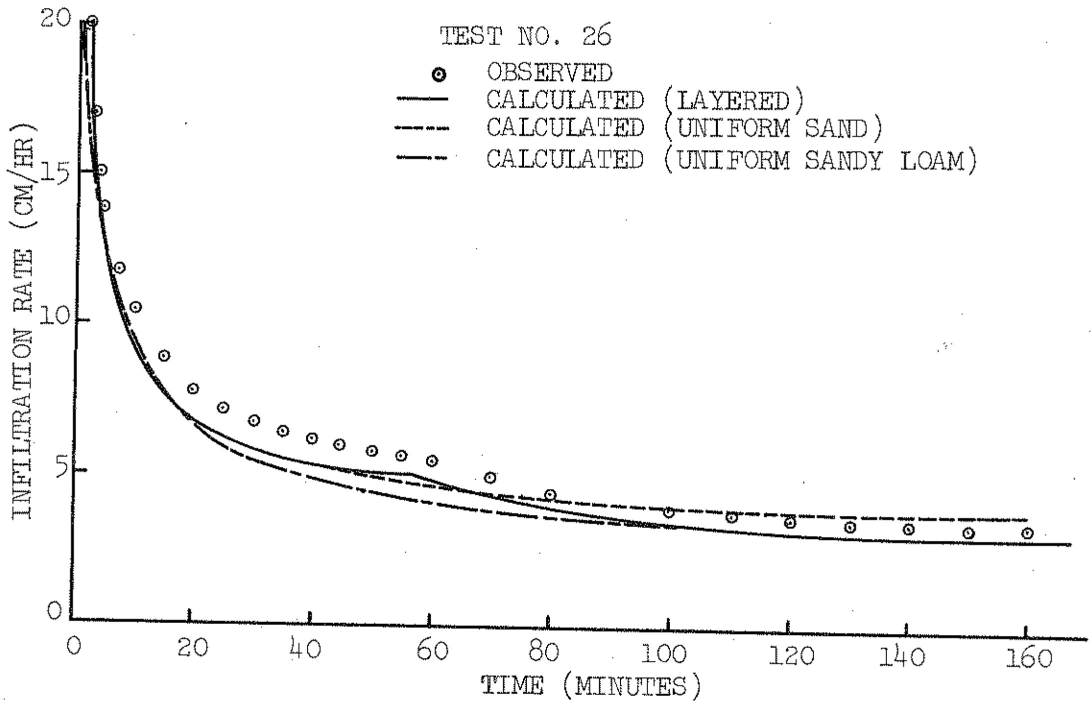



Figure 34. Influx Curves for a Layered Column of 12 Inches Sand Mixture Over 12 Inches Sandy Loam.

layered soils made up of a layer of sandy loam over a layer of sand mixture. Although the predicted influx curves for tests 25 and 26 (Figures 33 and 34) were in general agreement with observed results, the reliability of the numerical solutions was uncertain due to instabilities occurring when the wetting front reached the layer junction. An error made in packing the layer of sand mixture further confounded the results for these tests. Consequently, the results of tests 25 and 26 probably should not be used to judge the validity of the K(h) functions for the two soils.

# CHAPTER 6

## SUMMARY AND CONCLUSIONS

### Summary

An approximate method of determining the hydraulic conductivity function of unsaturated soil was proposed. The method was based on the assumption that the conductivity-pressure head relationship could be effectively represented by an empirical 3-parameter equation presented by Gardner (1958). Procedures were given for evaluating the equation parameters from measurements of the soil-water characteristic, the influx curve for an initially dry soil column, and K(0). A preliminary evaluation of the method was made using soil property data obtained from the literature for four soils.

An experimental investigation was conducted to further evaluate the proposed method. The conductivity functions of two artificially packed soils were determined. Infiltration rate-time relationships were then measured for infiltration into soil columns having different initial water content distributions. These relationships were compared to influx curves calculated with the use of the determined K(h) functions. Similar comparisons were made between computed and measured results for infiltration into layered columns of the two soils.

### Conclusions

On the basis of the results obtained in this study, the following conclusions were drawn:

The proposed method can be used to determine an effective hydraulic conductivity function for the soils analyzed in this study. The analysis made with soil property data from the literature showed that the proposed method gave a good approximation of the conductivity data for soils in which the K(h) relationship had the general form of Gardner's equation. Although the actual K(h) relationship for one soil was not of the form given by Gardner's equation, the function obtained still allowed reliable predictions of the infiltration rate-time relationships for a wide range of initial water contents.

The K(h) functions obtained for the two soils which were investigated experimentally gave reliable predictions of the influx curves for different initial water content distributions. Further evidence of the validity of the K(h) functions was given by the close agreement between the predicted and observed wetting front movement for the tests on dry soils. The results of experiments on layered soils were inconclusive due to errors made in packing the sand mixture layer and to difficulties encountered in obtaining numerical solutions for dry layered soils.

Errors in the soil-water characteristic can be partially compensated by the proposed method of determining the conductivity function. Infiltration predictions with the resulting $h(\theta)$ and K(h) relationships will not be as accurate as predictions based on some improved measurement of $h(\theta)$, but more accurate than when the functions are determined independently with the same error in $h(\theta)$.

With the exception of the $h(\theta)$ determination, which is necessary if theory is used to characterize infiltration regardless of the method for determining K(h), the measurements required by the proposed method may be made with much less time and effort than is required by conventional procedures. Approximately 2 minutes of CPU time on a CDC 6500 digital computer were required to determine $h_1$ in Gardner's equation.

Recommendations for Future Research

The results of this study indicated that the proposed method is a promising means of determining the effective hydraulic conductivity function of an unsaturated soil. Additional research is needed to further evaluate and refine the method.

In order to evaluate the range of applicability of the method it is suggested that investigations be conducted to determine the conductivity functions of additional soils. Although further experiments on packed soils would be useful, investigations on field cores would have greater utility to the eventual practical application of the method. The validity of the functions obtained could be tested by comparing them to measurements made with conventional procedures. However, differences in the two independent determinations should probably be evaluated based on predictions of infiltration for various boundary conditions. The validity of the functions should also be tested by comparing measured and calculated infiltration relationships for rainfall boundary conditions as well as those applied in the present study.

Investigations are also needed to determine the reliability with which infiltration can be predicted under field conditions using a conductivity function obtained from field cores with the proposed method.

Development and modifications of techniques to make *in situ* measurements of the variables required by the proposed method would also be a worthwhile objective of future investigations.

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APPENDICES

# APPENDIX A

## GAMMA-RAY ATTENUATION MEASUREMENTS

A gamma-ray attentuation apparatus was constructed to provide a nondestructive means of measuring the bulk density and water contents of soil columns. The apparatus was designed to have the capability of measuring changing water contents in a soil column undergoing infiltration or drainage. However, it was used in this study for measuring only steady state bulk density and initial water content distributions.

### Attenuation Equations

The principles and techniques for making gamma attenuation measurements have been described in detail by Gardner (1965). The basic equations given below follow the notation of Swartzendruber, et al. (1969). The intensity of a gamma beam passing through a soil column may be expressed as

$$I = I_o \exp\left(-U_s \rho_s - U_c \rho_c - U_w \theta\right) \tag{A1}$$

where $I_o$ and $I$ are the fluxes (counts per minute) incident on and passing through the soil column, respectively. $\rho_s$ and $\rho_c$ are the densities of the soil and column walls and $\theta$ is the volumetric water content. $U_s$ and $U_c$ represent products of the mass attenuation coefficients and effective thicknesses of the soil and column walls, respectively; $U_w$ is the product of the mass attenuation coefficient, effective soil thickness and the density of water.

For a dry soil ($\theta = 0$) equation A1 reduces to

$$I = I_d = I_o \exp\left(-U_s \rho_s - U_c \rho_c\right) \tag{A2}$$

and for an empty column ($\rho_s = 0$)

$$I = I_a = I_o \exp\left(-U_c \rho_c\right) \tag{A3}$$

For a column filled with water ($\rho_s = 0$, $\theta = 1$)

$$I = I_w = I_o \exp\left(-U_c \rho_c - U_w\right) \tag{A4}$$

Combining equations A1, A2, A3, and A4 and solving for $\theta$, $\rho_s$, and $U_w$ yields

$$\theta = \frac{1}{U_w} \ln\left(I_d/I\right) \tag{A5}$$

$$\rho_s = \frac{1}{U_s} \ln\left(I_a/I_d\right) \tag{A6}$$

$$U_w = \ln\left(I_a/I_w\right) \tag{A7}$$

By measuring $I_d$ at several points along the column, an average value of $U_s$ can be determined from equation A5 by

$$U_s = \overline{U}_s = \frac{1}{\overline{\rho}_s} \overline{\ln\left(I_a/I_d\right)} \tag{A8}$$

where $\overline{\rho}_s$ is the known bulk density of the soil and $\overline{\ln\left(I_a/I_d\right)}$ is the mean value of $\ln\left(I_a/I_d\right)$ for the points measured.

Due to instrument drift for very high count rates (greater than about 1,300,000 counts per minute), it was not possible to accurately determine $I_a$; consequently it was necessary to make a measurement additional to those outlined above. The count rate for the empty column was reduced by placing a brass bar 0.438 inches thick in series with the column. Then equations A3 and A4 take the forms

$$I_a = I_o \exp\left(-U_c \rho_c - U_b \rho_b\right) \tag{A9}$$

$$I_w = I_o \exp\left(-U_c \rho_c - U_b \rho_b - U_w\right) \tag{A10}$$

where $U_b$ and $\rho_b$ are constants for the brass bar. The product $U_b \rho_b$ was obtained by making gamma attenuation measurements on two different bar thicknesses. Substituting equations A9 and and A10 for equations A3 and A4, equation A6 becomes

$$\rho_s = \frac{1}{U_s} [\ln \frac{I_a}{I_d} + U_b\rho_b] \qquad (A11)$$

while equations A5 and A7 remain in their initial form.

Thus by measuring I, $I_a$, $I_w$, and $U_b\rho_b$, the bulk density and water content distributions of a soil column could be determined.

## Equipment and Procedure

A schematic diagram of the gamma-ray attenuation apparatus is given in Figure A1. The gamma rays were emitted from a 250-mc Cesium 137 encapsulated source which was shielded by embedding it in a lead cylinder 6 inches in diameter and 6 inches long.[1] The beam of gamma rays was collimated through a slit having a minimum height of 1.0 mm, a width of 1.0 cm, and a length (along the beam) of 8.1 cm. After passing through the soil column, the beam was received through a second collimator (1.0 mm high, 2.0 cm wide, and 5.1 cm long) and its intensity detected by a sodium iodide scintillation crystal. The light pulses generated by the scintillation crystal were detected by a photomultiplier tube and the signal was transmitted through a preamplifier and an amplifier to a pulse-height analyzer.[2] Pulses with energy levels greater than 0.5 Mev were counted by an electronic counter-timer[3] and the total number of counts for a preset time was recorded in digital form on a Teletype.[4]

---

[1] The source was selected and the shielding and collimators designed based on an apparatus developed in the Department of Agronomy, Purdue University, described by Swartzendruber, et al. (1969).

[2] The gamma-ray source and associated electronic equipment, with the exception of the electronic counter and Teletype, were supplied by Nuclear-Chicago Corp., Box 367, Des Plains, Illinois.

[3] The counter-timer and the interfacing to the Teletype were designed and constructed in the Department of Agricultural Engineering and are described in detail by Goodrich (1970).

[4] Teletype Corp., Skokie, Illinois.

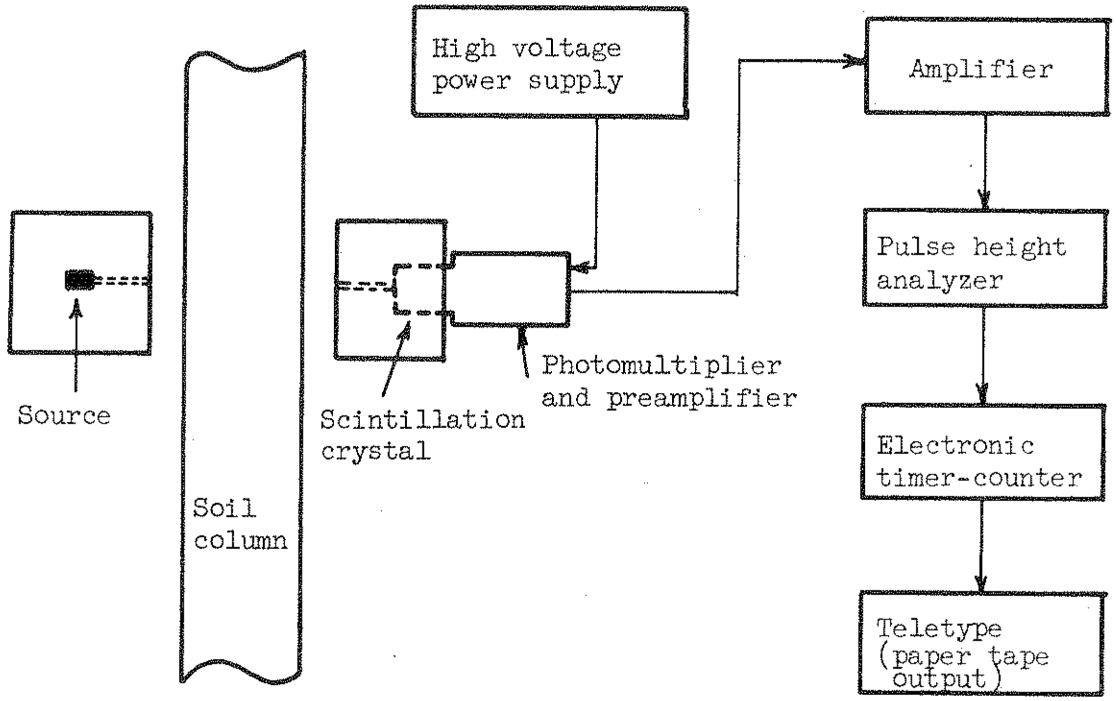


Figure A1. Schematic of Gamma-Ray Attenuation Equipment.

The lead shields containing the source and detector were rigidly fixed to an elevator platform which could be moved up and down the soil column. The elevator platform was situated in a 61-inch by 30-inch framework of 2-inch steel angle and was raised and lowered by three 0.75-inch diameter Acme screws which were connected by a chain drive to an electric motor. The applicator described in Chapter 4 was used as a reference in aligning columns in the apparatus. A picture of the gamma-ray attenuation apparatus and associated equipment is given in Figure A2.

Due to gradual drifts in the electronic components at high count rates, it was necessary to calibrate the apparatus prior to measuring the density or water content distribution in each column. This was accomplished by placing a brass bar 0.438 inches thick between the source and detector and adjusting the amplifier gain until a standard count rate was obtained.

