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A two-layer composite model of the vocal fold lamina propria for fundamental frequency regulation

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The mechanical properties of the vocal fold lamina propria, including the vocal fold cover and the vocal ligament, play an important role in regulating the fundamental frequency of human phonation. This study examines the equilibrium hyperelastic tensile deformation behavior of cover and ligament specimens isolated from excised human larynges. Ogden's hyperelastic model is used to characterize the tensile stress-stretch behaviors at equilibrium. Several statistically significant differences in the mechanical response differentiating cover and ligament, as well as gender are found. Fundamental frequencies are predicted from a string model and a beam model, both accounting for the cover and the ligament. The beam model predicts nonzero F_0 for the unstretched state of the vocal fold. It is demonstrated that bending stiffness significantly contributes to the predicted F_0, with the ligament contributing to a higher F_0, especially in females. Despite the availability of only a small data set, the model predicts an age dependence of F_0 in males in agreement with experimental findings. Accounting for two mechanisms of fundamental frequency regulation—vocal fold posturing (stretching) and extended clamping—brings predicted F_0 close to the lower bound of the human phonatory range. Advantages and limitations of the current model are discussed. [DOI: 10.1121/1.2749460]

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I. INTRODUCTION

The fundamental frequency of vocal fold vibration (F_0) is a central characteristic of human phonation, and is the primary variable affecting vocal pitch. An understanding of the dependence of F_0 on age and gender is thus of significant interest. Furthermore, this issue is of importance for the development of realistic computer models of phonation or the development of biomaterials for surgical applications.[1] Intrinsically, the fundamental frequency of phonation is dependent on the mechanical properties of the vocal fold lamina propria, including the vocal fold cover, i.e., the epithelium and the superficial layer of the lamina propria, and the vocal ligament, i.e., the intermediate layer and the deep layer of the lamina propria.

Constitutive models that can reliably describe the equilibrium stress-strain or stress-stretch response of components of the vocal fold lamina propria are critical to the prediction of the equilibrium F_0. Experimental data on the tensile stress-stretch response of human vocal fold cover and vocal ligament specimens have been obtained, clearly demonstrating a nonlinear relationship between stress and stretch.[1,2] As in our previous study,[3] Ogden's hyperelastic model[4] is applied to characterize the mechanical response of tissue specimens. Having the empirical data for both the cover and the ligament described by the same model allows for an examination of the differences in constitutive parameters between the cover and the ligament. Furthermore, age- and gender-related differences of the tissue response can be investigated.

The ideal string model[3,5] is a commonly used model for the prediction of F_0. This model considers a structural vibration of the vocal fold lamina propria in dependence of vocal fold length and tension. Alternatively, vocal fold vibration has been analyzed by beam models. In a beam model not only the tensile stiffness but also the bending stiffness is accounted for. Thus, unlike for the string model, the spatial direction of vibration relative to the beam axis (the anterior-posterior direction) has to be specified. Beam models[6–8] are of interest since, in general, string models tend to underpredict the fundamental frequency when compared to empirical speaking F_0 data.[9,10] Titze and Hunter[6] developed a beam model accounting for the vocal ligament as the dominant load-carrying component of the lamina propria, and idealized the tissue mechanical response as linear elastic. Descout *et al.*[7] developed a multilayer beam model also with linear elastic properties. Bickley[8] considered a homogeneous beam and developed a model without accounting for the effects of vocal fold stretch. It is to be noted that both the string and the beam models consider structural vibration as opposed to the mucosal wave model that considers phonation as the propagation of shear waves on the vocal fold surface in an inferior-to-superior direction.[5]

The present study describes enhanced string and beam models by accounting for the layered structure of the lamina propria. Both models account for the changes in cross-

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TABLE I. Summary of the constitutive model parameters and the geometrical properties of all specimens; μ_j—initial equilibrium shear modulus; α_j—dimensionless power (nonlinearity of the elastic response); $A_{0,j}$—initial cross-section area of the vocal fold cover or the vocal ligament; L—*in situ* length of the vocal fold; (c) —vocal fold cover; (l) —vocal ligament.

Gender	Age	μ_j [kPa]	α_j	$A_{0,j}$ [mm^2]	L [mm]	Gender	Age	μ_j [kPa]	α_j	$A_{0,j}$ [mm^2]	L [mm]
Male	17 (c)	1.37	8.8	14.55	14.1	Female	46 (l)	2.61	16.0	4.29	16.5
	19 (l)	1.37	11.0	10.46	16.3		54 (l)	2.28	15.5	9.14	16.9
	33 (c)	3.50	14.1	11.47	17.9		73 (c)	0.57	16.0	8.94	16.9
	33 (l)	4.40	15.7	8.09	17.9		73 (l)	0.64	14.3	6.07	16.9
	49A (l)	1.41	19.2	13.34	24.9		80 (c)	4.12	14.5	7.83	14.5
	49B (l)	0.80	20.0	13.49	19.8		80 (l)	1.18	15.5	8.77	14.5
	51 (c)	1.77	14.6	13.85	21.4		82 A (c)	3.63	15.4	7.49	14.9
	51 (l)	1.39	16.0	16.46	21.4		82A (l)	10.46	13.0	6.39	14.9
	54 (l)	1.64	14.7	5.25	20.5		82B (c)	0.43	19.2	8.47	14.6
	65 (c)	6.67	16.1	8.20	20.5		82B (l)	0.23	16.9	6.56	14.6
	65 (l)	3.48	24.7	6.08	20.5		83 (c)	1.31	19.5	6.89	15.1
	66 (c)	5.60	14.5	9.80	20.4		83 (l)	0.21	19.0	5.31	15.1
	66 (l)	1.22	24.5	11.90	20.4		85 (c)	1.43	16.8	8.23	12.4
	79 (l)	1.42	20.0	4.23	22.0		97 (c)	1.77	12.5	9.53	13.9
	88 (l)	3.26	19.2	5.18	20.7						
	99 (c)	6.39	14.4	7.31	17.9						
	99 (l)	1.51	22.0	9.43	17.9						

section areas with tensile deformation. Specifically, the beam model is developed for vibrations in the medial-lateral direction, i.e., the predominant direction of vocal fold vibration. While a string model will always predict zero F_0 in an unstretched state, the beam model as developed in this study can be used to predict fundamental frequencies for unstretched vocal fold.[11] Past beam models[6] based on Ref. 12 did not provide a solution for the unstretched state, but rather predicted infinite F_0 at that state. Numerical solutions to the beam vibration equations are compared to two closed form solutions, i.e., the solution for the unstretched state and the solution for the stretched state. In the present study, the fundamental frequency models are combined with the constitutive models characterizing the experimental stress-stretch response of both vocal fold cover and vocal ligament specimens.

The fundamental frequency models allow for investigations into the regulation of F_0. While aspects of neuromuscular control and sensorimotor feedback certainly contribute to the regulation of F_0,[13–15] the present study focuses on the effects of vocal fold length change on fundamental frequency. Two mechanisms are considered. The process of vocal fold posturing, i.e., the length changes due to activities of the cricothyroid muscle and the thyroarytenoid muscle,[5,16] not only modifies the vocal fold length but simultaneously changes the stiffness of the tissue due to the nonlinear stress-stretch response to tension.[1] In addition, F_0 could be changed by a process referred to as extended clamping,[6,17,18] where the posterior portions of the membranous vocal folds at the vocal processes are presumably pressed or "clamped" together by the arytenoid cartilages such that vibration over this portion is inhibited. This mechanism could become effective once vocal folds have been stretched,[19] and may reduce the effective length of the vocal fold in the stretched state.

