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# Decomposing Welfare Changes in the GTAP Model

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## *Decomposing Welfare Changes in the GTAP Model*

**Karen M. Huff**

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**GTAP Technical Paper No. 5**

**January, 2000**

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### *Abstract*

This paper develops a complete decomposition of the change in global welfare in the GTAP model. In particular, this money metric change is broken down into component parts, each of which relates to a quantity change interacting with a distortion in the model. This enables the user to assess, for example, how much of the gains from trade reform are attributable to a given commodity and/or a given region. The commodity and regionspecific changes in allocative efficiency can be further decomposed by transaction/tax instrument. We find that this greatly facilitates the presentation and analysis of results from GTAP.

We motivate the derivation of this decomposition with the case of a one region, three commodity analogue to the GTAP model. This permits us to focus on purely allocative efficiency effects (no terms of the trade changes). Extension to the multiregion model adds the prospect of terms of trade effects on regional EV, and the multiregion decomposition isolates the contribution of tradable price changes to regional welfare. This is demonstrated in a 3 region, 3 commodity example. Finally, we offer a more complete decomposition which takes into account the impact of changes in endowments and technology on regional welfare.

This revised (2001) version introduces a number of important changes to the original (1996) paper. In particular, we build on the new final demand structure for GTAP proposed by McDougall (2001). This includes explicit recognition of changes in the marginal utility of income, as well as a *per capita* decomposition. We also take account of version 5.0 changes in the standard GTAP model, including the introduction of multiple international margins commodities.



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# *Decomposing Welfare Changes in the GTAP Model Introduction*

## *1. Introduction*

This paper describes an extension to the theoretical structure of the GTAP model of the world economy to facilitate further analysis of welfare changes (see also, Huff, 1999). This is accomplished by implementing a decomposition of the equivalent variation (EV) welfare measure currently employed in the model. The revised (2001) version of this paper draws on a new final demand system for GTAP proposed by McDougall (2001).

The equations needed to decompose welfare changes have been treated as an "add-on" module which may be appended to the bottom of the TABLO code used to implement the standard GTAP model. [See Hertel and Tsigas (1996) for a complete description of the original GTAP model structure.] This new module does not affect the basic theory of the model, nor are any additions to the data base required. Rather, its role is to facilitate analysis of the sources of welfare gains in the GTAP model. Furthermore, since the theory embedded in the GTAP model is quite standard, this decomposition technique could also be applied — with appropriate modifications — to other AGE models. Indeed Hanslow (2001) has taken up this challenge. He provides a very general welfare decomposition which can be applied to a wide class of AGE models. His paper provides a useful companion to this technical paper.

## *2. Graphical Exposition*

The decomposition developed in this paper may be most easily understood in terms of a simple graphical example which has been adapted from Loo and Tower (1989). Figure 1 depicts a small, closed economy in which all economic activity has been divided into two sectors: A and B. Furthermore, there is only one mobile factor of production, labor. The two lines,  $aA^*$  and  $bB^*$ , portray the social marginal value product of labor in each of the two sectors. The optimal allocation of labor between A and B,  $L^*$ , is defined by the intersection of these two lines. By equating the social marginal value product of labor in the two sectors, this is the allocation which maximizes welfare in this simple economy.

However, the actual allocation of labor in figure 1 diverges from the optimal point  $e^*$ . This is due to the presence of an ad valorem tax on labor usage (with rate  $\tau$ ) in sector A. The marginal value product of labor, net of this tax, is represented by the line  $aA$ . This discourages the employment of labor in A, resulting in an equilibrium at point  $e$ . In the face of this market outcome, the economy experiences a deadweight loss equal to the shaded triangle in figure 1. With the endowment of labor fixed exogenously -- as is usually the case in comparative static simulations -- the only way to increase welfare in this economy is to reduce the excess burden associated with this distortion. In the GTAP model there are many such distortions, and a key contribution of this technical paper is to identify how these interact with simulation experiments in order to generate changes in welfare.

Figure 1. Excess Burden in a Two Sector Economy

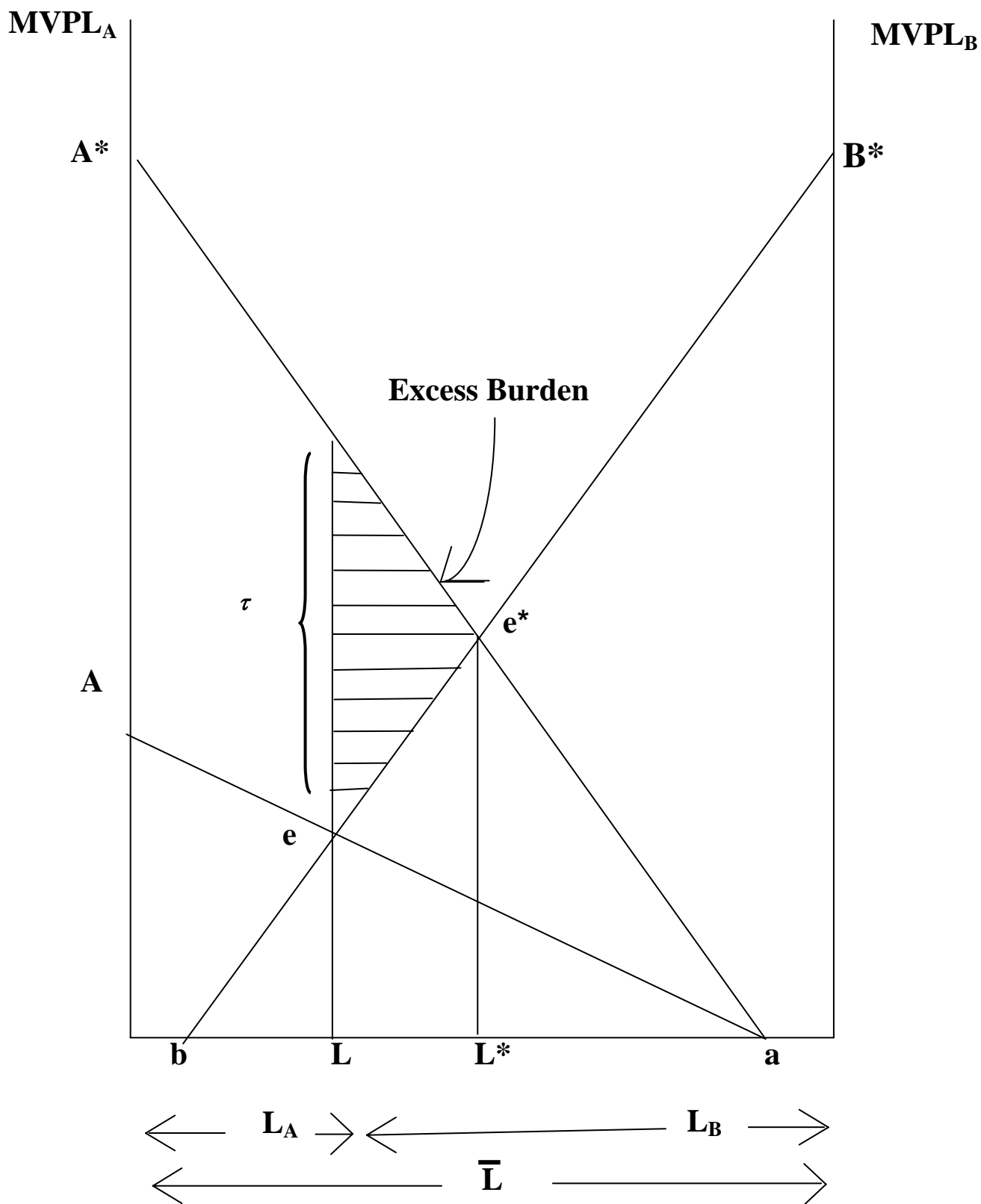


Figure 2 shows the outcome of a simulation experiment whereby the tax on labor in this two sector, closed economy is reduced from  $\tau$  to  $\tau'$ . This shifts the net of tax MVP schedule for sector A upwards to A' and accordingly changes the equilibrium from  $e$  to  $e'$ . The resulting reallocation of labor from sector B to sector A ( $dL$ ) reduces the excess burden associated with the labor tax, and generates an improvement in allocative efficiency equal to the shaded trapezoid. The size of this gain is seen to be a function of the size of the initial distortion ( $\tau$ ), the degree of reform ( $\tau-\tau'$ ), and the responsiveness of the labor market to this change ( $dL$ ).

Of course, most GTAP simulations only perturb a few of the taxes/subsidies in the model. Indeed, in some cases, none of the distortions are shocked. Instead, technology or endowments are perturbed. Nevertheless, welfare may change as a result of allocative efficiency effects. This point is illustrated in figure 3. Here, there is an improvement in the technology used to produce good A, resulting in an upward shift of both the social and net-of-tax MVP schedules. This has the effect of shifting the equilibrium allocation of labor in the economy to  $e'$ . Now the gains from this technological improvement may be decomposed into two parts. The first is the direct gain due to the use of improved technology to produce current levels of good A. However, there is also an indirect gain which results from the reallocation of labor between sectors in the face of the pre-existing labor market distortion. This allocative efficiency effect stems from the fact that any external shock which causes labor to be reallocated from the relatively low social MVP sector B to the higher social MVP sector A, will bring gains to the economy. Alternatively, this is the gain which is forgone, if for some reason the labor were prevented from moving from B to A in the wake of this technological improvement. This area is a function of the size of the pre-existing distortion ( $\tau$ ) and the amount of labor reallocated across this distortion as a result of the simulation ( $dL$ ).

The remainder of this paper outlines an approach to measuring the area of the trapezoid in Figure 2. The beauty of this numerical approach is that it generalizes to handle any arbitrary number of distortions, of which there are thousands in most GTAP applications.



Figure 2. Reduction in Excess Burden in the Two Sector Economy

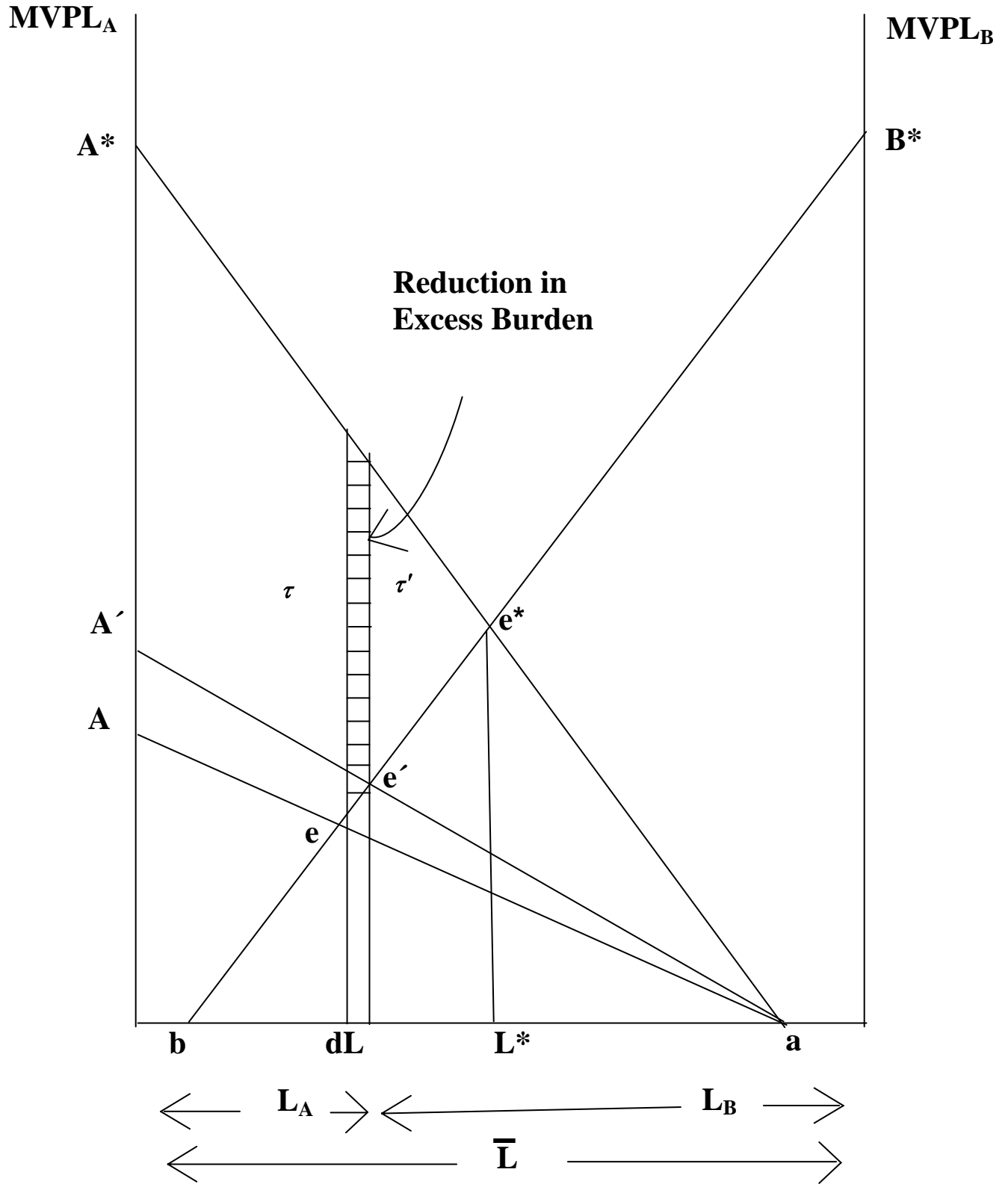
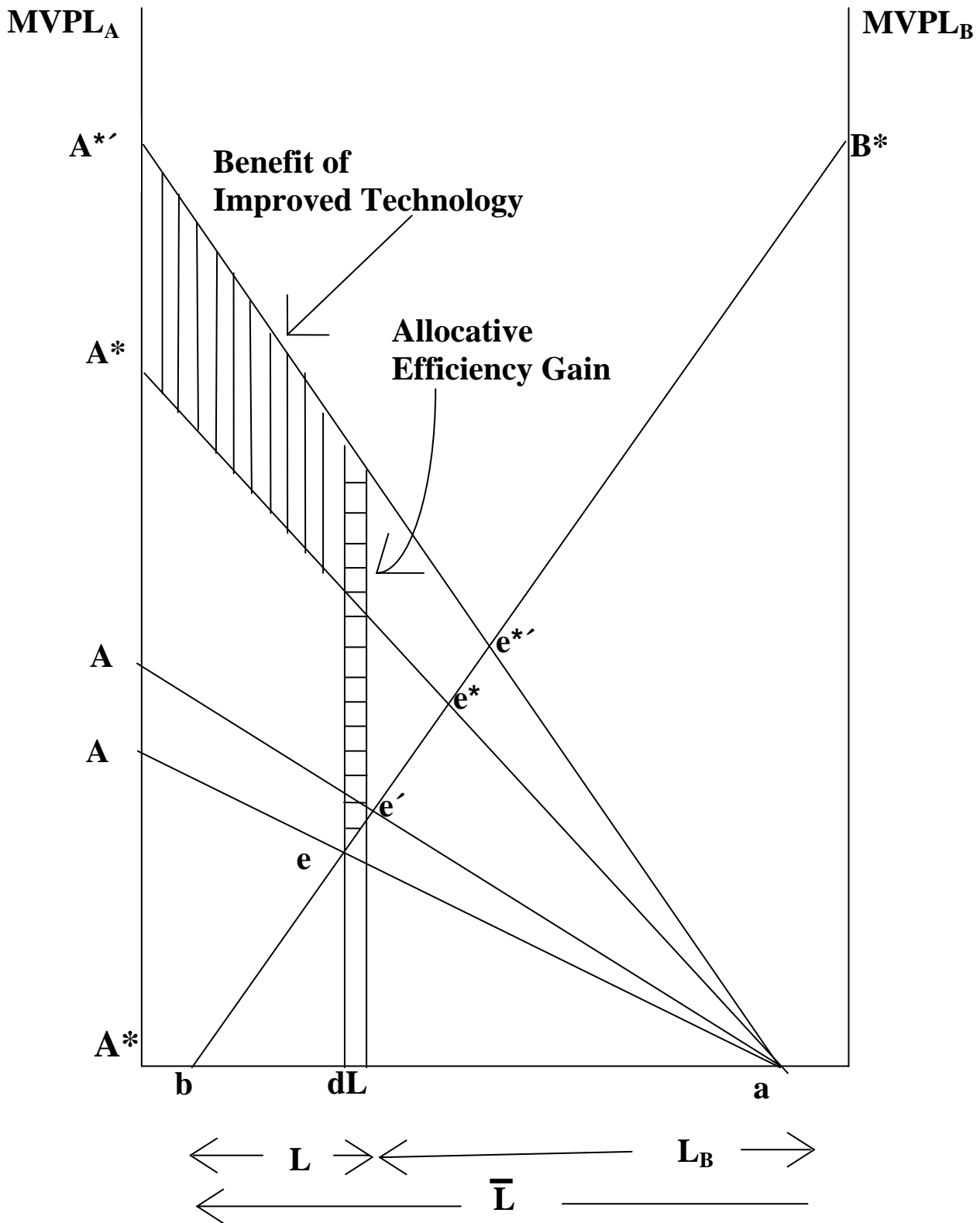


Figure 3. Allocative Efficiency Consequences of an Advancement in Technology in Sector A



### 3. Welfare Changes in the Single Region Model

The single region version of the GTAP model has proven to be very convenient for teaching and demonstration purposes, so the decomposition is first developed in this context and a simple example of its implementation is presented. The decomposition is then extended to the multiregion case.

