2013

Theoretical Foundations for Effective STEM Learning Environments

Aran W. Glancy
University of Minnesota - Twin Cities, aglancy@purdue.edu

Tamara J. Moore
Purdue University, tamara@purdue.edu

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Aran W. Glancy, The University of Minnesota

Tamara J. Moore, Purdue University

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The ideas of John Dewey, Zoltan Dienes, and Richard Lesh have influenced research and practice in science, mathematics, and engineering classrooms for quite some time. Experiential education, concrete manipulatives, and multiple representations are just some of the lasting ideas taken in part from these theorists that remain important components of current educational practice. Educators in mathematics, science, and engineering have each appropriated the ideas of these theorists, in different ways and to different degrees, for use in their separate classrooms. As integrated approaches to science, technology, engineering, and mathematics (STEM) become more common, the need to develop effective strategies in these cross-disciplinary environments becomes more urgent. As part of that development effort, it is worth revisiting the theories of Dewey, Dienes, and Lesh to see how they apply in integrated spaces. Building on the ideas of these theorists and those that have expanded on their work, this paper will describe the characteristics of effective STEM learning environments with a focus on the middle school level. Specifically, we will argue that for these instructional environments to be effective they should meaningfully integrate the STEM subjects, encourage collaboration, and provide students with authentic and realistic situations in which to engage with the STEM content. Furthermore, these experiences need to allow students multiple access points to the concepts and encourage them to engage with and express the concepts in multiple modes of representation. Dewey, Dienes, and Lesh provide the theoretical underpinnings not only to set up effective learning environments within each discipline but also to maximize the connections between the disciplines in integrated STEM classes or schools. Through both application and interpretation of theory and specific examples, we hope to make our vision of effective STEM learning environments clear.

The term STEM is used in a variety of contexts and brings with it many different connotations. Portions of this paper are devoted to explaining our interpretation of the term STEM, however, in the interest of clarity we offer a brief definition from our perspective. When we refer to STEM or STEM learning environments, we are specifically referring to classrooms or schools where conscious and overt efforts are made to coordinate the learning objectives and learning activities of two or more of the STEM disciplines. This can happen in one classroom where students pursue multiple learning objectives or in several different classrooms where teachers work to together to coordinate the learning activities. In this sense, integration is an essential characteristic of a learning environment if it is to be called STEM. The principals we describe in this paper apply whether the integration includes all of the STEM disciplines or just two, say mathematics and science, but classrooms that integrate all the disciplines, specifically through engineering activities as described by Moore, Stohlmann, Wang, Tank, Glancy, and Roehrig (in press) are extremely promising and exemplify the type of integration to which we refer.

STEM Integration

As described above, instruction in effective STEM learning environments begins with the meaningful integration of STEM disciplines. The natural connections between science, technology, engineering, and mathematics have caused them to be lumped together in local, state, and national conversations, but despite their proximity in discussions of policy and practice, they still remain largely isolated within the silo-structure of most schools. Recent efforts, however, have begun to shift the focus from simply identifying issues common to the disciplines and then tackling them separately, toward addressing those issues through meaningful integration of the subjects (for examples see Bossé, Lee, Swinson, & Faulconer, 2010; Furner &

Kumar, 2007; Moore et al., 2013; Nyaumwe & Brown, 2010; Redmond et al., 2007; Wang, Moore, Roehrig, & Park, 2011; Yarker & Park, 2012; Moore, Guzey, Roehrig, Davis, & Imbertson, 2013). According to Moore, Roehrig, Lesh, and Guzey (2010), "In order to prepare students to address the problems of our society, it is necessary to provide students with opportunities to understand the problems through rich, engaging, and powerful experiences that integrate the disciplines of STEM" (p. 4). Separating the disciplines sets up artificial divides that are not generally present outside of the classroom, while integration presents the disciplines in a more honest or realistic fashion.