In summary, we hypothesize that:

(1) The mechanical response of vocal fold tissue can be described by a hyperelastic constitutive model, and that the parameter values of the constitutive model can reveal statistically significant differences in tissue response depending on tissue type, gender and age;
(2) The two-layer composite beam model in conjunction with realistic material parameters can reveal the contributions of the vocal fold cover and the vocal ligament to F_0, with the model quantifying the effect of bending stiffness relative to tensile stiffness; and that fundamental frequencies can be calculated for unstretched vocal folds as well as for vocal folds subjected to posturing (stretching) and extended clamping;
(3) This modeling approach allows for predictions of gender- and age-related differences in F_0, and that it can be established whether these differences result from the geometrical features or the mechanical response of the tissue components in the vocal fold lamina propria.

II. METHODS

A. Measurements of tensile mechanical response of the vocal fold

The passive uniaxial tensile stress-stretch response of the vocal fold cover and the vocal ligament was measured by sinusoidal stretch-release deformation (loading-unloading), with the use of a dual-mode servo control lever system (Aurora Scientific Model 300B-LR, Aurora, ON, Canada).[1,3] Measurements of the displacement and force of the lever arm were made by the servo control lever system with a displacement accuracy of 1.0 μm and a force resolution of 0.3 mN. The servo control lever system possessed a displacement range of up to 8–9 mm in the frequency range of 1–10 Hz. Specimens were stretched to a fixed maximum length ℓ_{rev} at load reversal. The uniaxial stretch λ_u at load reversal is defined as $\lambda_{u,\text{rev}}, = \ell_{\text{rev}}/L$ with L being the *in situ* length, i.e., the initial length of the specimen. The experimental protocol was approved by the Institutional Review Board of UT Southwestern Medical Center.

Vocal fold cover and vocal ligament were dissected with instruments for phonomicrosurgery.[1] Specimens were dissected from 21 larynges excised within 24 h postmortem, procured from autopsy from human cadavers free of head and neck disease and laryngeal pathologies. All subjects were nonsmokers, and were Caucasians or Hispanics, although race was not a factor in the procurement. Tissue specimens were obtained from 12 male subjects (Table I), of which vocal fold cover specimens were obtained from six, whereas vocal ligament specimens were obtained from all but the youngest subject (age $Y=17$). Specimens were obtained from nine female subjects (Table I), of which vocal fold cover specimens were obtained from seven. Vocal ligament specimens were also obtained from seven subjects. Before dissection, the *in situ* vocal fold length, defined as the distance from the anterior commissure to the vocal process, i.e., the membranous vocal fold length in a relaxed, cadaveric state, was measured by digital calipers for each specimen. Throughout the dissection, each specimen remained attached to small portions of the thyroid and the arytenoid cartilages, allowing for the attachment of sutures for mechanical testing under natural boundary conditions. Exploiting the displacement range of the lever system, male specimens were loaded to stretches of up to $\lambda_{u,\mathrm{rev}}=1.35$ while female specimens were loaded to stretches of up to $\lambda_{u,\mathrm{rev}}=1.6$.

The experimental equilibrium response is obtained from the measured hysteretic tensile stress-stretch curves as the midpoint values of stresses at equal stretch for the loading and unloading portions of the hysteresis loop in the stabilized (preconditioned) state.[3]

B. Constitutive model for equilibrium response

The equilibrium response is characterized by a nonlinear hyperelastic constitutive model. The vocal fold lamina propria has been recognized as viscoelastic and anisotropic with a primarily parallel arrangement of collagen and elastin fibers, particularly for the vocal ligament.[20,21] In the present study, only the elastic tissue response in the anterior-posterior direction is of concern and effects of anisotropy are thus not considered. A first-order Ogden model[4] together with the assumption of incompressible material response allows for an appropriate description of both the vocal fold cover and the vocal ligament. The first-order Ogden model possesses two parameters, the initial shear modulus μ describing the tissue stiffness, and the power α describing the nonlinearity of the elastic response. It has been shown to be promising in characterizing the tensile behavior of other soft tissues.[22,23] This model is described by a strain energy density function w of the form[4]

$$w = \frac{2\mu}{\alpha^2}(\lambda_1^{\alpha} + \lambda_2^{\alpha} + \lambda_3^{\alpha} - 3). \tag{1}$$

Deformation is characterized through the principal stretches $\lambda_1, \lambda_2, \lambda_3$ which for incompressibility satisfy $\lambda_1\lambda_2\lambda_3=1$. For uniaxial loading of a specimen of *in situ* length L to a current length ℓ the principal stretches are $\lambda_1=\lambda_u$, $\lambda_2=\lambda_3=1/\sqrt{\lambda_u}$ where $\lambda_u=\ell/L$ is the stretch in the anterior-posterior direction. The nominal stress $\sigma_{\mathrm{nominal}}$ in the equilibrium response is obtained as derivative of the strain energy density w with respect to the applied stretch λ_u

$$\sigma_{\mathrm{nominal}} = \frac{\partial w}{\partial \lambda_u} = \frac{2\mu}{\alpha}\left(\lambda_u^{\alpha-1} - \lambda_u^{-\alpha/2-1}\right). \tag{2}$$

Under uniaxial tension, the Cauchy stress (true stress) σ can be expressed as the product of the nominal stress and the stretch in the longitudinal direction

$$\sigma = \lambda_u \cdot \frac{\partial w}{\partial \lambda_u} = \frac{2\mu}{\alpha}\left(\lambda_u^{\alpha} - \lambda_u^{-\alpha/2}\right). \tag{3}$$

The tangent Young's modulus E_t, i.e., the instantaneous stiffness at a given level of stretch, is obtained by differentiating the Cauchy stress σ with respect to the stretch λ_u

$$E_t = \frac{\partial \sigma}{\partial \lambda_u} = \mu\left(2\lambda_u^{\alpha-1} + \lambda_u^{-\frac{\alpha}{2}-1}\right). \tag{4}$$

For the stretch $\lambda_u=1$ and incompressible material response, the tangent Young's modulus $E_t=3\mu$.

The hyperelastic model is applied to characterize the equilibrium uniaxial tensile response of each specimen. A unique set of parameters μ and α can be determined for each specimen through curve fitting of Eq. (2) by achieving a local minimum of the sum of squares of the differences between the experimental data and the simulation results through the Levenberg–Marquardt method,[24,25] under the condition that both μ and α are positive.

C. Models of fundamental frequency regulation

1. Composite string model

Similar to our previous study,[3] the investigation of the fundamental frequency of phonation begins with a string model of phonation.[5] However, here the string is composed of the vocal fold cover and the vocal ligament. Figure 1(a) depicts a cross section of a typical vocal fold with the total initial cross-section area A_0 composed of the cover $A_{0,c}$ and the ligament $A_{0,l}$. The length of vocal fold cover and vocal ligament are assumed to be identical at all times. Thus, the longitudinal stretch of the vocal fold cover $\lambda_{u,c}$ and the vocal ligament $\lambda_{u,l}$ are identical

$$\lambda_u = \lambda_{u,c} = \lambda_{u,l}. \tag{5}$$

The partial differential equation governing the vibration of a flexible string with two fixed ends is[5]

$$T\frac{\partial^2 v}{\partial x^2} = \rho A \frac{\partial^2 v}{\partial t^2}, \tag{6}$$

where ρ is the density, T is the tensile force applied to the composite string and A is the current total cross-section area of the string, and v, the displacement from the equilibrium position (in the medial-lateral direction) of an infinitesimal part of the string with a distance x (anterior-posterior) from a string end point, is a function of x and time t, $v=v(x,t)$. Equation (6) can be simplified as

$$\bar{\sigma}\frac{\partial^2 v}{\partial x^2}=\rho\frac{\partial^2 v}{\partial t^2}, \tag{7}$$

where $\bar{\sigma}=T/A$ is the average, or homogenized longitudinal Cauchy stress. This quantity is related to the Cauchy stress in the vocal fold cover σ_c and the vocal ligament σ_l through

$$\bar{\sigma}=f_c\cdot\sigma_c+f_l\cdot\sigma_l, \tag{8}$$

where f_c and f_l are the current area ratios of the cover and the ligament, respectively, $f_j=A_j/(A_l+A_c)$, with the subscript j denoting cover ($j=c$) or ligament ($j=l$). The area ratios are assumed to be constant along the longitudinal axis of the vocal fold, as well as during the elongation process, $f_{0,j}=A_{0,j}/(A_{0,l}+A_{0,c})=f_j$. Assuming perfect bonding between the vocal fold cover and the vocal ligament and enforcing equilibrium between applied force and internal force resultants in the cover and the ligament, the Cauchy stresses in the cover and the ligament σ_c and σ_l can be calculated in dependence of the stretch λ_u and the constitutive parameters through Eq. (3).