The GTAP model features a representative regional household whose behavior is governed by an aggregate utility function specified over *per capita* private household consumption, *per capita* government spending and *per capita* savings<sup>1</sup>. The percentage change in aggregate *per capita* utility for a region [ $u(r)$ ] is the welfare change variable computed by the standard GTAP model during simulations<sup>2</sup>. The model also computes a money metric equivalent of this utility change and any change in population, [ $EV(r)$ ]. This convenient measure summarizes the regional welfare changes resulting from a policy shock in dollar values (\$US million) and is frequently reported in studies employing the GTAP model. The decomposition proposed in this paper is based on this measure.

#### 3.1 Regional Equivalent Variation in GTAP

Following McDougall (2001), we can obtain the equivalent variation (EV) associated with a perturbation to the GTAP model as follows. The regional household's EV is equal to the difference between the expenditure required to obtain the new (post-simulation) level of utility at initial prices ( $Y_{EV}$ ) and that available initially ( $\bar{Y}$ ):

$$EV = Y_{EV} - \bar{Y} \quad (1)$$

Differentiating we obtain:

$$dEV = (.01)Y_{EV} y_{EV} \quad (2)$$

where  $y_{EV}$  is the percentage change in  $Y_{EV}$ . Since McDougall's revised demand system for the regional household is on a *per capita* basis,  $y_{EV}$  can be broken into a percentage change in population ( $n$ ) and the percentage change in *per capita* expenditure ( $X_{EV}$ ) required to achieve new *per capita* utility at initial prices:

$$y_{EV} = n + X_{EV} \quad (3)$$

One of McDougall's contributions to the new regional demand system was to introduce the elasticity of expenditure with respect to utility,  $\Phi$ , which captures the impact of non-homothetic preferences for private consumption on *per capita* regional utility (see also Hanslow, 2001). Thus we have:

---

1. Savings enters the static utility function as a proxy for future consumption.

2. In line with standard GTAP notational conventions, this technical paper uses the following conventions: lower case lettering is used when referring to variables in the model representing percentage changes in underlying theoretical variables and upper case lettering is employed when referring to the levels of model variables or coefficients.

$$X = P + \Phi u \quad (4)$$

Holding prices constant at initial levels we obtain the *per capita* change associated with the EV measure:

$$X_{EV} = \Phi_{EV} u \quad (5)$$

Plugging (5) into (3) and the result into (2), we have the following expression for the change in regional welfare:

$$\begin{aligned} dEV &= (0.01) Y_{EV} (n + \Phi_{EV} u) \\ &= (0.01) Y_{EV} n + (0.01) Y_{EV} \Phi_{EV} u \end{aligned} \quad (6)$$

The ultimate goal in this paper<sup>3</sup> is to introduce a decomposition of real income, such that we can explain the sources of  $dEV$  deriving from the second term in (6).

A decomposition of total regional real income may be defined as:

$$D \equiv Y(y - p) \quad (7)$$

Noting that the percentage change in total regional expenditure equals the sum of the percentage changes in *per capita* expenditure and population,  $y = n + x$ , and using equation (4) to obtain  $u \Phi = (x - p)$ , we have the following:

$$u = \Phi^{-1} (y - p - n). \quad (8)$$

Plugging this into (6) results in the following decomposition of  $EV$  as a function of regional real income:

$$dEV = (.01) \left[ 1 - \frac{\Phi_{EV}}{\Phi} \right] Y_{EV} n + (.01) \frac{\Phi_{EV}}{\Phi} \frac{Y_{EV}}{Y} D \quad (9)$$

The first term in (9) captures the impact of changing population on regional  $EV$ . This is a straightforward scaling of the population growth rate. The second term is more interesting. It shows the link between the change in total real income in the region and the  $EV$ .

### 3.2 Basic Decomposition

As noted above, in a comparative static, AGE model, with fixed population, endowments and technology, the only means of increasing welfare is by reducing the excess burden owing to existing distortions. Furthermore, as was shown in figures 2 and 3, any change in allocative efficiency may be directly related to taxes (or tax changes) interacting with equilibrium quantity changes. Thus the following form for the single region, decomposition of real income is hardly surprising (a complete derivation will be given below):

$$\begin{aligned} D = & \quad (10) \\ & \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * \text{qo}(i)) - \text{VDEP} * \text{kb} \\ & + \text{sum}(i, \text{NSAV\_COMM}, \text{PTAX}(i) * \text{qo}(i)) \\ & + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \text{ETAX}(i, j) * \text{qfe}(i, j))) \end{aligned}$$

<sup>3</sup> Whereas the earlier (1996) version of this technical paper focused on decomposition of the aggregate, regional  $EV$ , this version follows McDougall's suggestion of decomposing *per capita* regional utility.

$$\begin{aligned}
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \text{DFTAX}(i, j) * \text{qf}(i, j))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{DPTAX}(i) * \text{qp}(i)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{DGTAX}(i) * \text{qg}(i))
\end{aligned}$$

The first term on the right-hand side of (10) corresponds to the change in income due to changing endowments, net of depreciation. It is typically zero in a comparative static simulation. The following five terms each correspond to a transaction tax instrument in the one region model with PTAX, ETAX, DFTAX, DPTAX and DGTAX representing a tax on output of good  $i$ , a tax on use of endowment  $i$  in industry  $j$ , a tax on use of intermediate good  $i$  in industry  $j$ , a tax on private household consumption of good  $i$  and a tax on government consumption of good  $i$ , respectively. Each tax (subsidy) is paired with the relevant quantity change, which defines the nature of the tax. For example,  $\text{qfe}(i,j)$  is the percentage change in derived demand by industry  $j$  for endowment commodity  $i$ . As mentioned above,  $\text{ETAX}(i,j)$  is the tax on endowment  $i$  usage in sector  $j$ . For those unfamiliar with GTAP notation, the other quantity changes are:  $\text{qo}(i)$  - change in supply of good  $i$ ,  $\text{qf}(i,j)$  - change in derived demand for intermediate good  $i$  by sector  $j$ ,  $\text{qp}(i)$  - change in consumer demand for good  $i$ , and  $\text{qg}(i)$ -change in government demand for good  $i$ .

The intuition behind this decomposition is straightforward in light of the discussion of figures 1 - 3. For example, it is welfare-improving to increase the level of a relatively highly taxed activity, since this involves the reallocation of a commodity or endowment from a low value use into a relatively high social marginal value usage. Conversely, if the simulation in question reduces the level of a subsidized activity, this will tend to benefit the economy in question, since it involves the reallocation of resources away from a relatively low social marginal value product use. Furthermore, note that if there are no taxes in the initial equilibrium, and the nature of the shock is something other than a tax/subsidy intervention, then there will be no allocative efficiency effect from the simulation. Equation (1) also shows why it is so important to utilize a non-linear solution procedure for this model, whereby tax revenues are updated over the course of the simulation. For example, if one starts out in a distortion-free setting (all tax terms in (1) equal zero), then the local approximation to the efficiency effect of introducing a distortion will necessarily be zero. However, for a non-infinitesimal tax, once the tax revenue term is updated, it will appear in (1), thereby interacting with the taxed quantity to generate a change in allocative efficiency.

The decomposition offered in (1) is designed to provide as much detail as possible on the sources of the welfare changes from policy experiments. Not only can it show that a portion of the overall welfare change has resulted from decreased output ( $\text{qo}$ ), but it also shows the components of the change in terms of output changes of specific commodities interacting with the output taxes or subsidies (PTAX) present in the model for each of the commodities in question. Likewise, if a model simulation resulted in an increase in the use of a intermediate input ( $\text{qf}$ ) that is taxed (DFTAX), the decomposition clearly shows how this contributes positively to the overall welfare change. Finally, note that summation of all of the various terms in the decomposition equals the overall change in real income ( $D$ ) from the policy simulation under study.

### ***3.3 Formal Derivation of Equation (10)***

Our derivation of equation (10) is inspired by the work of Keller (1980), who offered a similar decomposition which he implemented in an applied general equilibrium model of The Netherlands. The complete derivation is provided in Appendix A. In the text below we simply provide the basic idea behind this derivation.

The starting point is the expression for the change in household income as a function of primary factor payments net of depreciation (first two terms on the right hand side), and tax revenues net of subsidies (remaining terms on right hand side):

$$\begin{aligned}
\text{INCOME} * y = & \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * [\text{ps}(i) + \text{qo}(i)]) & (11) \\
& - \text{VDEP} * [\text{pcgds} + \text{kb}] \\
+ & \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i) * [\text{pm}(i) + \text{qo}(i)] \} \\
& - \{ \text{VOA}(i) * [\text{ps}(i) + \text{qo}(i)] \} ) \\
+ & \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i, j) * [\text{pfe}(i, j) + \text{qfe}(i, j)] \} \\
& - \{ \text{VFM}(i, j) * [\text{pm}(i) + \text{qfe}(i, j)] \} ) ) \\
+ & \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i, j) * [\text{pfe}(i, j) + \text{qfe}(i, j)] \} \\
& - \{ \text{VFM}(i, j) * [\text{pmes}(i, j) + \text{qfe}(i, j)] \} ) ) \\
+ & \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VFA}(i, j) * [\text{pf}(i, j) + \text{qf}(i, j)] \} \\
& - \{ \text{VFM}(i, j) * [\text{pm}(i) + \text{qf}(i, j)] \} ) ) \\
+ & \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VPA}(i) * [\text{pp}(i) + \text{qp}(i)] \} \\
& - \{ \text{VPM}(i) * [\text{pm}(i) + \text{qp}(i)] \} ) \\
+ & \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VGA}(i) * [\text{pg}(i) + \text{qg}(i)] \} \\
& - \{ \text{VGM}(i) * [\text{pm}(i) + \text{qg}(i)] \} ) .
\end{aligned}$$

The sets used in the single region model include: NSAV\_COMM, DEMD\_COMM, PROD\_COMM, ENDW\_COMM, ENDWS\_COMM, ENDWM\_COMM, and CGDS\_COMM. These represent the sets of non-savings commodities, derived demand commodities, produced commodities, endowment commodities, sluggish endowment commodities (typically land), mobile endowment commodities (typically labor and capital), and capital goods commodities (cgds), respectively.

The left hand side of (11) consists of household income (INCOME) multiplied by its percentage change (y). On the right hand side VOA, VFA, VPA and VGA represent value flows at agents' prices for output, firms' purchases of intermediate inputs, private and government household purchases, respectively, and VOM, VFM, VPM and VGM represent the same value flows, but at market prices. Thus, the difference between the value of sales at market and agents' prices represents output tax revenues, and the difference between the value of purchases at agents' prices and market prices represents input tax revenues. VDEP is the value of depreciation, while pcgds and kb are the price of capital goods and initial stock of capital goods, respectively. Each of the above-mentioned value flows is multiplied by the sum of the percentage changes in its associated price and quantity.

We proceed by substituting most of the equilibrium conditions in the model into expression (11). For example, total differentiation of the zero profits condition and use of the envelope theorem, yields the following relationship between output price and input prices:

$$\begin{aligned}
& (\text{all}, j, \text{PROD\_COMM}) \\
& \text{VOA}(j) * \text{ps}(j) & (12) \\
& = \text{sum}(i, \text{ENDW\_COMM}, \text{VFA}(i, j) * \text{pfe}(i, j)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VFA}(i, j) * \text{pf}(i, j)).
\end{aligned}$$

If the right hand side of (12) is substituted into the second term on the right-hand side of (11), then those portions of the third and fourth terms involving pfe(i) and pf(i,j) can be canceled. To simplify further, the following market clearing conditions for the traded goods and endowments are used:

$$(\text{all}, i, \text{TRAD\_COMM})$$

$$VOM(i) = \text{sum}(j, \text{PROD\_COMM}, VFM(i, j)) + VPM(i) + VGM(i). \quad (13)$$

$$\begin{aligned} & (\text{all}, i, \text{ENDW\_COMM}) \\ & VOM(i) = \text{sum}(j, \text{PROD\_COMM}, VFM(i, j)). \end{aligned}$$

Multiplying both sides of this equation by the percentage change in the market price of  $i$ ,  $pm(i)$ , and substituting into the expression for regional income gives:

$$\begin{aligned} \text{INCOME} * y &= \text{sum}(i, \text{ENDW\_COMM}, VOA(i) * [ps(i) + qo(i)]) & (11') \\ & \quad - \text{VDEP} * [pcgds + kb] \\ + \text{sum}(i, \text{NSAV\_COMM}, \{VOM(i) * qo(i)\} & \\ & \quad - \{VOA(i) * qo(i)\}) - VOA("cgds") * ps("cgds") \\ & \quad + VOA("cgds") * ps("cgds") + VOM("cgds") * pm("cgds") \\ + \text{sum}(i, \text{ENDWS\_COMM}, \{VOM(i) * pm(i)\}) & \\ - \text{sum}(i, \text{ENDW\_COMM}, \{VOA(i) * ps(i)\}) & \\ + \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{VFA(i, j) * qfe(i, j)\} & \\ & \quad - \{VFM(i, j) * qfe(i, j)\})) \\ + \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{VFA(i, j) * qfe(i, j)\} & \\ & \quad - \{VFM(i, j) * [pmes(i, j) + qfe(i, j)]\})) \\ + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{VFA(i, j) * qf(i, j)\} & \\ & \quad - \{VFM(i, j) * qf(i, j)\})) \\ + \text{sum}(i, \text{TRAD\_COMM}, \{VPA(i) * [pp(i) + qp(i)]\} - \{VPM(i) * qp(i)\}) & \\ + \text{sum}(i, \text{TRAD\_COMM}, \{VGA(i) * [pg(i) + qg(i)]\} - \{VGM(i) * qg(i)\}). & \end{aligned}$$

Several relationships may then be employed in the simplification of the expression for regional income. First, since there are no taxes on capital goods:

$$pcgds = ps("cgds") = pm("cgds"), \text{ and} \quad (14)$$

$$VOA("cgds") = VOM("cgds"). \quad (15)$$

The market prices of sluggish commodities can be related to their aggregate, as follows:

$$\begin{aligned} & (\text{all}, i, \text{ENDWS\_COMM}) \\ & VOM(i) * pm(i) = \text{sum}(j, \text{PROD\_COMM}, VFM(i, j) * pmes(i, j)). \end{aligned} \quad (16)$$

