Parametric Representation of Scroll Geometry with Variable Wall Thickness

Bryce R. Shaffer1* and Eckhard A. Groll2

1Air Squared Inc.
Broomfield, CO, USA

2Purdue University, Mechanical Engineering,
West Lafayette, IN, USA

* Corresponding Author: bryce@airsquared.com

ABSTRACT

In a scroll-type compressor, compression is achieved by relative contact between two spiral curves. Since the scroll invention by Leon Creux in (1905) multiple methods have been developed for calculating the scroll geometry and pocket volume. What can generally be considered the most classical method, is defining each scroll curve as the involute of a circle. The two sets of scroll curves are then closed by using the arc of a circle to form the tip of each spiral. In this paper, a new method to calculate the scroll geometry is introduced. By deriving each scroll curve from the radius of curvature parameterized with involute angle, a wide range of scroll involute geometries can be considered that are not included in the classical method. In addition, all possible tip conditions involved in a two arc configuration can be implemented to give a comprehensive scroll compressor geometry definition.

1. INTRODUCTION

Graveson (2001) proposed a more elegant definition for scroll geometry through the use of a natural equation. By using the natural equation, a base scroll curve can be generated which defines the shape of all four involute curves. By translating the base scroll curve as the outer wall of the orbiting scroll, one can obtain the inner wall of the fixed scroll where together they represent the first chamber of the scroll machine. By performing a proper reflection of this chamber, the second chamber is obtained and all four walls of the scroll machine are defined. In order to close the curves, extension geometry needs to be defined at the tip of the fixed and orbiting scrolls. Together, this gives all scroll geometry necessary to generate the required geometric properties of the compressor during operation.

2. BASE SCROLL CURVE

Before the geometry of the compressor can be considered, a base scroll curve needs to be defined. The base scroll curve will ultimately result as the outer wall of the orbiting scroll but it is used in delineating virtually all aspects of the scroll compressor geometry. The base scroll curve can be easily defined parametrically by its radius of curvature \( \rho(\varphi) \) as a function of \( \varphi \), the tangent direction.

In order to make the scroll base curve definition general, a 3rd order polynomial is used defined as the natural equation shown in (1). There are several constraints when defining the natural equation. First, the resulting curve must be without self-intersection and have the form of a spiral. The curve must also be a strictly increasing positive function. The coefficients \( a, b, c \) and \( d \) in equation (1) can be negative but they can’t prevent the function from strictly increasing

\[
\rho(\varphi) = a + b\varphi + c\varphi^2 + d\varphi^3, \quad \varphi \geq 0
\]
From the natural equation, the x and y coordinates can be determined by integration of the natural equation

\[ x_{10}(\varphi) = \int_0^\varphi (p(u) \cos u, p(u) \sin u) \, du. \] (3)

The vector \( x_{10} \) represents the x and y components of the base scroll. Integrating, equations (4) and (5) are obtained

\[
\begin{align*}
x_{101}(\varphi) &= [d\varphi^3 + c\varphi^2 + (b - 6d)\varphi - 2c + a] \sin \varphi \\
&\quad + [3d\varphi^2 + 2c\varphi + b - 6d]\cos \varphi + 6d - b \\
x_{10q}(\varphi) &= [3d\varphi^2 + 2c\varphi + b - 6d]\sin \varphi \\
&\quad + [-d\varphi^3 - c\varphi^2 + (6d - b)\varphi + 2c - a]\cos \varphi - 2c + a.
\end{align*}
\] (4) (5)

The base scroll can take a variety of forms which depend on the constants in the natural equation. Figure 1 shows the first order, first order with offset, second and third order scenarios of the base scroll along with each curve's resulting evolute. A first order natural equation will result in a circle evolute, and constant wall thickness. With higher order, a spiral evolute is observed. Constant wall thickness will result if all four scroll curves have a common simple, closed and convex evolute as in cases (a) and (b). Cases (c) and (d) will result in variable wall thickness leading to nonlinear compression with rotation angle.

![Figure 1](image)

**Figure 1**: First Order (a), First Order with Offset (b), Second Order (c) and Third Order (d) Scroll Curves.

### 3. THE MATING CURVE AND REFLECTION

#### 3.1 The Mating Curve

Defining the outer wall of the orbiting scroll as the base scroll curve translating about a closed circular path of radius R by angle \( \theta \) gives the following equations for \( x_{11} \).