The lowest allowed frequency, or the fundamental frequency for the string model F_0^{string}, can be expressed as a function of the current vocal fold length ℓ, the tissue density ρ, and the tissue homogenized longitudinal Cauchy stress $\bar{\sigma}$

$$F_0^{\text{string}}=\frac{1}{2\ell}\sqrt{\frac{\bar{\sigma}}{\rho}}. \tag{9}$$

The fundamental frequency is thus dependent on the magnitude of deformation applied as a result of vocal fold length changes due to posturing. Predictions of fundamental frequency can be obtained from Eq. (9) with use of the constitutive model once the material parameters are determined. For the hyperelastic tissue response the combination of Eqs. (3), (8), and (9) leads to a prediction of F_0^{string} in dependence of the constants of the hyperelastic constitutive model and stretch λ_u as

$$F_0^{\text{string}}=\frac{1}{L\sqrt{2\rho}}\sqrt{\frac{\mu_c f_c}{\alpha_c}(\lambda_u^{\alpha_c-2}-\lambda_u^{-\alpha_c/2-2})+\frac{\mu_l f_l}{\alpha_l}(\lambda_u^{\alpha_l-2}-\lambda_u^{-\alpha_l/2-2})}. \tag{10}$$

The fundamental frequency is thus dependence of the hyperelastic response and the area ratios of the cover and the ligament, the *in situ* vocal fold length, tissue density, and stretch. It should be noted here that Eqs. (9) and (10) are developed by use of the Cauchy stress, a factor not considered in our previous study[3] and other past work, e.g., Refs. 5–7.

2. Composite beam model

A beam model is proposed with the beam cross section accounting for the presence of the vocal fold cover and the vocal ligament. Following Fig. 1(b), the cover and the ligament are both approximated by rectangular shapes. The line connecting the geometrical center of the idealized vocal fold cover to that of the idealized vocal ligament is in the medial-lateral direction. The idealized rectangular cross-section dimensions are h_j and b_j as the thickness and transverse depth of the vocal fold cover and the vocal ligament, respectively. The aspect ratio m_j is defined as $m_j=h_j/b_j$ for the two cross-section components, Fig. 1(b). Here the aspect ratios are assumed to be identical for both the cover and the ligament, $m=m_c=m_l$.

When the composite beam is under tension the partial differential equation describing its vibrations becomes[12]

$$T\frac{\partial^2 v}{\partial x^2}-\bar{E}_t I\frac{\partial^4 v}{\partial x^4}=\rho A\frac{\partial^2 v}{\partial t^2}, \tag{11}$$

where $\bar{E}_t$, is the composite tangent Young's modulus and I is the area moment of inertia of the composite cross section. Equation (11) can be written into the following form:

$$\bar{\sigma}\frac{\partial^2 v}{\partial x^2}-\bar{E}_t\kappa^2\frac{\partial^4 v}{\partial x^4}=\rho\frac{\partial^2 v}{\partial t^2}, \tag{12}$$

where $\kappa=\sqrt{I/A}$ is the radius of gyration of the composite cross section. The value of $\bar{E}_t\kappa^2$ is a measure of the bending stiffness for the composite beam. It is determined from the following consideration. Following standard derivations of Euler Bernoulli beam theory for layered beams with rectangular cross-section geometry,[26] the bending moment of the vocal fold can be written as

$$M=\bar{E}_t I\frac{\partial^2 v}{\partial x^2}=\frac{A^2(E_{t,c}^2 f_c^3+E_{t,l}^2 f_l^3+6E_{t,c}E_{t,l}\sqrt{f_c^3 f_l^3}+4E_{t,c}E_{t,l}f_c f_l)}{12m(E_{t,c}f_c+E_{t,l}f_l)}\frac{\partial^2 v}{\partial x^2}, \tag{13}$$

where $E_{t,c}$, and $E_{t,l}$ are the tangent Young's moduli of the cover and the ligament given by Eq. (4), respectively. Under the assumption of incompressibility, the current cross-section area A is related to the initial cross-section area A_0 through $A=A_0/\lambda_u$. $\bar{E}_t\kappa^2$ then can be obtained as

$$\bar{E}_t\kappa^2 = \frac{\bar{E}_t I}{A}$$
$$= \frac{A_0(E_{t,c}^2 f_c^3 + E_{t,l}^2 f_l^3 + 6E_{t,c}E_{t,l}\sqrt{f_c^3 f_l^3} + 4E_{t,c}E_{t,l} f_c f_l)}{12 m\lambda_u (E_{t,c} f_c + E_{t,l} f_l)} \tag{14}$$

In Eq. (12), v is the time varying displacement from the equilibrium position in the medial-lateral direction, and is expressed as $v(x,t)=\xi(x)\cdot e^{-2\pi Ft}$ with F being the frequency of vibration, and $i=\sqrt{-1}$. Clamped boundary conditions at the vocal process (the posterior end point, $x=0$) and at the anterior commissure (the anterior end point, $x=\ell$) are assumed. The clamped boundary conditions imply that both the displacement $\xi(x)$ and its derivative $\partial\xi/\partial x$ need to vanish at the boundaries

$$\xi(0) = \xi(\ell) = 0$$
$$\left.\frac{\partial \xi}{\partial x}\right|_{x=0} = \left.\frac{\partial \xi}{\partial x}\right|_{x=\ell} = 0. \tag{15}$$

Substituting the presumed form of solutions of $v(x,t)$ into Eq. (12), one obtains

$$\frac{d^4\xi}{dx^4} - 8\pi^2\beta^2\frac{d^2\xi}{dx^2} - 16\pi^4\gamma^4\xi = 0 \tag{16}$$

with

$$\beta^2 = \frac{\bar{\sigma}}{8\pi^2\bar{E}_t\kappa^2} \quad \gamma^4 = \frac{\rho F^2}{4\pi^2\bar{E}_t\kappa^2}. \tag{17}$$

The parameter β characterizes the ratio between restoring forces due to tension and bending, while γ quantifies the ratio between inertia and bending. The solution to this ordinary differential equation [Eq. (16)] is

$$\xi(x) = D_1\cosh(2\pi\psi x) + D_2\sinh(2\pi\psi x) + D_3\cos(2\pi\zeta x) + D_4\sin(2\pi\zeta x) \tag{18}$$

in which

$$\psi = [(\beta^4+\gamma^4)^{1/2}+\beta^2]^{1/2} \quad \zeta = [(\beta^4+\gamma^4)^{1/2}-\beta^2]^{1/2}. \tag{19}$$

Applying the boundary conditions Eq. (15) to Eq. (18) yields

$$D_1 + D_3 = 0$$
$$D_1\cosh(2\pi\psi\ell) + D_2\sinh(2\pi\psi\ell) + D_3\cos(2\pi\zeta\ell) + D_4\sin(2\pi\zeta\ell) = 0$$
$$\psi D_2 + \zeta D_4 = 0$$
$$\psi D_1\sinh(2\pi\psi\ell) + \psi D_2\cosh(2\pi\psi\ell) - \zeta D_3\sin(2\pi\zeta\ell) + \zeta D_4\cos(2\pi\zeta\ell) = 0. \tag{20}$$