The bulk density distribution of a column of soil was determined by aligning the column in the gamma-ray apparatus and making three 30-second counts at 1.0 inch increments along the column. Initial water content

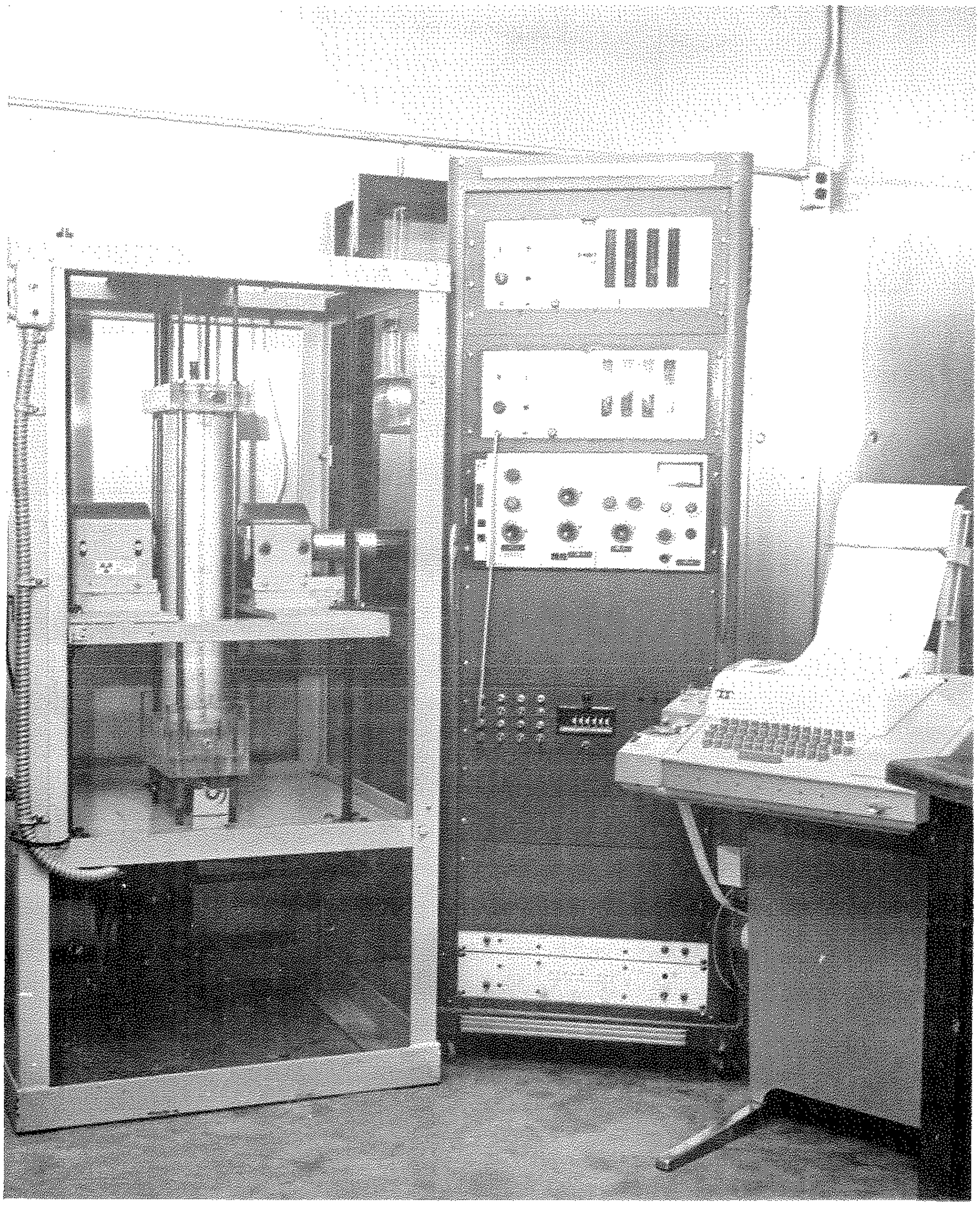

Figure A2. Gamma-Ray Attenuation Apparatus and Associated Equipment.

measurements were made by again taking three 30-second counts at the same positions that were "sampled" for the dry condition. Since differences in bulk density would have appeared as errors in the water content determinations, it was important to make the gamma measurements in as close to the same positions as possible for the wet and dry conditions.

## Results

The bulk density and initial water content distributions for all the tests conducted in this study are given in Figure A3. The method given by Gardner (1965) was used to calculate the uncertainty in the water content determinations. Calculations showed that measurements for the sand mixture at a water content of 0.12 $cm^3/cm^3$ were within $\pm$ 0.0033 and $\pm$ 0.0045 $cm^3/cm^3$ (99.5 percent confidence level) for the 4.25 inch and 3.5 inch columns, respectively. Corresponding determinations for the sandy loam at a water content of 0.30 $cm^3/cm^3$ were within $\pm$ 0.0030 and $\pm$ 0.0042 $cm^3/cm^3$. Additional errors, not included in the above calculations, were caused by slight drifts in the instrumentation, and by inadvertently positioning the gamma beam in slightly different positions for the dry and wet conditions.

The reliability of the gamma-ray attenuation measurements was tested experimentally by comparing water contents determined with the procedures described above to values obtained by taking gravimetric samples. The separate water content determinations agreed within 0.01 $cm^3/cm^3$ for 8 of the 9 columns tested. The independent measurements for the remaining column were within 0.02 $cm^3/cm^3$. Based on the results of these tests and the above calculations it was concluded that the gamma-ray attenuation apparatus and procedures described herein could be used to measure water contents accurately to within approximately $\pm$ 0.01 $cm^3/cm^3$ for the soils investigated. This accuracy was judged to be sufficient for the present study.

## Sample Calculations

The following sample calculations demonstrate the computations involved and the count rates obtained in determining the bulk density and initial water content for one point in a 4.25-inch column of sand mixture.

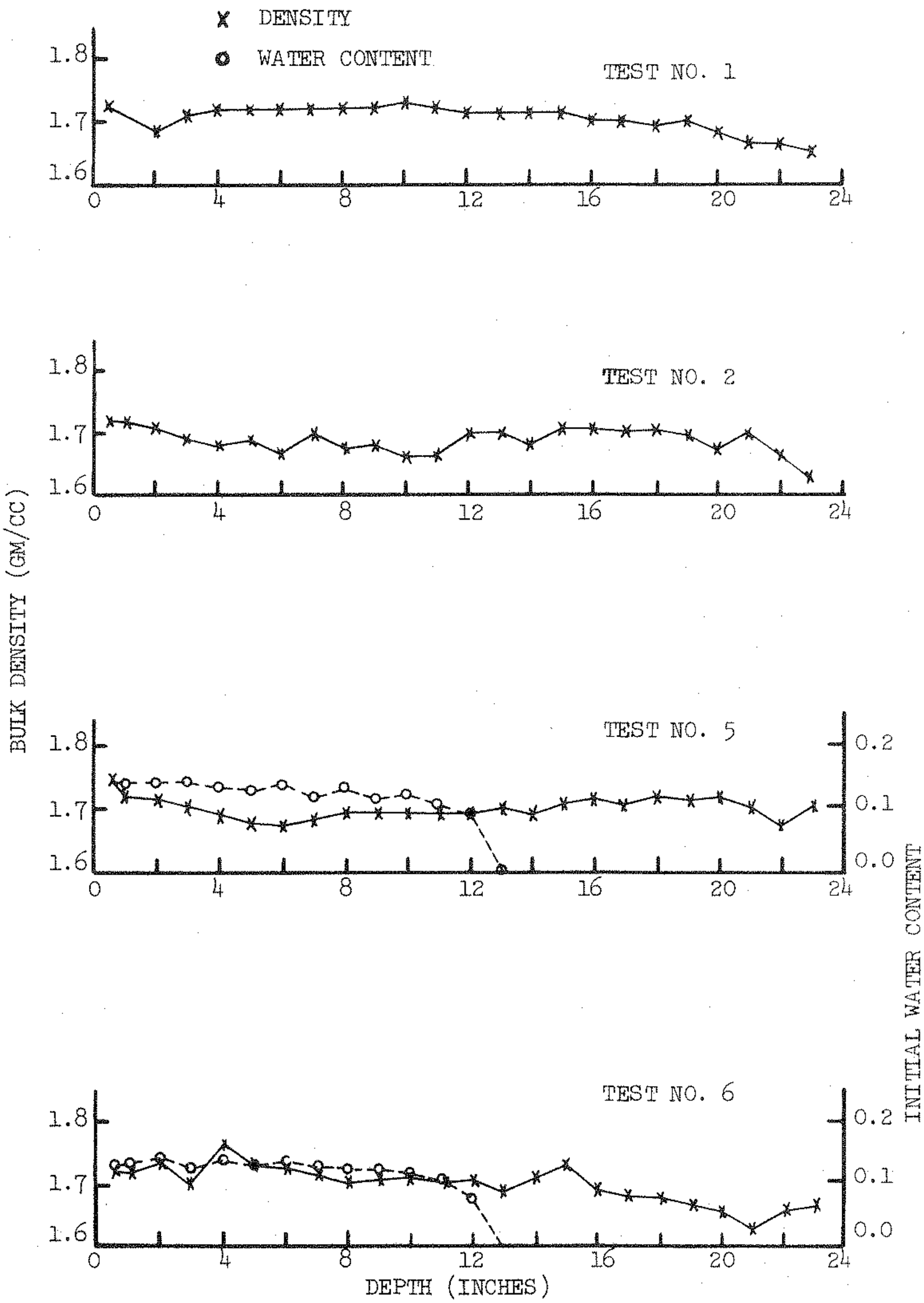


Figure A3. Bulk Density and Initial Water Content Distributions for All Tests Conducted.

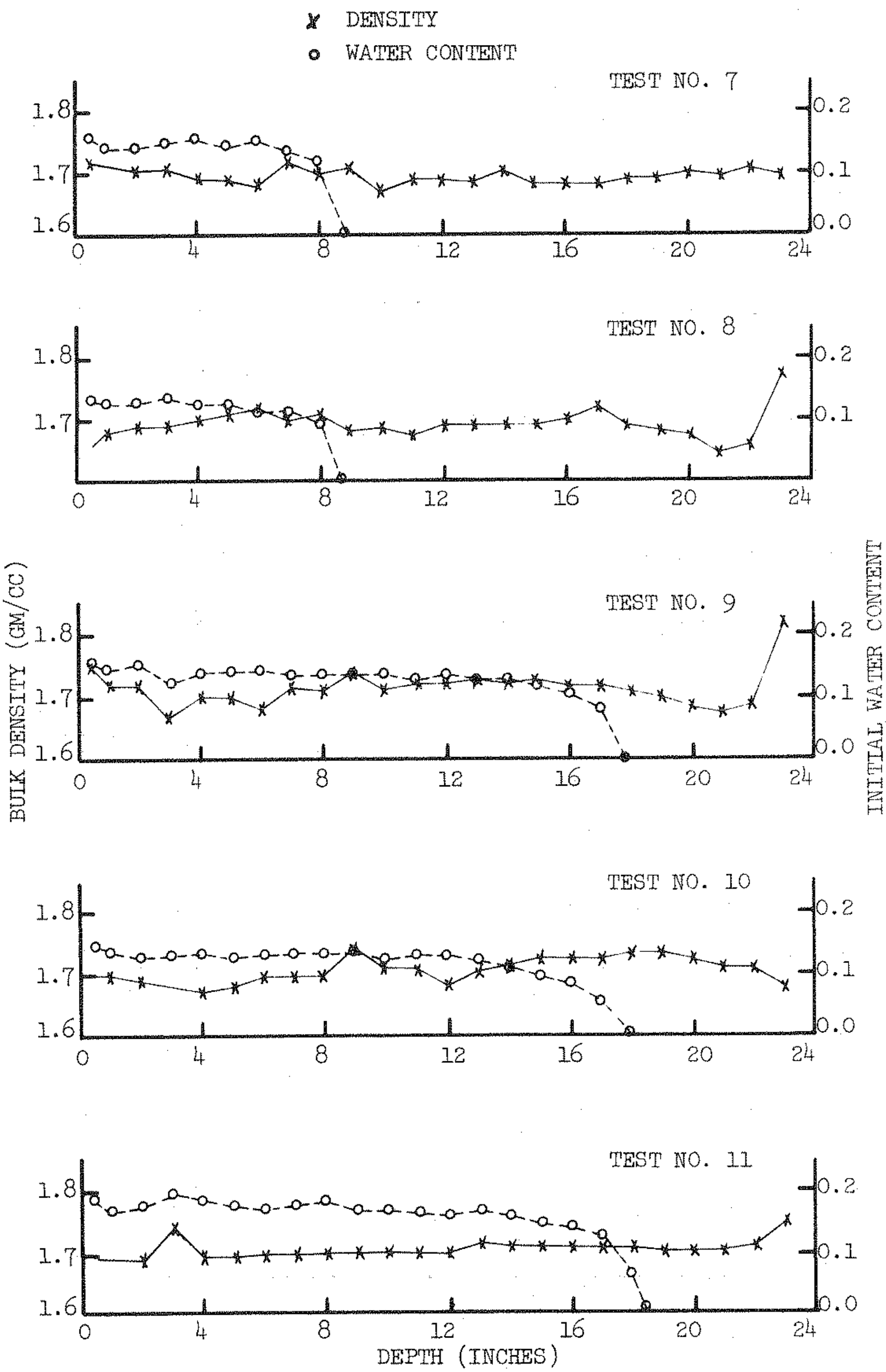


Figure A3, cont.

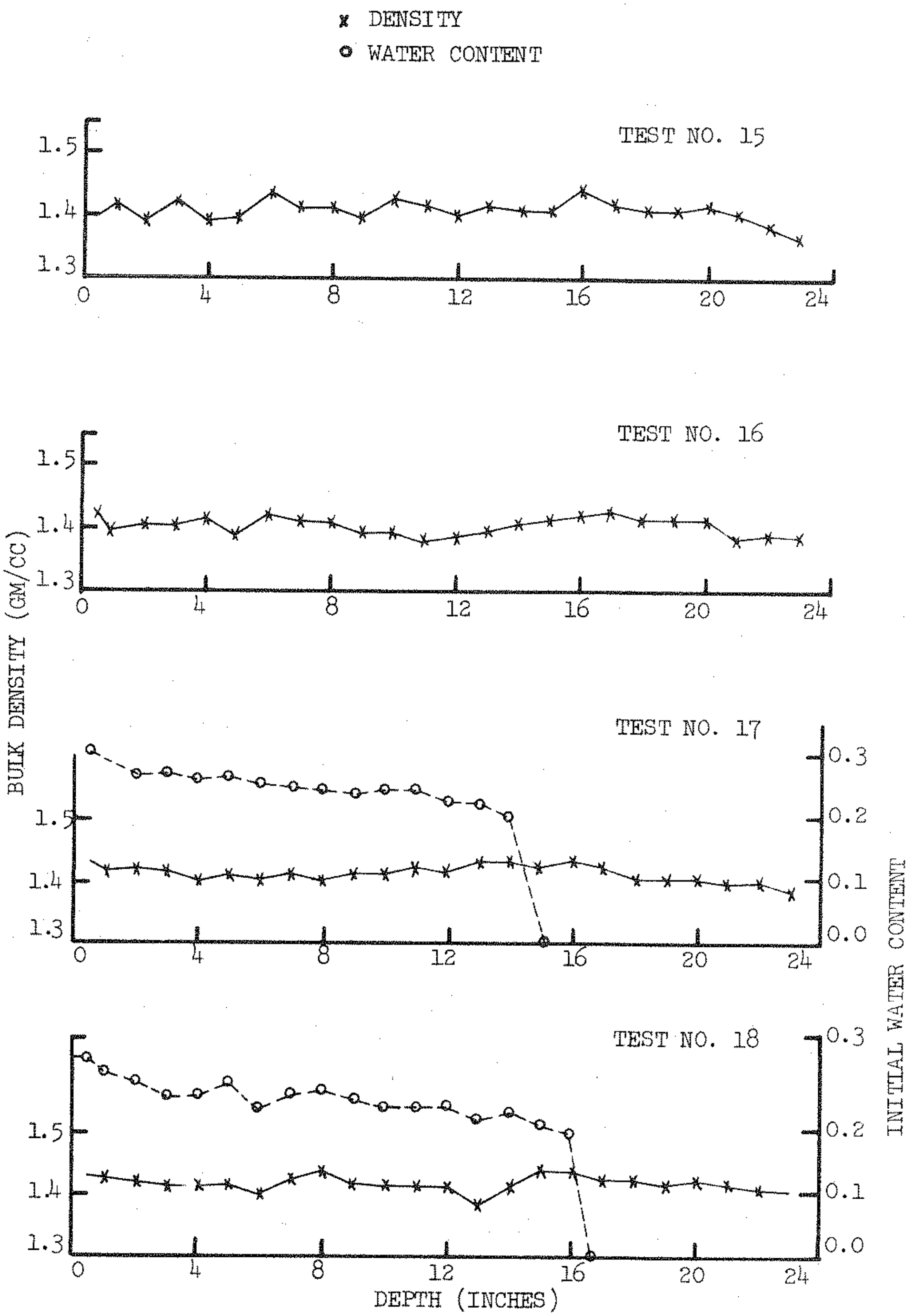


Figure A3, cont.