\[
\begin{align*}
x_{11}(\varphi, \theta) &= x_{101}(\varphi) + R \sin \theta \\
x_{11q}(\varphi, \theta) &= x_{10q}(\varphi) - R \cos \theta
\end{align*}
\] (6) (7)
Where \( x_{1i} \) and \( x_{i1} \) represent the x and y component of the outer wall of the orbiting scroll with tangent direction and orbiting angle. If one imposes the condition that every point along the base curve will have a unique contact position as the base scroll curve progresses about this path, the corresponding points of the mating curve can be collected by mapping these points from the contact points of the orbiting scroll. The resulting two equations are the definition of the inner wall of the fixed scroll

\[
\begin{align*}
y_{1i}(\varphi) &= x_{1i}(\varphi, \varphi) = x_{1i0}(\varphi) + R \sin \varphi \\
y_{i1}(\varphi) &= x_{i1}(\varphi, \varphi) = x_{i10}(\varphi) - R \cos \varphi \\
0 &\leq \varphi \leq \varphi_e.
\end{align*}
\]

Where \( \varphi_e \) represents the ending angle of the scroll wrap. This angle can be obtained from the following relation

\[
\varphi_e = 2\pi W.
\]

3.2 Reflection

At this point, two curves have been obtained that define one chamber for the scroll compressor. If this chamber is reflected and translated, the second chamber can be located. The easiest way to perform this it to first reflect and translate the outer wall of the fixed scroll \( Y_{1i} \), to locate the base scroll curve for the second compression chamber \( X_{2o} \), and then use this curve as the basis to generate both remaining fixed and orbiting scroll curves. This reflection and translation can be seen in Figure 2.

Equations (13) and (14) give the x and y components for the resulting base scroll curve for the second chamber

\[
\begin{align*}
x_{2o1}(\varphi) &= -y_{1i}(\varphi) + T_i \\
x_{2o1}(\varphi) &= -y_{i1}(\varphi) + T_i.
\end{align*}
\]

Where \( T_i \) and \( T_j \) represent the translation part. This variable can be determined by aligning two points of equal slope along the inner and outer wall of the orbiting scroll \( (X_2 \text{ and } X_1) \). Aligning horizontal slope positions \( p_1 \) and \( p_2 \) in the x direction and vertical slope positions \( p_3 \) and \( p_4 \) in the y-direction give the best results for both constant and variable wall thickness. For constant wall thickness this translation results in points of equal slope along either
curve being aligned tangentially with constant radial offset. For variable wall thickness this translation gives a quasi-linear increase in wall thickness with involute angle. Equations (15) and (16) give the resulting values for \( T_i \) and \( T_j \), respectively

\[
\begin{align*}
T_i &= x_{1oI}(\pi) + y_{1I}(0) \quad (15) \\
T_j &= x_{1oJ}\left(\frac{3\pi}{2}\right) + y_{1J}\left(\frac{\pi}{2}\right). \quad (16)
\end{align*}
\]

Applying the base scroll curve for the second compression chamber to both the fixed and orbiting scroll walls in the same manner presented earlier gives the resulting equations for the second compression chamber

\[
\begin{align*}
x_{2i}(\varphi, \theta) &= x_{2oI}(\varphi) + R \sin \theta \\
x_{2j}(\varphi, \theta) &= x_{2oJ}(\varphi) - R \cos \theta \quad (17, 18) \\
y_{2i}(\varphi) &= x_{2i}(\varphi, \varphi) = x_{2oI}(\varphi) + R \sin \varphi \quad (19) \\
y_{2j}(\varphi) &= x_{2j}(\varphi, \varphi) = x_{2oJ}(\varphi) - R \cos \varphi \quad (20)
\end{align*}
\]

\[0 \leq \varphi \leq \varphi_e, 0 \leq \theta \leq 2\pi. \quad (21)\]

### 3.3 Wall Thickness

Another parameter of interest is the minimum wall thickness; this parameter is the vertical distance between points \( p_1 \) and \( p_2 \) in Figure 2 and is obtained by subtracting the two vertical distances

\[
\delta_{\text{min}} = x_{1oI}(\pi) - x_{2oI}(0). \quad (22)
\]

When substituting in the appropriate coordinates and simplifying the result is a definition for the minimum wall thickness strictly based on the initial base curve and the orbiting radius

\[
\delta_{\text{min}} = x_{1oI}(\pi) - x_{1oJ}\left(\frac{3\pi}{2}\right) + x_{1oJ}(0) - R. \quad (23)
\]

Using equations (4) and (5) one can further reduce the wall thickness as a function of the orbit radius and coefficients from the natural equation

\[
\delta_{\text{min}} = \pi b + c(2\pi + \pi^2) + d(-6 + 6\pi^2 + \pi^4) - R. \quad (24)
\]

For constant wall thickness, the local wall thickness \( \delta_{\text{wall}}(\theta) \) along the scroll curve will remain at \( \delta_{\text{min}} \). Although, by taking the distance between the two scroll wraps as a function of involute location for both base scroll curves gives the local wall thickness for a variable wall compressor

\[
\delta_{\text{wall}}(\theta) = \sqrt{(x_{1oI}(\varphi + 2\pi) - x_{2oI}(\varphi))^2 + (x_{1oJ}(\varphi + 2\pi) - x_{2oJ}(\varphi))^2}. \quad (25)
\]