The following characteristic equation is derived by eliminating D_1 through D_4 from Eq. (20) with the help of the relationship $\psi^2-\zeta^2=2\beta^2$

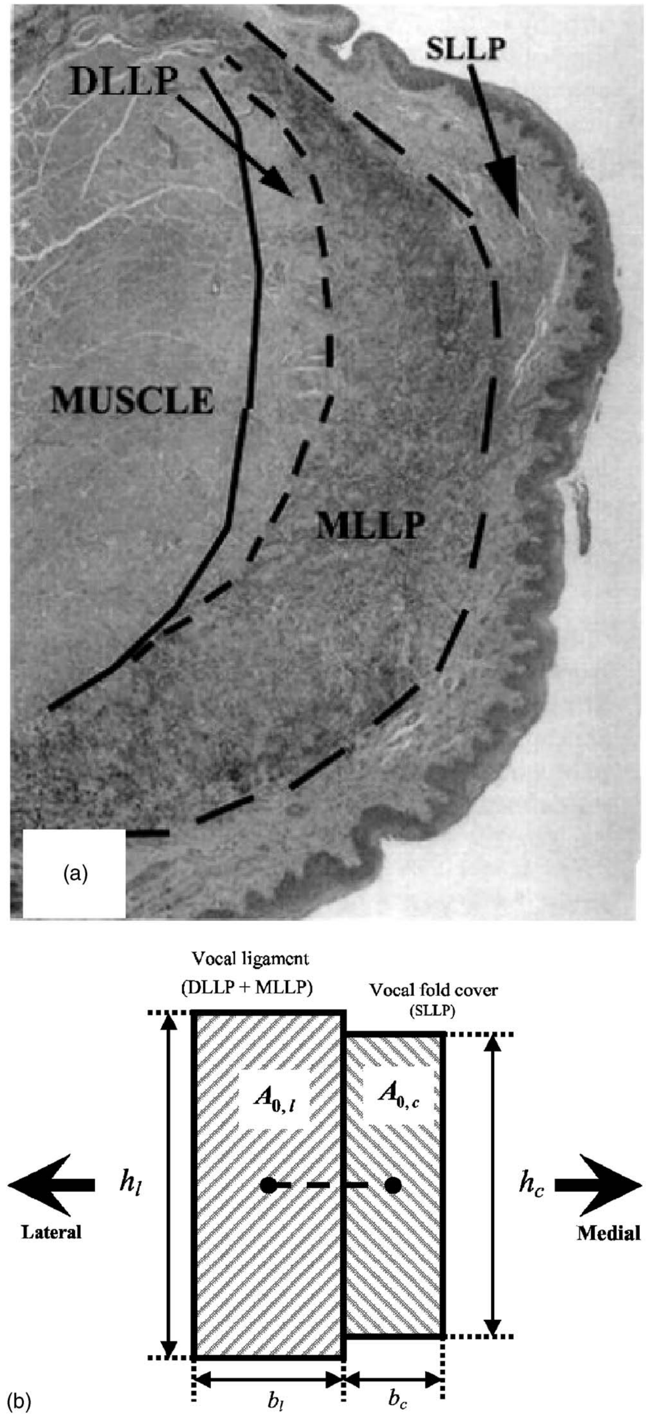

FIG. 1. (a) Layered structure of the human vocal fold illustrating the vocal fold cover, or superficial layer of the lamina propria (SLLP), and the vocal ligament, or middle and deep layers of the lamina propria (MLLP and DLLP) (mid-membranous vocal fold coronal section of a 43-year-old male stained for elastin); from Gray *et al.*[30] (reproduced with permission from Annals of Otology, Rhinology and Laryngology); (b) the geometrical approximation of the cross section of the two-layer composite beam model of the lamina propria with $A_{0,\ell}$ and $A_{0,c}$ indicating the cross-section areas of the ligament and the cover, respectively.

$$\tan(\pi\ell\zeta) = -\sqrt{1+\left(\frac{2\beta^2}{\xi^2}\right)}\tanh(\pi\ell\sqrt{\zeta^2+2\beta^2}). \tag{21}$$

The fundamental frequency, i.e., the lowest allowed frequency of vibration F, can then be obtained from Eqs. (17) and (19)

$$F_0 = 2\pi\gamma^2\sqrt{\frac{\bar{E}_t\kappa^2}{\rho}} = 2\pi\sqrt{(\zeta^2+\beta^2)^2-\beta^4}\sqrt{\frac{\bar{E}_t\kappa^2}{\rho}} = 2\pi\zeta\sqrt{(\zeta^2+2\beta^2)\frac{\bar{E}_t\kappa^2}{\rho}} \tag{22}$$

with ζ being the lowest positive value satisfying Eq. (21).

When β goes to zero, i.e., when the tension is zero and the vocal fold is at its original length L, the characteristic equation [Eq. (21)] reduces to

$$\tan(\pi L\zeta) + \tanh(\pi L\zeta) = 0 \tag{23}$$

with the lowest positive value $\zeta = 0.7528/L$ and the fundamental frequency F_0 is given from Eqs. (4), (14), and (22) as

$$F_0(\lambda_u = 1) = 2\pi\zeta^2\sqrt{\frac{\bar{E}_t\kappa^2}{\rho}} = \frac{1.7804}{L^2}\sqrt{\frac{A_0(\mu_c^2 f_c^3 + \mu_l^2 f_l^3 + 6\mu_c\mu_l\sqrt{f_c^3 f_l^3} + 4\mu_c\mu_l f_c f_l)}{m\rho(\mu_c f_c + \mu_l f_l)}}. \tag{24}$$

When β is large but not infinite, i.e., when tension dominates the restoring force of the oscillating beam, an approximate expression for the lowest allowed value of ζ can be obtained through expanding both sides of Eq. (21) and retaining only the first two terms in the series expansions as in[6,12]

$$\zeta \approx \frac{1}{2\ell}\left[1 + \frac{2}{\pi}\sqrt{B} + \left(\frac{4}{\pi^2} + 0.5\right)B\right] \quad \text{with } B = \frac{\pi^2\bar{E}_t\kappa^2}{\bar{\sigma}L^2\lambda_u^2}. \tag{25}$$

Substituting Eq. (25) into Eq. (22), the fundamental frequency F_0 then becomes

$$F_0 \approx 2\pi\zeta\sqrt{\frac{2\beta^2\bar{E}_t\kappa^2}{\rho}} = \frac{1}{2\ell}\sqrt{\frac{\bar{\sigma}}{\rho}}\left[1 + \frac{2}{\pi}\sqrt{B} + \left(\frac{4}{\pi^2} + 0.5\right)B\right]. \tag{26}$$

In Eq. (26) the parameter B is given from Eqs. (8), (14), and (25) as

$$B = \frac{\pi^2\bar{E}_t\kappa^2}{\bar{\sigma}L^2\lambda_u^2} = \frac{\pi^2 A_0(E_{t,c}^2 f_c^3 + E_{t,l}^2 f_l^3 + 6E_{t,c}E_{t,l}\sqrt{f_c^3 f_l^3} + 4E_{t,c}E_{t,l}f_c f_l)}{12m\lambda_u^3 L^2(E_{t,c}f_c + E_{t,l}f_l)(\sigma_c f_c + \sigma_l f_l)} \tag{27}$$

with σ_j and $E_{t,j}$ given by Eqs. (3) and (4), respectively.

In order to demonstrate the validity of Eqs. (24) and (26), numerical solutions to Eq. (21) were obtained through an algebraic equation solver and substituted into Eq. (22) to calculate values of F_0.