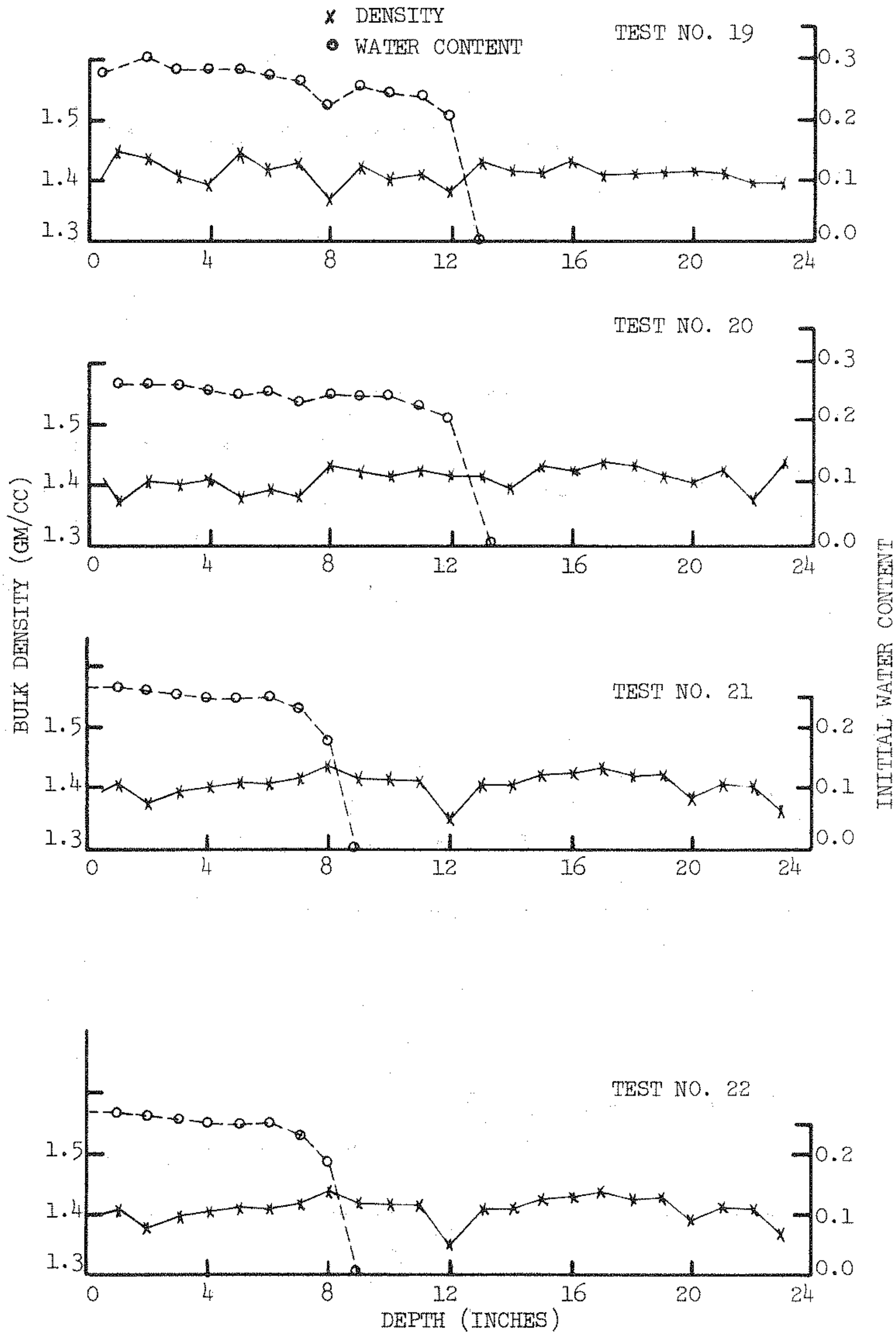


Figure A3, cont.

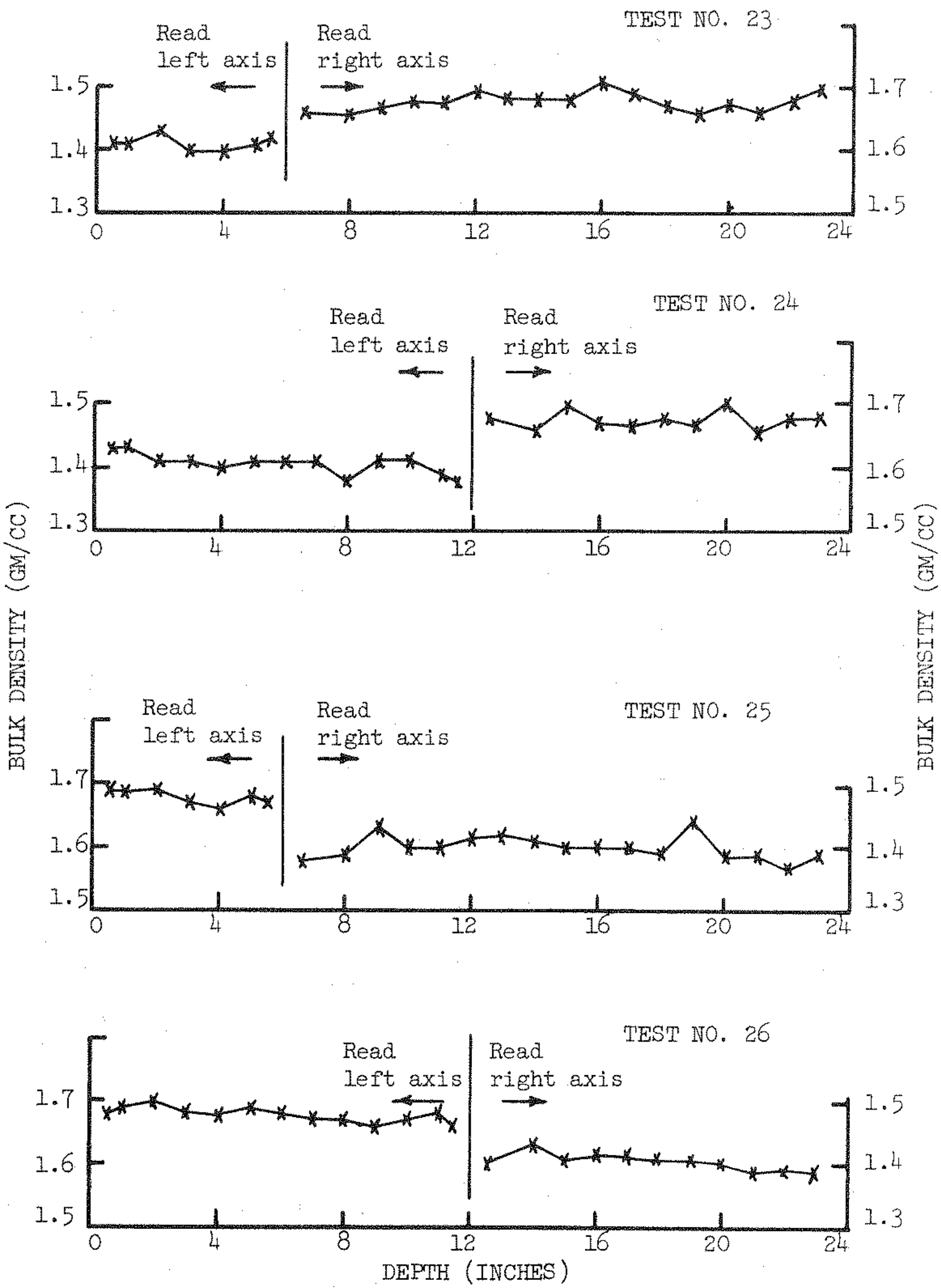


Figure A3, cont.

Test No. 6   Sand Mixture   $\bar{\rho}_s = 1.71$ gm/cc

| | |
|---|---|
| $I_a = 1{,}094{,}860$ counts per minute (CPM) | (through empty column plus 0.438-inch brass bar) |
| $I_w = 454{,}800$ CPM | (through column filled with water plus 0.438-inch brass bar) |
| $I_d = 539{,}458$ CPM | (through column filled with dry sand mixture at x = 6.0 inches) |
| $I = 479{,}256$ CPM | (through wet column at x = 6.0 inches) |

From the count rates at 24 positions in the column and $\bar{\rho}_s$, $U_s$ was determined by equation A8 to be $U_s = 0.770$ cm$^3$/gm. The value of the product $U_b\rho_b$ was determined by counting through two different thicknesses of brass. $U_b\rho_b = 0.621$

From equation A11

$$\rho_s = \frac{1}{0.770}\left[\ln \frac{1{,}094{,}860}{539{,}458} + 0.621\right]$$

$$\underline{\rho_s = 1.73 \text{ gm/cm}^3} \qquad (x = 6.0 \text{ inches})$$

From equation A7

$$U_w = \ln \frac{1{,}094{,}860}{454{,}800}$$

$$\underline{U_w = 0.88 \text{ cm}^3/\text{cm}^3}$$

Then using equation A5

$$\theta = \frac{1}{0.88} \ln \frac{539{,}458}{479{,}256}$$

$$\underline{\theta = 0.136 \text{ cm}^3/\text{cm}^3} \qquad (x = 6.0 \text{ inches})$$

# APPENDIX B

## COMPUTER PROGRAMS

The following pages contain listings of the computer programs, INFIL2 and INFIL3, developed to solve the $\theta$-based and h-based forms of the Richards equation for infiltration into unsaturated soils. Both programs were written in FORTRAN IV and were processed on a CDC 6500 computer.

### INFIL2

As described in Chapter 3, INFIL2 was developed to solve the $\theta$-based form of the Richards equation for infiltration into a uniform soil from a ponded surface of negligible depth. The version given here is also capable of applying a one-dimensional search to determine the parameter $h_1$ in Gardner's equation giving the minimum difference between the measured and calculated influx curves. Approximately 26,000 words of memory are required to process this program. The basic logic and essential features of solving the governing equation and conducting a Golden section search for the optimum $h_1$ are given in the main program, INFIL2.

Input data consist of: (1) the observed infiltration rate versus time relationship (these data are needed only if a search for the optimum $h_1$ value is desired); (2) basic variables describing the system, length of the soil column, number of depth increments, and the time for which the solution is desired; (3) the initial water content distribution; (4) the end points for the search $h_1$ and the number of trials to be made (not necessary if search is not desired); and (5) the soil-water characteristic and the conductivity function.

The purpose of the subroutine SETUP is to input tabular data for the soil-water characteristic and the parameters in Gardner's equation and establish in an array corresponding values of the water content, pressure head, conductivity, and diffusivity. Interpolation is used to define the array for increments of water content of 0.001 so that the soil properties can be rapidly recalled. Other versions of subroutine SETUP were used to input the conductivity and/or diffusivity functions in tabular form as well as the soil-water characteristic. The version given here was used in conjunction with the search procedure for determining the parameters in Gardner's equation.

The purpose of subroutine RETEN is to determine for an array of water contents (solution for water content distribution at a given time step) the corresponding values of pressure head, conductivity, and diffusivity.

A sample of input data and the output produced is given with the program listing. The physically significant programming symbols used are defined in the following list. Other symbols which appear in the programs were used for convenience and will not be further defined.

Programming Symbols

| | |
|---|---|
| A | Coefficient $\alpha$ defined in equation 24 ($cm^2/sec$). |
| ABSVOL | Area between measured and computed influx curves for one time increment (cm). |
| ACCIF | Calculated accumulative infiltration (cm). |
| AK | Hydraulic conductivity stored in an array with corresponding water content for rapid recall (cm/hr). |
| ALEN | Column length considered (cm). |
| AT | Time (minutes). |
| B | Coefficient $\beta$ defined in equation 25 ($cm^2/sec$). |
| C | Coefficient $\epsilon$ defined in equation 26 ($cm^2/sec$). |
| CLOCK | Time in the measured influx curve data (minutes). |
| CON | Hydraulic conductivity defined at each node. |
| CRIT | Iteration parameter. |
| D | Coefficient $\zeta$ defined in equation 27 ($cm^2/sec$). |
| DDT | Increase in time increment computed after each time step (sec). |
| DELX | Depth increment (cm). |
| DIFF | Soil-water diffusivity at each node point ($cm^2/sec$). |
| DT | Time increment (seconds). |