### 4. TIP GEOMETRY

#### 4.1 Two Arc Tip Geometry

As the tangent direction goes to zero, the scroll curves terminate. At this point some geometry is needed to close the scroll curves. There are infinite solutions to closing the scroll curves, for the definitions here, all solutions will be composed of some combination of two arcs and an internal circle tangent line as derived from Shaffer (2012). This way, the conditions above can be fulfilled while allowing a wide range of solutions for the tip geometry. Figure 3 shows the two arc tip geometry with a connecting tangent line for the orbiting scroll. The tangent line is represented by \( X_t \) while \( X_{te} \) and \( X_{2e} \) represent the arc extensions curves for the outer and inner walls with radiiuses \( R_b \) and \( R_a \).
For a given base scroll curve $X_{10}(\varphi)$, orbit radius $R$, and arc extension radiiuses $R_b$ and $R_a$ there will be unique tangency points $X_{1e0,\text{start}}$ and $X_{2e0,\text{start}}$ between the inner tangent line and two extension circles that represent the ignition of the extension curves. These points can be calculated as follows

$$dR = \left[(2b + 2\pi c + d(3\pi^2 - 12))^2 + (R_b - R - 2\varphi + 2\pi c + 6\pi^2 + R_b)^2\right]^{1/2}$$  \hspace{1cm} (26)

$$r_a = -2b - 2\pi c + d(12 - 3\pi^2)$$  \hspace{1cm} (27)

$$r_b = R + 2\varphi - 2\pi c - 6\pi^2 - R_a - R_b$$  \hspace{1cm} (28)

$$X_{1e0,\text{start}} = \frac{R_b(R_a + R_b)r_a}{dR^2} - \sqrt{R_b^2 - \left(\frac{R_b(R_a + R_b)}{dR}\right)^2} \frac{r_b}{dR}$$  \hspace{1cm} (29)

$$X_{1e0,\text{start}} = \frac{R_b(R_a + R_b)r_b}{dR^2} + \sqrt{R_b^2 - \left(\frac{R_b(R_a + R_b)}{dR}\right)^2} \frac{r_a}{dR}$$  \hspace{1cm} (30)

Where $dR$, $r_a$, and $r_b$ are used for simplification of Equations (29) and (30). The only requirement held between $R_a$ and $R_b$ is the following relation. This is derived from imposing that the term inside the radical from equations (29) and (30) remains positive

$$\frac{(R_a + R_b)}{dR} \leq 1.$$  \hspace{1cm} (31)

Another parameter of importance is the tangent direction at which the two extension arcs terminate. Because the extension arcs occur after termination of the scroll curves ($\varphi = 0$), this value is negative. This is the starting tangent direction $\varphi_s$

$$\varphi_s = \tan^{-1} \frac{X_{1e0,\text{start}} - R_b}{X_{1e0,\text{start}}} - \frac{\pi}{2}.$$  \hspace{1cm} (32)
This results in the following definitions for the base scroll curve extensions

\[ x_{1e0l}(\varphi) = R_b \cos(\varphi + \frac{3\pi}{2}) \]  
\[ x_{1e0o}(\varphi) = R_b + R_h \sin(\varphi + \frac{3\pi}{2}). \] (33) (34)

Orbiting about a closed circular path as before gives the following definitions for the outer wall of the orbiting scroll

\[ x_{1e1}(\varphi, \theta) = x_{1e0l}(\varphi) + R \sin \theta \]  
\[ x_{1e1j}(\varphi, \theta) = x_{1e0o}(\varphi) - R \cos \theta \]  
\[ \varphi_s \leq \varphi \leq 0, 0 \leq \theta \leq 2\pi. \] (35) (36)

The inner wall of the fixed scroll will have the following relations

\[ y_{1e1}(\varphi) = R_a \cos(\varphi + \frac{3\pi}{2}) \]  
\[ y_{1e1j}(\varphi) = R_a - R + R_a \sin(\varphi + \frac{3\pi}{2}) \]  
\[ \varphi_s \leq \varphi \leq 0. \] (37) (38) (39)