III. RESULTS

A. Characterization of the vocal fold cover and the vocal ligament

The geometrical features of the cover and the ligament specimens were obtained after dissection. Table I summarizes the *in situ* vocal fold lengths L and initial cross-section areas of the cover and the ligament in the undeformed state, $A_{0,c}$ and $A_{0,l}$. By definition the *in situ* lengths of the cover and the ligament are identical for each individual subject. Results from Mann-Whitney U tests investigating gender differences and differences between cover and ligament are given in Table II. The results indicated that the female specimens possess shorter *in situ* lengths than the male specimens at a level of statistical significance ($p=0.0002$). The *in situ* lengths of the male specimens were found in dependence of age as $L(\text{male})(Y) = 21.02[1.0 - \exp(-0.07331\cdot Y)]$ ($R^2 = 0.6025$). No age dependence could be determined for females due to the small age range of the samples available.

The cross-section areas of the cover and the ligament were found to be not statistically significantly different for either the male or the female specimens ($p=0.500$ and $p=0.063$, respectively). Gender-related differences in the geometrical characteristics of the specimens were also examined. No statistically significant differences were found for vocal fold cover specimens ($p=0.264$) as well as for vocal ligament specimens ($p=0.204$). In the subsequent analysis a tissue density of $\rho = 1040\ \text{kg/m}^3$ for both the vocal fold cover and the vocal ligament is assumed throughout the study.[20]

Figure 2 shows two examples of the tissue equilibrium response together with the description through Ogden's hyperelastic model. The stress-stretch curves for the two specimens shown are very similar qualitatively but differ significantly in stress level. This strong qualitative similarity is also found if all stress-stretch curves are compared, but there is a significant inter-subject variability that exists. A strong nonlinear dependence of stress on the applied stretch is observed, especially at higher stretch ($\lambda_u \geq 1.2$). This finding is similar to experimental findings on vocal fold tissues.[1–3] Ogden's model can characterize both stress-stretch curves well, despite the large inter-subject differences in the specific stress levels associated with the individual tissue response.

The material parameters μ_j, α_j ($j=c,l$) were determined for all specimens and respective values are given in Table I. First, the vocal fold cover and the vocal ligament are compared to each other. In order to conduct paired statistical tests and to make this comparison meaningful, only those subjects with both cover and ligament specimens available are selected for the analysis, including five male ($Y=33$, 51, 65, 66, 99 years) and five female subjects ($Y=73$, 80A, 82A, 82B, 83 years). For the paired Mann-Whitney U tests, only the differences between the cover and the ligament of the same subject are of concern. Table II summarizes the mean values and standard deviations of the parameters, as well as the results of these statistical tests. For males no statistically significant difference was found between μ_c(male) and μ_l(male)($p=0.093$), while α_c(male) and α_l(male) are statistically significantly different ($p=0.031$). For female subjects

TABLE II. Summary of the results of statistical tests conducted on the constitutive and geometrical parameters for the cover-ligament comparison and the gender difference.

		Cover			Ligament					
Gender	Parameter	Mean	SD	n	Mean	SD	n	U-test p value	Paired test?	Significantly different?
Male	μ [kPa]	4.79	2.09		2.40	1.45		0.093	Yes	No
	α	14.7	0.8	5	20.6	4.4	5	0.031	Yes	Yes
	A_0 [mm^2]	10.13	2.62		10.39	4.00		0.500	Yes	No
Female	μ [kPa]	2.01	1.74		2.54	4.44		0.406	Yes	No
	α	16.9	2.3	5	15.7	2.3	5	0.094	Yes	No
	A_0[mm^2]	7.92	0.81		6.62	1.29		0.063	Yes	No

		Male			Female					
Component	Parameter	Mean	SD	n	Mean	SD	n	U-test p value	Paired test?	Significantly different?
Cover	μ [kPa]	5.11	2.27		1.89	1.44		0.021	No	Yes
	α	14.9	0.8		16.3	2.5		0.158	No	No
	A_0 [mm^2]	9.79	2.90	4	8.20	0.89	7	0.264	No	No
	L [mm]	20.91	1.88		15.10	1.50		0.0002	No	Yes
Ligament	μ [kPa]	1.79	0.93		2.52	3.63		0.235	No	No
	α	20.0	3.4		15.7	1.9		0.006	No	Yes
	A_0 [mm^2]	9.48	4.49	9	6.65	1.75	7	0.204	No	No
	L [mm]	20.91	1.88		15.10	1.50		0.0002	No	Yes

no statistically significant differences between the vocal fold cover and the vocal ligament were observed. Paired Mann-Whitney U tests for parameters μ and α resulted in p-values of 0.406 and 0.094, respectively.

The dependence of the hyperelastic model parameters on gender is investigated for the cover and the ligament separately. In order to examine gender-related differences without the confounding effect of age, the statistical analyses with unpaired U tests were conducted for subjects of a similar age range, with the three youngest male subjects ($Y=17$, 19, 33) excluded. Table II summarizes the results of the statistical tests on gender difference. For the cover specimens a U test indicates the male-to-female difference in the mean values of equilibrium shear modulus to be statistically significant ($p=0.021$). A similar analysis was also conducted between α_c(male) and α_c(female) but indicated that differences in these parameters are statistically insignificant ($p=0.158$). For the ligament specimens the gender-related difference in the mean values of equilibrium shear modulus was not significant ($p=0.235$), while the difference between α_l(male) and α_l(female) was statistically significant with $p=0.006$.

For male vocal fold cover specimens a relationship between the tissue constitutive parameters and age was established, thereby expanding on previously published data for the male vocal fold cover.[3] The values for μ_c(male) and α_c(male) were expressed as $\mu_c(\text{male})(Y)=7.0[1.0-\exp(-0.01884\cdot Y)]$[kPa] (with the coefficient of determination $R^2=0.62$) and $\alpha_c(\text{male})(Y)=15.35[1.0-\exp(-0.05841\cdot Y)]$ ($R^2=0.87$). The same nonlinear regression analysis was conducted for the constitutive and geometrical parameters of the male ligament specimens, including μ_l(male) and α_l(male). It was found that the nonlinearity of the elastic response α_l(male) depends on age with $\alpha_l(\text{male})(Y)=22.59[1.0-\exp(-0.03521\cdot y)]$ ($R^2=0.58$). No fit describing the age dependence of μ_l(male) could be found, hence in further analysis an average, age-independent value of $\mu_l(\text{male})=1.99$ kPa was employed. Also, no changes in the area ratios f_j with age were observed.

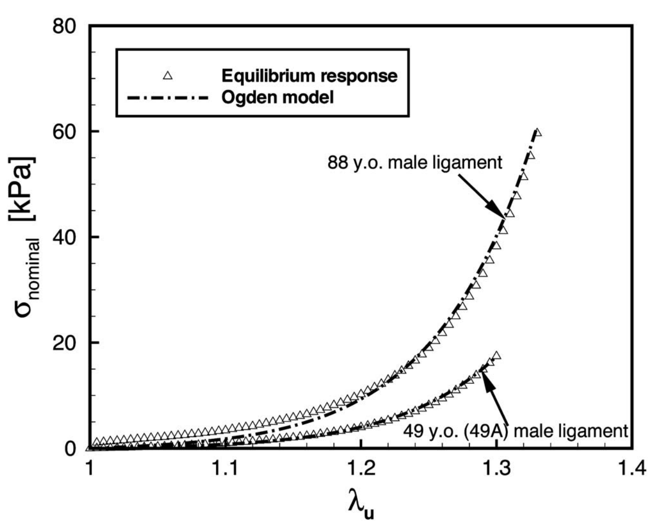

FIG. 2. Comparison between experimental data (the delta-shaped symbols) and simulation results (the dash-dot lines) of tensile equilibrium stress-stretch response at 1 Hz for two ligament specimens (88-year-old male and 49-year-old male).