Net investment is defined as:

$$\text{NETINV} = VOA("cgds") - \text{VDEP}. \quad (17)$$

Using (14) - (17) to simplify the expression for income, subtracting  $(\text{SAVE} * psave)$  from both sides, and rearranging, the following expression for the regional welfare decomposition is obtained:

$$\begin{aligned} \text{INCOME} * y & \\ - \text{SAVE} * psave & \quad (11'') \\ - \text{sum}(i, \text{TRAD\_COMM}, VPA(i) * pp(i)) & \\ - \text{sum}(i, \text{TRAD\_COMM}, VGA(i) * pg(i)) & \\ = \text{sum}(i, \text{ENDW\_COMM}, VOA(i) * qo(i)) & \\ + \text{sum}(i, \text{NSAV\_COMM}, \{VOM(i) - VOA(i)\} * qo(i)) & \\ + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, & \\ \quad \{VFA(i, j) - VFM(i, j)\} * qfe(i, j))) & \end{aligned}$$

$$\begin{aligned}
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i, j) - \text{VIFM}(i, j) \} * \text{qfm}(i, j))) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VFA}(i, j) - \text{VFM}(i, j) \} * \text{qf}(i, j))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VPA}(i) - \text{VPM}(i) \} * \text{qp}(i)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VGA}(i) - \text{VGM}(i) \} * \text{qg}(i)) \\
& + \text{NETINV} * \text{pcgds} \\
& - \text{SAVE} * \text{psave}.
\end{aligned}$$

The final step involves extracting INCOME from each term on the left-hand side of (11') to get the term:

$$\begin{aligned}
& \text{INCOME} * [y - \text{SAVE}/\text{INCOME} * \text{psave} \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VPA}(i)/\text{INCOME} * \text{pp}(i)) \\
& \quad - \text{sum}(i, \text{TRAD\_COMM}, \text{VGA}(i)/\text{INCOME} * \text{pg}(i)) ]
\end{aligned} \tag{18}$$

This expression gives INCOME times the change in "deflated income":  $Y(y - p) = D$

Next, substitute in the following tax instruments:

$$\begin{aligned}
& \text{VOM}(i) - \text{VOA}(i) = \text{PTAX}(i), \\
& \text{VFA}(i, j) - \text{VFM}(i, j) = \text{ETAX}(i, j) \text{ for endowment commodities } i, \\
& \text{VFA}(i, j) - \text{VFM}(i, j) = \text{DFTAX}(i, j), \\
& \text{VPA}(i) - \text{VPM}(i) = \text{DPTAX}(i),
\end{aligned} \tag{19}$$

and

$$\text{VGA}(i) - \text{VGM}(i) = \text{DGTAX}(i).$$

Also note that

$$\text{pcgds} = \text{psave}, \tag{20}$$

and, by virtue of Walras= Law:

$$\text{NETINV} = \text{SAVE} \tag{21}$$

This gives:

$$\begin{aligned}
D = & \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * \text{qo}(i)) - \text{VDEP} * \text{kb} \\
& + \text{sum}(i, \text{NSAV\_COMM}, \text{PTAX}(i) * \text{qo}(i)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \text{ETAX}(i, j) * \text{qfe}(i, j))) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \text{DFTAX}(i, j) * \text{qf}(i, j))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{DPTAX}(i) * \text{qp}(i)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{DGTAX}(i) * \text{qg}(i))
\end{aligned} \tag{22}$$

which is just equation (10) above.

### 3.4 Per Capita Decomposition of Income and EV

McDougall (2001) argues in favor of placing the EV decomposition in (9) on a per capita basis. This eliminates the interaction between population changes and changes in the elasticity of expenditure with respect to utility. To do this, simply replace equation (7) with:

$$D^* \equiv Y(x - p) \tag{23}$$

where  $x$ , is the percentage change in *per capita* expenditure.



Noting that  $u = \Phi = (x - p)$ , substitute this into (6) to get the *per capita* EV decomposition:

$$dEV = (0.01) Y_{EV} n + (0.01) \frac{\Phi_{EV}}{\Phi} \frac{Y_{EV}}{Y} D^* \quad (24)$$

In order to convert (10) to a *per capita* basis, deduct  $INCOME * pop$  from both sides to obtain:

$$\begin{aligned} D^* = & \\ & \text{sum}(i, ENDW\_COMM, VOA(i) * [qo(i)-pop]) - VDEP * [kb-pop] \quad (25) \\ & + \text{sum}(i, NSAV\_COMM, PTAX(i) * [qo(i)-pop]) \\ & + \text{sum}(i, ENDW\_COMM, \text{sum}(j, PROD\_COMM, ETAX(i, j) * [qfe(i, j)-pop])) \\ & + \text{sum}(j, PROD\_COMM, \text{sum}(i, TRAD\_COMM, DFTAX(i, j) * [qf(i, j)-pop])) \\ & + \text{sum}(i, TRAD\_COMM, DPTAX(i) * [qp(i)-pop]) \\ & + \text{sum}(i, TRAD\_COMM, DGTAX(i) * [qg(i)-pop]) \end{aligned}$$

Note that all of the quantity terms in (25) are deflated by population.

The final step in obtaining a usable decomposition is to substitute (25) into (24) to obtain:

$$\begin{aligned} dEV = EV\_ALT = & \\ & \{ .01 * EVSCALFACT \} * \quad (26) \\ & \{ \text{sum}(i, ENDW\_COMM, VOA(i) * [qo(i)-pop]) - VDEP * [kb-pop] \\ & + \text{sum}(i, NSAV\_COMM, PTAX(i) * [qo(i)-pop]) \\ & + \text{sum}(i, ENDW\_COMM, \text{sum}(j, PROD\_COMM, ETAX(i, j) * [qfe(i, j)-pop])) \\ & + \text{sum}(j, PROD\_COMM, \text{sum}(i, TRAD\_COMM, DFTAX(i, j) * [qf(i, j)-pop])) \\ & + \text{sum}(i, TRAD\_COMM, DPTAX(i) * [qp(i)-pop]) \\ & + \text{sum}(i, TRAD\_COMM, DGTAX(i) * [qg(i)-pop]) \} \\ & + .01 * INCOMEEV * pop ; \end{aligned}$$

where  $EVSCALFACT = \frac{\Phi_{EV}}{\Phi} \frac{Y_{EV}}{Y}$  and  $INCOMEEV = \frac{Y_{EV}}{Y}$

For the sake of convenience, we also define a set of terms which identify the contribution to regional EV of any given quantity change. For example, the contribution of changes in *per capita* output of good  $i$  to the welfare gain (loss) as measured by the EV would be given by:

$$CNT\_qoi(i) = [.01/INCRATIO] * PTAX(i) * [qo(i)-pop] \quad (27)$$

For convenience these components of the total  $EV\_ALT$  expression are routinely computed as part of the add-on module accompanying this paper. They are subsequently processed using the program, `DECOMP.TAB`, and presented in an easy-to-read, header array file: `DECOMP.HAR`

### 3.5 Empirical Examples

The use of the decomposition and demonstration of its practical value can be shown with a few simple examples using the one region version of the model. This version of the one region model is used in the GTAP preparatory course: [www.agecon.purdue.edu/gtap/gtaponline](http://www.agecon.purdue.edu/gtap/gtaponline). It features three produced commodities, namely, food, manufactures and services. The data associated with this model

contains no distortions initially, but taxes and subsidies can be introduced through simulation of the model. The distortions introduced in the simulation are then present in the updated data which can be used for subsequent policy experiments.

The first two experiments are intended to correspond roughly to figures 1 and 3 above. We begin with the undistorted data base and introduce a 50% *ad valorem* tax on labor used in the manufacturing sector. This reduces the demand for labor in manufacturing activity and drives it into other sectors until net of tax wages are equalized across sectors. The resulting change in volume of labor services (measured in \$US millions) is reported in Table 1. Most of this labor is absorbed by services, which is already far-and-away the largest employer in this one region model.

The final column in Table 1 reports the loss in welfare associated with the “Harberger triangle” created by the manufacturing labor tax. It amounts to \$102,014 million. Note that since there are no pre-existing labor taxes in food or services production, the movement of labor into these sectors does not make any direct contribution to the real *per capita* income ( $D^*$ ) and hence welfare.

Table 1. Impact of Tax and Labor Used in Manufacturing

Sector	Labor Tax Rates		Change in Employment (\$US mill.)	Welfare Contribution (\$US mill.)
	Initial	Final		
Food	0	0	13,307	0
Manufactures	0	50	-475,256	-102,014
Services	0	0	461,949	0
Total	n.a.	n.a.	0	-102,014

Next consider an example motivated by Figure 3. Here we begin with the data base created by the preceding labor tax experiment. Thus there is now a pre-existing distortion in the labor market. We then introduce an exogenous shock to technology such that labor moves into the manufacturing sector.<sup>4</sup> The results are shown in Table 2. Even though the tax on labor in manufactures is unchanged in this experiment, its presence interacts with the technology shock to produce a second-best effect. In particular, welfare rises as a result of the increased employment of labor in the taxed sector.

<sup>4</sup> Specifically, we introduce capital-augmenting technical change in the services sector (10% shock). Due to the price-inelastic demand for manufactures, any technical change in that sector results in labor moving out of manufactures.

**Table 2. Impact of Exogenous Technological Change in Presence of Pre-existing Labor Tax**

Sector	Labor Tax Rates		Change in Employment	Welfare Contribution
	Initial	Final	(\$US mill.)	(\$US mill.)
Food	0	0	15,136	0
Manufactures	50	50	81,928	41,911
Services	0	0	-97,064	0
Total	n.a.	n.a.	0	41,911.

The next experiment involves placing a 10% subsidy on the output of manufactures. This experiment is conducted twice. First, the subsidy is introduced in a first-best environment using the original, undistorted data. In the second case, the initial data base is first shocked to introduce a 10% subsidy to food output. The subsidy to manufactures is then introduced in this second-best environment (i.e. in the presence of a pre-existing subsidy on food). The analysis focuses on the differences in the welfare change from the same policy experiment conducted first without, and then with, pre-existing distortions.

Table 3 gives the percentage changes in output from the two experiments. They are quite similar. (i.e., starting from the different base for the second experiment did not change the output results much.) Food output rose by 0.61% and 0.65%, respectively, while manufacturing output rose by 3.20% and 3.21%, respectively. Services output fell by 0.74% for the first experiment and by 0.76% for the second experiment. The increase in manufacturing output is the result expected following the introduction of a subsidy on this activity. Typically, other sectors are expected to contract, as finite resources are bid away from these non-subsidized activities. This is indeed the case for services. However, food output actually rises in this simulation. This complementary relationship derives from two sources. First, food and manufactures are compensated complements in private household demand, in this 3 commodity aggregation. Secondly, because the food sector is heavily dependent on manufactured inputs, lower prices for manufactures tend to lower the price of food output as well, *ceteris paribus*.

**Table 3. Output Changes Following Subsidy on Manufactures Output (% change)**

	No Distortions	Pre-Existing Food Subsidy
Food	0.61	0.65
Manufactures	3.20	3.21
Services	-0.74	-0.76

Tables 4 and 5 report the output changes and welfare results for the introduction of a manufacturing subsidy both without and with pre-existing distortion respectively.

The final column of Table 4 reports the welfare decomposition terms associated with the output subsidies in the model. Because there are no interventions in food or services in the initial data

base, the terms are zero for these two sectors. However, the introduction of a subsidy on manufactures shows up in the second row of Table 4, since  $PTAX("mnc") < 0$  after the subsidy is introduced, and  $qo("mnc") > 0$ . The EV loss of US \$20,280 million is fully due to this intervention.

Since the output changes resulting from the two manufactures subsidy experiments are virtually the same, would one expect the welfare results to be the same? As table 5 shows, the answer to this question is definitely "no". The welfare loss is about \$5,000 million greater than for the first experiment. Why do the results differ? This is where the decomposition comes in handy. Table 5 shows that the direct welfare loss attributed to the manufacturing subsidy, \$20,395 million, is comparable to that in Table 4. However, there is now also an indirect effect owing to the interaction between  $PTAX("food")$  and  $qo("food")$ . The presence of the pre-existing subsidy on food output has an additional negative effect on welfare equal to US \$4,879 million.

The complementary relationship between food and manufacturing is crucial to the welfare outcome obtained. Since the increase in manufacturing output leads to an increase in food output, this positive change to  $qo("food")$  interacts with the negative  $PTAX("food")$  term. (This is negative since it represents a subsidy or negative tax.) The consequence is a negative contribution to EV. Had the two goods been substitutes, the welfare loss for the second experiment would have been smaller than for the first since the interaction between  $qo("food")$  and  $PTAX("food")$  would have produced a positive welfare result which would partially offset the loss due to increased manufacturing output.

From this very simple example of a policy experiment with the single region version of the GTAP model, the usefulness of the proposed welfare decomposition is apparent. It enables the user to pinpoint the exact sources of the model welfare changes thereby providing the correct explanation for the results obtained.

**Table 4. Impact of Manufactures Subsidy in Undistorted Economy**

Sector	Output subsidy Rates		Change in Output (\$US mill.)	Welfare Contribution (\$US mill.)
	Initial	Final		
Food	0	0	27,429	0
Manufactures	0	10	443,989	-20,280
Services	0	0	-177,945	0
Total	n.a.	n.a.	n.a.	-20,280

Table 5 Impact of Manufactures Subsidy in Presence of Pre-existing Food Subsidy

Sector	Output subsidy Rates		Change in Output (\$US mill.)	Welfare Contribution (\$US mill.)
	Initial	Final		
Food	20	20	24,126	-4,879
Manufactures	0	10	446,391	-20,395
Services	0	0	-182,410	0
Total	n.a.	n.a.	n.a.	-25,275

#### 4. Decomposition of the Multiregion EV

For policy applications, the multiregion version of the GTAP model is employed. The standard, multiregion model has many pre-existing distortions and general equilibrium relationships among the model sectors are more complex. In the single region case with few or no pre-existing distortions, it is fairly straightforward to predict welfare outcomes and to see the underlying model interactions behind them. With the multiregion trade model predicting experimental outcomes and being able to explain them is a much more difficult task. This makes the welfare decomposition very appealing as an aid for analyzing results from the standard GTAP modeling framework. For the multiregion version of the GTAP trade model, the equivalent variation can be expressed as follows:

$$\begin{aligned}
 &EV\_ALT(r) \tag{27} \\
 &= [0.01 * EVSCALFACT(r)] \\
 &* [\text{sum}\{i, ENDW\_COMM, VOA(i, r) * [qo(i, r) - pop(r)]\} \\
 &\quad - VDEP(r) * [kb(r) - pop(r)] \\
 &\quad + \text{sum}\{i, NSAV\_COMM, PTAX(i, r) * [qo(i, r) - pop(r)]\} \\
 &\quad + \text{sum}\{i, ENDW\_COMM, \text{sum}\{j, PROD\_COMM, ETAX(i, j, r) * [qfe(i, j, r) - pop(r)]\}\} \\
 &\quad + \text{sum}\{j, PROD\_COMM, \text{sum}\{i, TRAD\_COMM, IFTAX(i, j, r) * [qfm(i, j, r) - pop(r)]\}\} \\
 &\quad + \text{sum}\{j, PROD\_COMM, \text{sum}\{i, TRAD\_COMM, DFTAX(i, j, r) * [qfd(i, j, r) - pop(r)]\}\} \\
 &\quad + \text{sum}\{i, TRAD\_COMM, IPTAX(i, r) * [qpm(i, r) - pop(r)]\} \\
 &\quad + \text{sum}\{i, TRAD\_COMM, DPTAX(i, r) * [qpd(i, r) - pop(r)]\} \\
 &\quad + \text{sum}\{i, TRAD\_COMM, IGTAX(i, r) * [qgm(i, r) - pop(r)]\} \\
 &\quad + \text{sum}\{i, TRAD\_COMM, DGTAX(i, r) * [qgd(i, r) - pop(r)]\} \\
 &\quad + \text{sum}\{i, TRAD\_COMM, \text{sum}\{s, REG, XTAXD(i, r, s) * [qxs(i, r, s) - pop(r)]\}\} \\
 &\quad + \text{sum}\{i, TRAD\_COMM, \text{sum}\{s, REG, MTAX(i, s, r) * [qxs(i, s, r) - pop(r)]\}\} \\
 &\quad + \text{sum}\{i, TRAD\_COMM, \text{sum}\{s, REG, VXWD(i, r, s) * pfob(i, r, s)\}\} \\
 &\quad + \text{sum}\{m, MARG\_COMM, VST(m, r) * pm(m, r)\} \\
 &\quad - \text{sum}\{i, TRAD\_COMM, \text{sum}\{s, REG, VXWD(i, s, r) * pfob(i, s, r)\}\} \\
 &\quad - \text{sum}\{m, MARG\_COMM, VTMD(m, r) * pt(m)\} \\
 &\quad + NETINV(r) * pcgds(r) \\
 &\quad - SAVE(r) * psave(r) ] \\
 &+ 0.01 * INCOME EV(r) * pop(r);
 \end{aligned}$$

The right hand side of this expression is the real income decomposition derived for the multiregion GTAP model in Appendix B. One can see that the decomposition of the EV for the GTAP multiregion trade model is very similar to the single region version. The main differences involve

additional terms arising from the presence of trade taxes (on both imports (MTAX) and exports (XTAXD)) and terms to capture the effect of changes in regional terms of trade. The other significant difference is the added regional dimension of the decomposition. These differences result in the addition of three more sets, namely, REG, TRAD\_COMM, and MARG\_COMM. These are the sets of regions, traded commodities, and margins commodities, respectively.