This timely and compelling argument can trace its roots all the way back to the work of Dewey. In his 1899 lectures, Dewey (1966) made a similar argument against "teaching subjects isolatedly from each other" (p. 189). In his view, this approach deemphasized the relationships between the subjects and prevented the students from perceiving the unity of their pursuits. He states, "To introduce [the subjects] to the child as distinct from the start, is to disorganize and disintegrate, instead of coordinate and connect" (p. 193). Dewey continues, arguing that outside of the artificial setting in schools, our experiences are holistic and only upon reflection can we identify the distinct subjects within them. Specifically referring to mathematics, Lesh and Zawojewski (2007), make a similar argument saying that "the traditional topics serve as good descriptors of the work" (p. 781) but mask the fact that in realistic situations, the mathematics is "more complex, situated, and multidisciplinary than the conventional topic descriptions imply" (p. 781). For students who are just learning these subjects, those distinctions carry no meaning, and as Lesh and Zawojewski point out, they may not see the connection between what they learn in school and the situations they encounter out of school. This is especially relevant at the middle school level, where teachers (and thus their classes) begin to specialize more teaching only one subject rather than all subjects. Thus, the first step in building a successful STEM instructional environment at any level is the meaningful integration of the disciplines.

Realistic Problems

Once integrated, the problems within subjects more accurately resemble realistic situations. Dewey's (1916) alternative to separate disciplines was a learning environment structured on *vocations* or *occupations*, but he was very careful to explain that he was not advocating career training. As he states, "the only adequate training *for* occupations is training *through* occupations" (1916, p. 297). Instead, his use of the terms vocation or occupation referred to "a direction of life activities as renders them perceptibly significant to a person, because of the consequences they accomplish, and also useful to his associates" (1916, p. 294). In his view, activities in school should model those experiences outside of school that we find engaging and fulfilling, and these activities often have a social or civic component to them. Rather than focus on preparation for future careers, he argued that focusing on the present value of experiences would prepare students for continued growth, something needed in any career.

Dewey's conception of vocational education, although powerful, is open to interpretation. Some current manifestations of this approach come in the form of case-based or problem-based learning (Duch, Groh, & Allen, 2001; Savery & Duffy, 1995). In this approach, rather than challenging students with straightforward, simplified problems, students are given complex, realistic problems that are "simulations of real life experiences" (Lesh & Harel, 2003, p. 158). Within a STEM environment, those realistic experiences can be the jumping off point toward the science, technology, or engineering concepts. What is important is that the problems or activities are realistic or authentic and that, just as Dewey argued, the students see the purpose in engaging in them, not because of their future utility but because of their inherent value.

A thematic, interdisciplinary unit centered on an engineering design challenge is an example of an application of this approach. The “Wind Turbines” unit created by Dare, Pettis, and Moore (2013) provides the basis for one example. The challenge within the unit is to determine the optimal location for a wind turbine that will be placed on the grounds of the students’ school. While investigating this challenge, students explore design features of the windmill, relevant weather patterns, and potentially many other concepts. The context of this situation is not a problem of mathematics, science, or engineering, but a problem for the community. While investigating and solving their challenge, the students will use mathematics, science, and engineering, but the problem itself is interdisciplinary.

The Collaborative Nature of STEM

Lesh and Dewey agreed that students’ problems in school should be grounded in the real world, but all three believed students should work collaboratively. Each theorist, however, approached it from a slightly different perspective. For Dewey (1916, 1938) education was both social in nature and served a function within a democracy, thus students should act and be treated like members of a community with all the freedoms of the members of a democratic society. The pursuits Dewey envisioned for his students were community pursuits, requiring the students to work together as a community of learners. According to Lesh, realistic, interdisciplinary problems outside of school are usually tackled by teams, often where members have different areas of expertise (see e.g. Hamilton, Lesh, Lester, & Brilleslyper, 2008; Lesh, Hoover, Hole, Kelly, & Post, 2000). Because of this, it is logical for students to also approach their problems in teams. Teamwork also has the added benefit of encouraging communication and metacognition. Dienes was also supportive of group work, acknowledging the importance of the social aspect of learning (Sriraman & Lesh, 2007). As Sriraman and Lesh point out, Dienes’s activities often required group work even to the point where "the learner often is a group" (p. 73). Dienes, himself, says, "I emphasized small group work long before it became popular" (Sriraman & Lesh, 2007, p. 63). Although these arguments for collaboration and group work are qualitatively different, their superposition makes it clear that teamwork is another essential component of effective STEM learning environments.