B. Fundamental frequency prediction

Fundamental frequencies are predicted based on the average constitutive and geometrical parameter values for males (with the three youngest subjects excluded for the same reasons stated in Sec. III A) and females. An aspect

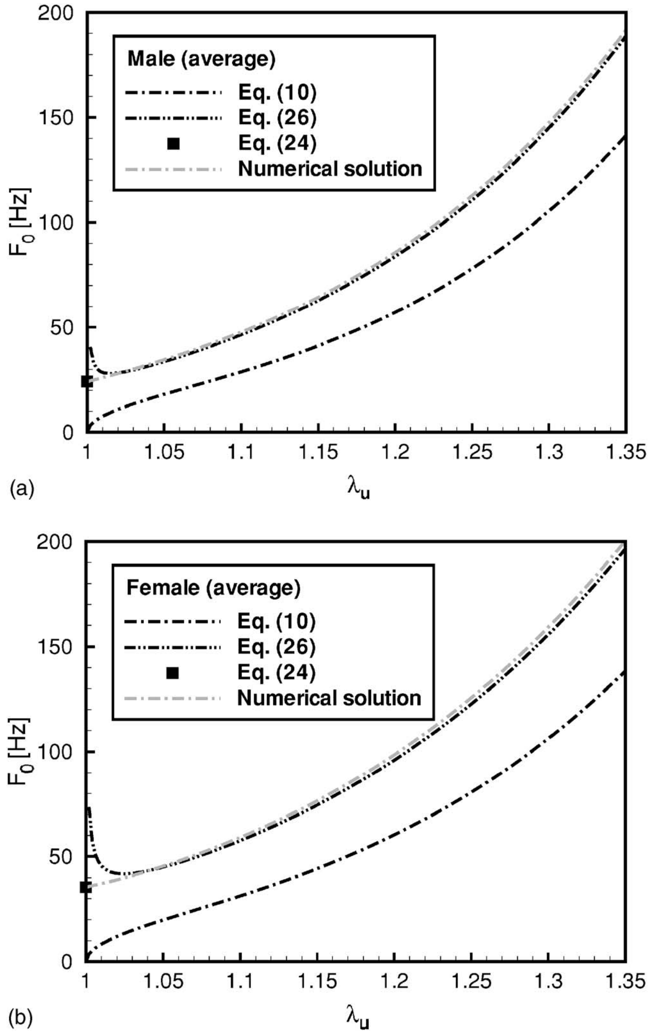

FIG. 3. Dependence of the fundamental frequency F_0 predicted by the string model and the beam model upon the stretch level for (a) the average male vocal fold and (b) the average female vocal fold.

ratio $m=3$ for both the vocal fold cover and the vocal ligament is assumed as an approximation. Following Fig. 1(a) an aspect ratio of $m=3$ is a reasonable geometrical approximation of the layered microstructure of the lamina propria.

Figure 3 summarizes the dependence of the predicted F_0 on vocal fold stretch, based on the following models considered in this study: (i) the composite string model, Eq. (10), (ii) the composite beam model for the unstretched vocal fold, Eq. (24), (iii) the composite beam model for the stretched vocal fold, Eq. (26), and (iv) the numerical solution to the composite beam model. The closed form solution derived from the beam model, Eq. (26), describes the vocal folds in a stretched state. Equation (26) is only valid when the vocal fold is stretched to the extent that tension dominates the restoring force, i.e., when β is much larger than ζ in Eq. (21). Predictions of F_0 through Eq. (26) are thus not valid at small values of stretch. The numerical solution demonstrates the range of validity of Eq. (26). Numerically predicted values of F_0 are found to agree very well with the predicted values of Eq. (24) at $\lambda_u=1.0$, as well as with the values of Eq. (26) for stretch beyond 1.05.

The string model predicts $F_0=0$ for the unstretched state ($\lambda_u=1.0$), whereas the new beam model, Eq. (24), predicts nonzero value of F_0 for the unstretched state: F_0(male) $=24.2$ Hz and F_0(female)$=35.4$ Hz. This unstretched state is, however, not physiologically relevant for phonation, whereas a stretch range of 1.2–1.3 is reasonable in speaking.[27] As the stretch is increased a nonlinear increase in F_0 is predicted. For males, at a stretch of 1.2, a typical magnitude of vocal fold elongation,[16,27] the F_0 predicted by the string model is 57.2 Hz, whereas the composite beam model predicts a fundamental frequency of 83.8 Hz for a stretch of 1.2. For adult males empirical speaking F_0 was found to be between around 90–150 Hz, depending on age, geometrical and anatomical variations, and other factors.[9,10] While the F_0 predicted by the string model is low compared to empirical data, the beam model [Eq. (26)] is capable of predicting F_0 values approaching the phonatory range at a stretch of 1.2. For females, at a stretch of 1.3, a typical magnitude of vocal fold elongation,[16,27] the F_0 predicted by the string model is 106.2 Hz, whereas the composite beam model predicts a fundamental frequency of 159.2 Hz for a stretch of 1.3. For adult females the empirical speaking F_0 ranges from around 150 to 250 Hz.[9,10] While the F_0 predicted by the string model is again low compared to empirical data, the beam model [Eq. (26)] is once again more capable of predicting F_0 values approaching the phonatory range at a stretch of 1.3.

While Fig. 3 considers fundamental frequency regulation through vocal fold stretch, Fig. 4 considers results for F_0 regulation through "extended clamping," i.e., reduction in the effective vocal fold length in the stretched state by a presumed compression of the arytenoid cartilages such that vibration is inhibited over a segment of the membranous vocal fold close to the vocal process. In the present model, the extent of activity of this mechanism is quantified by defining the effective length ratio as the quotient of the length of the freely vibrating vocal fold ℓ_{free} and the current vocal fold length $\ell=L\lambda_u$

$$\Phi = \frac{\ell_{\text{free}}}{\ell} = \frac{\ell_{\text{free}}}{L\lambda_u}. \tag{28}$$

Figure 4 demonstrates the dependence of F_0 on the effective length ratio, for males and females, respectively, at several stretch levels, and compares the predicted F_0 with empirical data. This diagram thus provides maps from which the effectiveness of the combined action of the two mechanisms of fundamental frequency regulation—posturing and extended clamping—can be assessed. For males, the lower bound on empirically obtained fundamental frequencies ($F_0=90$ Hz) can be reached by $\lambda_u>1.2$ at $\Phi=1.0$ (no extended clamping) or $\lambda_u>1.15$ at $\Phi=0.8$ (as extended clamping is activated). The upper bound on empirically obtained fundamental frequencies ($F_0=150$ Hz), however, would either require considerable stretch $\lambda_u>1.3$ at $\Phi=1.0$, or a combination of $\lambda_u>1.2$ and an activation of extended clamping with $\Phi=0.8$. For females, the lower bound on empirically obtained fundamental frequencies ($F_0=150$ Hz) requires higher stretch than for males, $\lambda_u>1.28$ at $\Phi=1.0$, or $\lambda_u>1.22$ at $\Phi=0.8$. The upper bound on empirically obtained fundamen-