Changes in welfare in the multiregion model are therefore attributed to the interactions between taxes (both pre-existing and newly introduced taxes) and quantity changes taking place over the course of the simulation, as well as the added effect of changes in regional terms of trade and changes in the relative prices of savings and investment. The contribution of the regional terms of trade effect on welfare is given by:

$$\begin{aligned}
 \text{CNTtotr}(r) = & \\
 & [0.01 * \text{EVSCALFACT}(r)] * \tag{28} \\
 & [\text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{VXWD}(i, r, s) * \text{pfob}(i, r, s)\}\} \\
 & \quad + \text{sum}\{m, \text{MARG\_COMM}, \text{VST}(m, r) * \text{pm}(m, r)\} \\
 & \quad - \text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{VXWD}(i, s, r) * \text{pfob}(i, s, r)\}\} \\
 & \quad - \text{sum}\{m, \text{MARG\_COMM}, \text{VTMD}(m, r) * \text{pt}(m)\}]
 \end{aligned}$$

Here VXWD represents the value of exports at world prices, VST represents margins export values and VTMD represents the value of margin of type m used in shipping for region r. The variables pfob and pcif are the percentage changes in the *fob* and *cif* prices respectively.

Those regions that are net suppliers of savings to the global bank ( $\text{SAVE}(r) > \text{NETINV}(r)$ ) benefit from a rise in the price of savings, relative to investment goods. This effect is captured in the next term of (27):

$$\text{CNTcgds}(r) = [0.01 * \text{EVSCALFACT}(r)] * [\text{NETINV}(r) * \text{pcdgs}(r) - \text{SAVE}(r) * \text{psave}(r)].$$

This component of regional welfare has been considerably muted in the current version of the GTAP model by permitting  $\text{psave}(r)$  to vary by region. In particular,  $\text{psave}(r)$  moves closely with  $\text{pcgds}(r)$  in order to capture the fact that the majority of savings is invested domestically.<sup>5</sup>

Since balance of payments equilibrium in the GTAP data base requires that the difference between exports and imports equal the difference between savings and investment, the coefficients of the terms in the  $\text{CNTtotr}$  and  $\text{CNTcgds}$  equations sum to zero. This means that we can deflate the prices in these two equations by any arbitrary price index without altering their combined total.<sup>6</sup> We deflate them by  $\text{pxworld}$ , the index of global exports. This prevents these terms from giving spurious individual results. A simple example of this would be when the numeraire is shocked. The formulation in (28) and (29) would give equal and offsetting, but non-zero, contributions to welfare (unless exports equaled imports), whereas the deflated version would report all individual contributions as zero.

The final term in (27) relates to the impact of population on regional EV. This is normally zero, since population is seldom shocked in standard policy simulations.

A simple, empirical example using the multiregion welfare decomposition is presented in the next section of this paper. For a more sophisticated use of this EV decomposition, see Arndt, Hertel,

<sup>5</sup> Readers interested in replicating results from the GTAP book will need to adopt the "GTAP book closure" by which all regional savings rates are fixed and equal to the average price of capital goods, worldwide.

<sup>6</sup> Our thanks to Kevin Hanslow for suggesting this.

Huff, and McDougall (1996). It also provides a good reference for how to present and use the results obtained for the welfare decomposition when analyzing policy experiments.

#### ***4.1 An Empirical Example***

The policy experiment presented in this section makes use of a 3x3 aggregation of the version 2 GTAP data base. It features three produced sectors: food, manufactures and services, and three regions: the United States (USA), the European Union (EU) and the rest of the world (ROW). The experiment involves the reduction of one distortion in the data base. The import tariff on food from USA to the European Union is reduced by 10%.<sup>7</sup>

Table 6 presents the percentage changes in output for the three regions. All of the output changes are small, with food output changing the most in all regions. The tariff reduction results in an increase of USA food production of 0.89% while it falls by 0.36% and 0.11%, respectively for EU and ROW. Manufacturing output falls by 0.15% in USA and rises by 0.09% and 0.03%, respectively for EU and ROW.

**Table 6. Output Changes from Multiregion Liberalization Experiment (percentage change)**

	USA	EU	ROW
Food	0.89	-0.36	-0.11
Manufactures	-0.15	0.09	0.03
Services	0	0.01	0

Table 7 provides a summary of the regional welfare changes that result from the experiment. USA and EU both experience a slight increase in welfare due to cheaper US food imports into the EU, while ROW experiences a small decrease in welfare. Welfare increased by 0.017% and 0.004%, respectively for USA and EU while it falls by 0.004% for ROW. In dollar terms, the equivalent variation measure of these welfare changes are \$887 US million and \$216 million for USA and EU, respectively. ROW loses \$440 US million. Columns 3-5 of Table 7 break down the total EV into component parts. The terms of trade component clearly dominates for USA. A much smaller, negative allocative efficiency effect in USA (\$-10 US million) has little impact on the overall EV, as does the \$40 million contribution due to changes in prices of investment goods and savings. In the EU, the allocative efficiency effects contribute \$801 US million to the EV. The deterioration in EU's terms of trade amount to a reduction in EV of \$556 US million, which offsets much of the allocative efficiency gains. In ROW, both the allocative efficiency effects and the terms of trade effects are negative, -\$127 US million and -\$295 US million, respectively.

<sup>7</sup> See also, chapter 2 of Hertel (1997). Replication of this experiment may be readily undertaken using the RunGTAP software (GTAP book, 3x3 aggregation) available at the GTAP website. See also the GTAP Short Course Hands-on document (Pearson et al., 2000) Note that the results from this simulation differ from those in the GTAP book due to the use of the region-specific price of savings.

Table 7. Welfare Results from Reducing Tariff on EU Imports of Food from USA (1992 \$ US million)

Region	% Change in Welfare	Aggregate Welfare Effect	Contribution of Allocative Effects	Contribution of TOT Effects	Contribution of I-S Effects
USA	0.017	887	-10	851	45
EU	0.004	216	801	-556	-28
ROW	-0.005	-440	-127	-295	-17
Total	n.a.	663	664	-1	0

Table 8 provides a decomposition of the allocative efficiency effects, by commodity for each of the regions. From equation (27), it can be seen that each of these effects results from summing by commodity, the products of distortions in individual markets and quantity changes for transactions under each distortion. For example, the largest figure in Table 8 is a \$771 US million contribution from transactions involving food to welfare in EU. This gain results from the combination of the decrease in output (-.35%) of the subsidized food sector (7.3% subsidy) and an increase in food imports (4.7%) which faces an average tariff of 37% (post-simulation value).

Table 8. Decomposition of the Regional Allocative Efficiency Effects by Community (1992 \$ US million)

	USA	EU	ROW
Food	-39	771	-254
Manufactures	35	17	113
Services	-5	13	13
TOTAL	-10	801	-127

Table 9 decomposes the allocative efficiency effects by tax instrument for the EU. Taxes or subsidies on output and import taxes contribute the most to the total. It is hardly surprising that import taxes represent the most important tax instrument, since this is where the tariff cut occurs.

Table 9 Decomposition of Allocative Efficiency Effect of Reducing Tariff on EU Imports of Food from USA by Tax Instrument(1992 \$ US million)

Tax Instrument	Contribution to Welfare in EU
Primary Factor Taxes	0
Output Taxes	341
Input Taxes	69
Taxes on Final Demand	-20
Export Taxes	-39
Import Taxes	449
Total	801



Since food and imports have been identified as the major source of efficiency gains in the EU as a result of the 10% cut in the tariff on US food imports, Table 10 focuses more narrowly on the contribution to allocative efficiency of food imports to the EU by source. Imports of food from USA to EU increased significantly (53.3% or \$4,565 US million) while food imports from ROW to EU declined by \$2,148 US million. Since imports from both regions face positive tariffs, the interaction between the change in imports and the tariffs is positive for USA (\$1,503 US million) and negative for ROW (-\$1,016 US million). The net result is a positive contribution to welfare of \$487 US million.

Table 10 Decomposition of Contribution of Import Taxes on Food in EU by Trading Source (1992 \$ US million)

	Tariff Equivalent <sup>a</sup>		% Change in Bilateral Imports	Contribution to Welfare
	Initial	Final		
USA	37	23	4565	1503
EU	0	0	0	0
ROW	42	42	-2148	-1016
Total	n.a.	n.a.		487

<sup>a</sup> Tariff equivalents for post-simulation data base.

As this simple experiment demonstrates, the multiregion welfare decomposition is a useful tool of analysis for users of the GTAP model. It allows the user to pinpoint the contributions of each economic transaction to the final welfare results of a policy simulation. The tables presented for this example represent only a subset of the possible information that terms of the decomposition provide on economic activities by region and commodity. Additional information is readily available from the DECOMP.TAB program which also accompanies this technical paper. This is run following any GTAP simulation and it organizes the EV contribution terms into an easily readable header array file: DECOMP.HAR. This offers a sequence of coefficients, beginning with the aggregate EV decomposition, and ending with detailed contributions from individual flows.

## ***5. Decomposition of the Multiregion EV in the Presence of Technical Change***

The final piece of the decomposition involves the addition of terms relating to technical change. These are exhaustively derived in Appendix B. However, only the final expression is produced here. You can see that each technical change term is premultiplied by the value of the associated economic flow.

$$\begin{aligned}
 &EV\_ALT(r) \\
 &= [0.01*EVSCALFACT(r)] \tag{29} \\
 &\quad * [\sum\{i,ENDW\_COMM, VOA(i,r)*[qo(i,r)-pop(r)]\}- VDEP(r)*[kb(r) - pop(r)] \\
 &\quad + \sum\{i,NSAV\_COMM, PTAX(i,r)*[qo(i,r) - pop(r)]\} \\
 &\quad + \sum\{i,ENDW\_COMM, \sum\{j,PROD\_COMM,ETAX(i,j,r)*[qfe(i,j,r) - pop(r)] \}\} \\
 &\quad + \sum\{j,PROD\_COMM, \sum\{i,TRAD\_COMM,IFTAX(i,j,r)*[qfm(i,j,r)- pop(r)] \}\} \\
 &\quad + \sum\{j,PROD\_COMM, \sum\{i,TRAD\_COMM,DFTAX(i,j,r)*[qfd(i,j,r)- pop(r)]\}\}
 \end{aligned}$$

$$\begin{aligned}
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{IPTAX}(i, r) * [\text{qpm}(i, r) - \text{pop}(r)]\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{DPTAX}(i, r) * [\text{qpd}(i, r) - \text{pop}(r)]\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{IGTAX}(i, r) * [\text{qgm}(i, r) - \text{pop}(r)]\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{DGTAX}(i, r) * [\text{qgd}(i, r) - \text{pop}(r)]\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{XTAXD}(i, r, s) * [\text{qxs}(i, r, s) - \text{pop}(r)]\}\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{MTAX}(i, s, r) * [\text{qxs}(i, s, r) - \text{pop}(r)]\}\} \\
& + \text{sum}\{i, \text{PROD\_COMM}, \text{VOA}(i, r) * \text{ao}(i, r)\} \\
& + \text{sum}\{j, \text{PROD\_COMM}, \text{VVA}(j, r) * \text{ava}(j, r)\} \\
& + \text{sum}\{i, \text{ENDW\_COMM}, \text{sum}\{j, \text{PROD\_COMM}, \text{VFA}(i, j, r) * \text{afe}(i, j, r)\}\} \\
& + \text{sum}\{j, \text{PROD\_COMM}, \text{sum}\{i, \text{TRAD\_COMM}, \text{VFA}(i, j, r) * \text{af}(i, j, r)\}\} \\
& + \text{sum}\{m, \text{MARG\_COMM}, \text{sum}\{i, \text{TRAD\_COMM}, \\
& \quad \text{sum}\{s, \text{REG}, \text{VTMFSD}(m, i, s, r) * \text{atmfSD}(m, i, s, r)\}\}\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{VXWD}(i, r, s) * \text{pfob}(i, r, s)\}\} \\
& + \text{sum}\{m, \text{MARG\_COMM}, \text{VST}(m, r) * \text{pm}(m, r)\} \\
& - \text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{VXWD}(i, s, r) * \text{pfob}(i, s, r)\}\} \\
& - \text{sum}\{m, \text{MARG\_COMM}, \text{VTMD}(m, r) * \text{pt}(m)\} \\
& + \text{NETINV}(r) * \text{pcgds}(r) \\
& - \text{SAVE}(r) * \text{psave}(r) ] \\
& + 0.01 * \text{INCOMEEV}(r) * \text{pop}(r) ;
\end{aligned}$$

In equation (29)  $ao$ ,  $afe$ ,  $ava$ ,  $af$  and  $atmfSD$  represent the following technical change variables: output augmenting technical change, primary factor  $i$  augmenting technical change, value-added augmenting technical change, composite intermediate input  $i$  augmenting technical change, and technical change in the transportation of tradable commodity  $i$  using mode  $m$ , from source  $r$  to destination  $s$ , respectively.  $VTMFSD(m, i, r, s)$  represents the value of services of mode  $m$ , used to transport good  $i$  from  $r$  to  $s$ .

The TABLO source code for the welfare decomposition is available in electronic form as an appendix to this paper. In order to make use of the proposed welfare decomposition, this code should be appended to the TABLO code for the standard GTAP model (versions 5.0 and higher). An electronic version of the tablo file combining the current version of the standard GTAP model with this decomposition code is available from the GTAP web site in conjunction with an electronic copy of this technical paper. The URL is: <http://www.agecon.purdue.edu/gtap/>

Readers interested in a similar welfare decomposition for CGE models that is not specific to GTAP are referred to the technical paper by Hanslow (2001). His paper offers a valuable companion to this technical paper.

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## ***Appendix A: Derivation of the Real Income Decomposition for Single Region Model***

The goal of this appendix is to derive the real income decomposition:  $Y(y - p) = D$  introduced in section 3.1.