| | |
|---|---|
| DWDH | Water capacity ($cm^{-1}$). |
| EXPI | Measured infiltration rates (cm/hr). |
| EXPVOL | Measured accumulative infiltration (cm). |
| GA | Exponent a in Gardner's equation (equation 18) for the conductivity-head relationship. |
| GB | Parameter b in Gardner's equation (sec/cm). |
| GH1 | Parameter $h_1$ in Gardner's equation (cm). |
| H | Pressure head (cm). |
| HH | Pressure head data read in tabular form (cm). |
| HEAD | Pressure head stored in an array with water content for rapid recall (cm). |
| ITTER | Number of iterations. |
| NEX | Number of infiltration rate-time data. |
| NI | Number of depth increments. |
| NN | Number of depth nodes. |
| NNCK | Node point at which water content is continuously checked in shortcut procedure discussed in Chapter 3. |
| NTRY | Number of trials to find the minimum R value. |
| NVOL | Number of EXPVOL data given. |
| R | Area between measured and calculated influx curves (cm). |
| RATE | Calculated infiltration rate using a three point forward difference approximation of the potential gradient at x = 0 (cm/hr). |
| RAT1 | Calculated infiltration rate using a two point forward difference approximation of the gradient at x = 0 (cm/hr). |
| T | Time (seconds). |
| TH | Water content data read in tabular form ($cm^3/cm^3$). |
| TSTOP | Time at which solution is terminated (minutes). |
| W | Water content ($cm^3/cm^3$). |
| W1 | Initial water content ($cm^3/cm^3$). |
| W2 | Water content for previous iteration ($cm^3/cm^3$). |
| X | Depth (cm). |
| XXA, XXB | End points between which search is conducted for optimum GH1 (cm). |
| Y | Depth for reading in initial water content distribution (cm). |
| Z | Initial water contents ($cm^3/cm^3$). |

```
C                              R. W. SKAGGS ** 04/10/70
C                                     INFIL2
      PROGRAM INFIL2(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT)
C
C         A PROGRAM TO SOLVE THE THETA-BASED RICHARDS EQUATION FOR
C         INFILTRATION INTO A UNIFORM SOIL WITH A PONDED SURFACE OF
C         NEGLIGIBLE DEPTH.  THE PROGRAM IS ALSO CAPABLE OF USING A
C         ONE-DIMENSIONAL SEARCH TO FIND THE PARAMETER GHI IN GARDNERS
C         EQUATION MINIMIZING THE OBJECTIVE FUNCTION.
C
      DIMENSION HEAD(750), AK(750), DX(750), H(100), W1(100), Z(25), Y(2
     15), X(100), A(100), B(100), C(1C0), D(100), E(100), F(100)
      DIMENSION W(100,2), CON(100,10), DIFF(100,10), T(2000), RATE(2000)
     1, AT(2000), ACCIF(2000), W2(100), R(20), XTRY(20)
      DIMENSION CLUCK(100), EXPI(100), EXPVOL(100), ABSVOL(100)
C         READ IN DATA FROM EXPERIMENTRL TEST
C         IF SEARCH IS NOT DESIRED, INPUT NEX=0
      READ (5,205) NEX,NVOL
      IF (NEX.EQ.0) GO TO 20
      READ (5,210) (CLOCK(I),EXPI(I),EXPVOL(I),I=1,NEX)
      IF (EXPVOL(NEX).GT.0.0) GO TO 15
      N=NVOL+1
      VOLUME=EXPVOL(NVOL)
      DO 10 I=N,NEX
          EXPVOL(I)=VOLUME+(EXPI(I)+EXPI(I-1))/2.0*(CLOCK(I)-CLOCK(I-1))/
     1    60.
10    VOLUME=EXPVOL(I)
15    WRITE (6,215)
      WRITE (6,220)
      WRITE (6,225)
      WRITE (6,230) (CLUCK(I),EXPI(I),EXPVOL(I),I=1,NEX)
20    CONTINUE
C         READ IN COLUMN LENGTH, NO OF INCREMENTS, POROSITY,AND STOP TIME
      READ (5,180) ALEN,NI,P,TSTOP
C         READ IN INITIAL WATER CONTENT DISTRIBUTION--MUST HAVE LAST
C         FIELD VACENT.
      K=1
      L=7
25    READ (5,175) (Z(I),Y(I),I=K,L)
      IF (Y(L).EQ.0.0) GO TO 30
      L=L+7
      K=K+7
      GO TO 25
30    CONTINUE
      SS1=0.0
C         READ IN END POINTS FOR GHI SEARCH AND NUMBER OF TRIALS FOR GHI
C         TO BE MADE.
      READ (5,240) XXA,XXB,NTRY
C         IF NO SEARCH IS TO BE MADE INPUT NTRY=0.0 AND XXA=GHI.
      KTRY=1
      XLONG=XXB-XXA
      XTRY(1)=XXA+0.6180340*XLONG
      XB=XTRY(KTRY)
      GHI=XTRY(KTRY)
      IF (NTRY.EQ.0.0) GHI=XXA
35    CALL SETUP (AK,HEAD,DX,SS1,GHI,GA,GB)
      WRITE (6,185)
      WRITE (6,195)
      WRITE (6,200)
      AMOUNT=0.0
      ANI=NI
      NN=NI+1
      NT=NN
C         DEFINE THE POSITION X
      X(1)=0.0
      DELX=ALEN/ANI
      CRIT=0.001
      NNCK=NN-5
      DO 40 I=2,NN
          IM=I-1
40    X(I)=X(IM)+DELX
      J=1
      M=1
      MP=2
      W(1,J)=Y(1)
      DO 45 I=2,NN
          IF (Z(MP).LT.X(I)) M=M+1
          MP=M+1
45    W(I,J)=((X(I)-Z(M))/(Z(MP)-Z(M))*(Y(MP)-Y(M))+Y(M))
      DO 50 I=1,NN
          W2(I)=W(I,1)
50    W1(I)=W(I,1)
      IF (NN.GT.(NI+1)) NN=NI+1
C         NOW THE INITIAL CONDITIONS HAVE BEEN EVALUATED.  THE NEXT STEP
```

```
C        IS TO EVALUATE THE COEFFICIENTS D AND K AT EACH NODE POINT.
      CALL RETEN (W,H,CON,DIFF,AK,DX,HEAD,J,NN,L)
      DT=0.05
      D1=DT
      JM=1
      JJ=1
      AT(1)=0.0
      TIME=0.0
      J=2
55    T(JJ)=TIME+DT
      NIS=NN-1
      L=1
      TIME=T(JJ)
C        AT REPRESENTS TIME IN MINUTES
      AT(JJ)=T(JJ)/60.0
60    DO 65 I=2,NIS
         DX2DT=DELX**2/DT
         IP=I+1
         IM=I-1
         DP=(DIFF(IP,L)-DIFF(IM,L))*0.25
         A(I)=DIFF(I,L)-DP
         B(I)=-2.0*DIFF(I,L)-DX2DT
         C(I)=DIFF(I,L)+DP
65    D(I)=.50*DELX*(CON(IP,L)-CON(IM,L))-W(I,JM)*DX2DT
      E(1)=0.0
      F(1)=W1(1)
      DO 70 I=2,NIS
         IM=I-1
         DIVID=B(I)+A(I)*E(IM)
         E(I)=-C(I)/DIVID
70    F(I)=(D(I)-A(I)*F(IM))/DIVID
      W(NN,J)=W(NN,1)
      DO 75 I=1,NIS
         K=NN-I
         KP=NN-I+1
         W(K,J)=F(K)+E(K)*W(KP,J)
         IF (W(K,J).GT.W(1,J)) W(K,J)=W(1,J)
75    CONTINUE
C        NOW HAVE DISTRIBUTION OF W FOR THIS TIME STEP SO REDIFINE
C        DIFF(I) AND K(I) AND TEST TO SEE IF ORIGIONAL ESTIMATE
C        WAS SUFFICIENT
      LM=L
      L=L+1
      TRY=0.0
      CALL RETEN (W,H,CON,DIFF,AK,DX,HEAD,J,NN,L)
      DO 80 I=1,NN
         IF (ABS(W(I,J)-W2(I)).GE.CRIT) TRY=1.
         W2(I)=W(I,J)
80    CONTINUE
C        IF THE SOLUTION HAS NOT CONVERGED AFTER 8 ITERATIONS, TIME
C        STEP IS REDUCED AND ANOTHER TRIAL IS MADE.
      IF (L.GE.8) GO TO 85
C        IF TRY=1 ANOTHER ITTERATION WILL BE MADE.
      IF (TRY.EQ.1.0) GO TO 60
      RATE(JJ)=(-DIFF(1,L)*(4.0*W(2,J)-3.0*W(1,J)-W(3,J))/(2.0*DELX)+CON
     1(1,L))*3600.0
      RAT1=(DIFF(1,L)*(W(1,J)-W(2,J))/DELX+CON(1,L))*3600.
      IF ((W(4,J)-0.10).LT.W1(4)) RATE(JJ)=RAT1
      IF (T(JJ).LT.60.0) RATE(JJ)=RAT1
      ACCIF(JJ)=AMOUNT+RATE(JJ)*DT/3600.
      AMOUNT=ACCIF(JJ)
C        RATE IN CM/HR
C        ACCIF IN CM
      ITTER=L-1
      XI=ITTER
      WRITE (6,190) AT(JJ),RATE(JJ),ACCIF(JJ),W(2,J),W(5,J),W(10,J),W(15
     1,J),W(20,J),W(25,J),W(30,J),W(35,J),W(75,J)
      GO TO 90
C        THE FOLLOWING STATEMENTS REDUCE DT BY 1/2 WHEN COULD NOT GET
C        CONVERGENCE.
C        ****************************************
85    CONTINUE
      TIME=TIME-DT
      DT=DT/2.0
      GO TO 55
C        ****************************************
90    CONTINUE
      IF (AT(JJ).GT.TSTOP) GO TO 105
      JJ=JJ+1
      DO 95 I=1,NN
         CON(I,1)=CON(I,L)
95    DIFF(I,1)=DIFF(I,L)
C        THE FOLLOWING STATEMENTS IMPLEMENT A PART SCANNER FEATURE.
C        THIS MEANS THAT THE ENTIRE COLUMN LENGTH WILL NOT BE CONSIDERED
C        IN THE SOLUTION FOR ALL OF THE TIME.
```

```
C        ****************************************
      IF (ABS((W(NNCK,J)-W1(NNCK))/W1(NNCK)).GT.0.001) NN=NN+5
      IF (NN.GT.NT) NN=NT
      DDT=5.0*(1.0-0.4*XI)
      DT=DT+DDT
      IF (DT.LE.0.0) DT=D1
      D1=DT
      IF (JJ.EQ.2) NN=NN/4
      NNCK=NN-5
C        ****************************************
      IF (J.EQ.1) GO TO 100
      JM=2
      J=1
      GO TO 55
100   JM=1
      J=2
      GO TO 55
105   CONTINUE
C        IF SEARCH IS NOT DESIRED, EXIT PROGRAM.
      IF (NEX.EQ.0.0) GO TO 170
C        THE FOLLOWING SECTION EVALUATES THE ABSOLUTE DIFFERENCE
C        BETWEEN THE EXPERIMINTAL RATES AND THE PREDICTED RATES
C        AND DETERMINES THE NEXT TRIAL VALUE OF GHI.
C        ****************************************
      ABSVOL(1)=0.0
      N=2
      TOTAL=ACCIF(1)
      DO 120 J=2,NEX
         DO 110 I=N,JJ
            IF (AT(I).GE.CLOCK(J)) GO TO 115
110      CONTINUE
115      APART=ACCIF(I-1)-TOTAL
         APART=APART+(CLOCK(J)-AT(I-1))/(AT(I)-AT(I-1))*(ACCIF(I)-ACCIF(
     1   I-1))
         EXPART=EXPVOL(J)-EXPVOL(J-1)
         ABSVOL(J)=ABS(EXPART-APART)
         TOTAL=TOTAL+APART
         N=I
120   CONTINUE
      SUM=0.0
      DO 125 K=1,NEX
125   SUM=SUM+ABSVOL(K)
      WRITE (6,235) SUM
      SS1=1.0
      R(KTRY)=SUM
      IF (KTRY.EQ.1) GO TO 130
      GO TO 135
130   XTRY(2)= XA+XXB-XB
      KTRY=KTRY+1
      RSMALL=R(1)
      GH1=XTRY(KTRY)
      GO TO 35
135   IF (R(KTRY).LT.RSMALL) GO TO 145
      IF (XTRY(KTRY).GT.XB) GO TO 140
      XXA=XTRY(KTRY)
      GO TO 160
140   XXB=XTRY(KTRY)
      GO TO 160
145   RSMALL=R(KTRY)
      IF (XTRY(KTRY).GT.XB) GO TO 150
      XXB=XB
      GO TO 155
150   XXA=XB
155   XB=XTRY(KTRY)
160   IF (KTRY.GE.NTRY) GO TO 165
      KTRY=KTRY+1
      XTRY(KTRY)=XXA+XXB-XB
      GH1=XTRY(KTRY)
      GO TO 35
C        ****************************************
165   WRITE (6,245) (XTRY(I),GS,GB,R(I),I=1,NTRY)
170   CONTINUE
      STOP
C
175   FORMAT (7(F4.2,F6.2))
180   FORMAT (F10.2,I10,2F10.2)
185   FORMAT (1H1)
190   FORMAT (5X,3F10.2,5X,9F10.3)
195   FORMAY (1H0/1H0,9X,4HTIME,6X,4HRATE,5X,6HVOLUME,48X,13HWATER CONTE
     1NT)
200   FORMAT (7X,9H(MINUTES),2X,7H(CM/HR),5X,4H(CM),12X,5H(X=2),5X,5H(X=
     15),5X,6H(X=10),4X,6H(X=15),4X,6H(X=20),4X,6H(X=25),4X,6H(X=30),4X,
     26H(X=35),4X,6H(X=45))
205   FORMAT (2I2)
210   FORMAT (3F10.2)
```

```
215   FORMAT  (1H1,19X,17HEXPERIMENTAL DATA)
220   FORMAT  (1H0/19X,4HTIME,6X,4HRATE,5X,6HVOLUME)
225   FORMAT  (17X,9H(MINUTES),2X,7H(CM/HR),5X,4H(CM))
230   FORMAT  (14X,3F10.2)
235   FORMAT  (1H0/1H0,15X,36HTHE ABSOLUTE DIFFERENCE BETWEEN THE /16X,50
     1HCALCULATED AND EXPERIMENTAL INFILTRATION CURVES IS,F10.4)
240   FORMAT  (2F10.2,I2)
245   FORMAT  (4F20.4)
      END


      SUBROUTINE RETEN (W,H,CON,DIFF,AK,DX,HEAD,J,NN,L)
C
C         THIS SUBROUTINE CONVERTS W ARRAY TO CORRESPONDING H,DIFF,AND
C         K ARRAYS.
C
      DIMENSION W(100,2), H(100), CON(100,10), DIFF(100,10), AK(750), DX
     1(750), HEAD(750)
      DO 5 I=1,NN
         AW=1000.0*W(I,J)
         IW=AW
         IF (IW.EQ.0) IW=1
         BW=IW
         IWP=IW+1
         COEFF=AW-BW
         H(I)=HEAD(IW)+COEFF*(HEAD(IWP)-HEAD(IW))
         DIFF(I,L)=DX(IW)+COEFF*(DX(IWP)-DX(IW))
         CON(I,L)=AK(IW)+COEFF*(AK(IWP)-AK(IW))
5     CONTINUE
      RETURN
      END


      SUBROUTINE SETUP (AK,HEAD,DX,SS1,GH1,GA,GB)
C
C         SUBROUTINE SETUP READS IN THE WATER RETENTION CURVE, COMPUTES
C         CONDUCTIVITIES AND DIFFUSIVITIES  AND PLACES THEM IN ARRAY
C         FORM SO THAT THEY CAN BE EASILY RECALLED IN SUBROUTINE RETEN.
C
      DIMENSION HEAD(750), AK(750), DX(750), DWDH(750), HH(100), TH(100)
     1, DS(100), CS(100), FC(101)
      IF (SS1.GT.0.0) GO TO 25
      READ (5,45) GH2,GA,GB
      READ (5,50) NUM
      READ (5,55) (TH(I),HH(I),I=1,NUM)
C         FIRST DETERMINE C AT EACH OF THE H-THETA POINTS READ IN.
      NUMM=NUM-1
      DO 5 I=2,NUMM
5     FC(I)=0.5*((TH(I+1)-TH(I))/(HH(I+1)-HH(I))+(TH(I)-TH(I-1))/(HH(I)-
     1HH(I-1)))
      FC(1)=(TH(2)-TH(1))/(HH(2)-HH(1))
      FC(NUM)=(TH(NUM)-TH(NUMM))/(HH(NUM)-HH(NUMM))
C     THE FOLLOWING SECTION INTERPOLATES TABLE READ ABOVE TO NEAREST .1
C     PERCENT WATER CONTENT AND ASSIGNS CORRESPONDING HEADS TO AN ARRAY
C     WHICH CAN BE CALLED BY USING 1000.0*W FOR THE SUBSCRIPT.
      I=1
      X1=1000.*TH(1)+0.00000001
      X2=1000.*TH(NUM)+0.00000001
      N1=X1
      N2=X2
      HEAD(N2)=HH(NUM)
      DWDH(N2)=FC(NUM)
      N2M=N2-1
      DO 20 J=N1,N2M
         AJ=J
         GO TO 15
10       I=I+1
15       CONTINUE
         ATH=1000.*TH(I)
         ATH1=1000.0*TH(I+1)
         IF (AJ.GE.ATH1) GO TO 10
         ITH=ATH
         ITH1=ATH1
         HEAD(J)=HH(I)+(AJ-ATH)*(HH(I+1)-HH(I))/(ATH1-ATH)
         A=(AJ-ATH)/(ATH1-ATH)
         DWDH(J)=FC(I)+A*(FC(I+1)-FC(I))
20    CONTINUE
      N1P=N1+1
25    CONTINUE
      CK1=(1.0/GH1)**GA
      DO 30 I=N1,N2
```

```
         AK(I)=1.0/(CK1*(-HEAD(I))**GA+GB)
30    DX(I)=AK(I)/DWDH(I)
      WRITE (6,40)
      WRITE (6,60) GH1,GA,GB
      WRITE (6,70)
      WRITE (6,75)
      WRITE (6,80)
      WRITE (6,65)
      WRITE (6,70)
      DO 35 I=N1,N2,10
         X=I
         AI=X/1000.0
35    WRITE (6,85) HEAD(I),AI,AK(I),DWDH(I))
      RETURN
C
40    FORMAT (1H1)
45    FORMAT (3E15.8)
50    FORMAT (I2)
55    FORMAT (F10.5,10X,F10.5)
60    FORMAT (15X,62HTHE RELATIONSHIP USED FOR CONDUCTIVITY VS. HEAD MAY
     1 BE WRITTEN//25X,9HK = ((H/ ,F5.2,5H )** ,F5.2,3H + ,F9.2,8H )**(-
     21))
65    FORMAT (1H /25X,4H(CM),15X,9H(PERCENT),12X,8H(CM/SEC),12X,8H(CM**-
     11))
70    FORMAT (1H0/1H0)
75    FORMAT (28X,24HTABLE OF SOIL PROPERTIES)
80    FORMAT (1H0/1H0/25X,4HHEAD,17X,5HTHETA,12X,12HCONDUCTIVITY,7X,14HW
     1ATER CAPACITY)
85    FORMAT (15X,4E20.8)
      END
```

## SAMPLE INPUT DATA

EXPERIMENTAL DATA

| TIME (MINUTES) | RATE (CM/HR) | VOLUME (CM) |
|---|---|---|
| 0.00 | 90.50 | 0.00 |
| .50 | 39.30 | .60 |
| 1.00 | 31.40 | .82 |
| 2.00 | 19.80 | 1.31 |
| 3.00 | 16.00 | 1.65 |
| 4.00 | 13.20 | 1.94 |
| 5.00 | 12.40 | 2.19 |
| 6.00 | 11.60 | 2.39 |
| 7.00 | 11.20 | 2.58 |
| 8.00 | 10.50 | 2.76 |
| 9.00 | 10.00 | 2.93 |
| 10.00 | 9.70 | 3.10 |
| 11.00 | 9.30 | 3.25 |
| 12.00 | 8.90 | 3.41 |
| 13.00 | 8.70 | 3.55 |
| 14.00 | 8.45 | 3.70 |
| 15.00 | 8.20 | 3.83 |
| 16.00 | 7.95 | 3.97 |
| 18.00 | 7.50 | 4.23 |
| 20.00 | 7.10 | 4.47 |
| 22.00 | 6.77 | 4.70 |
| 24.00 | 6.50 | 4.92 |
| 26.00 | 6.21 | 5.13 |
| 28.00 | 6.01 | 5.34 |
| 30.00 | 5.86 | 5.54 |

NEX = 25
NVOL = 07
ALEN = 60cm
NI = 60
TSTOP = 30 min
XXA = -6.0 cm
XXB = -8.0 cm
GA = 3.90
GB = 1385 sec/cm
NTRY = 08

Initial Water Content

| Z (cm) | Y ($cm^3/cm^3$) |
|---|---|
| 0.0 | 0.35 |
| 0.1 | 0.00 |
| 60.0 | 0.00 |

Tabular data for the soil-water characteristic are included in the output on the following page.