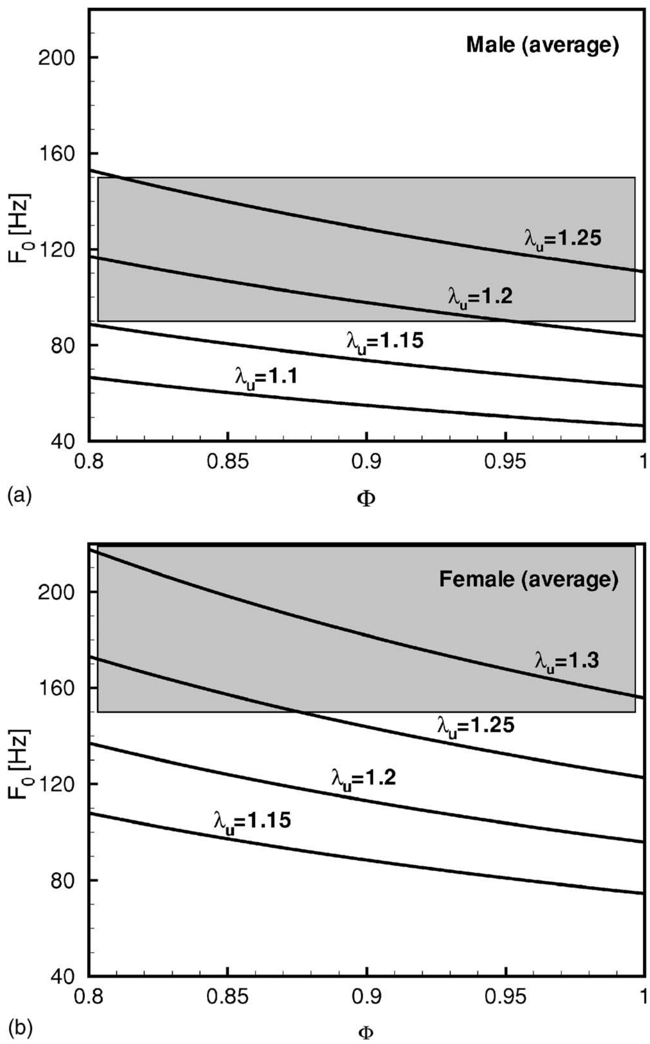

FIG. 4. Dependence of the fundamental frequency F_0 upon the effective length ratio Φ at several stretch levels λ_u for (a) the average male vocal fold and (b) the average female vocal fold. Shaded areas indicate empirical speaking fundamental frequencies.

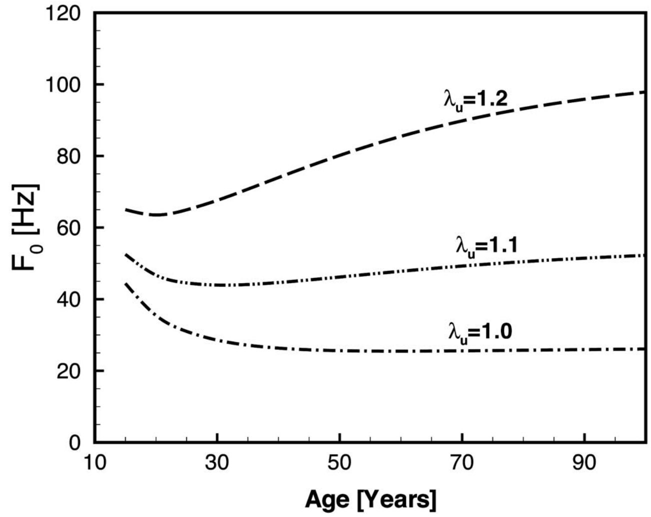

FIG. 5. Fundamental frequency F_0 predicted by the two-layer beam model for males in dependence of age at different stretch levels.

tal frequencies (F_0=220 Hz) would require very high stretch $\lambda_u > 1.5$ at $\Phi=1.0$, or a combination of $\lambda_u > 1.3$ and an activation of extended clamping with $\Phi=0.8$.

For male subjects, Eqs. (24) and (26) can be used to examine the dependence of F_0 on age, based upon the age dependence of the constitutive parameters and *in situ* length values as given in Sec. III A. For the cross-section area of vocal ligament and cover no age dependence could be established, and average values of all male subjects were used for these parameters, $\bar{A}_{0,c}=10.9\ \text{mm}^2, \bar{A}_{0,l}=9.5\ \text{mm}^2$. Figure 5 shows the age dependence of predicted F_0 at three different stretch levels (λ_u=1.0, 1.1 and 1.2). The predictions are characterized by a decrease of F_0 with age in early years until $Y \approx 20$, which is consistent with developmental lifespan changes and could be attributed to the significant increase of vocal fold length during puberty. Subsequently, for increasing age and λ_u=1.1 and 1.2, a gradual increase of F_0 is predicted, also consistent with empirical lifespan changes.[9,10]

In order to find out whether the constitutive parameters (μ_j, α_j) or the geometrical parameters (A_0, L, f_j) play a more significant role in this gender difference, we conduct a numerical experiment. Thereby, we exchange these two sets of parameters between males and females and calculate fundamental frequency values from the composite beam model [Eq. (26)] for two cases: case 1 with a vocal fold with male geometry and female material properties, and case 2 with female geometry and male material properties. At high stretch levels (λ_u=1.2) the fundamental frequencies for the numerical experiment are F_0(case 1)=63.3 Hz and F_0(case 2)=127.9 Hz compared to F_0(male)=83.8 Hz, F_0(female)=95.9 Hz. At high stretch level the change in geometry affects males more (53% increase for F_0(case 2) relative to F_0(male)) than females (33% decrease for F_0(case 1) relative to F_0(female)), but changes in material properties are more significant for females (33% increase for F_0(case 2) relative to F_0(female)) than for males (25% decrease in F_0(case 1) relative to F_0(male)). At low stretch levels (λ_u =1.05) the predicted frequencies are F_0(case 1)=27.9 Hz and F_0(case 2)=54.1 Hz, compared to F_0(male) =33.7 Hz, F_0(female)=45.0 Hz. Now, the change in geometry affects males even more strongly (63% increase for F_0(case 2) relative to F_0(male)) than females (38% decrease for F_0(case 1) relative to F_0(female)). Changes in material properties are now overall less significant and affect females (20% increase for F_0(case 2) relative to F_0(female)) on a similar level as males (18% decrease in F_0(case 1) relative to F_0(male)).The reduced influence of material properties on fundamental frequency predictions is due to the fact that at low levels of stretch the nonlinearity of the tissue mechanical response only influences results a little.

To investigate the contributions of the vocal ligament to F_0, the beam model is also applied to the vocal fold cover only. Figure 6 shows F_0 predictions for the two-layer beam model and the cover-only beam model for males and females, respectively. Fundamental frequencies predicted for the cover-only model are significantly below those of the composite model, especially for females. For males the present study did not find the difference between the composite model and the cover-only model to be significant, however, the number of samples considered is small. For

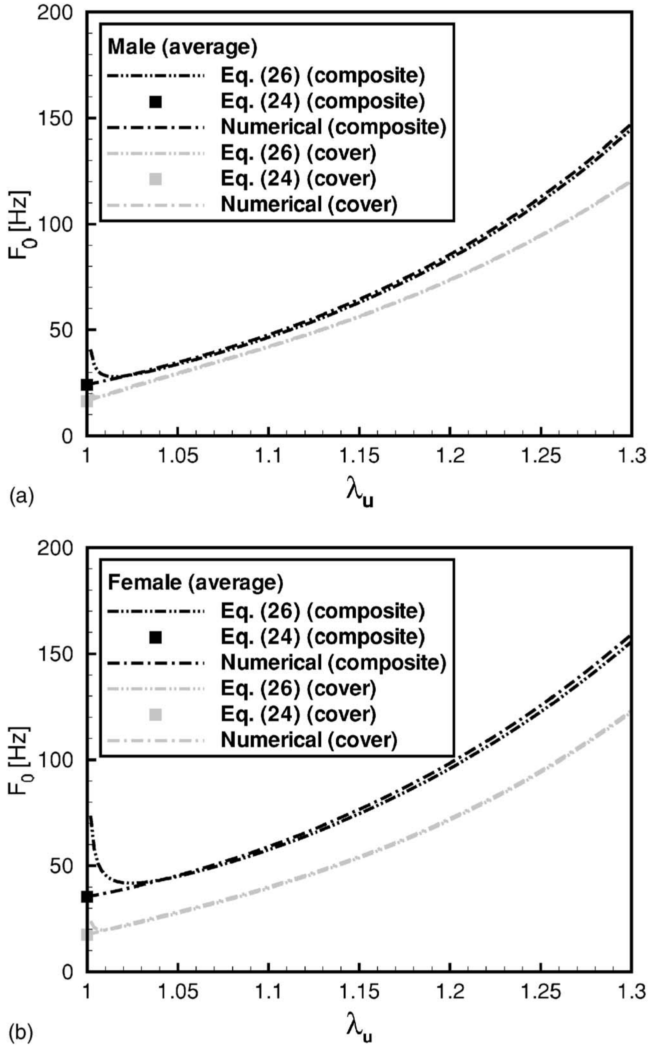

FIG. 6. Dependence of the fundamental frequency F_0 projected on the two-layer composite beam model and the cover-only beam model upon the stretch level for (a) the average male vocal fold and (b) the average female vocal fold.

females the presence of the ligament contributes to a much higher F_0. These predictions could reflect the finding of the initial shear modulus of the male cover being significantly larger than that of the female.