The decomposition starts with the GTAP equation for regional income which equals the sum of primary factor payments and tax receipts.

$$\begin{aligned}
 \text{INCOME} * y &= \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * [\text{ps}(i) + \text{qo}(i)] \\
 &\quad - \text{VDEP} * [\text{pcgds} + \text{kb}] \\
 &+ \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i) * [\text{pm}(i) + \text{qo}(i)] \} \\
 &\quad - \{ \text{VOA}(i) * [\text{ps}(i) + \text{qo}(i)] \} ) \\
 &+ \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
 &\quad \{ \text{VFA}(i,j) * [\text{pfe}(i,j) + \text{qfe}(i,j)] \} \\
 &\quad - \{ \text{VFM}(i,j) * [\text{pm}(i) + \text{qfe}(i,j)] \} ) ) \\
 &+ \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
 &\quad \{ \text{VFA}(i,j) * [\text{pfe}(i,j) + \text{qfe}(i,j)] \} \\
 &\quad - \{ \text{VFM}(i,j) * [\text{pmes}(i,j) + \text{qfe}(i,j)] \} ) ) \\
 &+ \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
 &\quad \{ \text{VFA}(i,j) * [\text{pf}(i,j) + \text{qf}(i,j)] \} \\
 &\quad - \{ \text{VFM}(i,j) * [\text{pm}(i) + \text{qf}(i,j)] \} ) ) \\
 &+ \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VPA}(i) * [\text{pp}(i) + \text{qp}(i)] \} \\
 &\quad - \{ \text{VPM}(i) * [\text{pm}(i) + \text{qp}(i)] \} ) \\
 &+ \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VGA}(i) * [\text{pg}(i) + \text{qg}(i)] \} \\
 &\quad - \{ \text{VGM}(i) * [\text{pm}(i) + \text{qg}(i)] \} ).
 \end{aligned}$$

To simplify the above expression, take the levels form of the zero profits condition for all  $j$  in the set  $\text{PROD\_COMM}$ :

$$\text{PROFITS}(j) = \text{VOA}(j) - \text{sum}(i, \text{DEMD\_COMM}, \text{VFA}(i,j)).$$

When this is totally differentiated, the change in profits is zero and the quantity changes cancel due to the model assumptions of constant returns to scale and cost minimization. The resulting expression is:

$$\begin{aligned}
 &\text{VOA}(j) * \text{ps}(j) \\
 &= \text{sum}(i, \text{ENDW\_COMM}, \text{VFA}(i,j) * \text{pfe}(i,j)) \\
 &+ \text{sum}(i, \text{TRAD\_COMM}, \text{VFA}(i,j) * \text{pf}(i,j)).
 \end{aligned}$$

Next substitute in the zero profits condition without canceling out the terms involving the capital goods commodity:

$$\begin{aligned}
 &\text{VOA}(\text{"cgds"}) * \text{ps}(\text{"cgds"}) = \\
 &\quad \text{sum}(i, \text{ENDW\_COMM}, \text{VFA}(i, \text{"cgds"}) * \text{pfe}(i, \text{"cgds"})) \\
 &\quad + \text{sum}(i, \text{TRAD\_COMM}, \text{VFA}(i, \text{"cgds"}) * \text{pf}(i, \text{"cgds"})),
 \end{aligned}$$

and the expression for regional income becomes:

$$\begin{aligned}
\text{INCOME} * y = & \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * [\text{ps}(i) + \text{qo}(i)]) \\
& - \text{VDEP} * [\text{pcgds} + \text{kb}] \\
& + \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i) * [\text{pm}(i) + \text{qo}(i)] \} \\
& \quad - \{ \text{VOA}(i) * \text{qo}(i) \} ) \\
& - \text{sum}(i, \text{ENDW\_COMM}, \{ \text{VOA}(i) * \text{ps}(i) \} ) \\
& - \text{VOA}(\text{"cgds"}) * \text{ps}(\text{"cgds"}) \\
& + \text{VOA}(\text{"cgds"}) * \text{ps}(\text{"cgds"}) \\
& + \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \{ \text{VFA}(i,j) * \text{qfe}(i,j) \} \\
& \quad - \{ \text{VFM}(i,j) * [\text{pm}(i) + \text{qfe}(i,j)] \} ) ) \\
& + \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \{ \text{VFA}(i,j) * \text{qfe}(i,j) \} \\
& \quad - \{ \text{VFM}(i,j) * [\text{pmes}(i,j) + \text{qfe}(i,j)] \} ) ) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VFA}(i,j) * \text{qf}(i,j) \} \\
& \quad - \{ \text{VFM}(i,j) * [\text{pm}(i) + \text{qf}(i,j)] \} ) ) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VPA}(i) * [\text{pp}(i) + \text{qp}(i)] \} \\
& \quad - \{ \text{VPM}(i) * [\text{pm}(i) + \text{qp}(i)] \} ) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VGA}(i) * [\text{pg}(i) + \text{qg}(i)] \} \\
& \quad - \{ \text{VGM}(i) * [\text{pm}(i) + \text{qg}(i)] \} ).
\end{aligned}$$

To simplify further, take the market clearing condition for the traded goods market in its linearized form for all  $i$  in the set of  $\text{TRAD\_COMM}$ :

$$\begin{aligned}
\text{qo}(i) = & \text{sum}(j, \text{PROD\_COMM}, \text{SHRFM}(i,j) * \text{qf}(i,j)) \\
& + \text{SHRPM}(i) * \text{qp}(i) + \text{SHRGM}(i) * \text{qg}(i),
\end{aligned}$$

Now express it in the levels form:

$$\text{VOM}(i) = \text{sum}(j, \text{PROD\_COMM}, \text{VFM}(i,j)) + \text{VPM}(i) + \text{VGM}(i).$$

Multiply this by market price,  $\text{pm}(i)$ , and substitute into the expression for regional income giving:

$$\begin{aligned}
\text{INCOME} * y = & \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * [\text{ps}(i) + \text{qo}(i)]) \\
& - \text{VDEP} * [\text{pcgds} + \text{kb}] \\
& + \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i) * \text{qo}(i) \} \\
& \quad - \{ \text{VOA}(i) * \text{qo}(i) \} ) \\
& - \text{VOA}(\text{"cgds"}) * \text{ps}(\text{"cgds"}) \\
& + \text{VOA}(\text{"cgds"}) * \text{ps}(\text{"cgds"}) \\
& + \text{VOM}(\text{"cgds"}) * \text{pm}(\text{"cgds"}) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \{ \text{VOM}(i) * \text{pm}(i) \} ) \\
& - \text{sum}(i, \text{ENDW\_COMM}, \{ \text{VOA}(i) * \text{ps}(i) \} )
\end{aligned}$$

$$\begin{aligned}
& + \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \{ \text{VFA}(i,j) * \text{qfe}(i,j) \} \\
& - \{ \text{VFM}(i,j) * [\text{pm}(i) + \text{qfe}(i,j)] \} )) \\
& + \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \{ \text{VFA}(i,j) * \text{qfe}(i,j) \} \\
& - \{ \text{VFM}(i,j) * [\text{pmes}(i,j) + \text{qfe}(i,j)] \} )) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VFA}(i,j) * \text{qf}(i,j) \} \\
& - \{ \text{VFM}(i,j) * \text{qf}(i,j) \} )) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VPA}(i) * [\text{pp}(i) + \text{qp}(i)] \} \\
& \quad - \{ \text{VPM}(i) * \text{qp}(i) \} ) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VGA}(i) * [\text{pg}(i) + \text{qg}(i)] \} \\
& \quad - \{ \text{VGM}(i) * \text{qg}(i) \} ).
\end{aligned}$$

The next step utilizes the market clearing condition for mobile endowments,

$$\text{VOM}(i) * \text{qo}(i) = \text{sum}(j, \text{PROD\_COMM}, \text{VFM}(i, j) * \text{qfe}(i, j)).$$

In the levels this becomes:

$$\text{VOM}(i) = \text{sum}(j, \text{PROD\_COMM}, \text{VFM}(i, j)).$$

for all  $i$  in the set of mobile endowment commodities ( $\text{ENDW\_COMM}$ ).

This is multiplied by market price,  $\text{pm}(i)$  and substituted into the expression for regional income yielding:

$$\begin{aligned}
\text{INCOME} * y & = \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * [\text{ps}(i) + \text{qo}(i)] \\
& \quad - \text{VDEP} * [\text{pcgds} + \text{kb}] \\
& + \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i) * \text{qo}(i) \} \\
& \quad - \{ \text{VOA}(i) * \text{qo}(i) \} ) \\
& - \text{VOA}(\text{"cgds"}) * \text{ps}(\text{"cgds"}) \\
& + \text{VOA}(\text{"cgds"}) * \text{ps}(\text{"cgds"}) \\
& + \text{VOM}(\text{"cgds"}) * \text{pm}(\text{"cgds"}) \\
& + \text{sum}(i, \text{ENDWS\_COMM}, \{ \text{VOM}(i) * \text{pm}(i) \} ) \\
& - \text{sum}(i, \text{ENDW\_COMM}, \{ \text{VOA}(i) * \text{ps}(i) \} ) \\
& + \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i,j) * \text{qfe}(i,j) \} \\
& \quad - \{ \text{VFM}(i,j) * \text{qfe}(i,j) \} )) \\
& + \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \{ \text{VFA}(i,j) * \text{qfe}(i,j) \} \\
& - \{ \text{VFM}(i,j) * [\text{pmes}(i,j) + \text{qfe}(i,j)] \} )) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VFA}(i,j) * \text{qf}(i,j) \} \\
& \quad - \{ \text{VFM}(i,j) * \text{qf}(i,j) \} ))
\end{aligned}$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VPA}(i) * [\text{pp}(i) + \text{qp}(i)] \} \\ - \{ \text{VPM}(i) * \text{qp}(i) \})$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VGA}(i) * [\text{pg}(i) + \text{qg}(i)] \} \\ - \{ \text{VGM}(i) * \text{qg}(i) \}).$$

The following relationships are to be employed in the simplification of the expression for regional income:

$$\text{pcgds} = \text{ps}(\text{"cgds"}) = \text{pm}(\text{"cgds"}),$$

$$\text{and } \text{VOA}(\text{"cgds"}) = \text{VOM}(\text{"cgds"}).$$

The expression for regional income becomes:

$$\text{INCOME} * y = \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * \text{ps}(i))$$

$$+ \text{VOA}(\text{"cgds"}) * \text{pcgds}$$

$$- \text{VDEP} * \text{pcgds}$$

$$+ \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * \text{qo}(i))$$

$$- \text{VDEP} * \text{kb}$$

$$+ \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i) * \text{qo}(i) \} \\ - \{ \text{VOA}(i) * \text{qo}(i) \})$$

$$+ \text{sum}(i, \text{ENDWS\_COMM}, \{ \text{VOM}(i) * \text{pm}(i) \})$$

$$- \text{sum}(i, \text{ENDW\_COMM}, \{ \text{VOA}(i) * \text{ps}(i) \})$$

$$+ \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i,j) * \text{qfe}(i,j) \} \\ - \{ \text{VFM}(i,j) * \text{qfe}(i,j) \}))$$

$$+ \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\ \{ \text{VFA}(i,j) * \text{qfe}(i,j) \} \\ - \{ \text{VFM}(i,j) * [\text{pmes}(i,j) + \text{qfe}(i,j)] \}))$$

$$+ \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VFA}(i,j) * \text{qf}(i,j) \} \\ - \{ \text{VFM}(i,j) * \text{qf}(i,j) \}))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VPA}(i) * [\text{pp}(i) + \text{qp}(i)] \} \\ - \{ \text{VPM}(i) * \text{qp}(i) \})$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VGA}(i) * [\text{pg}(i) + \text{qg}(i)] \} \\ - \{ \text{VGM}(i) * \text{qg}(i) \}).$$

The next simplification takes advantage of some price relationships in the model. First the equation that generates the composite price for sluggish endowments is used:

$$\text{pm}(i) = \text{sum}(j, \text{PROD\_COMM}, \text{REVSHR}(i,j) * \text{pmes}(i,j)).$$

$$\text{REVSHR}(i,j) = \text{VFM}(i,j) / \text{sum}(k, \text{PROD\_COMM}, \text{VFM}(i,k)),$$

which also can be written as:

$$\text{REVSHR}(i,j) = \text{VFM}(i,j) / \text{VOM}(i).$$

Also note that:

$$\text{NETINV} = \text{VOA}(\text{"cgds"}) - \text{VDEP},$$

and finally subtract  $\text{SAVE} * \text{psave}$  from both sides and rearrange to yield the following expression for the regional real income decomposition:

$$\begin{aligned} & \text{INCOME} * y \\ & - \text{SAVE} * \text{psave} \\ & - \text{sum}(i, \text{TRAD\_COMM}, \text{VPA}(i) * \text{pp}(i)) \\ & - \text{sum}(i, \text{TRAD\_COMM}, \text{VGA}(i) * \text{pg}(i)) = \\ & \quad \text{sum}(i, \text{NSAV\_COMM}, \{\text{VOM}(i) - \text{VOA}(i)\} * \text{qo}(i)) \\ & + \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i) * \text{qo}(i)) \\ & - \text{VDEP} * \text{kb} \\ & + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\ & \quad \{\text{VFA}(i,j) - \text{VFM}(i,j)\} * \text{qfe}(i,j))) \\ & + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\ & \quad \{\text{VFA}(i,j) - \text{VFM}(i,j)\} * \text{qf}(i,j))) \\ & + \text{sum}(i, \text{TRAD\_COMM}, \{\text{VPA}(i) - \text{VPM}(i)\} * \text{qp}(i)) \\ & + \text{sum}(i, \text{TRAD\_COMM}, \{\text{VGA}(i) - \text{VGM}(i)\} * \text{qg}(i)) \\ & + \text{NETINV} * \text{pcgds} \\ & - \text{SAVE} * \text{psave}. \end{aligned}$$

To simplify the LHS of the expression for regional welfare decomposition, divide and multiply through by  $\text{INCOME}$ .

$$\begin{aligned} & \text{INCOME} * \\ & ( y \\ & - \text{SAVE}/\text{INCOME} * \text{psave} \\ & - \text{sum}(i, \text{TRAD\_COMM}, \text{VPA}(i)/\text{INCOME} * \text{pp}(i)) \\ & - \text{sum}(i, \text{TRAD\_COMM}, \text{VGA}(i)/\text{INCOME} * \text{pg}(i)) ) = Y(y - p) = D \end{aligned}$$

Hence we have achieved our decomposition of real income.

Finally, substitute in the following tax instruments on the right-hand side of the decomposition:

$$\text{VOM}(i) - \text{VOA}(i) = \text{PTAX}(i),$$

$$\text{VFA}(i,j) - \text{VFM}(i,j) = \text{ETAX}(i,j) \text{ for all } i \text{ endowment commodities,}$$

$$\text{VFA}(i,j) - \text{VFM}(i,j) = \text{DFTAX}(i,j),$$

$$\text{VPA}(i) - \text{VPM}(i) = \text{DPTAX}(i),$$

and

$$\text{VGA}(i) - \text{VGM}(i) = \text{DGTAX}(i).$$

Also note that:



pcgds = psave,

and NETINV=SAVE . The final expression for the decomposition becomes:

D =

$$\begin{aligned} & \text{sum}(i, \text{ENDW\_COMM}, \text{VOM}(i) * \text{qo}(i)) - \text{VDEP} * \text{kb} \\ & + \text{sum}(i, \text{NSAV\_COMM}, \text{PTAX}(i) * \text{qo}(i)) \\ & + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \text{ETAX}(i,j) * \text{qfe}(i,j))) \\ & + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \text{DFTAX}(i,j) * \text{qf}(i,j))) \\ & + \text{sum}(i, \text{TRAD\_COMM}, \text{DPTAX}(i) * \text{qp}(i)) \\ & + \text{sum}(i, \text{TRAD\_COMM}, \text{DGTAX}(i) * \text{qg}(i)) \end{aligned}$$

This is equation (10) in the text. It can be placed on a *per capita* basis, as discussed in Section 3.4.

## ***Appendix B: Derivation of EV Decomposition for the Multiregion Model***

[Note: This derivation applies to version 5.0 of GTAP.TAB. This includes a region-specific price of savings and multiple margins commodities.]