# SAMPLE OUTPUT

THE RELATIONSHIP USED FOR CONDUCTIVITY VS. HEAD MAY BE WRITTEN

K = ((H/ 7.24 )** 6.50 + 1385.00 )**(-1)

## TABLE OF SOIL PROPERTIES

| HEAD (CM) | THETA (PERCENT) | CONDUCTIVITY (CM/SEC) | WATER CAPACITY (CM**-1) |
|---|---|---|---|
| -1.45000000E+03 | 0. | 1.09112660E-15 | 2.00000000E-05 |
| -9.50000000E+02 | 1.00000000E-02 | 1.70436937E-14 | 2.00000000E-05 |
| -4.50000000E+02 | 2.00000000E-02 | 2.19222848E-12 | 1.01052632E-04 |
| -1.81000000E+02 | 3.00000000E-02 | 8.16311526E-10 | 2.58626566E-04 |
| -1.43000000E+02 | 4.00000000E-02 | 3.77642306E-09 | 4.92721805E-04 |
| -1.05000000E+02 | 5.00000000E-02 | 2.81202902E-08 | 7.26817043E-04 |
| -9.66000000E+01 | 6.00000000E-02 | 4.83489172E-08 | 1.03132832E-03 |
| -8.82000000E+01 | 7.00000000E-02 | 8.73310779E-08 | 1.33583960E-03 |
| -8.12000000E+01 | 8.00000000E-02 | 1.49473103E-07 | 1.56135531E-03 |
| -7.56000000E+01 | 9.00000000E-02 | 2.37811263E-07 | 1.70787546E-03 |
| -7.00000000E+01 | 1.00000000E-01 | 3.92097422E-07 | 1.85439560E-03 |
| -6.48000000E+01 | 1.10000000E-01 | 6.47351357E-07 | 1.99725275E-03 |
| -5.96000000E+01 | 1.20000000E-01 | 1.11428353E-06 | 2.14010989E-03 |
| -5.50000000E+01 | 1.30000000E-01 | 1.87620234E-06 | 2.37637363E-03 |
| -5.10000000E+01 | 1.40000000E-01 | 3.05996555E-06 | 2.70604396E-03 |
| -4.70000000E+01 | 1.50000000E-01 | 5.18799142E-06 | 3.03571429E-03 |
| -4.42000000E+01 | 1.60000000E-01 | 7.70658539E-06 | 3.44805519E-03 |
| -4.14000000E+01 | 1.70000000E-01 | 1.17261058E-05 | 3.85389610E-03 |
| -3.89000000E+01 | 1.80000000E-01 | 1.74372326E-05 | 4.10129870E-03 |
| -3.67000000E+01 | 1.90000000E-01 | 2.51780916E-05 | 4.18701299E-03 |
| -3.45000000E+01 | 2.00000000E-01 | 3.69915455E-05 | 4.27272727E-03 |
| -3.20000000E+01 | 2.10000000E-01 | 5.84308784E-05 | 5.57142857E-03 |
| -3.06000000E+01 | 2.20000000E-01 | 7.60722347E-05 | 5.35714286E-03 |
| -2.78000000E+01 | 2.30000000E-01 | 1.30078560E-04 | 3.78571429E-03 |
| -2.53000000E+01 | 2.40000000E-01 | 2.08290496E-04 | 4.38095238E-03 |
| -2.32000000E+01 | 2.50000000E-01 | 3.00306302E-04 | 4.88095238E-03 |
| -2.12000000E+01 | 2.60000000E-01 | 4.05273389E-04 | 4.28571429E-03 |
| -1.84000000E+01 | 2.70000000E-01 | 5.50639299E-04 | 3.86904762E-03 |
| -1.60000000E+01 | 2.80000000E-01 | 6.41525218E-04 | 4.58333333E-03 |
| -1.40000000E+01 | 2.90000000E-01 | 6.85891434E-04 | 4.58333333E-03 |
| -1.16000000E+01 | 3.00000000E-01 | 7.10990435E-04 | 4.00641026E-03 |
| -9.00000000E+00 | 3.10000000E-01 | 7.19875724E-04 | 4.42307692E-03 |
| -7.00000000E+00 | 3.20000000E-01 | 7.21601691E-04 | 5.00000000E-03 |
| -5.00000000E+00 | 3.30000000E-01 | 7.21974497E-04 | 5.00000000E-03 |
| -3.00000000E+00 | 3.40000000E-01 | 7.22019956E-04 | 4.16666667E-03 |
| -0. | 3.50000000E-01 | 7.22021661E-04 | 3.33333333E-03 |

| TIME (MINUTES) | RATE (CM/HR) | VOLUME (CM) | WATER CONTENT (X=2) | (X=5) | (X=10) | (X=15) | (X=20) | (X=25) | (X=30) | (X=35) | (X=60) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| .00 | 275.52 | .00 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| .16 | 130.62 | .48 | .186 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| .28 | 72.52 | .64 | .260 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| .41 | 65.02 | .79 | .270 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| .61 | 42.27 | .96 | .299 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| .68 | 36.46 | 1.01 | .307 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| .77 | 32.80 | 1.06 | .311 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| .88 | 31.31 | 1.12 | .313 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 1.02 | 30.66 | 1.19 | .314 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 1.15 | 30.29 | 1.26 | .314 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 1.26 | 29.32 | 1.31 | .316 | .005 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 1.39 | 26.14 | 1.37 | .320 | .026 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 1.54 | 21.02 | 1.43 | .323 | .056 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 1.76 | 20.66 | 1.50 | .324 | .114 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 1.94 | 20.38 | 1.56 | .325 | .169 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 2.08 | 20.28 | 1.61 | .325 | .220 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 2.14 | 20.23 | 1.63 | .326 | .233 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 2.25 | 20.10 | 1.67 | .326 | .240 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 2.42 | 18.89 | 1.72 | .328 | .244 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 2.69 | 17.33 | 1.80 | .330 | .250 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 2.89 | 16.60 | 1.86 | .330 | .255 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 3.08 | 16.08 | 1.91 | .331 | .263 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 3.26 | 15.47 | 1.96 | .332 | .268 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 3.51 | 14.75 | 2.02 | .333 | .272 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 3.80 | 14.14 | 2.09 | .334 | .276 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 4.05 | 13.67 | 2.15 | .334 | .279 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 4.16 | 13.47 | 2.17 | .334 | .281 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 4.23 | 13.33 | 2.19 | .335 | .282 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 4.44 | 12.90 | 2.24 | .335 | .285 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 4.79 | 12.31 | 2.31 | .336 | .287 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 5.13 | 11.90 | 2.38 | .336 | .289 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 5.37 | 11.64 | 2.42 | .337 | .291 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 5.48 | 11.51 | 2.45 | .337 | .292 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 5.66 | 11.29 | 2.48 | .337 | .293 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 5.99 | 10.90 | 2.54 | .338 | .295 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 6.37 | 10.61 | 2.61 | .338 | .297 | .001 | .000 | .000 | .000 | .000 | .000 | .000 |
| 6.71 | 10.41 | 2.67 | .339 | .298 | .009 | .000 | .000 | .000 | .000 | .000 | .000 |
| 6.84 | 10.33 | 2.69 | .339 | .299 | .019 | .000 | .000 | .000 | .000 | .000 | .000 |
| 6.97 | 10.25 | 2.71 | .339 | .299 | .033 | .000 | .000 | .000 | .000 | .000 | .000 |
| 7.19 | 10.08 | 2.75 | .339 | .301 | .059 | .000 | .000 | .000 | .000 | .000 | .000 |
| 7.60 | 9.76 | 2.82 | .340 | .302 | .117 | .000 | .000 | .000 | .000 | .000 | .000 |
| 7.99 | 9.53 | 2.88 | .340 | .303 | .181 | .000 | .000 | .000 | .000 | .000 | .000 |
| 8.31 | 9.36 | 2.93 | .340 | .304 | .223 | .000 | .000 | .000 | .000 | .000 | .000 |
| 8.48 | 9.28 | 2.96 | .340 | .305 | .228 | .000 | .000 | .000 | .000 | .000 | .000 |
| 8.66 | 9.19 | 2.98 | .341 | .305 | .229 | .000 | .000 | .000 | .000 | .000 | .000 |
| 8.85 | 9.08 | 3.01 | .341 | .306 | .229 | .000 | .000 | .000 | .000 | .000 | .000 |
| 9.17 | 8.91 | 3.06 | .341 | .307 | .233 | .000 | .000 | .000 | .000 | .000 | .000 |
| 9.60 | 8.75 | 3.12 | .341 | .308 | .241 | .000 | .000 | .000 | .000 | .000 | .000 |
| 9.97 | 8.64 | 3.18 | .341 | .309 | .251 | .000 | .000 | .000 | .000 | .000 | .000 |
| 10.14 | 8.59 | 3.20 | .341 | .309 | .255 | .000 | .000 | .000 | .000 | .000 | .000 |
| 10.35 | 8.52 | 3.23 | .342 | .309 | .257 | .000 | .000 | .000 | .000 | .000 | .000 |
| 10.53 | 8.47 | 3.26 | .342 | .310 | .257 | .000 | .000 | .000 | .000 | .000 | .000 |
| 10.70 | 8.41 | 3.28 | .342 | .310 | .258 | .000 | .000 | .000 | .000 | .000 | .000 |
| 11.07 | 8.29 | 3.33 | .342 | .311 | .259 | .000 | .000 | .000 | .000 | .000 | .000 |
| 11.57 | 8.16 | 3.40 | .342 | .312 | .263 | .000 | .000 | .000 | .000 | .000 | .000 |

Note: Above variables were printed out for every 4th time step.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11.97 | 8.06 | 3.45 | .342 | .313 | .267 | .000 | .000 | .000 | .000 | .000 | .000 |
| 12.18 | 8.01 | 3.48 | .342 | .313 | .268 | .000 | .000 | .000 | .000 | .000 | .000 |
| 12.35 | 7.96 | 3.50 | .342 | .314 | .269 | .000 | .000 | .000 | .000 | .000 | .000 |
| 12.57 | 7.90 | 3.53 | .343 | .314 | .269 | .000 | .000 | .000 | .000 | .000 | .000 |
| 12.77 | 7.85 | 3.56 | .343 | .314 | .270 | .000 | .000 | .000 | .000 | .000 | .000 |
| 13.18 | 7.75 | 3.61 | .343 | .315 | .271 | .000 | .000 | .000 | .000 | .000 | .000 |
| 13.62 | 7.66 | 3.67 | .343 | .316 | .273 | .000 | .000 | .000 | .000 | .000 | .000 |
| 13.77 | 7.63 | 3.69 | .343 | .316 | .274 | .000 | .000 | .000 | .000 | .000 | .000 |
| 13.93 | 7.60 | 3.71 | .343 | .316 | .275 | .000 | .000 | .000 | .000 | .000 | .000 |
| 14.15 | 7.55 | 3.74 | .343 | .317 | .276 | .000 | .000 | .000 | .000 | .000 | .000 |
| 14.37 | 7.51 | 3.76 | .343 | .317 | .277 | .000 | .000 | .000 | .000 | .000 | .000 |
| 14.61 | 7.46 | 3.79 | .343 | .317 | .277 | .000 | .000 | .000 | .000 | .000 | .000 |
| 15.01 | 7.37 | 3.84 | .343 | .318 | .278 | .001 | .000 | .000 | .000 | .000 | .000 |
| 15.58 | 7.26 | 3.91 | .343 | .319 | .280 | .014 | .000 | .000 | .000 | .000 | .000 |
| 15.82 | 7.22 | 3.94 | .344 | .319 | .281 | .028 | .000 | .000 | .000 | .000 | .000 |
| 16.05 | 7.19 | 3.97 | .344 | .319 | .281 | .045 | .000 | .000 | .000 | .000 | .000 |
| 16.25 | 7.15 | 3.99 | .344 | .319 | .282 | .063 | .000 | .000 | .000 | .000 | .000 |
| 16.50 | 7.11 | 4.02 | .344 | .320 | .283 | .087 | .000 | .000 | .000 | .000 | .000 |
| 16.76 | 7.06 | 4.05 | .344 | .320 | .284 | .114 | .000 | .000 | .000 | .000 | .000 |
| 17.11 | 7.00 | 4.09 | .344 | .321 | .284 | .157 | .000 | .000 | .000 | .000 | .000 |
| 17.63 | 6.92 | 4.16 | .344 | .321 | .285 | .209 | .000 | .000 | .000 | .000 | .000 |
| 17.96 | 6.88 | 4.19 | .344 | .321 | .286 | .224 | .000 | .000 | .000 | .000 | .000 |
| 18.22 | 6.84 | 4.22 | .344 | .322 | .287 | .225 | .000 | .000 | .000 | .000 | .000 |
| 18.44 | 6.82 | 4.25 | .344 | .322 | .288 | .226 | .000 | .000 | .000 | .000 | .000 |
| 18.70 | 6.78 | 4.28 | .344 | .322 | .288 | .227 | .000 | .000 | .000 | .000 | .000 |
| 18.97 | 6.75 | 4.31 | .344 | .322 | .289 | .228 | .000 | .000 | .000 | .000 | .000 |
| 19.27 | 6.71 | 4.34 | .344 | .323 | .289 | .231 | .000 | .000 | .000 | .000 | .000 |
| 19.73 | 6.65 | 4.39 | .344 | .323 | .290 | .239 | .000 | .000 | .000 | .000 | .000 |
| 20.09 | 6.60 | 4.43 | .344 | .324 | .291 | .246 | .000 | .000 | .000 | .000 | .000 |
| 20.32 | 6.58 | 4.46 | .345 | .324 | .291 | .249 | .000 | .000 | .000 | .000 | .000 |
| 20.55 | 6.55 | 4.48 | .345 | .324 | .291 | .251 | .000 | .000 | .000 | .000 | .000 |
| 20.81 | 6.52 | 4.51 | .345 | .324 | .292 | .252 | .000 | .000 | .000 | .000 | .000 |
| 21.09 | 6.49 | 4.54 | .345 | .324 | .293 | .253 | .000 | .000 | .000 | .000 | .000 |
| 21.39 | 6.46 | 4.57 | .345 | .325 | .293 | .253 | .000 | .000 | .000 | .000 | .000 |
| 21.79 | 6.41 | 4.62 | .345 | .325 | .294 | .255 | .000 | .000 | .000 | .000 | .000 |
| 22.32 | 6.36 | 4.67 | .345 | .326 | .294 | .258 | .000 | .000 | .000 | .000 | .000 |
| 22.53 | 6.34 | 4.69 | .345 | .326 | .295 | .260 | .000 | .000 | .000 | .000 | .000 |
| 22.77 | 6.31 | 4.72 | .345 | .326 | .295 | .261 | .000 | .000 | .000 | .000 | .000 |
| 23.04 | 6.29 | 4.75 | .345 | .326 | .295 | .263 | .000 | .000 | .000 | .000 | .000 |
| 23.34 | 6.26 | 4.78 | .345 | .326 | .296 | .263 | .000 | .000 | .000 | .000 | .000 |
| 23.50 | 6.24 | 4.80 | .345 | .326 | .296 | .264 | .000 | .000 | .000 | .000 | .000 |
| 23.86 | 6.21 | 4.83 | .345 | .327 | .297 | .264 | .000 | .000 | .000 | .000 | .000 |
| 24.29 | 6.17 | 4.88 | .345 | .327 | .297 | .265 | .000 | .000 | .000 | .000 | .000 |
| 24.75 | 6.13 | 4.93 | .345 | .327 | .298 | .267 | .000 | .000 | .000 | .000 | .000 |
| 25.03 | 6.10 | 4.95 | .345 | .328 | .298 | .268 | .000 | .000 | .000 | .000 | .000 |
| 25.32 | 6.08 | 4.98 | .345 | .328 | .298 | .269 | .000 | .000 | .000 | .000 | .000 |
| 25.62 | 6.05 | 5.01 | .345 | .328 | .299 | .270 | .000 | .000 | .000 | .000 | .000 |
| 25.83 | 6.04 | 5.03 | .345 | .328 | .299 | .270 | .000 | .000 | .000 | .000 | .000 |
| 26.20 | 6.01 | 5.07 | .345 | .328 | .299 | .271 | .001 | .000 | .000 | .000 | .000 |
| 26.57 | 5.98 | 5.11 | .345 | .328 | .300 | .271 | .004 | .000 | .000 | .000 | .000 |
| 27.15 | 5.93 | 5.17 | .345 | .329 | .300 | .272 | .021 | .000 | .000 | .000 | .000 |
| 27.46 | 5.91 | 5.20 | .346 | .329 | .301 | .273 | .038 | .000 | .000 | .000 | .000 |
| 27.76 | 5.89 | 5.23 | .346 | .329 | .301 | .274 | .058 | .000 | .000 | .000 | .000 |
| 28.05 | 5.87 | 5.25 | .346 | .329 | .301 | .274 | .080 | .000 | .000 | .000 | .000 |
| 28.36 | 5.84 | 5.28 | .346 | .329 | .302 | .275 | .106 | .000 | .000 | .000 | .000 |
| 28.63 | 5.82 | 5.31 | .346 | .330 | .302 | .276 | .130 | .000 | .000 | .000 | .000 |
| 29.00 | 5.80 | 5.35 | .346 | .330 | .302 | .276 | .168 | .000 | .000 | .000 | .000 |
| 29.40 | 5.77 | 5.39 | .346 | .330 | .303 | .277 | .201 | .000 | .000 | .000 | .000 |
| 29.87 | 5.74 | 5.43 | .346 | .330 | .303 | .277 | .220 | .000 | .000 | .000 | .000 |