Performing the same reflection and translation as previously defined for the scroll curves gives the following relations for the remaining tip geometry

\[ x_{2e0l}(\varphi) = -y_{1e1}(\varphi) + T_i \]  
\[ x_{2e0j}(\varphi) = -y_{1e1j}(\varphi) + T_i \]  
\[ x_{2e1}(\varphi, \theta) = x_{2e0l}(\varphi) + R \sin \theta \]  
\[ x_{2e1j}(\varphi, \theta) = x_{2e0j}(\varphi) - R \cos \theta \]  
\[ y_{2e1}(\varphi) = x_{2e1}(\varphi, \varphi) = x_{2e0l}(\varphi) + R \sin \varphi \]  
\[ y_{2e1j}(\varphi) = x_{2e1j}(\varphi, \varphi) = x_{2e0j}(\varphi) - R \cos \varphi \]  
\[ \varphi_s \leq \varphi \leq 0, 0 \leq \theta \leq 2\pi. \] (40) (41) (42) (43) (44) (45) (46)

4.2 Perfect Meshing

The mating curve for the orbiting scroll is calculated by collecting the contact points as the curve orbits about its imposed path. This results in perfect meshing between the two scroll curves. If the same operation is performed on the two arcs that complete the tip of the scroll, perfect meshing in the tip region is obtained as well. This leads to an extended compression process and ultimately a higher volume ratio for a given compressor.
Figure 4 shows the extension arc for the outer wall (2012) of the orbiting scroll $X_{1e}$, (red) as it orbits from position 1 to position 2 about the black dotted line. In order for $X_{1e}$ to maintain contact with the inner wall of the fixed scroll $Y_{1e}$ (blue), the following dependence between $R_a$ and $R_b$ needs to be satisfied. Imposing this condition, a family of solutions with perfect meshing is obtained

$$R_a = R + R_b$$  \hspace{1cm} (47)  

$$0 < R_a, 0 < R_b.$$  \hspace{1cm} (48)

Earlier it was stated that the curves must be selected in a way to prevent contact between both the inner and outer scroll wraps. It the case of perfect meshing, it can be noticed that if the size of $R$ or $R_b$ is increased beyond its calculated value from Equation (48), $X_{1e}$ will be pushed into contact with $Y_{1e}$. This concludes that the perfect meshing condition is also an upper limit on $R_a$. The perfect meshing condition can be seen in Figure 6 (b)

$$R_a > R + R_b.$$  \hspace{1cm} (49)

### 4.3 Elimination of Tangent Line

Utilizing Equation (31) as an equality, one can obtain the limit at which the tangent line length goes to zero. At this limit, the tip geometry is completed by a two arcs only, with a point of inflection at the arc intersection as shown in Figure 6 (c). This condition also satisfies continuity of the first derivative

$$R_a + R_b = dR.$$  \hspace{1cm} (50)

A particular case arises when $R_b$ is taken to be much smaller than $R_a$. At this limit the solution approaches a one arc tip geometry, where the radius $R_a$ can be taken as approximately $dR$. This scenario is shown in Figure 6 (e).

### 4.4 Tip Designs

Plotting the “No tangent line” limit and “Perfect meshing” limit with $R_a$ and $R_b$, all possible tip designs can be visualized as shown in Figure 5 in gray. The top side of the gray triangle represents the “No tangent line” limit while the bottom side represents the “Perfect meshing” limit. Figure 6 shows all five tip geometries discussed in this section while Figure 5 gives the appropriate location of these designs on a $R_a$ vs $R_b$ plot.

Holding the orbiting scroll radius $R_b$ constant while approaching both limits gives tip geometries (b) and (c). Taking the intersection point of both limits gives tip geometry (d) which conforms to both perfect meshing and the
no tangent line condition. The location of $R_b$ for this condition can be analytically determined as a function of initial design data. This is shown in the following relationship:

$$R_b = \frac{(2b + 2\pi c + d(3\pi^2 - 12))^2 + (-R - 2a + 2\pi c + 6d\pi^2)^2}{4(R + 2a - 2\pi c - 6d\pi^2)} - \frac{R}{2}$$

(51)

Figure 5: Limits of $R_a$ and $R_b$

5. CONCLUSION

This paper presented a new method to calculate the geometry of scroll compressors through the use of the natural equation. By deriving each scroll curve from the radius of curvature parameterized with involute angle, a wide range of scroll involute geometries can be considered that are not included in the classical method, which is based on defining each scroll curve as the involute of a circle. In addition, it has been shown how all possible tip conditions involved in a two arc configuration can be implemented to give a comprehensive scroll compressor geometry definition. Finally, the geometries of variable wall thickness scroll compressors can be more readily generated using the new method.
Figure 6: Five Types of Tip Geometry
### NOMENCLATURE

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<th>Symbol</th>
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### Greek

- δ_{min} Minimum Wall Thickness (m)
- δ_{wall} Wall Thickness (m)
- θ Orbit Location (rad)
- ρ Radius of Curvature (m)
- φ Tangent Direction (rad)
- φ_{e} Ending Tangent Direction (rad)
- φ_{s} Starting Tangent Direction (rad)

### REFERENCES