The multiregion real income welfare decomposition,  $Y(r) [y(r) - p(r)] = D(r)$ , starts with the GTAP equation for regional income which equals the sum of primary factor payments and tax receipts:

$$\begin{aligned}
 \text{INCOME}(r) * y(r) = & \\
 & \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i,r) * [\text{ps}(i,r) + \text{qo}(i,r)]) \\
 & \quad - \text{VDEP}(r) * [\text{pcgds}(r) + \text{kb}(r)] \\
 & + \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i,r) * [\text{pm}(i,r) + \text{qo}(i,r)] \} \\
 & \quad - \{ \text{VOA}(i,r) * [\text{ps}(i,r) + \text{qo}(i,r)] \} ) \\
 & + \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
 & \quad \{ \text{VFA}(i,j,r) * [\text{pfe}(i,j,r) + \text{qfe}(i,j,r)] \} \\
 & \quad - \{ \text{VFM}(i,j,r) * [\text{pm}(i,r) + \text{qfe}(i,j,r)] \} ) ) \\
 & + \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
 & \quad \{ \text{VFA}(i,j,r) * [\text{pfe}(i,j,r) + \text{qfe}(i,j,r)] \} \\
 & \quad - \{ \text{VFM}(i,j,r) * [\text{pmes}(i,j,r) + \text{qfe}(i,j,r)] \} ) ) \\
 & + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
 & \quad \{ \text{VIFA}(i,j,r) * [\text{pfm}(i,j,r) + \text{qfm}(i,j,r)] \} \\
 & \quad - \{ \text{VIFM}(i,j,r) * [\text{pim}(i,r) + \text{qfm}(i,j,r)] \} ) ) \\
 & + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
 & \quad \{ \text{VDFA}(i,j,r) * [\text{pfd}(i,j,r) + \text{qfd}(i,j,r)] \} \\
 & \quad - \{ \text{VDFM}(i,j,r) * [\text{pm}(i,r) + \text{qfd}(i,j,r)] \} ) ) \\
 & + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIPA}(i,r) * [\text{ppm}(i,r) + \text{qpm}(i,r)] \} \\
 & \quad - \{ \text{VIPM}(i,r) * [\text{pim}(i,r) + \text{qpm}(i,r)] \} ) \\
 & + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDPA}(i,r) * [\text{ppd}(i,r) + \text{qpd}(i,r)] \} \\
 & \quad - \{ \text{VDPM}(i,r) * [\text{pm}(i,r) + \text{qpd}(i,r)] \} )
 \end{aligned}$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIGA}(i,r) * [\text{pgm}(i,r) + \text{qgm}(i,r)] \}$$

$$- \{ \text{VIGM}(i,r) * [\text{pim}(i,r) + \text{qgm}(i,r)] \})$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDGA}(i,r) * [\text{pgd}(i,r) + \text{qgd}(i,r)] \}$$

$$- \{ \text{VDGM}(i,r) * [\text{pm}(i,r) + \text{qgd}(i,r)] \})$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG},$$

$$\{ \text{VXWD}(i,r,s) * [\text{pfob}(i,r,s) + \text{qxs}(i,r,s)] \}$$

$$- \{ \text{VXMD}(i,r,s) * [\text{pm}(i,r) + \text{qxs}(i,r,s)] \}))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG},$$

$$\{ \text{VIMS}(i,s,r) * [\text{pms}(i,s,r) + \text{qxs}(i,s,r)] \}$$

$$- \{ \text{VIWS}(i,s,r) * [\text{pcif}(i,s,r) + \text{qxs}(i,s,r)] \})).$$

To simplify the above expression, take the levels form of the zero profits condition (for all  $j$  in the set of  $\text{PROD\_COMM}$ ) :

$$\text{PROFITS}(j,r) = \text{VOA}(j,r) - \text{sum}(i, \text{DEMD\_COMM}, \text{VFA}(i,j,r)).$$

When this is totally differentiated, and the model is in equilibrium, the change in profits is zero due to the envelope result and the quantity changes cancel due to the model assumptions of constant returns to scale and cost minimization. The potential for technical change is included in this formulation. The resulting expression is:

$$\text{VOA}(j,r) * [\text{ps}(j,r) + \text{ao}(j,r)]$$

$$= \text{sum}(i, \text{ENDW\_COMM}, \text{VFA}(i,j,r) *$$

$$[\text{pfe}(i,j,r) - \text{afe}(i,j,r) - \text{ava}(j,r)])$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \text{VFA}(i,j,r) * [\text{pf}(i,j,r) - \text{af}(i,j,r)]).$$

Note the following:

$$\text{VFA}(i, j, r) = \text{VDFA}(i, j, r) + \text{VIFA}(i, j, r),$$

and

$$\text{pf}(i, j, r) = \text{FMSHR}(i, j, r) * \text{pfm}(i, j, r) + [1 - \text{FMSHR}(i, j, r)] * \text{pfd}(i, j, r),$$

with

$$\text{FMSHR}(i,j,r) = \text{VIFA}(i,j,r) / \text{VFA}(i,j,r).$$

The final expression for zero profits becomes:

$$\text{VOA}(j,r) * [\text{ps}(j,r) * \text{ao}(j,r)]$$

$$\begin{aligned}
&= \text{sum}(i, \text{ENDW\_COMM}, \text{VFA}(i, j, r) * \\
&\quad [\text{pfe}(i, j, r) - \text{afe}(i, j, r) - \text{ava}(j, r)]) \\
&+ \text{sum}(i, \text{TRAD\_COMM}, \text{VIFA}(i, j, r) * [\text{pfm}(i, j, r) - \text{af}(i, j, r)]) \\
&+ \text{sum}(i, \text{TRAD\_COMM}, \text{VDFA}(i, j, r) * [\text{pfd}(i, j, r) - \text{af}(i, j, r)]).
\end{aligned}$$

If the following term is both added and subtracted to the expression for regional income,  $\text{sum}(i, \text{TRAD\_COMM}, \text{VOA}(i, r) * \text{ao}(i, r))$ , and the zero profits condition is substituted in without canceling out the terms involving the capital goods commodity:

$$\begin{aligned}
&\text{VOA}(\text{"cgds"}, r) * \text{ps}(\text{"cgds"}, r) = i \\
&\quad \text{sum}(i, \text{ENDW\_COMM}, \text{VFA}(i, \text{"cgds"}, r) * \text{pfe}(i, \text{"cgds"}, r)) \\
&\quad + \text{sum}(i, \text{TRAD\_COMM}, \text{VDFA}(i, \text{"cgds"}, r) * \text{pfd}(i, \text{"cgds"}, r)) \\
&\quad + \text{VIFA}(i, \text{"cgds"}, r) * \text{pfm}(i, \text{"cgds"}, r),
\end{aligned}$$

the expression for regional income becomes:

$$\begin{aligned}
&\text{INCOME}(r) * y(r) = \\
&\quad \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i, r) * [\text{ps}(i, r) + \text{qo}(i, r)]) \\
&\quad - \text{VDEP}(r) * [\text{pcgds}(r) + \text{kb}(r)] \\
&\quad + \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i, r) * [\text{pm}(i, r) + \text{qo}(i, r)] \} \\
&\quad - \{ \text{VOA}(i, r) * \text{qo}(i, r) \} ) \\
&\quad - \text{sum}(i, \text{ENDW\_COMM}, \{ \text{VOA}(i, r) * \text{ps}(i, r) \} ) \\
&\quad - \text{VOA}(\text{"cgds"}, r) * \text{ps}(\text{"cgds"}, r) \\
&\quad + \text{VOA}(\text{"cgds"}, r) * \text{ps}(\text{"cgds"}, r) \\
&\quad + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i, j, r) * \text{qfe}(i, j, r) \} \\
&\quad - \{ \text{VFM}(i, j, r) * [\text{pm}(i, r) + \text{qfe}(i, j, r)] \} ))
\end{aligned}$$

$$\begin{aligned}
& + \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i, j, r) * \text{qfe}(i, j, r) \} \\
& \quad - \{ \text{VFM}(i, j, r) * [\text{pmes}(i, j, r) + \text{qfe}(i, j, r)] \} )) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIFA}(i, j, r) * \text{qfm}(i, j, r) \} \\
& \quad - \{ \text{VIFM}(i, j, r) * [\text{pim}(i, r) + \text{qfm}(i, j, r)] \} )) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDFA}(i, j, r) * \text{qfd}(i, j, r) \} \\
& \quad - \{ \text{VDFM}(i, j, r) * [\text{pm}(i, r) + \text{qfd}(i, j, r)] \} )) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VOA}(i, r) * \text{ao}(i, r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \text{VFA}(i, j, r) * [\text{afe}(i, j, r) + \text{ava}(j, r)])) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i, j, r) + \text{VDFA}(i, j, r) \} * \text{af}(i, j, r))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIPA}(i, r) * [\text{ppm}(i, r) + \text{qpm}(i, r)] \} \\
& \quad - \{ \text{VIPM}(i, r) * [\text{pim}(i, r) + \text{qpm}(i, r)] \} ) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDPA}(i, r) * [\text{ppd}(i, r) + \text{qpd}(i, r)] \} \\
& \quad - \{ \text{VDPM}(i, r) * [\text{pm}(i, r) + \text{qpd}(i, r)] \} ) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIGA}(i, r) * [\text{pgm}(i, r) + \text{qgm}(i, r)] \} \\
& \quad - \{ \text{VIGM}(i, r) * [\text{pim}(i, r) + \text{qgm}(i, r)] \} ) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDGA}(i, r) * [\text{pgd}(i, r) + \text{qgd}(i, r)] \} \\
& \quad - \{ \text{VDGM}(i, r) * [\text{pm}(i, r) + \text{qgd}(i, r)] \} ) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VXWD}(i, r, s) * [\text{pfob}(i, r, s) + \text{qxs}(i, r, s)] \} \\
& \quad - \{ \text{VXMD}(i, r, s) * [\text{pm}(i, r) + \text{qxs}(i, r, s)] \} )) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VIMS}(i, s, r) * [\text{pms}(i, s, r) + \text{qxs}(i, s, r)] \} \\
& \quad - \{ \text{VIWS}(i, s, r) * [\text{pcif}(i, s, r) + \text{qxs}(i, s, r)] \} ))).
\end{aligned}$$

The following terms in this expression capture the impact of technical change on regional welfare:

$$\begin{aligned}
& \text{sum}(i, \text{TRAD\_COMM}, \text{VOA}(i, r) * \text{ao}(i, r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \text{VFA}(i, j, r) * [\text{afe}(i, j, r) + \text{ava}(j, r)])) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i, j, r) + \text{VDFA}(i, j, r) \} * \text{af}(i, j, r))).
\end{aligned}$$

To simplify further, the market clearing condition for the traded goods market in its linearized form (for all  $i$  in the set  $\text{TRAD\_COMM}$ ) :

$$\begin{aligned}
\text{VOM}(i, r) * \text{qo}(i, r) &= \text{VDM}(i, r) * \text{qds}(i, r) \\
& + \text{VST}(i, r) * \text{qst}(i, r) \\
& + \text{sum}(s, \text{REG}, \text{VXMD}(i, r, s) * \text{qxs}(i, r, s)),
\end{aligned}$$

is expressed in the levels as:

$$\text{VOM}(i, r) = \text{VDM}(i, r) + \text{VST}(i, r) + \text{sum}(s, \text{REG}, \text{VXMD}(i, r, s)).$$

Likewise, the market clearing condition for domestic output,

$$qds(i,r) = \text{sum}(j, \text{PROD\_COMM}, \text{SHRDFM}(i,j,r) * qfd(i,j,r)) \\ + \text{SHRDPM}(i,r) * qpd(i,r) + \text{SHRDGM}(i,r) * qgd(i,r),$$

is expressed in the levels form (OVER ALL TRAD\_COMM AND ALL REG) as:

$$\text{VDM}(i,r) = \text{sum}(j, \text{PROD\_COMM}, \text{VDFM}(i,j,r)) + \text{VDPM}(i,r) + \text{VDGM}(i,r).$$

Combining the two expressions in levels form yields (OVER ALL TRAD\_COMM):

$$\text{VOM}(i,r) = \text{VDPM}(i,r) + \text{VDGM}(i,r) + \text{sum}(j, \text{PROD\_COMM}, \text{VDFM}(i,j,r)) \\ + \text{VST}(i,r) + \text{sum}(s, \text{REG}, \text{VXMD}(i,r,s)).$$

Multiply this by market price,  $pm(i,r)$ , and substitute into the expression for regional income giving:

$$\begin{aligned} \text{INCOME}(r) * y(r) = & \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i,r) * [\text{ps}(i,r) + \text{qo}(i,r)]) \\ & - \text{VDEP}(r) * [\text{pcgds}(r) + \text{kb}(r)] \\ & + \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i,r) * \text{qo}(i,r) \} \\ & - \{ \text{VOA}(i,r) * \text{qo}(i,r) \} \\ & - \text{VOA}(\text{"cgds"},r) * \text{ps}(\text{"cgds"},r) \\ & + \text{VOA}(\text{"cgds"},r) * \text{ps}(\text{"cgds"},r) \\ & + \text{VOM}(\text{"cgds"},r) * \text{pm}(\text{"cgds"},r) \\ & + \text{sum}(i, \text{ENDW\_COMM}, \{ \text{VOM}(i,r) * \text{pm}(i,r) \}) \\ & - \text{sum}(i, \text{ENDW\_COMM}, \{ \text{VOA}(i,r) * \text{ps}(i,r) \}) \\ & + \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i,j,r) * \text{qfe}(i,j,r) \} \\ & - \{ \text{VFM}(i,j,r) * [\text{pm}(i,r) + \text{qfe}(i,j,r)] \})) \\ & + \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i,j,r) * \text{qfe}(i,j,r) \} \\ & - \{ \text{VFM}(i,j,r) * [\text{pmes}(i,j,r) + \text{qfe}(i,j,r)] \})) \\ & + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIFA}(i,j,r) * \text{qfm}(i,j,r) \} \\ & - \{ \text{VIFM}(i,j,r) * [\text{pim}(i,r) + \text{qfm}(i,j,r)] \})) \\ & + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDFA}(i,j,r) * \text{qfd}(i,j,r) \} \\ & - \{ \text{VDFM}(i,j,r) * \text{qfd}(i,j,r) \})) \\ & + \text{sum}(i, \text{TRAD\_COMM}, \text{VOA}(i,r) * \text{ao}(i,r)) \\ & + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\ & \quad \text{VFA}(i,j,r) * [\text{afe}(i,j,r) + \text{ava}(j,r)])) \\ & + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\ & \quad \{ \text{VIFA}(i,j,r) + \text{VDFA}(i,j,r) \} * \text{af}(i,j,r)) \\ & + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIPA}(i,r) * [\text{ppm}(i,r) + \text{qpm}(i,r)] \} \\ & - \{ \text{VIPM}(i,r) * [\text{pim}(i,r) + \text{qpm}(i,r)] \}) \\ & + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDPA}(i,r) * [\text{ppd}(i,r) + \text{qpd}(i,r)] \}) \end{aligned}$$

$$\begin{aligned}
& - \{VDPM(i,r) * qpd(i,r)\} \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{VIGA(i,r) * [pgm(i,r) + qgm(i,r)]\} \\
& \quad - \{VIGM(i,r) * [pim(i,r) + qgm(i,r)]\}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{VDGA(i,r) * [pgd(i,r) + qgd(i,r)]\} \\
& \quad - \{VDGM(i,r) * qgd(i,r)\}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{VXWD(i,r,s) * [pfob(i,r,s) + qxs(i,r,s)]\} \\
& \quad - \{VXMD(i,r,s) * qxs(i,r,s)\})) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{VIMS(i,s,r) * [pms(i,s,r) + qxs(i,s,r)]\} \\
& \quad - \{VIWS(i,s,r) * [pcif(i,s,r) + qxs(i,s,r)]\})) \\
& + \text{sum}(i, \text{TRAD\_COMM}, VST(i,r) * pm(i,r)).
\end{aligned}$$

The next simplification uses the market clearing condition for tradeable commodities entering each region in its linearized form,

$$\begin{aligned}
qim(i,r) = & \text{sum}(j, \text{PROD\_COMM}, \text{SHRIFM}(i,j,r) * qfm(i,j,r)) \\
& + \text{SHRIPM}(i,r) * qpm(i,r) + \text{SHRIGM}(i,r) * qgm(i,r),
\end{aligned}$$

and converts it to the levels expression (for all  $i$  in  $\text{TRAD\_COMM}$  and all  $r$  in  $\text{REG}$ ):

$$VIM(i,r) = \text{sum}(j, \text{PROD\_COMM}, VIFM(i,j,r)) + VIPM(i,r) + VIGM(i,r).$$