The area between the calculated and experimental infiltration curves is 0.834 cm. The next extimate of GH1 is -6.76 cm.

INFIL3

INFIL3 was developed to solve the h-based form of the Richards equation for infiltration into a layered soil with arbitrary surface and initial conditions. Approximately 30,000 words of memory are required to process this program.

Input data consist of: (1) basic variables describing the system, the depth of each layer, the total number of depth increments, the real time at which the solution starts (used when it is necessary to start the solution with an initial head distribution from the θ-based solution due to convergence problems), and the time for which a solution is desired; (2) the initial head distribution; and (3) the soil properties for both soils.

The properties for the two soils are read and placed in arrays by the subroutine SETUP. Tabular data are read in for the water content and pressure head. The conductivity can be input in tabular form, as the coefficients in Gardner's equation, or it can be computed from tabular data for diffusivity. SETUP computes the water capacity and places it along with water content and hydraulic conductivity in an array which can be recalled by integer values of pressure for pressure heads from 0 to -2000 cm of water for each soil.

Subroutine RETEN takes the pressure head distribution at a given time (an array) and recalls the corresponding water contents, water capacities, and hydraulic conductivities for each soil.

A sample of input data and output produced is given with the program listing. Some of the programming symbols are the same as was used in INFIL2 and have already been defined. The remainder of the physically significant symbols are defined in the following list.

Programming Symbols

| | |
|---|---|
| AK | Hydraulic conductivity -- an array that can be recalled for integer pressure heads (cm/hr). |
| C | Water capacity ($cm^{-1}$). |
| CC | Coefficient ϵ defined in equation 26 ($cm^2$/sec). |
| CO1 | Hydraulic conductivity in the top layer corresponding to the pressure head at the layer junction (cm/hr). |

| | |
|---|---|
| CO2 | Hydraulic conductivity in the bottom layer corresponding to the pressure head at the layer junction (cm/hr). |
| DEEP1 | Depth of top layer (cm). |
| DEEP2 | Depth of bottom layer (cm). |
| DEPTH | Length of layered column (cm). |
| DER | Derivative of pressure head with respect to water content -- an array stored with corresponding pressure heads for rapid recall (cm). |
| DPDW | Derivative of pressure head with respect to water content -- an array computed from the input soil-water characteristic data and used to define DER by interpolation (cm). |
| GA,GB,GH1 | Parameters in Gardner's equation for the top layer. |
| H1 | Pressure head for previous iteration (cm). |
| H2 | Initial pressure head (cm). |
| N1S | Node above which saturated conditions exist. |
| NN1 | Node at the layer junction. |
| NUM1 | Number of soil property data points for layer 1. |
| NUM2 | Number of soil property data points for layer 2. |
| PRES | Pressure head data input in tabular form (cm). |
| TSTART | Time at start of solution (minutes). |
| THETA | Water content array stored with corresponding pressure heads for rapid recall ($cm^3/cm^3$). |
| WATER | Water content ($cm^3/cm^3$). |
| WAT | Water content data input in tabular form ($cm^3/cm^3$). |
| WA,WB,WH1 | Parameters in Gardner's equation for the bottom layer. |

```
C                          R. W. SKAGGS ** 04/10/70
C                                  INFIL3
      PROGRAM INFIL3(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT)
C
C        A PROGRAM TO SOLVE THE H-BASED RICHARDS EQUATION FOR INFILTRATION
C        INTO A TWO-LAYERED SOIL WITH ARBITRARY BOUNDARY AND INITIAL
C        CONDITIONS.
C
      DIMENSION Z(50), Y(50), X(101), H(101,2), C(101), CON(101), A(101)
     1, B(101), CC(101), D(101), E(101), F(101), RATE(2000), AT(2000), T
     2(2000)
      DIMENSION THETA(4000), DER(4000), AK(4000), WATER(101), H2(101), A
     1CCIF(2000)
      CRIT=0.01
C        READ IN AND ARRANGE SOIL PROPERTY  DATA.
      CALL SETUP (THETA,DER,AK)
C        READ IN NO. OF INCREMENTS, LAYER DEPTHS, AND TIMES FOR START AN
C        STOP.
      READ (5,95) NI,DEEP1,DEEP2,TSTOP,TSTART
C        READ IN INITIAL HEAD DISTRIBUTION
      K=1
      L=8
10    READ (5,100) (Z(I),Y(I),I=K,L)
      IF (Z(L).EQ.0.0) GO TO 15
      K=K+8
      L=L+8
      GO TO 10
15    CONTINUE
      DEPTH=DEEP1+DEEP2
      J=1
      L=1
      ANI=NI
      NN=NI+1
      NT=NN
      DELX=DEPTH/NI
      NN1=DEEP1/DELX+1.0
C        DEFINE THE POSITION X
      X(1)=0.0
      DO 20 I=2,NN
20    X(I)=X(I-1)+DELX
C        DETERMINE THE PRESSURE HEAD AT EACH NODE
      M=1
      H(1,J)=Y(1)
      DO 25 I=2,NN
         IF (Z(M+1).LT.X(I)) M=M+1
25    H(I,J)=((X(I)-Z(M))/(Z(M+1)-Z(M))*(Y(M+1)-Y(M))+Y(M))
      NS=1
      CALL RETEN (THETA,DER,AK,H,C,CON,NN1,NN,J,NS,WATER,CO1,CO2)
      DT=0.01
      JM=1
      JJ=1
      AT(1)=0.0
      TIME=TSTART
      WRITE (6,105)
      WRITE (6,120) TIME,AT(JJ)
      WRITE (6,110)
      WRITE (6,130)
      WRITE (6,115) (X(I),H(I,J),WATER(I),I=1,NN)
      DO 30 I=1,NN
30    H2(I)=H(I,JM)
      J=2
35    T(JJ)=TIME+DT
      KP=JJ/20
      KPRINT=KP*20
      NIS=NN-1
      L=1
      TIME=T(JJ)
      AT(JJ)=T(JJ)/60.0
40    DO 45 I=2,NI
         AP=CON(I)*DT/DELX**2
         BP=DT*(CON(I+1)-CON(I-1))/(4.0*DELX**2)
         A(I)=BP-AP
         B(I)=2.0*AP+C(I)
         CC(I)=-AP-BP
45    D(I)=C(I)*H(I,JM)-2.0*DELX*BP
      A(NN1)=CO1
      CC(NN1)=CO2
      B(NN1)=-(CO1+CO2)
      D(NN1)=(CO2-CO1)*DELX
      NNP=NN1+1
      BP=DT*(CON(NNP+1)-CO2)/(4.0*DELX**2)
      AP=CON(NNP)*DT/DELX**2
      A(NNP)=BP-AP
      B(NNP)=2.0*AP+C(NNP)
```

```
      CC(NNP)=-AP-BP
      D(NNP)=C(NNP)*H(NNP,JM)-2.0*DELX*BP
      NNM=NN1-1
      BP=DT*(CO1-CON(NNM-1))/(4.0*DELX**2)
      AP=CON(NNM)*DT/DELX**2
      A(NNM)=BP-AP
      B(NNM)=2.0*AP+C(NNM)
      CC(NNM)=-AP-BP
      D(NNM)=C(NNM)*H(NNM,JM)-2.0*DELX*BP
      E(1)=0
      F(1)=H(1,JM)
      DO 50 I=2,NI
         DIVID=B(I)+A(I)*E(I-1)
         E(I)=-CC(I)/DIVID
50    F(I)=(D(I)-A(I)*F(I-1))/DIVID
      H(NN,J)=H(NN,JM)
      H(1,J)=H(1,JM)
      DO 55 I=1,NI
         K=NN-I
         MK=K+1
55    H(K,J)=E(K)*H(MK,J)+F(K)
C        NOW HAVE DISTRIBUTION OF H FOR THE TIME STEP,NEXT CHECK THESE
C        VALUES AGAINST THOSE THAT WERE ASSUMED TO EVALUATE THE PARAMETE
C        CON AND C. IF THE PERCENT DIFFERENCE IS GREATER THAN CRIT,
C        RECOMPUTE.
      L=L+1
      TRY=0.0
      NS=1
      DO 65 I=2,NN
         IF (H(I,J).EQ.0.0) GO TO 60
         CHECK=(H(I,J)-H2(I))/H(I,J)
         CHECK=ABS(CHECK)
         IF (CHECK.GT.CRIT) TRY=1.0
60       CONTINUE
         H2(I)=H(I,J)
         IF (H(I,J).GT.0.0) NS=I
65    CONTINUE
C        IF TRY IS GREATER THAN 0.0 ANOTHER ITERATION WILL BE MADE.
      CALL RETEN (THETA,DER,AK,H,C,CON,NN1,NN,J,NS,WATER,CO1,CO2)
      IF (L.GE.8) GO TO 75
      IF (TRY.EQ.1.) GO TO 40
      DO 70 I=2,NN
70    CONTINUE
      RATE(JJ)=-CON(1)*(4.0*H(2,J)-3.0*H(1,J)-H(3,J)-2.0*DELX)/(2.*DELX)
     1*3600.
      RAT1=-CON(1)*(H(2,J)-H(1,J)-DELX)/DELX*3600.0
      ACCIF(JJ)=AMOUNT+RATE(JJ)*DT/3600.
      AMOUNT=ACCIF(JJ)
C        RATE  IN CM/HR , ACCIF IN CM.
      ITTER=L-1
      WI=ITTER
      IF (KPRINT.NE.JJ) GO TO 80
      WRITE (6,105)
      WRITE (6,120) AT(JJ)
      WRITE (6,125) RATE(JJ)
      WRITE (6,110)
      WRITE (6,130)
      WRITE (6,115) (X(I),H(I,J),WATER(I),I=1,NN)
      GO TO 80
C        THE FOLLOWING REDUCES DT WHEN COULD NOT GET CONVERGENCE ABOVE.
75    TIME=TIME-DT
      DT=DT/2.0
      GO TO 35
80    CONTINUE
      IF (AT(JJ).GE.TSTOP) GO TO 90
      JJ=JJ+1
      DDT=1.0*(1.0-0.4*WI)
      DT=DT+DDT
      IF (DT.LT.0.0) DT=D1
      D1=DT
      IF (J.EQ.1) GO TO 85
      JM=2
      J=1
      GO TO 35
85    JM=1
      J=2
      GO TO 35
90    CONTINUE
      WRITE (6,105)
      WRITE (6,135)
      WRITE (6,110)
      WRITE (6,140)
      WRITE (6,145)
      WRITE (6,150)
      WRITE (6,155) (AT(I),T(I),ACCIF(I),RATE(I),I=1,JJ)
```

```
      STOP
C
95    FORMAT (I3,7X,4F10.5)
100   FORMAT (8(F4.2,F6.0))
105   FORMAT (1H1)
110   FORMAT (1H0/1H0)
115   FORMAT (20X,3F20.3)
120   FORMAT (15X,46HDISTRIBUTION OF PRESSURE AND WATER CONTENT FOR//15X
     1,5HTIME=,F10.4,7HMINUTES)
125   FORMAT (1H0/1H0/25X,24HTHE INFILTRATION RATE IS,F10.4)
130   FORMAT (28X,8HPOSITION,12X,8HPRESSURE,9X,13HWATER CONTENT)
135   FORMAT (34X,51HA TABLE OF INFILTRATION VOLUME AND RATE VERSUS TIME
     1)
140   FORMAT (39X,4HTIME,32X,12HINFILTRATION)
145   FORMAT (32X,7HMINUTES,13X,7HSECONDS,13X,6HVOLUME,15X,4HRATE)
150   FORMAT (69X,13HCENTIMETERS,10X,7H(CM/HR))
155   FORMAT (15X,4F20.2)
      END


      SUBROUTINE SETUP (THETA,DER,AK)
      DIMENSION THETA(4000), DER(4000), AK(4000), WAT(101), PRES(101), D
     1PDW(101), COND(101), D(101)
      READ (5,65) NUM1,READ
C         IF READ =0, DIFFUSIVITY VALUES WILL BE READ IN, AND
C         CONDUCTIVITY VALUES COMPUTED FROM THEM.  IF READ
C         IS GREATER THAN 0.0 COND. VALUES WILL BE COMPUTED FROM
C         GARDNERS EQUATION.
      READ (5,70) (WAT(I),D(I),PRES(I),I=1,NUM1)
C         SET UP A TABLE OF WAT,PRES,DPDW,AND,IF READ=0,COND, AND LINEARL
C         INTERPOLATE FOR VALUES OF PARAMETERS ASSOCIATED WITH INTERGER
C         VALUES OF PRESSURE.
C         FIRST SHIFT THE ORIGN SO THAT H=-1 WHEN SATURATED.
      DO 5 I=1,NUM1
5     PRES(I)=PRES(I)-1.
      NUMM1=NUM1-1
      DO 10 I=2,NUMM1
10    DPDW(I)=((PRES(I+1)-PRES(I))/(WAT(I+1)-WAT(I))+(PRES(I)-PRES(I-1))
     1/(WAT(I)-WAT(I-1)))*0.5
      DPDW(1)=(PRES(2)-PRES(1))/(WAT(2)-WAT(1))
      DPDW(NUM1)=(PRES(NUM1)-PRES(NUMM1))/(WAT(NUM1)-WAT(NUMM1))
C         LINEAR INTERPOLATION FOLLOWS
      J=1
      DO 15 I=1,2000
         PI=-I
         IF (PI.LE.PRES(J)) J=J+1
         COEFF=(PI-PRES(J-1))/(PRES(J)-PRES(J-1))
         THETA(I)=WAT(J-1)+COEFF*(WAT(J)-WAT(J-1))
         DER(I)=DPDW(J-1)+COEFF*(DPDW(J)-DPDW(J-1))
         DIFF=D(J-1)+COEFF*(D(J)-D(J-1))
15    AK(I)=DIFF/DER(I)
      IF (READ.EQ.0) GO TO 25
      READ (5,75) GH1,GA,GB
      CK1=(1.0/GH1)**GA
      DO 20 I=1,2000
         AI=I-1
20    AK(I)=1.0/(CK1*AI**GA+GB)
25    CONTINUE
      WRITE (6,80)
      IF (READ.GT.0) WRITE (6,85) GH1,GA,GB
      WRITE (6,90)
      WRITE (6,115)
      WRITE (6,95)
      WRITE (6,100)
      WRITE (6,120)
      WRITE (6,90)
      DO 30 I=1,2000,10
         AI=-I+1
         DIFF=AK(I)*DER(I)
         CAP=1.0/DER(I)
30    WRITE (6,105) AI,THETA(I),AK(I),CAP
C         NOW READ IN THE PROPERTIES OF LAYER 2.
      READ (5,65) NUM2,READ
      READ (5,70) (WAT(I),D(I),PRES(I),I=1,NUM2)
C         SET UP A TABLE AS WAS DONE FOR LAYER 1
C         FOR THIS CASE SHIFT THE ORGIN SO THAT H=-2001 WHEN SATURATED.
      DO 35 I=1,NUM2
35    PRES(I)=PRES(I)-2001.
      NUMM2=NUM2-1
      DO 40 I=2,NUMM2
40    DPDW(I)=((PRES(I+1)-PRES(I))/(WAT(I+1)-WAT(I))+(PRES(I)-PRES(I-1))
     1/(WAT(I)-WAT(I-1)))*0.5
```

```
      DPDW(1)=(PRES(2)-PRES(1))/(WAT(2)-WAT(1))
      DPDW(NUMM2)=(PRES(NUM2)-PRES(NUMM2))/(WAT(NUM2)-WAT(NUMM2))
C        NOW LINEARLY INTERPOLATE AND SHIFT ORGIN SO THAT H=-2001 WHEN
C        HAVE SATURATION.
      J=1
      DO 45 I=2001,4000
         PI=-I
         IF (PI.LE.PRES(J)) J=J+1
         COEFF=(PI-PRES(J-1))/(PRES(J)-PRES(J-1))
         THETA(I)=WAT(J-1)+COEFF*(WAT(J)-WAT(J-1))
         DER(I)=DPDW(J-1)+COEFF*(DPDW(J)-DPDW(J-1))
         DIFF=D(J-1)+COEFF*(D(J)-D(J-1))
45    AK(I)=DIFF/DER(I)
      IF (READ.EQ.0) GO TO 55
      READ (5,75) WH1,WA,WB
      CK2=(1.0/WH1)**WA
      DO 50 I=2001,4000
         AI=I
50    AK(I)=1.0/(CK2*AI**WA+WB)
55    CONTINUE
      WRITE (6,80)
      IF (READ.GT.0) WRITE (6,85) WH1,WA,WB
      WRITE (6,90)
      WRITE (6,110)
      WRITE (6,95)
      WRITE (6,100)
      WRITE (6,120)
      WRITE (6,90)
      DO 60 I=2001,4000,10
         AI=-I+2001
         DIFF=AK(I)*DER(I)
         CAP=1.0/DER(I)
60    WRITE (6,105) AI,THETA(I),AK(I),CAP
      RETURN
65    FORMAT (I2,F5.2)
70    FORMAT (3E10.2)
75    FORMAT (3F10.4)
80    FORMAT (1H1)
85    FORMAT (15X,62HTHE RELATIONSHIP USED FOR CONDUCTIVITY VS. HEAD MAY
     1 BE WRITTEN//25X,9HK = ((H/ ,F5.2,5H )** ,3H + ,F9.2,8H )**(-1))
90    FORMAT (1H0/1H0)
95    FORMAT (22X,30HA TABLE OF THE SOIL PROPERTIES)
100   FORMAT (1H0/1H0/25X,4HHEAD,17X,5HTHETA,12X,12HCONDUCTIVITY,7X,14HW
     1ATER CAPACITY)
105   FORMAT (15X,4E20.3)
110   FORMAT (45X,7HLAYER 2)
115   FORMAT (45X,7HLAYER 1)
      END
```

```
      SUBROUTINE RETEN (THETA,DER,AK,H,C,CON,NN1,NN,J,NS,WATER,CO1,CO2)
C
C        THIS SUBROUTINE USES THE H ARRAYS TO OBTAIN CORRESPONDING
C        THETA, C, AND K ARRAYS FOR BOTH LAYERS.
C
      DIMENSION THETA(4000), DER(4000), AK(4000), C(101), CON(101), H(10
     11,2), WATER(101)
C        DEFINE H AS A POSITIVE QUANITY WITH THE ORGIN AT H= 1 FOR SAT.
      DO 5 I=1,NN1
5     H(I,J)=-H(I,J)+1.0
      IF (NS.GT.1) GO TO 20
10    DO 15 I=NS,NN1
         AH=H(I,J)
         IF (H(I,J).LT.1.0) H(I,J)=1.0
         IH=H(I,J)
         HI=IH
         WATER(I)=THETA(IH)+(H(I,J)-HI)*(THETA(IH+1)-THETA(IH))
         DHDT=DER(IH)+(H(I,J)-HI)*(DER(IH+1)-DER(IH))
         C(I)=1.0/DHDT
         CON(I)=AK(IH)+(H(I,J)-HI)*(AK(IH+1)-AK(IH))
15    H(I,J)=AH
      GO TO 30
20    DO 25 I=1,NN1
         WATER(I)=THETA(1)
         C(I)=0.0
25    CON(I)=AK(1)
      IF (NS.LT.NN1) GO TO 10
30    CC1=CON(NN1)
      C1=C(NN1)
      DO 35 I=1,NN1
35    H(I,J)=-H(I,J)+1.
C        FOR LAYER 2 DEFINE ORGIN AT H=2001
```

```
      DO 40 I=NN1,NN
40    H(I,J)=-H(I,J)+2001.
      MM=NN1
      IF (NS.GT.MM) MM=NS
      DO 45 I=MM,NN
         IH=H(I,J)
         HI=IH
         COEF=H(I,J)-HI
         WATER(I)=THETA(IH)+COEF*(THETA(IH+1)-THETA(IH))
         DHDT=DER(IH)+COEF*(DER(IH+1)-DER(IH))
         C(I)=1.0/DHDT
45    CON(I)=AK(IH)+COEF*(AK(IH+1)-AK(IH))
      IF (NS.LT.NN1) GO TO 55
      DO 50 I=NN1,NS
         WATER(I)=THETA(2001)
         C(I)=0.0
50    CON(I)=AK(2001)
55    CO2=CON(NN1)
      C2=C(NN1)
      DO 60 I=NN1,NN
60    H(I,J)=-H(I,J)+2001.
      RETURN
C
      END
```