Personal Experience

For Dewey and Lesh, it is also important for students to make a personal connection to the experience. Returning to the "Wind Turbine" example, by situating the task at the students' own school and making them the decision makers, this activity shifts from an academic exercise with no context to an individually and socially useful experience, or as Dewey describes, an "activity which renders service to others and engages personal powers in behalf of the accomplishment of results" (Dewey, 1916, pp. 306–307). Furthermore, this activity satisfies what Lesh, Hoover, Hole, Kelly, and Post (2000) call the "reality principle" which states that "it is important for students to try to make sense of the situation based on extensions of their own personal knowledge and experiences" (p. 614). Outside of school, problems are complex and "involve human preferences, values, and social dynamics" (Hamilton et al., 2008, p. 2), and the problems we challenge students with in school should exhibit these same characteristics. Lesh et al. (2000) go on to say that "the key to satisfying the reality principle is not for the problem to be 'real' in an absolute sense" (p. 615) as long it is realistic in nature and complexity. Although the "Wind Turbine" problem may not actually be real, the situation is feasible and believable.

All problems need not be as personal as "Wind Turbines" to satisfy Dewey and Lesh's requirement of meaningful experiences. One characteristic of engineering design tasks is that they involve a client, and by choosing compelling situations and clients with real needs the teacher can provide problems to which the students can relate and engage (Diefes-Dux, Moore, Zawojewski, Imbrie, & Follman, 2004). The same principles apply not just in engineering, but for problematic situations focused on mathematics, science, or technology. However in all these cases, the problems need to be rich enough to approximate problems that students would encounter in their lives outside of school.

Although Dienes acknowledged that focusing on personally or socially relevant, realistic problems is engaging to students (Sriraman & Lesh, 2007), the realistic context of problems was less central to his approach. If framed in a playful manner where students can explore patterns and relationships, he argued, activities can be engaging without necessitating a realistic context. That being said, meaningfully connecting activities to actual experience was just as important to Dienes as it was to Lesh and Dewey. Dienes (1960) describes mathematics saying that it "is based on experience; it is the crystallization of relationships into beautifully regular structure, distilled from our actual contacts with the real world" (p. 11). For example, students' experiences with the correlation between height and shoes size help those students form early conceptions of proportional relationships and proportional reasoning. Lesh and Doerr (2003) capitalized on this specific experience in the Big Foot Problem, in which middle school students are challenged to create a mathematical model of this exact relationship. By crafting and executing activities that illuminate the structures the students have experienced, we make activities at least potentially meaningful, and through uncovering those structures, learners build up their own conceptual understanding. Again, we can extend Dienes principle to the sciences as well. Students build their understanding of the natural world from their experiences with it. Concepts of force and motion begin forming when children move (or try to move) objects of different sizes. Instruction falls short when it fails to take into considerations the conceptions students bring with them from their experiences. For Dienes, connecting learning to the real world was not simply about engagement but was essential to abstracting mathematical structure, essential to learning. And these connections are essential for abstracting any structure, not just mathematical. For Dienes, the job of the teacher was to “accelerate the growth of the concepts by putting the most suitable experiences in the children’s way” (1960, p. 42).

In the sense that experiences ground a learner’s conceptual structures, all three theorists agree. Dienes calls this embodied knowledge, “where knowledge and abilities are organized around experience” (Sriraman & Lesh, 2007, p. 73). As Sriraman and Lesh explain, this view is consistent with (but predates) the theory of situated cognition. Dewey attributed similar value to the vocations around which his approach was built. As he says, “the vocation acts as both magnet to attract and as glue to hold. Such organization of knowledge is vital, because it has reference to needs; it is so expressed and readjusted in action that it never becomes stagnant” (1916, p. 297). Although Lesh distances himself slightly from the theory of situated cognition (Lesh, Doerr, Carmona, & Hjalmarson, 2003; Lesh & Harel, 2003), he does so only in the implications of this theory, not in the premise. According to Lesh et al. (2003), context is critically important and learners’ mental models are in fact situated in specific contexts; however, he and his colleagues “are more concerned with how knowledge is developed and structured to interpret specific contexts” (p. 223).

Multiple Representations

Although still very similar in many ways, the mechanism by which experiences support learning is explained in slightly different ways by Dewey, Dienes, and Lesh. For Dewey (1938), the power of vocation in learning is explained by his theory of experience. This theory is understood through the principles of continuity and interaction. Continuity refers to the way in which a student's present experiences affect her future experiences, and interaction refers to how a student's situation affects his current experience. Through continuity and interaction, all experiences are educative in that they influence future experiences, however some experiences support future growth while others inhibit it. The job then of the teacher is "to select the kind of present experiences that live fruitfully and creatively in subsequent experiences" (Dewey, 1938, p. 28). For Dewey, the types of experiences that support future learning are those that engage students in a community of citizen learners and encourage them to think creatively and independently. In the context of the "Wind Turbine" unit, the activity marks one experience (or set of experiences), but in implementing this activity, the teacher must consider the previous experiences of the students as well as those that come after to maximize the benefits of the unit.