This expression is then multiplied by  $pim(i,r)$  and substituted into the expression for regional income yielding:

$$\begin{aligned}
\text{INCOME}(r) * y(r) = & \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i,r) * [ps(i,r) + qo(i,r)] \\
& \quad - \text{VDEP}(r) * [\text{pcgds}(r) + \text{kb}(r)]) \\
& + \text{sum}(i, \text{NSAV\_COMM}, \{VOM(i,r) * qo(i,r)\} \\
& \quad - \{VOA(i,r) * qo(i,r)\}) \\
& - \text{VOA}(\text{"cgds"}, r) * \text{ps}(\text{"cgds"}, r) \\
& + \text{VOA}(\text{"cgds"}, r) * \text{ps}(\text{"cgds"}, r) \\
& + \text{VOM}(\text{"cgds"}, r) * \text{pm}(\text{"cgds"}, r) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \{VOM(i,r) * pm(i,r)\}) \\
& - \text{sum}(i, \text{ENDW\_COMM}, \{VOA(i,r) * ps(i,r)\}) \\
& + \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{VFA(i,j,r) * qfe(i,j,r)\} \\
& \quad - \{VFM(i,j,r) * [pm(i,r) + qfe(i,j,r)]\})) \\
& + \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{VFA(i,j,r) * qfe(i,j,r)\} \\
& \quad - \{VFM(i,j,r) * [pmes(i,j,r) + qfe(i,j,r)]\})) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{VIFA(i,j,r) * qfm(i,j,r)\} \\
& \quad - \{VIFM(i,j,r) * qfm(i,j,r)\})) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{VDFa(i,j,r) * qfd(i,j,r)\}
\end{aligned}$$

$$\begin{aligned}
& - \{VDFM(i,j,r) * qfd(i,j,r)\}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VOA}(i,r) * \text{ao}(i,r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \text{VFA}(i,j,r) * [\text{afe}(i,j,r) + \text{ava}(j,r)])) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{\text{VIFA}(i,j,r) + \text{VDFA}(i,j,r)\} * \text{af}(i,j,r))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{\text{VIPA}(i,r) * [\text{ppm}(i,r) + \text{qpm}(i,r)]\} \\
& \quad - \{\text{VIPM}(i,r) * \text{qpm}(i,r)\}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{\text{VDPA}(i,r) * [\text{ppd}(i,r) + \text{qpd}(i,r)]\} \\
& \quad - \{\text{VDPM}(i,r) * \text{qpd}(i,r)\}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{\text{VIGA}(i,r) * [\text{pgm}(i,r) + \text{qgm}(i,r)]\} \\
& \quad - \{\text{VIGM}(i,r) * \text{qgm}(i,r)\}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{\text{VDGA}(i,r) * [\text{pgd}(i,r) + \text{qgd}(i,r)]\} \\
& \quad - \{\text{VDGM}(i,r) * \text{qgd}(i,r)\}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{\text{VXWD}(i,r,s) * [\text{pfob}(i,r,s) + \text{qxs}(i,r,s)]\} \\
& \quad - \{\text{VXMD}(i,r,s) * \text{qxs}(i,r,s)\})) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{\text{VIMS}(i,s,r) * [\text{pms}(i,s,r) + \text{qxs}(i,s,r)]\} \\
& \quad - \{\text{VIWS}(i,s,r) * [\text{pcif}(i,s,r) + \text{qxs}(i,s,r)]\})) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VST}(i,r) * \text{pm}(i,r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VIM}(i,r) * \text{pim}(i,r)).
\end{aligned}$$

The next step utilizes the market clearing condition for mobile endowments:

$$\begin{aligned}
\text{VOM}(i,r) * \text{qo}(i,r) = & \text{sum}(j, \text{PROD\_COMM}, \text{VFM}(i,j,r) * \text{qfe}(i,j,r)) \\
& + \text{VOM}(i,r) * \text{endwslack}(i,r).
\end{aligned}$$

In the levels this becomes (for all  $i$  in  $\text{ENDWM\_COMM}$  and all  $r$  in  $\text{REG}$ ):

$$\text{VOM}(i,r) = \text{sum}(j, \text{PROD\_COMM}, \text{VFM}(i,j,r)).$$

This is multiplied by market price,  $\text{pm}(i, r)$  and substituted into the expression for regional income yielding:

$$\begin{aligned}
\text{INCOME}(r) * \text{y}(r) = & \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i,r) * [\text{ps}(i,r) + \text{qo}(i,r)]) \\
& - \text{VDEP}(r) * [\text{pcgds}(r) + \text{kb}(r)] \\
& + \text{sum}(i, \text{NSAV\_COMM}, \{\text{VOM}(i,r) * \text{qo}(i,r)\} \\
& \quad - \{\text{VOA}(i,r) * \text{qo}(i,r)\}) \\
& - \text{VOA}(\text{"cgds"}, r) * \text{ps}(\text{"cgds"}, r) \\
& + \text{VOA}(\text{"cgds"}, r) * \text{ps}(\text{"cgds"}, r)
\end{aligned}$$



$$\begin{aligned}
& + \text{VOM}(\text{"cgds"},r) * \text{pm}(\text{"cgds"},r) \\
& + \text{sum}(i,\text{ENDWS\_COMM}, \{ \text{VOM}(i,r) * \text{pm}(i,r) \}) \\
& - \text{sum}(i,\text{ENDW\_COMM}, \{ \text{VOA}(i,r) * \text{ps}(i,r) \}) \\
& + \text{sum}(i,\text{ENDWM\_COMM}, \text{sum}(j,\text{PROD\_COMM}, \{ \text{VFA}(i,j,r) * \text{qfe}(i,j,r) \} \\
& \quad - \{ \text{VFM}(i,j,r) * \text{qfe}(i,j,r) \})) \\
& + \text{sum}(i,\text{ENDWS\_COMM}, \text{sum}(j,\text{PROD\_COMM}, \{ \text{VFA}(i,j,r) * \text{qfe}(i,j,r) \} \\
& \quad - \{ \text{VFM}(i,j,r) * [\text{pms}(i,j,r) + \text{qfe}(i,j,r)] \})) \\
& + \text{sum}(j,\text{PROD\_COMM}, \text{sum}(i,\text{TRAD\_COMM}, \{ \text{VIFA}(i,j,r) * \text{qfm}(i,j,r) \} \\
& \quad - \{ \text{VIFM}(i,j,r) * \text{qfm}(i,j,r) \})) \\
& + \text{sum}(j,\text{PROD\_COMM}, \text{sum}(i,\text{TRAD\_COMM}, \{ \text{VDFA}(i,j,r) * \text{qfd}(i,j,r) \} \\
& \quad - \{ \text{VDFM}(i,j,r) * \text{qfd}(i,j,r) \})) \\
& + \text{sum}(i,\text{TRAD\_COMM}, \text{VOA}(i,r) * \text{ao}(i,r)) \\
& + \text{sum}(i,\text{ENDW\_COMM}, \text{sum}(j,\text{PROD\_COMM}, \\
& \quad \text{VFA}(i,j,r) * [\text{afe}(i,j,r) + \text{ava}(j,r)])) \\
& + \text{sum}(j,\text{PROD\_COMM}, \text{sum}(i,\text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i,j,r) + \text{VDFA}(i,j,r) \} * \text{af}(i,j,r)) \\
& + \text{sum}(i,\text{TRAD\_COMM}, \{ \text{VIPA}(i,r) * [\text{ppm}(i,r) + \text{qpm}(i,r)] \} \\
& \quad - \{ \text{VIPM}(i,r) * \text{qpm}(i,r) \}) \\
& + \text{sum}(i,\text{TRAD\_COMM}, \{ \text{VDPA}(i,r) * [\text{ppd}(i,r) + \text{qpd}(i,r)] \} \\
& \quad - \{ \text{VDPM}(i,r) * \text{qpd}(i,r) \}) \\
& + \text{sum}(i,\text{TRAD\_COMM}, \{ \text{VIGA}(i,r) * [\text{pgm}(i,r) + \text{qgm}(i,r)] \} \\
& \quad - \{ \text{VIGM}(i,r) * \text{qgm}(i,r) \}) \\
& + \text{sum}(i,\text{TRAD\_COMM}, \{ \text{VDGA}(i,r) * [\text{pgd}(i,r) + \text{qgd}(i,r)] \} \\
& \quad - \{ \text{VDGM}(i,r) * \text{qgd}(i,r) \}) \\
& + \text{sum}(i,\text{TRAD\_COMM}, \text{sum}(s,\text{REG}, \\
& \quad \{ \text{VXWD}(i,r,s) * [\text{pfob}(i,r,s) + \text{qxs}(i,r,s)] \} \\
& \quad - \{ \text{VXMD}(i,r,s) * \text{qxs}(i,r,s) \})) \\
& + \text{sum}(i,\text{TRAD\_COMM}, \text{sum}(s,\text{REG}, \\
& \quad \{ \text{VIMS}(i,s,r) * [\text{pms}(i,s,r) + \text{qxs}(i,s,r)] \} \\
& \quad - \{ \text{VIWS}(i,s,r) * [\text{pcif}(i,s,r) + \text{qxs}(i,s,r)] \})) \\
& + \text{sum}(i,\text{TRAD\_COMM}, \text{VST}(i,r) * \text{pm}(i,r)) \\
& - \text{sum}(i,\text{TRAD\_COMM}, \text{VIM}(i,r) * \text{pim}(i,r)).
\end{aligned}$$

The following relationships are to be employed in the simplification of the expression for regional income:

$$\text{pcgds}(r) = \text{ps}(\text{"cgds"},r) = \text{pm}(\text{"cgds"},r),$$

and  $\text{VOA}(\text{"cgds"},r) = \text{VOM}(\text{"cgds"},r)$ .

The expression becomes:

$$\begin{aligned}
& \text{INCOME}(r) * y(r) = \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i,r) * \text{ps}(i,r)) \\
& + \text{VOA}(\text{"cgds"},r) * \text{pcgds}(r) \\
& - \text{VDEP}(r) * \text{pcgds}(r) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i,r) * \text{qo}(i,r)) \\
& - \text{VDEP}(r) * \text{kb}(r) \\
& + \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i,r) * \text{qo}(i,r) \\
& \quad - \{ \text{VOA}(i,r) * \text{qo}(i,r) \} \}) \\
& + \text{sum}(i, \text{ENDWS\_COMM}, \{ \text{VOM}(i,r) * \text{pm}(i,r) \}) \\
& - \text{sum}(i, \text{ENDW\_COMM}, \{ \text{VOA}(i,r) * \text{ps}(i,r) \}) \\
& + \text{sum}(i, \text{ENDWM\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i,j,r) * \text{qfe}(i,j,r) \} \\
& \quad - \{ \text{VFM}(i,j,r) * \text{qfe}(i,j,r) \})) \\
& + \text{sum}(i, \text{ENDWS\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \{ \text{VFA}(i,j,r) * \text{qfe}(i,j,r) \} \\
& \quad - \{ \text{VFM}(i,j,r) * [\text{pmes}(i,j,r) + \text{qfe}(i,j,r)] \})) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIFA}(i,j,r) * \text{qfm}(i,j,r) \} \\
& \quad - \{ \text{VIFM}(i,j,r) * \text{qfm}(i,j,r) \})) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDFA}(i,j,r) * \text{qfd}(i,j,r) \} \\
& \quad - \{ \text{VDFM}(i,j,r) * \text{qfd}(i,j,r) \})) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VOA}(i,r) * \text{ao}(i,r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \text{VFA}(i,j,r) * [\text{afe}(i,j,r) + \text{ava}(j,r)])) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i,j,r) + \text{VDFA}(i,j,r) \} * \text{af}(i,j,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIPA}(i,r) * [\text{ppm}(i,r) + \text{qpm}(i,r)] \} \\
& \quad - \{ \text{VIPM}(i,r) * \text{qpm}(i,r) \}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDPA}(i,r) * [\text{ppd}(i,r) + \text{qpd}(i,r)] \} \\
& \quad - \{ \text{VDPM}(i,r) * \text{qpd}(i,r) \}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIGA}(i,r) * [\text{pgm}(i,r) + \text{qgm}(i,r)] \} \\
& \quad - \{ \text{VIGM}(i,r) * \text{qgm}(i,r) \}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDGA}(i,r) * [\text{pgd}(i,r) + \text{qgd}(i,r)] \} \\
& \quad - \{ \text{VDGM}(i,r) * \text{qgd}(i,r) \}) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VXWD}(i,r,s) * [\text{pfob}(i,r,s) + \text{qxs}(i,r,s)] \} \\
& \quad - \{ \text{VXMD}(i,r,s) * \text{qxs}(i,r,s) \}))
\end{aligned}$$

$$\begin{aligned}
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VIMS}(i, s, r) * [\text{pms}(i, s, r) + \text{qxs}(i, s, r)] \} \\
& \quad - \{ \text{VIWS}(i, s, r) * [\text{pcif}(i, s, r) + \text{qxs}(i, s, r)] \} )) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VST}(i, r) * \text{pm}(i, r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VIM}(i, r) * \text{pim}(i, r)).
\end{aligned}$$

The following portion of the expression for regional income shows the impact of changes in factor endowments and the initial capital stock on welfare:

$$\begin{aligned}
& \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i, r) * \text{qo}(i, r)) \\
& - \text{VDEP}(r) * \text{kb}(r).
\end{aligned}$$

The next part takes advantage of some price relationships in the model. First the equation that generates the composite price for sluggish endowments is used:

$$\text{pm}(i, r) = \text{sum}(j, \text{PROD\_COMM}, \text{REVSHR}(i, j, r) * \text{pms}(i, j, r)).$$

The revenue share equals:

$$\text{REVSHR}(i, j, r) = \text{VFM}(i, j, r) / \text{sum}(k, \text{PROD\_COMM}, \text{VFM}(i, k, r)),$$

which also can be written as:

$$\text{REVSHR}(i, j, r) = \text{VFM}(i, j, r) / \text{VOA}(i, r).$$

Then the equation that generates a price for aggregate imports is used:

$$\text{pim}(i, s) = \text{sum}(k, \text{REG}, \text{MSHRS}(i, k, s) * \text{pms}(i, k, s)).$$

The import share equals:

$$\text{MSHRS}(i, r, s) = \text{VIMS}(i, r, s) / \text{sum}(k, \text{REG}, \text{VIMS}(i, k, s)),$$

which can also be written as:

$$\text{MSHRS}(i, r, s) = \text{VIMS}(i, r, s) / \text{VIM}(i, s).$$

Also note that:

$$\text{NETINV}(r) = \text{VOA}(\text{"cgds"}, r) - \text{VDEP}(r),$$

and finally subtract  $\text{SAVE}(r) * \text{psave}(r)$  from both sides and rearrange to yield the following expression for the regional real income decomposition:

$$\begin{aligned}
& \text{INCOME}(r) * y(r) \\
& - \text{SAVE}(r) * \text{psave}(r) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VIPA}(i, r) * \text{ppm}(i, r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VDPA}(i, r) * \text{ppd}(i, r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VIGA}(i, r) * \text{pgm}(i, r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VDGA}(i, r) * \text{pgd}(i, r)) = \\
& \quad \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i, r) - \text{VOA}(i, r) \} * \text{qo}(i, r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i, r) * \text{qo}(i, r))
\end{aligned}$$

$$\begin{aligned}
& - \text{VDEP}(r) * \text{kb}(r) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \{ \text{VFA}(i,j,r) - \text{VFM}(i,j,r) \} * \text{qfe}(i,j,r))) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i,j,r) - \text{VIFM}(i,j,r) \} * \text{qfm}(i,j,r))) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VDFA}(i,j,r) - \text{VDFM}(i,j,r) \} * \text{qfd}(i,j,r))) \\
& \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VOA}(i,r) * \text{ao}(i,r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \text{VFA}(i,j,r) * [\text{afe}(i,j,r) + \text{ava}(j,r)])) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i,j,r) + \text{VDFA}(i,j,r) \} * \text{af}(i,j,r))) \\
& \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIPA}(i,r) - \text{VIPM}(i,r) \} * \text{qpm}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDPA}(i,r) - \text{VDPM}(i,r) \} * \text{qpd}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIGA}(i,r) - \text{VIGM}(i,r) \} * \text{qgm}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDGA}(i,r) - \text{VDGM}(i,r) \} * \text{qgd}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VXWD}(i,r,s) - \text{VXMD}(i,r,s) \} * \text{qxs}(i,r,s))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VIMS}(i,s,r) - \text{VIWS}(i,s,r) \} * \text{qxs}(i,s,r))) \\
& \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \{ \text{VXWD}(i,r,s) * \text{pfob}(i,r,s) \})) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VST}(i,r) * \text{pm}(i,r)) \\
& + \text{NETINV}(r) * \text{pcgds}(r) \\
& \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \{ \text{VIWS}(i,s,r) * \text{pcif}(i,s,r) \})) \\
& - \text{SAVE}(r) * \text{psave}(r)
\end{aligned}$$