## SAMPLE INPUT DATA

The sample input data and computer output were obtained for infiltration into a layered soil made up of 10 cm of Sarpy Loam over 39 cm of Geary silt loam. The soil properties used were given by Hanks and Bowers (1962). Other input data are given below.

NI = 98
DEEP1 = 10 cm
DEEP2 = 39 cm
TSTOP = 50 min
TSTART = 0

Initial Head Distribution

| Z (cm) | Y (cm) |
|---|---|
| 0.0 | 0.0 |
| 0.1 | -447 |
| 49.0 | -447 |

## SAMPLE OUTPUT

DISTRIBUTION OF PRESSURE AND WATER CONTENT FOR

TIME= 3.3200MINUTES

THE INFILTRATION RATE IS 16.8353 CM/HR

| POSITION-CM | PRESSURE | WATER CONTENT |
|---|---|---|
| 0.000 | 0.000 | .410 |
| .500 | -1.178 | .406 |
| 1.000 | -2.363 | .402 |
| 1.500 | -3.579 | .398 |
| 2.000 | -4.881 | .394 |
| 2.500 | -6.307 | .389 |
| 3.000 | -7.847 | .385 |
| 3.500 | -9.487 | .381 |
| 4.000 | -11.191 | .377 |
| 4.500 | -12.897 | .373 |
| 5.000 | -14.685 | .368 |
| 5.500 | -17.043 | .362 |
| 6.000 | -21.141 | .352 |
| 6.500 | -28.034 | .335 |
| 7.000 | -38.569 | .311 |
| 7.500 | -54.856 | .276 |
| 8.000 | -79.275 | .239 |
| 8.500 | -233.461 | .150 |
| 9.000 | -431.872 | .102 |
| 9.500 | -446.792 | .100 |
| 10.000 | -447.716 | .288 |
| 10.500 | -447.333 | .288 |
| 11.000 | -447.124 | .288 |
| 11.500 | -447.038 | .288 |
| 12.000 | -447.010 | .288 |
| 12.500 | -447.002 | .288 |
| 13.000 | -447.000 | .288 |
| 13.500 | -447.000 | .288 |
| 14.000 | -447.000 | .288 |
| 14.500 | -447.000 | .288 |
| 15.000 | -447.000 | .288 |
| 15.500 | -447.000 | .288 |
| 16.000 | -447.000 | .288 |
| 16.500 | -447.000 | .288 |
| 17.000 | -447.000 | .288 |
| 17.500 | -447.000 | .288 |
| 18.000 | -447.000 | .288 |
| 18.500 | -447.000 | .288 |
| 19.000 | -447.000 | .288 |
| 19.500 | -447.000 | .288 |
| 20.000 | -447.000 | .288 |
| 20.500 | -447.000 | .288 |
| 21.000 | -447.000 | .288 |
| 21.500 | -447.000 | .288 |
| 22.000 | -447.000 | .288 |
| 22.500 | -447.000 | .288 |
| 23.000 | -447.000 | .288 |
| 23.500 | -447.000 | .288 |
| 24.000 | -447.000 | .288 |
| 24.500 | -447.000 | .288 |
| 25.000 | -447.000 | .288 |
| 25.500 | -447.000 | .288 |

DISTRIBUTION OF PRESSURE AND WATER CONTENT FOR

TIME= 13.9441MINUTES

THE INFILTRATION RATE IS 9.2661 CM/HR

| POSITION-CM | PRESSURE | WATER CONTENT |
|---|---|---|
| 0.000 | 0.000 | .410 |
| .500 | -.424 | .409 |
| 1.000 | -.847 | .407 |
| 1.500 | -1.269 | .406 |
| 2.000 | -1.688 | .404 |
| 2.500 | -2.100 | .403 |
| 3.000 | -2.506 | .402 |
| 3.500 | -2.902 | .400 |
| 4.000 | -3.293 | .399 |
| 4.500 | -3.686 | .398 |
| 5.000 | -4.082 | .396 |
| 5.500 | -4.477 | .395 |
| 6.000 | -4.869 | .394 |
| 6.500 | -5.252 | .392 |
| 7.000 | -5.624 | .391 |
| 7.500 | -5.980 | .390 |
| 8.000 | -6.314 | .389 |
| 8.500 | -6.619 | .388 |
| 9.000 | -6.892 | .388 |
| 9.500 | -7.127 | .387 |
| 10.000 | -7.322 | .445 |
| 10.500 | -14.771 | .430 |
| 11.000 | -23.309 | .417 |
| 11.500 | -34.162 | .402 |
| 12.000 | -48.527 | .387 |
| 12.500 | -70.587 | .373 |
| 13.000 | -99.600 | .357 |
| 13.500 | -138.244 | .342 |
| 14.000 | -192.451 | .325 |
| 14.500 | -256.573 | .310 |
| 15.000 | -322.413 | .301 |
| 15.500 | -373.697 | .295 |
| 16.000 | -406.855 | .292 |
| 16.500 | -425.777 | .290 |
| 17.000 | -435.851 | .289 |
| 17.500 | -441.117 | .288 |
| 18.000 | -443.884 | .288 |
| 18.500 | -445.351 | .288 |
| 19.000 | -446.132 | .288 |
| 19.500 | -446.547 | .288 |
| 20.000 | -446.767 | .288 |
| 20.500 | -446.881 | .288 |
| 21.000 | -446.941 | .288 |
| 21.500 | -446.971 | .288 |
| 22.000 | -446.986 | .288 |
| 22.500 | -446.993 | .288 |
| 23.000 | -446.997 | .288 |
| 23.500 | -446.999 | .288 |
| 24.000 | -446.999 | .288 |
| 24.500 | -447.000 | .288 |
| 25.000 | -447.000 | .288 |
| 25.500 | -447.000 | .288 |

DISTRIBUTION OF PRESSURE AND WATER CONTENT FOR

TIME= 38.8810MINUTES

THE INFILTRATION RATE IS 1.9945 CM/HR

| POSITION-CM | PRESSURE | WATER CONTENT |
|---|---|---|
| 0.000 | 0.000 | .410 |
| .500 | .301 | .410 |
| 1.000 | .602 | .410 |
| 1.500 | .904 | .410 |
| 2.000 | 1.205 | .410 |
| 2.500 | 1.506 | .410 |
| 3.000 | 1.807 | .410 |
| 3.500 | 2.108 | .410 |
| 4.000 | 2.410 | .410 |
| 4.500 | 2.711 | .410 |
| 5.000 | 3.012 | .410 |
| 5.500 | 3.313 | .410 |
| 6.000 | 3.614 | .410 |
| 6.500 | 3.915 | .410 |
| 7.000 | 4.217 | .410 |
| 7.500 | 4.518 | .410 |
| 8.000 | 4.819 | .410 |
| 8.500 | 5.120 | .410 |
| 9.000 | 5.421 | .410 |
| 9.500 | 5.723 | .410 |
| 10.000 | 6.024 | .460 |
| 10.500 | 3.674 | .460 |
| 11.000 | 1.324 | .460 |
| 11.500 | -1.026 | .458 |
| 12.000 | -3.359 | .453 |
| 12.500 | -5.673 | .449 |
| 13.000 | -7.971 | .444 |
| 13.500 | -10.261 | .439 |
| 14.000 | -12.599 | .435 |
| 14.500 | -15.043 | .430 |
| 15.000 | -17.691 | .426 |
| 15.500 | -20.697 | .421 |
| 16.000 | -24.037 | .416 |
| 16.500 | -27.613 | .411 |
| 17.000 | -31.471 | .406 |
| 17.500 | -35.711 | .400 |
| 18.000 | -40.647 | .395 |
| 18.500 | -46.816 | .389 |
| 19.000 | -54.196 | .383 |
| 19.500 | -62.523 | .378 |
| 20.000 | -71.890 | .372 |
| 20.500 | -82.308 | .366 |
| 21.000 | -93.619 | .360 |
| 21.500 | -106.342 | .354 |
| 22.000 | -121.625 | .348 |
| 22.500 | -139.810 | .341 |
| 23.000 | -160.532 | .335 |
| 23.500 | -183.488 | .328 |
| 24.000 | -208.944 | .321 |
| 24.500 | -237.095 | .315 |
| 25.000 | -267.470 | .309 |
| 25.500 | -298.065 | .305 |

# APPENDIX C

# EXPERIMENTAL DATA

## Pressure Plate Data

A summary of the experimental pressure plate data for rep 2 of the sandy loam soil is given in Table C1. Pressures less than 150 cm of water were measured to ± 0.1 cm of water. The water content was determined by dividing the accumulative imbibition by the sample volume. The sample volume was measured to within ± 0.50 cc and the inflow to within 0.50 and 0.1 cc for pressures of 1 and 309 cm of water, respectively. The uncertainty of the water content measurements was determined by using Kline and McClintock's (1953) equation 4 which equates the absolute maximum uncertainty of a variable to a weighted sum of the absolute values of the uncertainties of its components. The absolute maximum uncertainties of the water content measurements at pressures of 1 and 309 cm of water were, respectively, ± 0.023 and ± 0.007 $cm^3/cm^3$. These values represent ± 9 and ± 5 percent of the water contents given in Table C1. The uncertainties of measurements on the sand mixture were of about the same magnitude.