Although, Dewey was very explicit about the ways in which experiences interact and influence each other, his theory stopped short of explaining how certain experiences support learning. Dienes and Lesh went a step further by examining how specific activities and the connections between them facilitate generalization, and abstraction. For Dienes, especially when working with elementary and middle school children, the real world experiences to which he referred were centered around actual physical objects (Dienes & Golding, 1971). These objects are also called concrete manipulatives because students need to have the opportunity to physically manipulate them as they explore their structure. In the context of the "Wind Turbine" problem, while investigating the effect of blade size and configuration on the power of the windmill, students are given table-top models of the windmills, and they are able to manipulate the number and configuration of blades, as well as the shape, size, and material of the blades. Additionally, while investigating the optimal gearing between the turbine and the motor, the students work with and explore real gears that they can connect and rearrange in different configurations. As they explore the materials, provided they do so with sufficient curiosity, the students notice patterns. Asymmetric blade configurations wobble; more blades don't necessarily mean more power; the angle of the blades effects the power; the direction that the last gear spins depends on the number of gears; the number of revolutions is related the number of teeth; etc. Thus they begin to see the structure underlying the activity.

Simply exploring these physical manipulatives, however, is not enough according to Dienes. As he says, "it is practically impossible to abstract from one set of experiences" (1960, p. 54). The ideas are not extracted from the concrete objects themselves but, on the contrary, in the relationships that working with the manipulatives exposes. Dienes (1960) describes four principles for structuring activities to lead to concept development and abstraction: the constructivity principle, the dynamic principle, and the principles of mathematical and perceptual variability. These principles together describe how activities can lead to abstract understanding of concepts, and they provide direction for developing activities to achieve that goal.

The constructivity principle and the dynamic principles (Dienes, 1960) are modeled in the activities with windmills and gears described above. The constructivity principle states that children's knowledge must be built upon previous knowledge and experience. The dynamic principle states that the structure behind a manipulative (or sets of manipulatives) does not become visible unless the system is allowed to change. Without being able to vary the number or angle of the blades, for example, the importance of those aspects of the system would never become apparent to the students.

For concept development to continue, however, the activities must also address the principles of mathematical and perceptual variability (Dienes & Golding, 1971; Dienes, 1960). For any given concrete experience or manipulative, there are many variables that can be adjusted, for example the number of teeth on the gears or the number of gears in a chain. The specific number of teeth is not important to the underlying structure, but the relationships between the number of teeth on different gears is. Where the dynamic principle states that students must be able to explore relationships, the principle of mathematical variability requires that systematic variation of the important variables be use to expose those structures. To bring out the structure behind the gears, the numbers of teeth need to be systematically varied within the exploration. Although Dienes calls this mathematical variability, in this case the word "mathematical" is referring to the underlying structure, so in applying this principle more broadly to STEM learning environments, we would call this the principle of "structural variability." Systematically varying the blade set-up to examine the relationship to the power output on the windmill is another example of structural variability centered around science concepts.

The remaining principle, the principle of perceptual variability, is the only one not addressed directly through the examples given from the "Wind Turbine" unit. Dienes and Golding (1971) refer to this principle both as the principle of perceptual variability and as the principle of multiple embodiments. By whichever name, the principle states that "every concept should be presented in as many different ways as possible" (p. 55) and these embodiments "should be as varied as possible perceptually, while aiming towards abstraction of the same concept" (p. 56). The gear investigation is an embodiment of the concept of ratio, but for students to see the concept as anything but an attribute of gears, they must experience it through other embodiments, or modes of representation. If the teacher were pursuing purely mathematical concepts they could follow the gear activity immediately with an investigation of scaling in geometry, or the Big Foot activity (Lesh & Doerr, 2003), or perhaps an exploration of the relationship of the lengths of shadows cast on the wall by a flashlight and the corresponding object distances. Similarly, if another focus of the windmill investigation were energy (as opposed to weather patterns, fluid flow, torque, etc.), the teacher would want to incorporate other experiences that embody energy. Students might also analyze a hydroelectric generator or look at the relationship between potential and kinetic energy in a roller coaster. The more embodiments, and the more varied the representations of the concept, the more likely the students are to understand it beyond the specific examples of windmills and gear ratios.