Since technical change in the shipping activity is also possible, this is incorporated through the following substitution:

$$\text{pcif}(i,r,s) = \text{FOBSHR}(i,r,s) * \text{pfob}(i,r,s) + \text{TRNSHR}(i,r,s) * \text{ptrans}(i,r,s),$$

$$\text{where } \text{ptrans}(i,r,s) = \text{sum}(m, \text{MARG\_COMM}, \text{VTFSD\_MSH}(i,r,s) * [\text{pt}(m) - \text{atmfsd}(m,i,r,s)]).$$

Therefore, we now have:

$$\begin{aligned}
& \text{INCOME}(r) * \text{y}(r) \\
& - \text{SAVE}(r) * \text{psave} \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VIPA}(i,r) * \text{ppm}(i,r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VDPA}(i,r) * \text{ppd}(i,r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VIGA}(i,r) * \text{pgm}(i,r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VDGA}(i,r) * \text{pgd}(i,r)) =
\end{aligned}$$

$$\begin{aligned}
& \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i,r) - \text{VOA}(i,r) \} * \text{qo}(i,r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i,r) * \text{qo}(i,r)) \\
& - \text{VDEP}(r) * \text{kb}(r) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \{ \text{VFA}(i,j,r) - \text{VFM}(i,j,r) \} * \text{qfe}(i,j,r)) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i,j,r) - \text{VIFM}(i,j,r) \} * \text{qfm}(i,j,r)) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VDFA}(i,j,r) - \text{VDFM}(i,j,r) \} * \text{qfd}(i,j,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VOA}(i,r) * \text{ao}(i,r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \text{VFA}(i,j,r) * [\text{afe}(i,j,r) + \text{ava}(j,r)])) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i,j,r) + \text{VDFA}(i,j,r) \} * \text{af}(i,j,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIPA}(i,r) - \text{VIPM}(i,r) \} * \text{qpm}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDPA}(i,r) - \text{VDPM}(i,r) \} * \text{qpd}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIGA}(i,r) - \text{VIGM}(i,r) \} * \text{qgm}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDGA}(i,r) - \text{VDGM}(i,r) \} * \text{qgd}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VXWD}(i,r,s) - \text{VXMD}(i,r,s) \} * \text{qxs}(i,r,s)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VIMS}(i,s,r) - \text{VIWS}(i,s,r) \} * \text{qxs}(i,s,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \{ \text{VXWD}(i,r,s) * \text{pfob}(i,r,s) \})) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VST}(i,r) * \text{pm}(i,r)) \\
& + \text{NETINV}(r) * \text{pcgds}(r) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \text{VIWS}(i,s,r) * \text{FOBSHR}(i,s,r) * \text{pfob}(i,s,r)) \\
& + \text{TRNSHR}(i,s,r) * [\text{sum}(m, \text{MARG\_COMM}, \text{VTMFS}_M\text{SD} \\
& \quad (i, s, r) * \text{pt}(m) - \text{atmfsd}(m, i, s, r) ] ] ) \\
& - \text{SAVE}(r) * \text{psave}(r).
\end{aligned}$$

This may be simplified by using the coefficients for the Value of Transport Margin usage by Margin, Freight, Source and Destination,  $\text{VTMFS}_M\text{SD}(m, i, r, s)$  and the Value of aggregate Transport Margins used in shipments to Destination  $r$ ,  $\text{VTMD}(m, r)$ .

$$\begin{aligned}
& \text{INCOME}(r) * y(r) \\
& - \text{SAVE}(r) * \text{psave}(r) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VIPA}(i,r) * \text{ppm}(i,r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VDPA}(i,r) * \text{ppd}(i,r)) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VIGA}(i,r) * \text{pgm}(i,r))
\end{aligned}$$

$$\begin{aligned}
& - \text{sum}(i, \text{TRAD\_COMM}, \text{VDGA}(i,r) * \text{pgd}(i,r)) = \\
& \text{sum}(i, \text{NSAV\_COMM}, \{ \text{VOM}(i,r) - \text{VOA}(i,r) \} * \text{qo}(i,r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{VOA}(i,r) * \text{qo}(i,r)) \\
& - \text{VDEP}(r) * \text{kb}(r) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \{ \text{VFA}(i,j,r) - \text{VFM}(i,j,r) \} * \text{qfe}(i,j,r))) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i,j,r) - \text{VIFM}(i,j,r) \} * \text{qfm}(i,j,r))) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VDFA}(i,j,r) - \text{VDFM}(i,j,r) \} * \text{qfd}(i,j,r))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VOA}(i,r) * \text{ao}(i,r)) \\
& + \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, \\
& \quad \text{VFA}(i,j,r) * [\text{afe}(i,j,r) + \text{ava}(j,r)])) \\
& + \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, \\
& \quad \{ \text{VIFA}(i,j,r) + \text{VDFA}(i,j,r) \} * \text{af}(i,j,r))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIPA}(i,r) - \text{VIPM}(i,r) \} * \text{qpm}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDPA}(i,r) - \text{VDPM}(i,r) \} * \text{qpd}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VIGA}(i,r) - \text{VIGM}(i,r) \} * \text{qgm}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \{ \text{VDGA}(i,r) - \text{VDGM}(i,r) \} * \text{qgd}(i,r)) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VXWD}(i,r,s) - \text{VXMD}(i,r,s) \} * \text{qxs}(i,r,s))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \\
& \quad \{ \text{VIMS}(i,s,r) - \text{VIWS}(i,s,r) \} * \text{qxs}(i,s,r))) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \{ \text{VXWD}(i,r,s) * \text{pfob}(i,r,s) \})) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VST}(i,r) * \text{pm}(i,r)) \\
& + \text{NETINV}(r) * \text{pcgds}(r) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \{ \text{VXWD}(i,s,r) * \text{pfob}(i,s,r) \})) \\
& - \text{sum}(m, \text{MARG\_COMM}, \text{VTMD}(m, r) * \text{pt}(m)) \\
& - \text{SAVE}(r) * \text{psave}(r) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \text{sum}(m, \text{MARG\_COMM}, \text{VTMFSD}(m,i,s,r) * \\
& \quad \text{atmfSD}(m, i, s, r) ) ) ) .
\end{aligned}$$

To simplify the LHS of the expression for the regional real income decomposition, divide and multiply through by  $\text{INCOME}(r)$  :

$$\begin{aligned}
& \text{INCOME}(r) * \\
& [ ( y(r)
\end{aligned}$$

- $\text{SAVE}(r)/\text{INCOME}(r) * \text{psave}(r)$
- $\text{sum}(i, \text{TRAD\_COMM}, \text{VIPA}(i,r)/\text{INCOME}(r) * \text{ppm}(i,r))$
- $\text{sum}(i, \text{TRAD\_COMM}, \text{VDPA}(i,r)/\text{INCOME}(r) * \text{ppd}(i,r))$
- $\text{sum}(i, \text{TRAD\_COMM}, \text{VIGA}(i,r)/\text{INCOME}(r) * \text{pgm}(i,r))$
- $\text{sum}(i, \text{TRAD\_COMM}, \text{VDGA}(i,r)/\text{INCOME}(r) * \text{pgd}(i,r)) = Y(r) * [y(r) - p(r)] = D(r)$

Next substitute the following tax instruments into the expression:

$$VOM(i,r) - VOA(i,r) = PTAX(i,r),$$

$$VFA(i,j,r) - VFM(i,j,r) = ETAX(i,j,r) \text{ for all } i \text{ ENDW\_COMM},$$

$$VIFA(i,j,r) - VIFM(i,j,r) = IFTAX(i,j,r),$$

$$VDFA(i,j,r) - VDFM(i,j,r) = DFTAX(i,j,r),$$

$$VIPA(i,r) - VIPM(i,r) = IPTAX(i,r),$$

$$VDPA(i,r) - VDPM(i,r) = DPTAX(i,r),$$

$$VIGA(i,r) - VIGM(i,r) = IGTAX(i,r),$$

$$VDGA(i,r) - VDGM(i,r) = DGTAX(i,r),$$

$$VXWD(i,r,s) - VXMD(i,r,s) = XTAXD(i,r,s),$$

and

$$VIMS(i,s,r) - VIWS(i,s,r) = MTAX(i,s,r).$$

The decomposition expression becomes:

$$D(r) =$$

$$\text{sum}(i, \text{NSAV\_COMM}, PTAX(i,r) * qo(i,r))$$

$$+ \text{sum}(i, \text{ENDW\_COMM}, VOA(i,r) * qo(i,r))$$

$$- \text{VDEP}(r) * kb(r)$$

$$+ \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM}, ETAX(i,j,r) * qfe(i,j,r)))$$

$$+ \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, IFTAX(i,j,r) * qfm(i,j,r)))$$

$$+ \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM}, DFTAX(i,j,r) * qfd(i,j,r)))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, VOA(i,r) * ao(i,r))$$

$$+ \text{sum}(i, \text{ENDW\_COMM}, \text{sum}(j, \text{PROD\_COMM},$$

$$VFA(i,j,r) * [afe(i,j,r) + ava(j,r)]))$$

$$+ \text{sum}(j, \text{PROD\_COMM}, \text{sum}(i, \text{TRAD\_COMM},$$

$$\{VIFA(i,j,r) + VDFA(i,j,r)\} * af(i,j,r)))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, IPTAX(i,r) * qpm(i,r))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, DPTAX(i,r) * qpd(i,r))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, IGTAX(i,r) * qgm(i,r))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, DGTAX(i,r) * qgd(i,r))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, XTAXD(i,r,s) * qxs(i,r,s)))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, MTAX(i,s,r) * qxs(i,s,r)))$$

$$+ \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \{VXWD(i,r,s) * pfob(i,r,s)\}))$$



$$\begin{aligned}
& + \text{sum}(i, \text{TRAD\_COMM}, \text{VST}(i, r) * \text{pm}(i, r)) \\
& + \text{NETINV}(r) * \text{pcgds}(r) \\
& - \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \{ \text{VXWD}(i, s, r) * \text{pfob}(i, s, r) \}) \\
& - \text{sum}(m, \text{MARG\_COMM}, \text{VTMD}(m, r) * \text{pt}(m)) \\
& - \text{SAVE}(r) * \text{psave}(r) \\
& + \text{sum}(i, \text{TRAD\_COMM}, \text{sum}(s, \text{REG}, \text{sum}(m, \text{MARG\_COMM}, \text{VTMFSD}(m, i, s, r) * \\
& \quad \text{atmfsd}(m, i, s, r) \} \})).
\end{aligned}$$

This decomposition may be placed on a *per capita* basis in the same manner as in Section 3.4. Substituting the resulting *per capita* decomposition,  $D^*$ , into the *EV* decomposition given in equation (24), and adding regional indexes to all the terms in that equation, we obtain the final welfare decomposition for the multiregion model:

$$\begin{aligned}
\text{EV\_ALT}(r) & = [0.01 * \text{EVSCALFACT}(r)] \\
& * [ \text{sum}\{i, \text{ENDW\_COMM}, \text{VOA}(i, r) * [\text{qo}(i, r) - \text{pop}(r)]\} - \text{VDEP}(r) * [\text{kb}(r) - \text{pop}(r)] \\
& + \text{sum}\{i, \text{NSAV\_COMM}, \text{PTAX}(i, r) * [\text{qo}(i, r) - \text{pop}(r)]\} \\
& + \text{sum}\{i, \text{ENDW\_COMM}, \text{sum}\{j, \text{PROD\_COMM}, \text{ETAX}(i, j, r) * [\text{qfe}(i, j, r) - \text{pop}(r)]\}\} \\
& + \text{sum}\{j, \text{PROD\_COMM}, \text{sum}\{i, \text{TRAD\_COMM}, \text{IFTAX}(i, j, r) * [\text{qfm}(i, j, r) - \text{pop}(r)]\}\} \\
& + \text{sum}\{j, \text{PROD\_COMM}, \text{sum}\{i, \text{TRAD\_COMM}, \text{DFTAX}(i, j, r) * [\text{qfd}(i, j, r) - \text{pop}(r)]\}\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{IPTAX}(i, r) * [\text{qpm}(i, r) - \text{pop}(r)]\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{DPTAX}(i, r) * [\text{qpd}(i, r) - \text{pop}(r)]\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{IGTAX}(i, r) * [\text{qgm}(i, r) - \text{pop}(r)]\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{DGTAX}(i, r) * [\text{qgd}(i, r) - \text{pop}(r)]\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{XTAXD}(i, r, s) * [\text{qxs}(i, r, s) - \text{pop}(r)]\}\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{MTAX}(i, s, r) * [\text{qxs}(i, s, r) - \text{pop}(r)]\}\} \\
& + \text{sum}\{i, \text{PROD\_COMM}, \text{VOA}(i, r) * \text{ao}(i, r)\} \\
& + \text{sum}\{j, \text{PROD\_COMM}, \text{VVA}(j, r) * \text{ava}(j, r)\} \\
& + \text{sum}\{i, \text{ENDW\_COMM}, \text{sum}\{j, \text{PROD\_COMM}, \text{VFA}(i, j, r) * \text{afe}(i, j, r)\}\} \\
& + \text{sum}\{j, \text{PROD\_COMM}, \text{sum}\{i, \text{TRAD\_COMM}, \text{VFA}(i, j, r) * \text{af}(i, j, r)\}\} \\
& + \text{sum}\{m, \text{MARG\_COMM}, \text{sum}\{i, \text{TRAD\_COMM}, \\
& \quad \text{sum}\{s, \text{REG}, \text{VTMFSD}(m, i, s, r) * \text{atmfsd}(m, i, s, r)\}\}\} \\
& + \text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{VXWD}(i, r, s) * \text{pfob}(i, r, s)\}\} \\
& + \text{sum}\{m, \text{MARG\_COMM}, \text{VST}(m, r) * \text{pm}(m, r)\} \\
& + \text{NETINV}(r) * \text{pcgds}(r) \\
& - \text{sum}\{i, \text{TRAD\_COMM}, \text{sum}\{s, \text{REG}, \text{VXWD}(i, s, r) * \text{pfob}(i, s, r)\}\} \\
& - \text{sum}\{m, \text{MARG\_COMM}, \text{VTMD}(m, r) * \text{pt}(m)\} \\
& - \text{SAVE}(r) * \text{psave}(r) ] \\
& + 0.01 * \text{INCOME EV}(r) * \text{pop}(r);
\end{aligned}$$

This decomposition of regional EV is composed of the allocative effects which are given by the various *per capita* quantity change terms multiplied by initial taxes, the terms of trade effects, the effects of technical change, and the effects of *per capita* endowment and population change.<sup>8</sup> For

<sup>8</sup> Note that a new value flow  $\text{VVA}(j, r)$  corresponding to the Value of Value Added in sector  $j$  of region  $r$  has been added to match up with value-added augmented technical change.

example, the contribution of *per capita* changes in output of good  $i$  in region  $r$  to its welfare gain (loss) as measured by the EV would be given by:

$$\text{CONT\_EV\_qoir}(i,r) = .01 * \text{EVSCALFACT}(r) * \text{PTAX}(i,r) * [\text{qo}(i,r) - \text{pop}].$$

This concludes the formal derivation of the welfare decomposition used in the GTAP model.