A signal detection experiment with limited number of trials.

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Abstract: Signal detection theory has been well accepted in vision science to measure human sensitivity to stimuli in a Psychophysical experiment. The theory is formulated so that the measured sensitivity is independent from a response bias (criterion). The formulation is based on an assumption that number of trials in the experiment is infinite but this assumption cannot be satisfied in practice. The assumption came from two normal distributions used in the formulation. The distributions respectively represent a set of signal trial and that of noise trials in the experiment. In this study, I will show how the violation of the assumption affects results in a signal detection experiment testing some sensitivity and a way to derive a likelihood function of the sensitivity based on measured sensitivity and criterion. The likelihood function allows us to use results of the signal detection experiment efficiently for Bayesian inference. I will also discuss how the criterion affects the results of the experiment under the violation of the assumption.

Equations: Assume sensitivity d_0 of the visual system for some task and a criterion c_0 used by a subject are given. Theoretical hit rate H_0 and false-alarm rate F_0 of the subject can be computed from these given values:

$$H_0 = H(d_0, c_0) = \Phi(-c_0 + d_0/2)$$

$$F_0 = F(d_0, c_0) = \Phi(-c_0 - d_0/2)$$

where Φ is the cumulative distribution function of the standard normal distribution. Consider testing the sensitivity of the subject in a signal detection experiment composed of a set of n signal trials and that of n noise trials. These sets can be regarded as two independent sets of Bernoulli trials. Then, hits and false-alarms in the experiment follow the binomial distribution:

$$\mathrm{P}(k_H;\ n, H_0) = \binom{n}{k_H} {H_0}^{k_H} (1 - H_0)^{n-k_H}$$

$$\mathrm{P}(k_F;\ n, F_0) = \binom{n}{k_F} {F_0}^{k_F} (1 - F_0)^{n-k_F}$$

where k_H and k_F are numbers of the hits and the false-alarms. Note that measured sensitivity is $d_m = \Phi^{-1}(k_H/n) - \Phi^{-1}(k_F/n)$ and a measured criterion is $c_m = -\Phi^{-1}(k_H/n)/2 - \Phi^{-1}(k_F/n)/2$. From these two distributions, a probability distribution $\mathrm{P}(d_m, c_m \mid d_0, c_0)$ can be computed. Conversely, the likelihood function of sensitivity d' of the visual system for d_m and c_m can be also derived from the distributions as well:

$$\mathrm{L}(d'|d_m, c_m) = \int \mathrm{P}\big(k_{Hm};\ n, \mathrm{H}(d', c)\big)\mathrm{P}\big(k_{Fm};\ n, \mathrm{F}(d', c)\big)dc$$

where $k_{Hm} = n\mathrm{H}(d_m, c_m)$ and $k_{Fm} = n\mathrm{F}(d_m, c_m)$ are measured numbers of hit and false-alarm trials respectively.