Since the activities described are embedded in a larger unit centered on the context of wind turbine design and placement, these extension activities might not follow the original activity immediately. From Dienes's perspective, this is not ideal, but Lesh and Harel (2003) point out that as long as the activities are situated in meaningful contexts, the structures that students identify (or the mental models they develop) will be persistent, if not fully formed. Thus, as long as future activities are structured to connect to previous learning, the conceptions students develop will still be available for reevaluation and revision.

In describing the development of manipulatives, however, Dienes advocated "controlling the extent of irrelevant qualities in the aids themselves, or in the situation in which they are used" (p. 54). With that in mind, he would most likely recommend even more direct and simplified embodiments for investigation than these. On the other hand, the more complex and realistic Big Foot problem (for example) is more consistent with the vocational principle of Dewey and satisfies the realistic quality suggested by Lesh. In a single classroom in which the STEM disciplines are integrated, managing this conflict can be a challenging balance. In "Wind Turbines," the students encounter several big ideas from mathematics, science, and engineering. To develop each of these ideas through experiences with multiple embodiments takes time, so teachers must choose the activities carefully to ensure that they are significant enough to warrant the investment. In a situation in which teachers from mathematics, science, engineering, and technology classes are collaborating; however, each teacher can use his or her time to allow the students to explore the relevant concepts, in essence developing the ideas in parallel.

The work of Dienes (and several of his contemporaries) cemented concrete manipulatives as staples in the mathematics classroom, yet Dienes's principles for instruction seem to have had little effect on classroom practice (Lesh, Post, & Behr, 1987). Lesh et al. (1987) conjecture that, despite Dienes's clear emphasis on the relational and operational nature of his approach, many overlook this and focus only on the physical embodiments of the concept without encouraging learners to look for the relationships between them. Furthermore, according to Lesh et al., many teachers do not share Dienes's view of mathematics "because they tend to view mathematics simply as a collection of isolated rules for manipulating symbols" (p. 653).

In part to help teachers broaden their conceptions of mathematics and as a means of extending Dienes's work (Cramer, 2003), Lesh proposed a translational model, which is now commonly known as the Lesh Translation Model (LTM). As seen in Figure 1, the LTM consists of five nodes indicating distinct representations, or embodiments and the translations between or within the representation. A translation is a connection or reformulation of a concept from one representation to another. For example, if a student, while working a word problem on potential energy draws a picture of the change in height of the object, she has performed a translation from written symbols to diagrams or pictures. The LTM is an ideal framework for thinking about conceptual understanding. Dienes's (1960) theory of multiple embodiments is grounded in the idea that structure is revealed through the connections between different manifestations of a concept. Dienes even defines mathematics, itself, not as concepts or facts but as "*actual* structural relationships between concepts" (p. 31). But if structure is revealed in the connections between modes of representation, how do we reveal those connections to learners? The LTM makes it explicit: connections are revealed when learners are asked to translate between representations, and the fluency with which an individual can translate between modes of representation is a measure of his conceptual understanding.