## Infiltration Rate-Time Relationship

A condensed plot of the strip chart recording of infiltration volume versus time for test No. 2 (dry sand mixture) is given in Figure C1. A weight loss of 50 grams was recorded on the 5 cm chart width and the chart was manually re-zeroed producing the traces shown in Figure C1. The actual time scale was 1 mm/sec so the time axis of Figure C1 is compressed by a factor of 12. Data points taken from the strip chart are given in Table C2. Infiltration rates were calculated by simple finite difference techniques and are also given in Table C2.

By using standard weights to calibrate the system, it was found that reproducible measurements of the inflow could be obtained to

Table C1. Summary of Pressure Plate Data, Sandy Loam Rep No. 2.

Pressure Plate #4 — Bulk Density = 1.71 gm/cc

Sample Volume = 230.0 cc — Temperature = 70°F

| Date | Pressure (cm water) | Burette Reading ($cm^3$) | | Imbibition ($cm^3$) | Accumulative Imbibition ($cm^3$) | Water Content ($cm^3/cm^3$) |
|---|---|---|---|---|---|---|
| 10-15-69 | 2000 | 0.8 | | (Initial conditions set) | | |
| 10-21 | 2000 | 13.6 | | 12.6 | 12.6 | 0.056 |
| 10-26 | 1360 | 14.7 | | 1.1 | 13.9 | 0.060 |
| 10-31 | 950 | 15.9 | | 1.2 | 15.1 | 0.065 |
| 11-10 | 510 | 17.8 | | 1.9 | 17.0 | 0.074 |
| 11-13 | 422 | 18.7 | | 0.9 | 17.9 | 0.078 |
| 11-21 | 309 | 21.6 | | 2.9 | 20.8 | 0.090 |
| 12-2 | 194 | 27.8 | | 6.2 | 27.0 | 0.117 |
| 12-7 | 150 | 30.9 | | 3.1 | 30.1 | 0.131 |
| 12-13 | 100 | 35.8 | | 5.0 | 35.0 | 0.152 |
| 12-13 | 100(Refill) | 0.8 | +35.0 | | | |
| 12-16 | 73.5 | 4.8 | | 4.0 | 39.0 | 0.170 |
| 12-21 | 50.0 | 12.2 | | 7.4 | 46.4 | 0.202 |
| 12-26 | 29.0 | 24.45 | | 12.25 | 58.65 | 0.255 |
| 1-3-70 | 20.2 | 33.2 | | 8.75 | 67.40 | 0.293 |
| 1-12 | 10.2 | 46.4 | | 13.2 | 80.6 | 0.351 |
| 1-12 | 10.2(Refill) | 1.0 | +45.4 | | | |
| 1-19 | 3.8 | 11.0 | | 10.0 | 91.6 | 0.396 |
| 1-31 | 1.0 | 23.1 | | 12.1 | 103.1 | 0.448[1] |

[1] Gravimetric water content samples taken at the end of run gave $\theta = 0.446$.

Table C2. Infiltration Data for Test No. 2. (Column Cross-sectional Area = 60.1 $cm^2$.)

| Time (minutes) | Inflow ($cm^3$) | Rate (cm/hr) | Time (minutes) | Inflow ($cm^3$) | Rate (cm/hr) |
|---|---|---|---|---|---|
| 0. | 0 | 54.0 | 16.0 | 39.0 | 7.9 |
| 0.167 | 9.0 | 51.5 | 17.0 | 46.7 | 7.6 |
| 0.333 | 17.0 | 46.5 | 18.0 | 54.3 | 7.4 |
| 0.50 | 23.5 | 36.0 | 19.0 | 5.8 | 7.1 |
| 0.667 | 29.0 | 31.5 | 20.0 | 12.9 | 7.1 |
| 0.833 | 34.0 | 28.5 | 22.5 | 30.6 | 7.0 |
| 1.0 | 38.5 | 27.0 | 25.0 | 47.8 | 6.7 |
| 1.5 | 50.5 | 24.0 | 27.5 | 0.6 | 6.2 |
| 2.0 | 4.0 | 19.0 | 30.0 | 16.0 | 6.0 |
| 2.5 | 13.5 | 18.3 | 32.5 | 30.7 | 6.0 |
| 3.0 | 22.3 | 17.3 | 35.0 | 45.4 | 5.9 |
| 3.5 | 30.8 | 15.2 | 37.5 | Refill of Water Reservoir | |
| 4.0 | 37.5 | 13.4 | 40.0 | 0.5 | 5.4 |
| 4.5 | 44.2 | 13.2 | 42.5 | 14.0 | 5.4 |
| 5.0 | 50.7 | 12.7 | 45.0 | 27.5 | 5.2 |
| 5.5 | 56.9 | 12.2 | 47.5 | 3.5 | 5.2 |
| 5.5 | 1.0 | | 50.0 | 16.6 | 5.1 |
| 6.0 | 6.8 | 11.4 | 52.5 | 29.2 | 5.0 |
| 6.5 | 12.4 | 11.0 | 55.0 | 41.6 | 5.0 |
| 7.0 | 17.7 | 10.6 | 57.5 | 54.1 | 5.0 |
| 7.5 | 23.0 | 10.5 | 60.0 | 11.1 | 4.9 |
| 8.0 | 28.2 | 10.4 | 62.5 | 23.3 | 4.8 |
| 9.0 | 38.6 | 10.1 | 65.0 | 34.9 | 4.7 |
| 10.0 | 48.4 | 9.6 | 67.5 | 46.6 | 4.7 |
| 11.0 | 57.9 | 9.2 | 70.0 | 58.5 | |
| 12.0 | 5.5 | 9.0 | 70.0 | 0.8 | 4.7 |
| 13.0 | 14.5 | 8.9 | 75.0 | 24.9 | 4.6 |
| 14.0 | 22.3 | 8.2 | 80.0 | 48.5 | 4.5 |
| 15.0 | 30.1 | 8.2 | 85.0 | 16.8 | 4.5 |
| | | | 90.0 | 39.1 | |

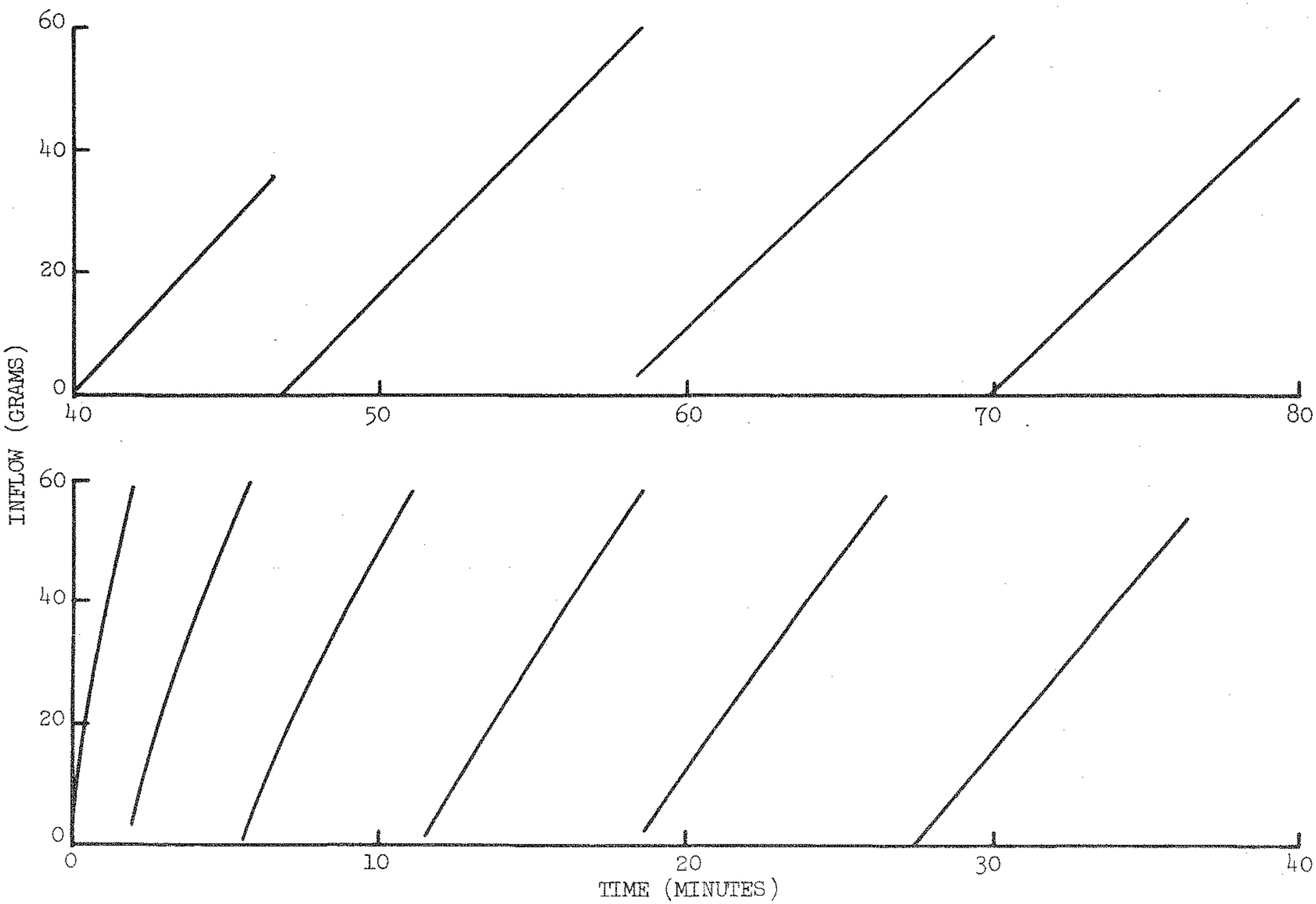


Figure C1. Plot of Infiltration Volume Versus Time for Test No. 2. Time Scale Condensed by a Factor of 12 from Original Strip Chart Record.

within ± 0.3 grams (or ± 0.3 cc). This was consistent with the strip chart recorder specifications which gave ± 0.35 gm for the settings used. The time increment between the points taken from the chart was determined within approximately 1 percent. Using these limits as the maximum uncertainties in the measurements, the maximum uncertainty of the infiltration rate was determined from equation 4 of Kline and McClintock. The uncertainties are given in Table C3.

Table C3. Uncertainty of Infiltration Rates for Test No. 2.

| Time (minutes) | Rate (cm/hr) | Uncertainty, Absolute (cm/hr) | Uncertainty, Percent |
|---|---|---|---|
| 2.5 | 18.3 | ± 0.83 | 8.3 |
| 10.0 | 9.6 | ± 0.41 | 4.1 |
| 30.0 | 6.0 | ± 0.18 | 3.0 |
| 65.0 | 4.7 | ± 0.18 | 3.8 |

Thus the uncertainty in the infiltration rate measurement is greatest at small times where the time increment is necessarily short. Errors in the infiltration rate measurements, which are estimated by the uncertainties given above, cause a plot of the determined infiltration rates to be scattered rather than lying on a smooth curve as one would expect for a natural process such as infiltration. Since the infiltration rates were determined at relatively short time increments, an improved estimate of the rates was obtained by plotting the infiltration rate versus time and drawing a smooth curve through the points. The data from Table C2 are plotted in Figure C2. The continuous curve is a visual fit to the data; this curve was used as input data in the method discussed in Chapter 2 to determine the conductivity function.

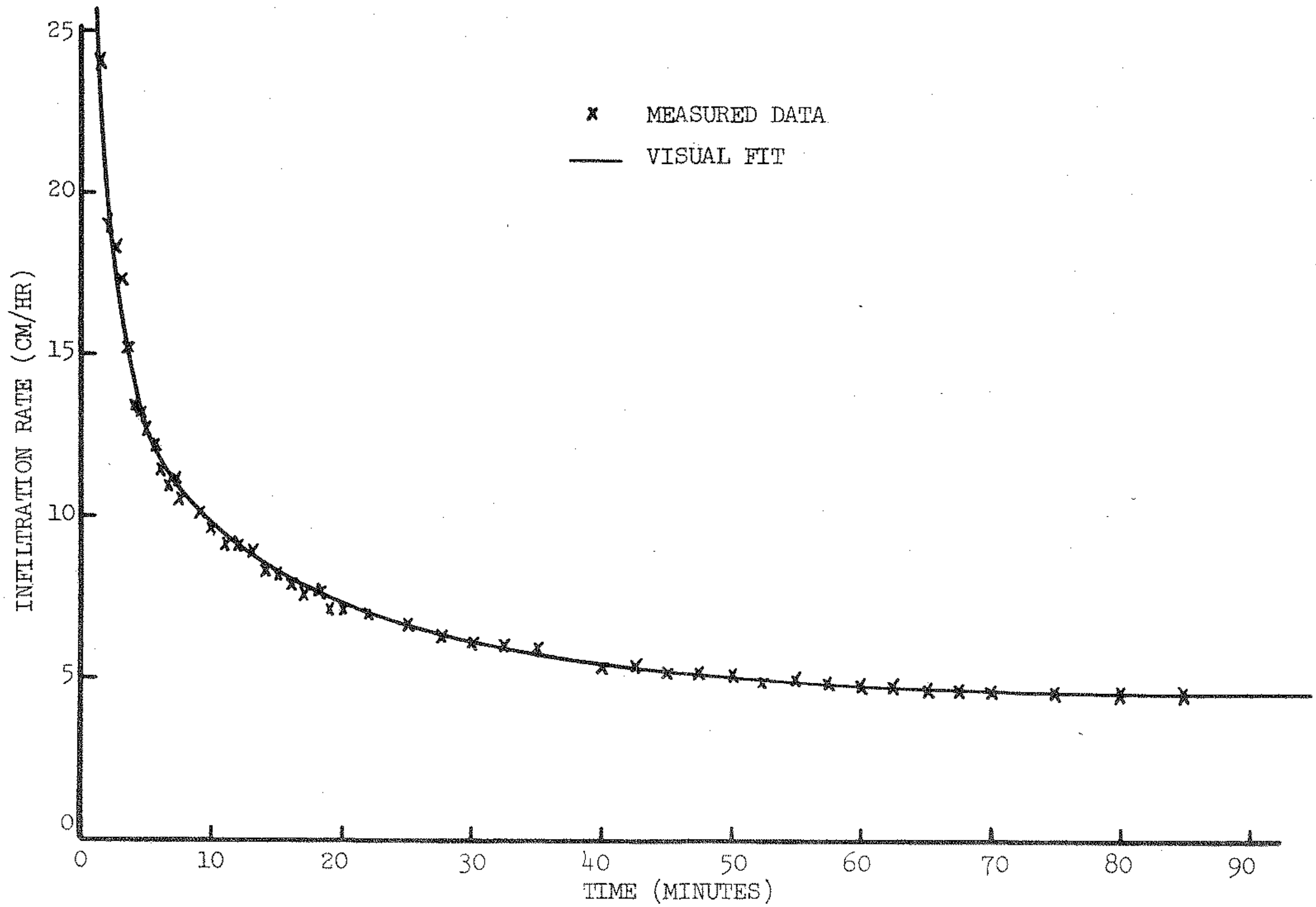


Figure C2. Visual Fit to Measured Infiltration Rate Data for Test No. 2.