IV. DISCUSSION

The present study is motivated by the premise that a better understanding of fundamental frequency regulation and F_0 predictions can only be possible if an accurate description of the tissue, i.e., its mechanical response and its geometric features, is combined with an appropriate mechanical model of vocal fold vibration.

The present results show that for males the nonlinearity of the elastic response, as characterized by the parameter α, of the ligament is significantly higher than that of the cover, whereas there appeared to be no significant difference in the initial equilibrium shear modulus μ between the cover and the ligament. This combination of parameters implies that when subjected to the same amount of stretch the ligament would carry a larger tensile stress. This may allow the vocal fold cover to remain relatively loose for facilitating the propagation of the mucosal wave, and also prevent it from being damaged from excessive tension by allowing the vocal ligament to assume most of the tensile stress. Interestingly, this may not apply for females as no statistically significant differences in the constitutive parameters were found between the cover and the ligament.

As for the gender dependence of the constitutive parameters, it was found that for vocal fold cover specimens the initial equilibrium shear modulus μ of males is significantly larger than that of females, whereas for vocal ligament specimens the nonlinearity of the elastic response α for males is significantly larger than that for females. These findings may suggest that the homogenized or average stress in the vocal fold of the average male would be larger than that of the average female at equal stretch. In the present study no direct correlation between the model parameters and the underlying histological structure of the tissues is made. However, qualitatively the present results agree with findings in previous studies which have investigated the age and gender dependence of vocal fold histological structure in terms of collagen and elastin content.[1,28–30] Tissues with higher stiffness—that of males relative to females, and those of older males relative to younger males—were shown to possess higher collagen content.[28] The age-related increase in F_0 for males, Fig. 5, could be attributed to the stiffening of vocal fold tissue with increasing age as related to increased collagen content with age.[28] The gradually increasing F_0 values predicted for λ_u $=1.1$ and 1.2 agree well with empirical lifespan changes where F_0 gradually increases with age in males above 40 years old.[9,10] It must be noted that any extrapolation of age dependence to include children cannot be considered at this stage of investigation. For children no age dependence of the constitutive and geometrical parameters has been determined in the present study and it is to be expected that significant changes in vocal fold structure occur during development and puberty.[5,8]

Fundamental frequencies also depend on the geometric features of the vocal fold. Female vocal folds have been found to be generally shorter than male vocal folds.[5] This is again confirmed in the present investigation, with the lengths of female vocal folds only 72% of those of male vocal folds on average. It has been demonstrated that female vocal folds are subjected to a higher stretch level during speaking than male vocal folds.[27] Shorter vocal folds and higher stretch levels may both compensate for the lower elastic modulus or a lower degree of nonlinearity, contributing to a higher overall F_0 for females compared to males. In the beam model predicted F_0 also depends on the aspect ratio of the beam cross section. The layered microstructure of the lamina propria motivates the use of rectangular cross-section geometries in the present study. In comparison, Titze and Hunter[6] employed a square cross section in their beam model. It is noted that all equations for F_0 presented in this study are in a general form such that different values of cross-section aspect ratio m can be employed in future studies.

From the predictions of the beam model it is found that the bending stiffness of the lamina propria alone can account for the restoring force of vibration when the vocal fold is fully relaxed. Thus the beam model can predict nonzero values of fundamental frequency in the unstretched state. For vocal folds in the stretched state, Fig. 3 indicates that the effects of bending stiffness are significant at any level of stretch. While the relative difference between F_0 predicted from the string and the beam model certainly decreases with increased stretch, the absolute difference between predictions of the string and the beam model in fact increases. On average, the contribution of bending stiffness is found to be larger for females than for males. From investigating, based on Fig. 3, an easily computed estimate for the predicted F_0 can be proposed. The sum of F_0 as predicted from the string model [Eq. (10)] and the beam model in the unstretched state [Eq. (24)] provides an approximation of the solution to the beam model [Eq. (26)]:

$$F_0(\lambda_u) \approx F_0(\lambda_u = 1) + F_0^{\text{string}} = \frac{1.7804}{L^2}\sqrt{\frac{A_0(\mu_c^2 f_c^3 + \mu_l^2 f_l^3 + 6\mu_c\mu_l\sqrt{f_c^3 f_l^3} + 4\mu_c\mu_l f_c f_l)}{m\rho(\mu_c f_c + \mu_l f_l)}} + \frac{1}{L\sqrt{2\rho}}\sqrt{\frac{\mu_c f_c}{\alpha_c}\left(\lambda_u^{\alpha_c-2} - \lambda_u^{-\alpha_c/2-2}\right) + \frac{\mu_l f_l}{\alpha_l}\left(\lambda_u^{\alpha_l-2} - \lambda_u^{-\alpha_l/2-2}\right)}. \quad (29)$$

It can be seen from Fig. 3 that the fundamental frequency models predict higher F_0 for the average female vocal fold than for the average male vocal fold, consistent with empirical data on human speaking F_0.

V. CONCLUSION

This study combines constitutive models of the vocal fold tissue with structural vibration models in order to investigate the dependence of the fundamental frequency of human phonation upon tissue properties, gender, age and F_0 regulation mechanisms. The mechanical responses of vocal fold cover and vocal ligament specimens are characterized, and it is shown that some statistically significant differences in the mechanical properties of these tissues exist in addition to the well established length difference between males and females. Fundamental frequencies calculated with the proposed two-layer composite beam model with vibration in the medial-lateral direction are significantly higher than those predicted by the string model. The fundamental frequency models presented differ from previous models in that vocal fold tissue is treated as incompressible, such that changes in vocal fold cross-section area during elongation are accounted for while previous models[6] assumed a constant cross-section area. Furthermore, both the vocal fold cover and the vocal ligament are integrated in the prediction of F_0 whereas previous studies projected the model predictions upon either the ligament[6] or the cover.[3] The beam model predicts nonzero fundamental frequency for the unstretched vocal fold [Eq. (24)], whereas the string model always predicts a zero fundamental frequency. It is proposed that—despite the nonlinear tissue response—a simple but reasonable prediction of F_0 can be obtained by summation of the F_0 predicted from the string model and the beam model for the relaxed vocal fold. Accounting for the bending stiffness of the vocal folds leads to significantly higher predicted F_0 than the string model over a wide range of stretch relevant to phonation. At vocal fold stretch levels deemed physiologically relevant the beam model is capable of predicting F_0 values approaching the lower bound of the phonatory range. Accounting for vocal fold stretch and extended clamping, i.e., the effective vocal fold length, the current two-layer beam model is capable of predicting F_0 values consistent with empirical data.[9,10]

Further additions and improvements can be developed based on the material parameters and the vibration model provided. It is believed that the vocalis muscle regulates the vibration of the vocal fold to some extent.[5] The two-layer composite beam model could be improved by placing the beam onto an elastic foundation representing the vocalis muscle. Such an additional constraint will lead to predictions of a higher F_0, and also introduce an additional mechanism of F_0 regulation through control of the muscle stiffness. Analytical solutions to the beam model could also be derived for cases with irregular cross-section geometries, and would then account for the macula flavae.[6] With such additions a more realistic and complete model would arise leading to more accurate predictions of the speaking F_0 at a physiological level.

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