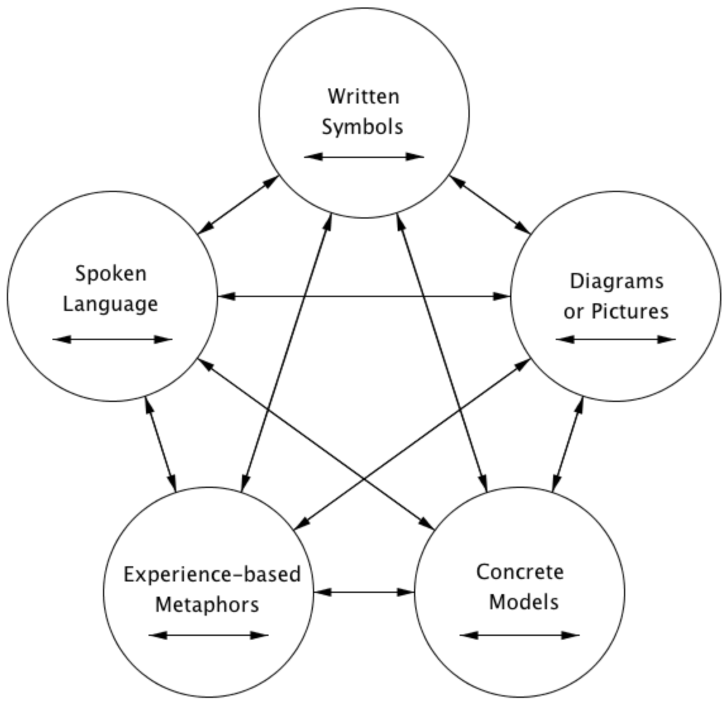

Figure 1. The Lesh Translation Model. Adapted from Lesh and Doerr (2003), this model shows five modes of representaiton (in circles) and the translations between or within them (double arrows)

The LTM is not only a model of conceptual understanding, but also a framework for guiding instruction. In order to maximize students' conceptual understanding of big ideas, STEM teachers can structure tasks and activities to require translations between modes of representation. For example, while analyzing the blade configurations in the windmill activity students could be asked to diagram their plans or observations (concrete to picture translation). Placing students in groups encourages them to communicate verbally about concepts (concrete to spoken language), and a teacher could ask students to relate personal experiences of when they felt the varying strength of the wind (spoken language to experience based metaphor). Furthermore, an understanding of the LTM will protect the teacher from assuming that competency in a single mode of representation, for example symbolic manipulation, is indicative of conceptual understanding. On the contrary, teachers who are mindful of representational fluency will have a window into the specific structural connections that students do and do not understand.

The STEM Translation Model

The framework established by the LTM can also serve as a model for yet another system: integrated STEM learning itself. If we consider integrated STEM thinking or learning to be made up of the concepts, skills, and higher order thinking that bind or link the STEM disciplines together, then the STEM acronym takes on a meaning beyond simply the sum of its parts. Certainly, deductive reasoning in mathematics, design thinking in engineering, inquiry in the sciences, and computational thinking in the fields of technology are distinct and independent approaches to problem solving, and cultivating these capabilities in students should be a primary goal of any STEM program. Each one, however, has its strengths and weaknesses, and each is especially well suited to a specific type of problem. But, as discussed above, real-world problems are complex and integrated. Tackling such problems requires not just the ability to use design thinking or inquiry (for example), but also the ability to choose the best approach or combination of approaches that capitalize on the strengths of each way of thinking. From this perspective, STEM encompasses not just the content, skills, and ways of thinking of each of the disciplines, but it also includes an understanding of the interactions between the disciplines and the ways they support and complement each other.

Thinking in this way of STEM as a whole, the individual disciplines become analogous to the modes of representation in the LTM. For example, mathematical or scientific problems and their solutions become particular ways in which STEM problems manifest themselves. Complex, integrated problems like the wind turbine unit are STEM problems requiring students to apply their skills from multiple disciplines. Once the students branch off to explore the specific mathematical or scientific principles in the gear ratio and blade design activities respectively, however, the lessons begin to look like effective lessons in the separate disciplines. Connecting the lessons throughout the unit back to the central problem adds context, meaning, and value to the unit and keeps them grounded in the overall STEM problem. However, the fact that those connections are in the curriculum does not guarantee that students will perceive them or that they will enhance learning. If students are unaware or unable to see and make these connections on their own, much of the value added through integration may be lost on them.

Just as the LTM argues that students develop deeper understanding of concepts when they are asked to translate between representational modes, students can be encouraged to make connections between the STEM disciplines by asking them to translate ideas between those different disciplines. Asking students to apply the ideas, skills, or techniques from one discipline in the course of tackling a problem in another, in essence *translating* between the disciplines, should help students perceive the relationships, similarities, and differences between the disciplines. This in turn should help them develop more sophisticated concepts of each discipline individually, and STEM as a whole. Combining this emphasis on the connections and translations between disciplines with the idea that the disciplines themselves are manifestations of the broader concept of STEM leads to the STEM Translation Model (Figure 2), which we propose as a way for conceptualizing STEM thinking and STEM learning.

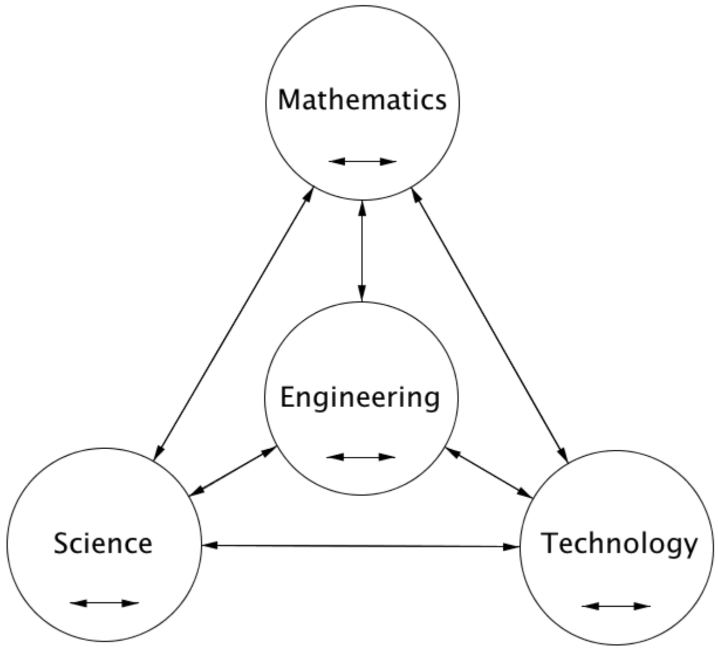

Figure 2: STEM Translation Model. STEM is the combination of the individual disciplines along with the translations that connect them.

With the STEM Translation Model as a framework, it follows that integrated STEM lessons and activities are at their best when they encourage students to make translations between the ideas of multiple disciplines. In the wind turbine activity, changing the angle of the blade changes both the magnitude and direction of the forces on the blades from the wind. Students will hopefully discover this at least implicitly while investigating the optimal angle for their engineering design challenge. This, in and of itself, however, does not necessarily guarantee that the students will make the connections to the science concepts of force and motion. In this case, a teacher who is mindful of the STEM Translation Model might ask the students to draw a force diagram to describe what they learned through the course of their engineering challenge. This translation should help students build a deeper conception of both force and motion but also of the connection between optimization in engineering and the predictive power of science.

The STEM Translation Model also provides a way of thinking about individual concepts that span multiple STEM disciplines. Vectors in mathematics and science, although in most ways very similar, are not identical. Mathematicians and scientists with many years of experience and training have no trouble relating and distinguishing the use of the term *vector* in a variety of different ways. To students just encountering these concepts for the first time, however, the subtle differences between how their math teachers and science teachers use the term can be confusing. The students see the concept in its two different manifestations (in math class and in science class) but they are unable to make the connection between them. To the contrary, however, Rich, Leatham and Wright (2012), have found that conditions are right, students learn concepts better in interdisciplinary settings through a process they call *convergent cognition*. The conditions for convergent cognition require a recognition of the "core concepts and processes" that bind the two uses together. As Rich et al. describe, "this synergistic relationship wherein combining two objects reveals a more complex object is what we believe may occur when a learner connects a core concept from two different domains" (p. 442). By first acknowledging the differences between the way vectors are used in math and in science, and then by helping students to make the connections between the different uses, teachers can support the development of deep conceptual understanding. And those connections can be developed through translations between the different representations.

Furthermore, a teacher who is considering concepts from the perspective of the STEM Translation Model will perceive student difficulties differently. For example, when a student is unable to apply the concepts of *mean* or *median* to a data set in science class, a teacher might assume that the student hadn't mastered those concepts in math class. On the contrary, however, the student may have mastered the concepts in mathematics, but his or her difficulty lies in translating that understanding to the application in science class. Helping this student does not require remediation on the concepts of mean and median as might previously have been thought, but support in making the translation to the new domain.

The idea of integrated teaching and learning traces its roots all the way back to the ideas of Dewey at the turn of the century if not before that, but even today within the current climate of STEM focus, we continue to work toward more effective integrated instruction. The arc of development from Dewey's theory of experience, to Dienes's multiple embodiments, culminating in the Lesh Translation Model leads to way of conceptualizing STEM integration through the connections between the disciplines. With the exception of Lesh, these theorists didn't speak directly about STEM, but their ideas are just as applicable in an integrated STEM environment. The ideas of multidisciplinary problem solving, teamwork and collaboration, connecting learning to personal experience, multiple embodiments, and representational fluency provide the theoretical foundations for effective STEM learning environments.

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